Red Brand Canners (RBC)

Production Planning, Revenue Analysis & Optimization

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Made By:

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(DNSC 6307 Optimization I)

1. Model for Canned Tomato Products Production

To create the most optimal model we first define the objective function. Given that we want to increase the sum of profit contribution from each product we outline the objective function as follows:

Maximize:
Contribution from Whole tomatoes + Contribution from Juice + Contribution from Paste

Contribution from Each Product:
Profit Contribution from selling each 1000 lbs unit of products:

a. Whole Tomatoes: $247.67
b. Tomato Juice: $198
c. Tomato Paste: $222

Total Profit Contribution from each Product:
Profit Contribution From selling each 1000 lbs unit of product * amount produced in 1000 lbs

a. Whole Tomatoes Contribution: 247.67 x amount whole produced in 1000lbs
b. Tomato Juice Contribution: 198 x amount juice produced in 1000lbs
c. Tomato Paste Contribution: 222 x amount paste produced in 1000lbs

Constraints:
To maximize the objective above we need to satisfy the following constraints:

a. **Demand Constraints:**
   Do not exceed the demand for each product:
   - Demand for Whole Tomatoes <= 14000
   - Demand for Tomato Juice <= 1000
   - Demand for Tomato Paste <= 2000
b. **Supply Constraints:**
   Do not exceed the supply constraints for each grade of tomato
   - A grade used for paste + A grade used for Juice + A grade used for Whole <= 600
   - B grade used for paste + B grade used for Juice + B grade used for Whole <= 2400
c. **Quality Constraints:**
   Ensure the quality of the produced product by making sure the mix of A and B produces the right net quality required by the product:
      - Whole: 9*(A-grade used for whole)+5*(B grade used for whole)/total used for whole >= 8
      - Juice: 9* (A grade used for juice)+5*(B grade used for juice)/total used for juice >= 6

In order to maximize the revenue given the constraints on quality, supply and demand it is recommended that they allocate the supply as follows in order to achieve the overall optimal profit contribution. Our optimization model (Appendix 1A,1B) recommends the distribution outlined in exhibit 1A for tomatoes used for the production of each product to maximize the total profit contribution.

Exhibit 1A: Breakdown of Canned Tomato Products Production					
	Amount (in 1000 lbs)				
Product	Grade A Tomatoes Used	Grade B Tomatoes Used	Total Product Produced	Total Demand for Product	Unmet Demand
Whole Tomatoes	525	175	700	14400	13700
Tomato Juice	75	225	300	1000	700
Tomato Paste	0	2000	2000	2000	0
Total	600	2400	3000		

Using the production plan above we can determine the product contribution for each product and the total as follows in exhibit 1B.

Exhibit 1B: Profit Contribution			
Product	Profit Contribution per 1000 lb	Amount Produced (in 1000 lbs)	Total Profit Contribution from Product
Whole Tomatoes	246.67	700	172669
Tomato Juice	198	300	59400
Tomato Paste	222	2000	444000
Total Profit Contribution			676067

Following the allocation and production plan our model has recommended the best possible value of the objective (i-e the overall profit contribution) is $676,067.

2. Updating Model with Additional supply of A tomatoes

2.1 Decision to buy the additional A grade tomatoes at 25.5 cents/lb

With the additional availability of 80,000 lbs of grade A tomatoes we can add it to our model as a separate new supply with separate constraints and decide the allocation to each of the products as shown in appendix 2B. see if it increases the profit contribution while also adjusting the cost of the tomatoes we buy.

Exhibit 2A: Updated Supply			
	Amount of Fruit (1000 pounds)		
Supply	Grade A	Grade B	Total
Mixed Grade Tomatoes	600	2400	3000
Additional A Grade	80	0	80
Total	680	2400	3080

With the additional grade A tomatoes we updated our model (Appendix 2A), the recommended allocation (Appendix 2B) is outlined below in exhibit 2B:

Exhibit 2B: Breakdown of Production (With Additional Grade A Supply Available)						
	Amount (in 1000 lbs)					
Product	Previous Supply Grade A Tomatoes Used	Previous Supply Grade B Tomatoes Used	New Supply Grade A Tomatoes Used	Total Product Produced	Demand for Product	Unmet Demand
Whole Tomatoes	535	205	80	820	14400	13580

Tomato Juice	65	195	0	260	1000	740
Tomato Paste	0	2000	0	2000	2000	0
Total	600	2400	80	3080		

The model recommends that we should buy the additional grade A tomatoes costing 25.5 cents per pound as it improves our maximum profit contribution by an amount of $1,280 as outlined below in exhibit 2C:

Exhibit 2C: New Profit Contribution (With additional grade A supply)			
Product	Profit Contribution (per 1000 lb)	Amount Produced (in 1000 lbs)	Net Profit Contribution from Product
Whole Tomatoes	$246.67	820	$202,267
Tomato Juice	$198	260	$51,480
Tomato Paste	$222	2000	$444,000
Cost Incurred From Additional Grade A Tomato Supply			($20,400)
Total Profit Contribution (With additional Grade A tomatoes)			$677,347
Total Profit Contribution (Without additional Grade A tomatoes)			$676,067
Difference in Total Profit Contribution			$1,280

This shows that if we increase the availability of grade A tomatoes by 80,000 lbs and adjust for the incurred cost of the fruit, we still get a positive difference to the overall profit contribution after adjusting for the cost incurred so it is recommended that we buy the additional supply.

2.2 Allocation of the new grade A tomatoes

The best allocation for the new supply of grade A tomatoes to achieve maximum profit contribution as recommended by the new model (Appendix 2B) is that we should utilize all of the 80,000 lbs of new grade A tomatoes to produce whole tomatoes as shown in exhibit 2B.

2.3 Another Optimal allocation

Since both the existing supply of grade A tomatoes and the new supply of A tomatoes produce the same results (or have the same quality) there is another optimal allocation solution that results in the same profit contribution as solution in 2.3. The optimal solution (Appendix 2C) outlined in Exhibit 2D and 2E shows that if we maintain the total amount of whole and juice produced as before by changing the new A supply with exactly the same amount of old A supply between whole and juice we get the same profit contributions.

Exhibit 2D: Alternative Breakdown of Production (With Additional Grade A Supply Available)						
	Amount (in 1000 lbs)					
Product	Previous Supply Grade A Tomatoes Used	Previous Supply Grade B Tomatoes Used	New Supply Grade A Tomatoes Used	Total Product Produced	Demand for Product	Unmet Demand
Whole Tomatoes	600	205	15	820	14400	13580
Tomato Juice	0	195	65	260	1000	740
Tomato Paste	0	2000	0	2000	2000	0
Total	600	2400	80	3080		

Exhibit 2E: Alternative Optimal Solution Profit Contribution (With additional grade A supply)			
Product	Profit Contribution (per 1000 lb)	Amount Produced (in 1000 lbs)	Net Profit Contribution from Product

Whole Tomatoes	$246.67	820	$202,267
Tomato Juice	$198	260	$51,480
Tomato Paste	$222	2000	$444,000
Cost Incurred From Additional Grade A Tomato Supply			($20,400)
Total Profit Contribution (With additional Grade A tomatoes)			$677,347
Total Profit Contribution (Without additional Grade A tomatoes)			$676,067
Difference in Total Profit Contribution			$1,280

2.4 Maximum Price for Additional Tomatoes

In sensitivity analysis for this model (Appendix 2B) we can see that the shadow price for the 80,000 lbs of new grade A supply we buy 16, meaning in the dual problem of minimizing our price to purchase this supply has room to be increased by $16 per 1000 lb to maintain the same profit contribution as when we didn't have the new supply available. Meaning that we can pay up to a maximum of $255 + $16 = $271 per 1000 lb for the additional supply. If we change the cost of the additional supply of grade tomatoes in the model we can get the same profit as before the new supply (shown in Appendix 2D). If we pay the max of $271 per 1000 lb for the new grade A tomatoes, the maximum profit contribution with optimal allocation we get is $676,067.

3. Enhancing Demand with Advertising

3.1 Which Product to Advertise for?

From the Sensitivity Analysis in Appendix 2B we can see that the demand for juice and whole tomatoes has a slack of 7333.33 and 13,600 respectively, meaning that we are not meeting the demand for these products with the existing optimal solution while on the other hand the demand tomato paste has a slack of zero, meaning that we are completely meeting the demand. So keeping this in mind we can suggest that if we increase the demand for paste through advertising which could possibly result in a new optimal allocation that might improve the total profit contribution. To test the model under advertising criteria (Appendix 3A), we test improved demand cases for all three products in the next section.

3.2 Which Product to Advertise For?

Holding the option to buy the additional supply of tomatoes gives a Profit Contribution of $677,347 (Appendix 2B). With the Marketing Department of Red Brand Canners feeling it could increase demand for any of their three tomato products by 5,000 cases through advertising for this scenario, we test 3 different models of Sensitivity Analysis to help us reach a conclusion. Firstly, we added the advertising demand for each product (Whole Tomatoes, Tomato Juice, Tomato Paste) which is a value deprived from 5000 multiplied by each product's pounds per case (18, 20, 25) divided by 1000. When adding in the advertising demand for our first model on Whole Tomatoes, the Profit Contribution came out to be the same. Similarly, when adding in the advertising demand for our second model on Tomato Juice, we found that the Profit Contribution came out to be the same as well. When we added in the advertising demand for our third model on Tomato Paste, we saw that the Profit Contribution increased from $677,347 to $683,388 with the difference being a profit increase of $6,041.

3.3 Maximum Amount to Pay for Advertising

We are now able to conclude that Red Brand Canners should be willing to pay about $6,041 dollars for the advertising campaign. We are also able to conclude that Tomato Paste is the only product that Red Brand Canners's advertising campaign should be directed toward as increasing the demand for Whole Tomatoes and Tomato Juice do not have any effect on the Profit Contribution at all.

Exhibit 3A: Difference in Profit after Advertising			
	Advertising for Whole Tomatoes	Advertising for Tomato Juice	Advertising for Tomato Paste
Total Profit Contribution Before Advertising	$677,347	$677,347	$677,347
Total Profit Contribution After Advertising	$677,347	$677,347	$683,388
Difference	0	0	$6,041

4. Additional supply of B tomatoes

4.1 Is the Additional Stock Worth Buying?

Gordon is offered additional Grade B tomatoes for 18 cents per pound but he refuses to buy the tomatoes claiming that the 18 cents per pound is based on the fact that the incoming crop normally contains a mix of grade A and grade B tomatoes. If we continue with the same model as in 3 which includes the option of buying additional Grade A tomatoes, in addition to the increase in demand because of advertising. It is not a sensible decision to purchase additional B tomatoes because within the existing model we have a lower price for the same resource.

4.2 Does the price 18.8 cents / lb improve our profit contribution?

From the model in appendix 3A, we increase the demand for tomato paste with advertising and from appendix 3C we can see that the shadow price for b_supply is 173 meaning that for every unit increase for b_supply will increase the objective function or the maximum profit contribution by 173 units. But the cost of this resource is 180 per unit meaning it costs more than it contributes to the profit so we end up losing money and therefore should not buy the additional B tomatoes.

5.Closing down production lines?

Starting up the production line for each of the products entails a substantial set-up cost. The estimated set-up costs are $50,000 per line. This means that if RBC decides not to start up a particular line, they would save $50,000, but they would not be able to produce that particular product.

5.1 Removing Paste Line

If we decide not to start up the paste line, we will see a decrease in our total profit, the explanation for the same is as follows. Initially, if we set up all the production lines for whole, juice and paste we receive a profit contribution of $676,067 and after deducting the setup cost of 150,000 from this gives us a total profit of $526,067. After excluding only the paste line, the profit contribution that we saw from the optimization and sensitivity analysis (Appendix 4A) after running this code was $313,111 and if we subtract the setup costs of $100,000 for whole and juice from this we get $213,111 which is the total profit of this combination. As we can clearly see, $213,111 is way lower than the actual total profit of $526,067 and so if we decide not to start up the Paste line, it will lower our total profit.

Exhibit 5A: Profit Contribution if all lines started (Adjusted for Startup Costs)			
Product	Profit Contribution per 1000 lb	Amount Produced (in 1000 lbs)	Total Profit Contribution from Product
Whole Tomatoes	246.67	700	172669
Tomato Juice	198	300	59400
Tomato Paste	222	2000	444000
Profit Contribution			676067
Setup Cost			(150000)
Total Profit Contribution			626067

5.2 Which production lines should RBC to start up

We have checked and tested for various scenarios (Appendix 4A,4B, 4C) to find out which lines would fetch us more profit when setup. From our results, we can say that starting the production lines for juice and paste by excluding the whole tomato production line. The profit contribution for this combination was 642,000 and after removing the set-up costs for juice and paste we will have 542,000 as the total profit which is greater than the original total profit 526,067. By removing juice and starting whole, paste lines we got 541,333 which is still lower than 542,000 which we got for juice and paste lines.

Exhibit 5B: Profit Contribution (After Removing Whole Tomato Line)			
Product	Profit Contribution per 1000 lb	Amount Produced (in 1000 lbs)	Total Profit Contribution from Product
Tomato Juice	198	1000	198000
Tomato Paste	222	2000	444000

Profit Contribution	642000
Set-up Cost	(100000)
Total Profit Contribution	542000

Exhibit 5C: Profit Contribution (After Removing Paste Line)			
Product	Profit Contribution per 1000 lb	Amount Produced (in 1000 lbs)	Total Profit Contribution from Product
Tomato Whole	246.67	466.667	115111
Tomato Juice	198	1000	198,000
Profit Contribution			313111
Set-up Cost			(100000)
Total Profit Contribution			213111

Exhibit 5D: Profit Contribution (After Removing Juice Line)			
Product	Profit Contribution per 1000 lb	Amount Produced (in 1000 lbs)	Total Profit Contribution from Product
Tomato Whole	246.67	800	197333
Tomato Paste	222	2000	444000
Profit Contribution			641333
Set-up Cost			(100000)
Total Profit Contribution			541333

6.One year later

An acceptable price would be 20 cents per pound for the next year, they could order up to 13 million pounds of tomatoes. Demand is pretty much the same as last year, i.e. they would be able to sell as many whole canned tomatoes as they can make, and demand for juice and paste is again limited to 50,000 and 80,000 cases, respectively. The only uncertainty is the quality of the crop. Looking at historical data, in the past there were basically three types of years.

- In “wet” years, like last year, the crop was quite poor, and contained only about 20% grade A.
- In “sunny” years, the percentage of A-tomatoes was around 60%.
- In “normal” years, the crop contained approximately 50% grade A tomatoes.

25% of the years were “sunny”, 50% “normal” years and 25% “poor”.

6.1 Purchase and Allocation Decisions for Sunny Year

As per the optimal decision variables in Appendix 5B, we should order the whole 13,000,000 lbs of tomatoes given it is a sunny year and allocate the resources as outlined in exhibit 6A.

Exhibit 6A: Breakdown of Production (For Sunny Year)				
	Amount (in 1000 lbs)			
Product	Grade A Tomatoes Used	Grade B Tomatoes Used	Total Product Produced	Profit Contribution
Whole Tomatoes	7525	2525	10,050	$2,489,084
Tomato Juice	225	675	900	$178,200
Tomato Paste	0	2000	2000	$444,000
Total Profit Contribution				$3,133,533
Cost Incurred for Purchasing				($2,600,000)
Net Profit				$513,533

6.2 Purchase and Allocation Decisions for Normal Year

For a normal year, the recommended order quantity (Appendix 5C) is 8,000,000 lbs of tomatoes and the allocation i outlined in Exhibit 6B:

Exhibit 6B: Breakdown of Production (For Normal Year)				
	Amount (in 1000 lbs)			
Product	Grade A Tomatoes Used	Grade B Tomatoes Used	Total Product Produced	Profit Contribution
Whole Tomatoes	3750	1250	5000	$1,238,350
Tomato Juice	250	750	1000	$198,000
Tomato Paste	0	2000	2000	$444,000
Total Profit Contribution				$1,875,833
Cost Incurred for Purchasing				($1,600,000)
Net Profit				$275,333

6.3 Purchase and Allocation Decisions for Poor Year

For a normal year, the recommended order quantity (Appendix 5D) is 2,727,270 lbs of tomatoes and the allocation i outlined in Exhibit 6A:

Exhibit 6C: Breakdown of Production (For Poor Year)	
	Amount (in 1000 lbs)

Product	Grade A Tomatoes Used	Grade B Tomatoes Used	Total Product Produced	Profit Contribution
Whole Tomatoes	545.455	181.818	727.273	$180,124
Tomato Juice	0	0	0	0
Tomato Paste	0	2000	2000	$444,000
Total Profit Contribution				$623,393
Cost Incurred for Purchasing				($545,454)
Net Profit				$77,939

6.4 Sunny Season Order Scenario Analysis

In our model from appendix 5A, we change the constraint for order quantity to be binding == 13000 and change the weather to get the optimal profit (Appendix 5E, 5F).
Exhibit 6D compares the difference in each scenario:

Exhibit 6D: Breakdown of Production (Order Quantity = 13,000,000 lbs)			
	Net Revenue		
Year	Good Year	Normal Year	Poor Year
Revenue Generated	$3,133,533	$2,697,556	$1,414,890
Cost	($2,600,000)	($2,600,000)	($2,600,000)
Net Profit	$513,533	$97,556	($1,185,110)

Looking at the scenario analysis above, if we order according to a sunny year and get a normal year we still make some profit while if it were a poor year RBC would end up losing money.

6.5 Expected Value Analysis

The probability for each type of weather is as follows:

- Sunny = 0.25
- Normal = 0.5
- Poor = 0.25

The average profit overall based on the probabilities would be:

Exhibit 6E: Expected Value Analysis (Order Quantity = Optimized for Season)			
	Net Revenue		
Year	Good Year	Normal Year	Poor Year
Revenue Generated	$3,133,533	$1,875,833	$623,393
Cost	($2,600,000)	($1,600,000)	($545,454)
Net Profit	$513,533	$275,333	$77,939
Probability	0.25	0.5	0.25
Expected Value	=0.25*$513,533 + 0.5*$275,333 + 0.25*$77,939 = $285,535		

6.6 Final Purchase Recommendation:

Given the different analyses above, the best course of action recommended for purchase is to buy according to the normal year production so the chances of profit would be the largest

Appendices

1A: Model 1

```
# Model m =
gp.Model("RBC")

# Create decision variables for tomatoes usage
aw = m.addVar(name="aw") aj =
m.addVar(name="aj") ap = m.addVar(name="ap")
bw = m.addVar(name="bw") bj =
m.addVar(name="bj") bp = m.addVar(name="bp")

# The objective is to maximize the profit contribution
obj = (4.44/18*1000)*(aw+bw)+198*(aj+bj)+222*(ap+bp)
m.setObjective(obj, GRB.MAXIMIZE)

# Demand constraints con1 =
m.addConstr(aw+bw<=14400, name='w_dem') con2 =
m.addConstr(aj+bj<=1000, name='j_dem') con3 =
m.addConstr(ap+bp<=2000, name='p_dem')

# Supply constraints con4 =
m.addConstr(aw+aj+ap<=600, name='a_sup') con5 =
m.addConstr(bw+bj+bp<=2400, name='b_sup')

# Quality constraints con6 =
m.addConstr(9*aw+5*bw>=8*(aw+bw), name='w_qual') con7
= m.addConstr(9*aj+5*bj>=6*(aj+bj), name='j_qual')

# Solve
m.optimize()
```

1B: Model 1 Optimal Decision Variables and Sensitivity Report

```
Optimal objective 6.760666667e+05

Profit contribution: 676067

Decision
variables: aw =
525 aj = 75 ap = 0
bw = 175 bj = 225
bp = 2000


  Name Final Value Reduced Cost Obj Coeff Upper Range Lower Range 0 aw
525.0 0.000000 246.666667 709.777778 181.777778
1 aj 75.0 0.000000 198.000000 262.888889 -265.111111 2 ap 0.0 -97.333333
222.000000 319.333333 -inf 3 bw 175.0 0.000000 246.666667 1636.000000
181.777778 4 bj 225.0 0.000000 198.000000 240.962963 43.629630
5   bp       2000.0      0.000000 222.000000          inf   173.666667


      Name Shadow Price RHS Coeff         Slack Upper Range Lower Range
0 w_dem 0.000000 14400 13700.0 inf 700.000000 1 j_dem 0.000000 1000
700.0 inf 300.000000 2 p_dem 48.333333 2000 0.0 2200.000000 1533.333333
3 a_sup 271.000000 600 0.0 1200.000000 133.333333 4 b_sup 173.666667
2400 0.0 2866.666667 2200.000000 5 w_qual -24.333333 0 0.0 466.666667
-600.000000
6 j_qual     -24.333333          0          0.0 1400.000000 -200.000000
```

2A: Model 2

```
# Model m_2 =
gp.Model("RBC_2")

# Create decision variables for tomatoes usage
aw = m_2.addVar(name="aw") aj =
m_2.addVar(name="aj") ap =
m_2.addVar(name="ap") bw =
m_2.addVar(name="bw") bj =
m_2.addVar(name="bj")
```

```
bp = m_2.addVar(name="bp")
AAw = m_2.addVar(name="AAw")
AAp = m_2.addVar(name="AAp")
AAj = m_2.addVar(name="AAj")

adDemandW = 0
adDemandJ = 0
adDemandP = 0

costAA = 255

obj_2 =
(4.44/18*1000)*(AAw+aw+bw)+198*(AAj+aj+bj)+222*(AAp+ap+bp)-costAA*(AAw+A
Aj+AAp) m_2.setObjective(obj_2, GRB.MAXIMIZE)

# Demand constraints con11 =
m_2.addConstr(aw+bw+AAw<=14400+adDemandW, name='w_dem') con22
= m_2.addConstr(aj+bj+AAj<=1000+adDemandJ, name='j_dem') con33
= m_2.addConstr(ap+bp+AAp<=2000+adDemandP, name='p_dem')

# Supply constraints con44 =
m_2.addConstr(aw+aj+ap<=600, name='a_sup') con55 =
m_2.addConstr(bw+bj+bp<=2400, name='b_sup') con66 =
m_2.addConstr(AAw+AAj+AAp<=80, name='AA_sup')

# Quality constraints con77 =
m_2.addConstr(9*aw+9*AAw+5*bw>=8*(aw+bw+AAw), name='w_qual') con88 =
m_2.addConstr(9*aj+9*AAj+5*bj>=6*(aj+bj+AAj), name='j_qual')

# Solve
m_2.optimize()
```

2B: Model 2 Optimal Decisions Variables and Sensitivity Report

```
Optimal objective 6.773466667e+05

Profit contribution: 677347

Decision variables:
aw = 535
```

```
aj = 65 ap
= 0 bw =
205 bj =
195 bp =
2000 AAw =
80
AAp = 0
AAj = 0


  Name  Final  Value  Reduced  Cost  Obj  Coeff  Upper  Range  Lower  Range  0  aw
535.0 0.000000e+00 246.666667 246.666667 181.777778
1   aj      65.0 0.000000e+00 198.000000 262.888889  198.000000
2   ap      0.0 -9.733333e+01 222.000000 319.333333  -inf
3   bw      205.0 0.000000e+00 246.666667 1636.000000     204.000000
4   bj      195.0 0.000000e+00 198.000000      240.666667  43.629630
5   bp      2000.0 0.000000e+00 222.000000     inf   173.666667
6   AAw     80.0 0.000000e+00-8.333333   inf   -8.333333
7   AAj     0.0 1.065814e-14 -57.000000  -57.000000  -inf
8   AAp     0.0 -9.733333e+01 -33.000000 64.333333   -inf


      Name Shadow Price RHS Coeff      Slack Upper Range Lower Range
0   w_dem   0.000000    14400 13580.0     inf   820.000000
1   j_dem   0.000000    1000  740.0 inf   260.000000
2   p_dem   48.333333   2000  0.0 2173.333333 1506.666667
3   a_sup   271.000000  600   0.0 1120.000000  124.444444
4   b_sup   173.666667  2400  0.0 2893.333333 2226.666667
5   AA_sup  16.000000   80    0.0   600.000000  0.000000
6   w_qual  -24.333333  0     0.0   546.666667  546.666667
7   j_qual  -24.333333  0     0.0 1426.666667 1426.666667
```

2C: Alternative Optimal Solution for Model 2

```
Optimal objective 6.773466667e+05

Profit contribution: 677347

Decision variables:
aw = 600
aj = 0

ap = 0

bw = 205

bj = 195

bp = 2000
```

```
AAw = 15
AAp = 0
AAj = 65
```

	Name	Final Value	Reduced Cost	Obj Coeff	Upper Range	Lower Range
0	aw	600.0	0.000000	246.666667	inf	246.666667
1	aj	0.0	-0.000000	198.000000	198.000000	-inf
2	ap	0.0	-97.333333	222.000000	319.333333	-inf
3	bw	205.0	0.000000	246.666667	1636.000000	204.000000
4	bj	195.0	0.000000	198.000000	240.666667	43.629630
5	bp	2000.0	0.000000	222.000000	inf	173.666667
6	AAw	15.0	0.000000	-8.333333	-8.333333	-22.555556
7	AAj	65.0	0.000000	-57.000000	7.888889	-57.000000
8	AAp	0.0	-97.333333	- 33.000000	64.333333	-inf

	Name	Shadow Price	RHS Coeff	Slack	Upper Range	Lower Range
0	w_dem	0.000000	14400	13580.0	inf	820.000000
1	j_dem	0.000000	1000	740.0	inf	260.000000
2	p_dem	48.333333	2000	0.0	2173.333333	1960.000000
3	a_sup	271.000000	600	0.0	1120.000000	480.000000
4	b_sup	173.666667	2400	0.0	2440.000000	2226.666667
5	AA_sup	16.000000	80	0.0	600.000000	66.666667
6	w_qual	-24.333333	0	0.0	120.000000	120.000000
7	j_qual	-24.333333	0	0.0	40.000000	40.000000

2D: Maximum Price Paid for New Supply

```
Optimal objective 6.760666667e+05

Profit contribution: 676067

Decision variables:
aw = 600
aj = 0

ap = 0

bw = 200

bj = 200

bp = 2000

AAw = 0
AAp = 0
AAj = 66.6667
```

	Name	Final Value	Reduced Cost	Obj Coeff	Upper Range	Lower Range
0	aw	600.000000	0.000000e+00	246.666667	inf	246.666667
1	aj	0.000000	-5.684342e-14	198.000000	198.000000	-inf
2	ap	0.000000	-9.733333e+01	222.000000	319.333333	-inf
3	bw	200.000000	0.000000e+00	246.666667	246.666667	246.666667
4	bj	200.000000	0.000000e+00	198.000000	198.000000	198.000000
5	bp	2000.000000	0.000000e+00	222.000000	inf	173.666667
6	AAw	0.000000	7.105427e-14	-24.333333	-24.333333	-inf
7	AAj	66.666667	0.000000e+00	-73.000000	-0.000000	-73.000000
8	AAp	0.000000	-9.733333e+01	-49.000000	48.333333	-inf

	Name	Shadow Price	RHS Coeff	Slack	Upper Range	Lower Range
0	w_dem	0.000000	14400	13600.000000	inf	800.000000
1	j_dem	0.000000	1000	733.333333	inf	266.666667
2	p_dem	48.333333	2000	0.000000	2200.0	1960.000000
3	a_sup	271.000000	600	0.000000	1200.0	480.000000
4	b_sup	173.666667	2400	0.000000	2440.0	2200.000000
5	AA_sup	0.000000	80	13.333333	inf	66.666667
6	w_qual	-24.333333	0	0.000000	120.0	120.000000
7	j_qual	-24.333333	0	0.000000	40.0	40.000000

3A: Model with Advertising

```
# Model m_2 =
gp.Model("RBC_2")

# Create decision variables for tomatoes usage
aw = m_2.addVar(name="aw") aj =
m_2.addVar(name="aj") ap =
m_2.addVar(name="ap") bw =
m_2.addVar(name="bw") bj =
m_2.addVar(name="bj") bp =
m_2.addVar(name="bp")
AAw = m_2.addVar(name="AAw")
AAp = m_2.addVar(name="AAp")
AAj = m_2.addVar(name="AAj")

adDemandW = 0
adDemandJ = 0
adDemandP = 0

costAA = 255
#adDemandW = (5000*18)/1000
#adDemandJ = (5000*20)/1000
#adDemandP = (5000*25)/1000
```

```
obj_2 =
(4.44/18*1000)*(AAw+aw+bw)+198*(AAj+aj+bj)+222*(AAp+ap+bp)-costAA*(AAw+A
Aj+AAp) m_2.setObjective(obj_2, GRB.MAXIMIZE)

# Demand constraints con11 =
m_2.addConstr(aw+bw+AAw<=14400+adDemandW, name='w_dem') con22
= m_2.addConstr(aj+bj+AAj<=1000+adDemandJ, name='j_dem') con33
= m_2.addConstr(ap+bp+AAp<=2000+adDemandP, name='p_dem')

# Supply constraints con44 =
m_2.addConstr(aw+aj+ap<=600, name='a_sup') con55 =
m_2.addConstr(bw+bj+bp<=2400, name='b_sup') con66 =
m_2.addConstr(AAw+AAj+AAp<=80, name='AA_sup')

# Quality constraints con77 =
m_2.addConstr(9*aw+9*AAw+5*bw>=8*(aw+bw+AAw), name='w_qual') con88 =
m_2.addConstr(9*aj+9*AAj+5*bj>=6*(aj+bj+AAj), name='j_qual')

# Solve
m_2.optimize()
```

3B: Advertising for Whole Tomatoes

```
Profit contribution: 677347

Decision variables:
aw = 535
aj = 65

ap = 0

bw = 205

bj = 195

bp = 2000

AAw = 80
AAp = 0
AAj = 0


   Name  Final Value   Reduced Cost    Obj Coeff   Upper Range   Lower Range
0    aw           535.00.000000e+00  246.666667   246.666667   181.777778
```

1	aj	65.0	0.000000e+00	198.000000	262.888889	198.000000
2	ap	0.0	-9.733333e+01	222.000000	319.333333	-inf
3	bw	205.0	0.000000e+00	246.666667	1636.000000	204.000000
4	bj	195.0	0.000000e+00	198.000000	240.666667	43.629630
5	bp	2000.0	0.000000e+00	222.000000	inf	173.666667
6	AAw	80.0	0.000000e+00	-8.333333	inf	-8.333333
7	AAj	0.0	1.065814e-14	-57.000000	-57.000000	-inf
8	AAp	0.0	-9.733333e+01	-33.000000	64.333333	-inf

	Name	Shadow Price	RHS Coeff	Slack	Upper Range	Lower Range
0	w_dem	0.000000	14490.0	13670.0	inf	820.000000
1	j_dem	0.000000	1000.0	740.0	inf	260.000000
2	p_dem	48.333333	2000.0	0.0	2173.333333	1506.666667
3	a_sup	271.000000	600.0	0.0	1120.000000	124.444444
4	b_sup	173.666667	2400.0	0.0	2893.333333	2226.666667
5	AA_sup	16.000000	80.0	0.0	600.000000	0.000000
6	w_qual	-24.333333	0.0	0.0	546.666667	546.666667
7	j_qual	-24.333333	0.0	0.0	1426.666667	1426.666667

3C: Advertising for Tomato Juice

```
Optimal objective 6.773466667e+05

Profit contribution: 677347

Decision variables:
aw = 535 aj = 65 ap =
0 bw = 205 bj = 195
bp = 2000 AAw = 80
AAp = 0
AAj = 0


  Name  Final  Value  Reduced  Cost  Obj  Coeff  Upper  Range  Lower  Range  0  aw
535.0 0.000000e+00 246.666667 246.666667 181.777778
1   aj      65.0 0.000000e+00 198.000000 262.888889  198.000000
2   ap      0.0 -9.733333e+01 222.000000 319.333333  -inf
3   bw      205.0 0.000000e+00 246.666667 1636.000000     204.000000
4   bj      195.0 0.000000e+00 198.000000     240.666667  43.629630
5   bp      2000.0 0.000000e+00 222.000000    inf   173.666667
6   AAw     80.0 0.000000e+00-8.333333   inf   -8.333333
7   AAj     0.0 1.065814e-14 -57.000000  -57.000000  -inf
8   AAp     0.0 -9.733333e+01 -33.000000 64.333333   -inf


      Name Shadow Price RHS Coeff      Slack Upper Range Lower Range
0   w_dem   0.000000    14400.0 13580.0  inf   820.000000
1   j_dem   0.000000    1100.0      840.0 inf   260.000000

 2   p_dem     48.333333     2000.0      0.0  2173.333333  1440.000000
 3   a_sup    271.000000      600.0      0.0  1120.000000   124.444444
 4   b_sup    173.666667     2400.0      0.0  2960.000000  2226.666667
 5  AA_sup     16.000000       80.0      0.0   600.000000     0.000000
 6  w_qual    -24.333333        0.0      0.0   546.666667   546.666667
 7  j_qual    -24.333333        0.0      0.0  1426.666667  1426.666667
```

3D: Advertising for Tomato Paste

```
Optimal objective 6.833883333e+05

Profit contribution: 683388

Decision variables:
aw = 581.875
aj = 18.125
ap = 0
bw =
220.625 bj
= 54.375 bp
= 2125 AAw
= 80
AAp = 0
AAj = 0


  Name Final Value Reduced Cost Obj Coeff Upper Range Lower Range 0 aw
581.875 0.000000e+00 246.666667 246.666667 181.777778
1   aj     18.125 0.000000e+00 198.000000    262.888889  198.000000
2   ap     0.000 -9.733333e+01 222.000000    319.333333  -inf
3   bw     220.625 0.000000e+00 246.666667 1636.000000   204.000000
4   bj     54.375 0.000000e+00 198.000000    240.666667  43.629630
5   bp     2125.000 0.000000e+00 222.000000   inf   173.666667
6   AAw    80.000 0.000000e+00    -8.333333   inf   -8.333333
7   AAj    0.000 1.065814e-14 -57.000000     -57.000000  -inf
8   AAp    0.000 -9.733333e+01 -33.000000    64.333333   -inf


      Name Shadow Price RHS Coeff         Slack Upper Range Lower Range
0   w_dem   0.000000   14400.0 13517.5  inf   882.500000
1   j_dem   0.000000   1000.0      927.5 inf   72.500000
2   p_dem   48.333333  2125.0      0.0 2173.333333 1506.666667
3   a_sup   271.000000  600.0 0.0   745.000000  82.777778
4   b_sup   173.666667  2400.0      0.0 3018.333333 2351.666667
5   AA_sup  16.000000   80.0  0.0   225.000000  0.000000
6   w_qual  -24.333333  0.0   0.0   588.333333  588.333333
7   j_qual  -24.333333  0.0   0.0 1551.666667 1551.666667
```

4A: Optimal Solution with Startup Costs (Removing Paste Line)

```
Optimal objective 3.131111111e+05

Profit contribution: 313111

Decision variables:
aw = 350 aj = 250
bw = 116.667 bj =
750


Name Final Value Reduced Cost        Obj Coeff Upper Range    Lower Range
0   aw      350.000000  0.0 246.666667   709.777778 -8.222222e+01
1   ap      250.000000  0.0 198.000000   526.888889 -2.651111e+02
2   bw      116.666667  0.0 246.666667 1636.000000 -2.842171e-14
3   bp      750.000000  0.0 198.000000   inf 4.362963e+01


      Name Shadow Price RHS Coeff           Slack Upper Range Lower
Range
0   w_dem   0.000000    14000 13933.333333     2400.0
466.666667
1   j_dem   115.777778  1000  0.000000    2400.0
0.000000
2   a_sup   328.888889  600   0.000000    5200.0
250.000000
3   b_sup   0.000000    2400  1533.333333 inf
866.666667
4   w_qual  -82.222222  0     0.000000    350.0
866.666667
5   j_qual  -82.222222  0     0.000000    1400.0
-1000.000000
```

4B: Optimal Solution with Startup Costs (Removing Juice Line)

```
Optimal objective 6.413333333e+05

Profit contribution: 641333

Decision variables:
aw = 600
ap = 0

bw = 200

bp = 2000


  Name Final Value Reduced Cost      Obj Coeff  Upper Range     Lower Range
0    aj        600.0        0.000000246.666667          inf -8.222222e+01
1    ap          0.0     -328.888889246.666667  550.888889            -inf
```

2	bj	200.0	0.000000	198.000000	inf	-2.842171e-14
3	bp	2000.0	0.000000	198.000000	inf	-0.000000e+00

	Name	Shadow Price	RHS Coeff	Slack	Upper Range	Lower Range
0	w_dem	0.000000	14400	13600.0	inf	800.0
1	p_dem	222.000000	2000	0.0	2200.0	0.0
2	a_sup	328.888889	600	0.0	1200.0	0.0
3	b_sup	0.000000	2400	200.0	inf	2200.0
4	w_qual	-82.222222	0	0.0	600.0	-600.0

4C: Optimal Solution with Startup Costs (Removing Whole Line)

Optimal objective 6.42000000 0e+05

Profit contribution: 642000

Decision variables:
aj = 250
ap = 350

bj = 750

bp = 1650

	Name	Final Value	Reduced Cost	Obj Coeff	Upper Range	Lower Range
0	aj	250.0	0.0	198	198.0	-594.0
1	ap	350.0	0.0	222	222.0	222.0
2	bj	750.0	0.0	198	inf	198.0
3	bp	1650.0	0.0	222	222.0	222.0

	Name	Shadow Price	RHS Coeff	Slack	Upper Range	Lower Range
0	j_dem	198.0	1000	0.0	1000.0	533.333333
1	p_dem	222.0	2000	0.0	2000.0	1650.000000
2	a_sup	0.0	600	0.0	inf	600.000000
3	b_sup	0.0	2400	0.0	2750.0	2400.000000
4	j_qual	-0.0	0	0.0	1400.0	-1000.000000

5A: Model with Purchase Quantity Decision and Weather

```
# Model m =

gp.Model("RBC")


# Create decision variables for tomatoes usage
aw = m.addVar(name="aw") aj =
m.addVar(name="aj")
```

```
ap = m.addVar(name="ap") bw =
m.addVar(name="bw") bj =
m.addVar(name="bj") bp =
m.addVar(name="bp") order_quant =
m.addVar(name="order_quant")

weather_pred = 'S'

# The objective is to maximize the profit contribution
obj = (4.44/18*1000)*(aw+bw) + 198*(aj+bj) + 222*(ap+bp)
-
200*(order_quant)
m.setObjective(obj, GRB.MAXIMIZE)

# Demand constraints

con2 = m.addConstr(aj+bj<=1000, name='j_dem')
con3 = m.addConstr(ap+bp<=2000, name='p_dem')

# Quality constraints con4 =
m.addConstr(9*aw+5*bw>=8*(aw+bw), name='w_qual') con5
= m.addConstr(9*aj+5*bj>=6*(aj+bj), name='j_qual')

#Purchase Constraint con6 = m.addConstr(order_quant <= 13000,
name='purchase_quantity')

#Supply Constraint for wet year
if weather_pred == 'S':
    A_prop = 0.6
    B_prop = 0.4
    P_W = 0.25
#Supply Constraint for wet year
elif weather_pred == 'N':
    A_prop = 0.5
    B_prop = 0.5
    P_W = 0.5
```

```
#Supply Constraint for wet year
elif weather_pred == 'P':
    A_prop = 0.2
    B_prop = 0.8
    P_W = 0.25
```

```
con7 = m.addConstr(aw+aj+ap<=A_prop*order_quant) con8 =
m.addConstr(bw+bj+bp<=B_prop*order_quant)

# Solve
m.optimize()
```

5B: Optimal Decision for Sunny Year

```
Optimal objective 5.135333333e+05

Profit contribution: 513533

Decision variables:
aw = 7575 aj = 225 ap
= 0 bw = 2525 bj =
675 bp = 2000
order_quant = 13000


           Name  Final Value Reduced Cost     Obj Coeff Upper Range \
0          aw    7575.0      0.000000 246.666667    709.777778
1          aj    225.0 0.000000 198.000000    262.888889
2          ap    0.0   -97.333333 222.000000  319.333333
3          bw    2525.0      0.000000 246.666667 1636.000000
4          bj    675.0 0.000000 198.000000    240.962963
5          bp    2000.0      0.000000 222.000000    inf
6          order_quant      13000.0     0.000000 -200.000000   inf

   Lower Range
0         185.587302
1         -229.555556
2         -inf
3         181.777778
4         55.481481
5         173.666667
6         -232.066667


              Name  Shadow Price RHS Coeff Slack     Upper Range     Lower
Range
0          j_dem 0.000000    1000.0 100.0      inf
900.000000
1          p_dem 48.333333   2000.0      0.0 2.600000e+03
1933.333333
```

```
2          w_qual    -24.333333        0.0     0.0   2.000000e+02
```

```
-1800.000000
3          p_qual    -24.333333        0.0    0.0  2.000000e+02
-600.000000 4
order_quantity        32.066667    13000.0    0.0  1.333333e+04
10000.000000
5           a_sup    271.000000     7800.0    0.0  1.800000e+03
-200.000000
6           b_sup    173.666667     5200.0    0.0  6.666667e+01
-600.000000
```

5C: Optimal Decision for Normal Year

```
Optimal objective 2.753333333e+05

Profit contribution: 275333

Decision variables:
aw = 3750 aj = 250
ap = 0 bw = 1250
bj = 750 bp = 2000
order_quant = 8000


           Name Final Value Reduced Cost     Obj Coeff Upper Range \
0           aw   3750.0      0.000000 246.666667    317.777778
1           aj   250.0 0.000000 198.000000    384.666667
2           ap   0.0   -186.666667 222.000000 408.666667
3           bw   1250.0      0.000000 246.666667    460.000000
4           bj   750.0 0.000000 198.000000    inf
5           bp   2000.0      0.000000 222.000000    inf
6           order_quant      8000.0      0.000000 -200.000000 -164.444444

   Lower Range
0         187.111111
1         19.333333
2         -inf
3         153.333333
4         138.444444
5         106.666667
6         -222.333333


              Name Shadow Price RHS Coeff     Slack Upper Range Lower
Range
0           j_dem     44.666667     1000.0      0.0 3500.000000
0.000000
```

```
1           p_dem    115.333333     2000.0      0.0  3666.666667
 0.000000
2          w_qual    -46.666667        0.0      0.0  2500.000000
 -5000.000000
                     -46.666667        0.0      0.0  3000.000000
```

```
3          p_qual

-1000.000000 4
order_quantity         0.000000    13000.0  5000.0          inf
8000.000000
5           a_sup    293.333333     6500.0     0.0  5000.000000
-2500.000000         106.666667     6500.0     0.0  2500.000000
6           b_sup
-1666.666667
```

5D: Optimal Decision for Poor Year

```
Optimal objective 7.793939394e+04

Profit contribution: 77939.4

Decision variables:
aw = 545.455
aj   0
=
ap   0
=
bw   181.818
=
bj   0
=
bp   2000
=
order_quant = 2727.27


           Name Final Value  Reduced Cost   Obj Coeff  Upper Range  \
0           aw   545.454545      0.000000  246.666667   917.777778

1           aj     0.000000      0.000000  198.000000   222.969697

2           ap     0.000000    -84.848485  222.000000   306.848485

3           bw   181.818182      0.000000  246.666667  2260.000000

4           bj     0.000000     -8.323232  198.000000   206.323232

5           bp  2000.000000      0.000000  222.000000          inf

6  order_quant  2727.272727      0.000000 -200.000000  -193.133333
   Lower Range

0   184.444444
```

```
1         -inf

2         -inf

3   188.333333

4         -inf

5   183.030303

6 -228.577778
```

```
              Name Shadow Price  RHS Coeff         Slack   Upper Range  \
0            j_dem     0.000000     1000.0  1000.000000           inf
1            p_dem    38.969697     2000.0     0.000000  9.533333e+03
2           w_qual   -21.212121        0.0     0.000000  5.000000e+02
3           p_qual   -23.292929        0.0     0.000000  1.500000e+03
4 order_quantity       0.000000    13000.0 10272.727273           inf
5            a_sup   267.878788     2600.0     0.000000  2.260000e+04
6            b_sup   183.030303    10400.0     0.000000  2.000000e+03

     Lower Range
0    0.000000 1
     0.000000
2     -
     22600.00000
     0
3     -0.000000
4     2727.272727
5     -500.000000
6     -
     7533.333333
```

5E: Order for Sunny But Actual Weather Poor

```
Optimal objective -1.185111111e+06

Profit contribution: -1.18511e+06

Decision variables:
aw = 2350
aj =  250

ap =  0

bw =  783.333

bj =  750
```

```
bp = 2000
order_quant = 13000

          Name  Final Value  Reduced Cost   Obj Coeff  Upper Range  \
0           aw  2350.000000      0.000000  246.666667   709.777778

1           aj   250.000000      0.000000  198.000000   526.888889

2           ap     0.000000   -328.888889  222.000000   550.888889

3           bw   783.333333      0.000000  246.666667  1636.000000

4           bj   750.000000      0.000000  198.000000          inf

5           bp  2000.000000      0.000000  222.000000          inf

6 order_quant 13000.000000       0.000000 -200.000000          inf

    Lower Range
0 -8.222222e+01
1 -2.651111e+02
2          -inf
3 -2.842171e-14
4  4.362963e+01
5 -0.000000e+00
6          -inf


             Name  Shadow Price  RHS Coeff        Slack   Upper Range  \
0           j_dem    115.777778     1000.0     0.000000  1.040000e+04

1           p_dem    222.000000     2000.0     0.000000  8.866667e+03

2          w_qual    -82.222222        0.0     0.000000  2.350000e+03

3          p_qual    -82.222222        0.0     0.000000  3.000000e+03

4  order_quantity   -134.222222    13000.0     0.000000           inf

5           a_sup    328.888889     2600.0     0.000000  2.060000e+04

6           b_sup      0.000000    10400.0  6866.666667           inf

    Lower Range
0      0.000000
```

```
1       0.000000
2       -
        20600.000000
3       -1000.000000
4       3636.363636
5       -2350.000000
6       -6866.666667
```

5F: Order for Sunny But Actual Weather Normal

```
Optimal objective 9.755555556e

Profit contribution: 97555.6

Decision variables:
aw = 6250
aj = 250

ap = 0

bw = 2083.33

bj = 750

bp = 2000
order_quant = 13000


          Name   Final Value  Reduced Cost   Obj Coeff  Upper Range  \
0             aw 6250.000000      0.000000  246.666667   709.777778

1             aj  250.000000      0.000000  198.000000   526.888889

2             ap    0.000000   -328.888889  222.000000   550.888889

3             bw 2083.333333      0.000000  246.666667  1636.000000

4             bj  750.000000      0.000000  198.000000          inf

5             bp 2000.000000      0.000000  222.000000          inf

6 order_quant 13000.000000        0.000000 -200.000000          inf

    Lower Range
0 -8.222222e+01
1          -2.651111e+02
2          -inf
3          -2.842171e-14
```

```
4    4.362963e+01
5    0.000000e+00
6            -inf


              Name  Shadow Price  RHS Coeff        Slack  Upper Range  \
0            j_dem    115.777778     1000.0     0.000000  3500.000000

1            p_dem    222.000000     2000.0     0.000000  3666.666667

2           w_qual    -82.222222        0.0     0.000000  6250.000000

3           p_qual    -82.222222        0.0     0.000000  3000.000000

4   order_quantity    -35.555556    13000.0     0.000000          inf

5            a_sup    328.888889     6500.0     0.000000  5000.000000

6            b_sup      0.000000     6500.0  1666.666667          inf

   Lower Range
0     0.000000

1     0.000000
2     -5000.000000
3     -1000.000000
4     8000.000000
5     -6250.000000
6     -1666.666667
```