Data-Driven Optimization and Forecasting: A Case Study for WARP Shoe Company

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Abstract

Production schedules play a substantial role in retail company profitability, and their optimal utilization is conducive to a company's growth. The WARP Shoe company has come to us for our expertise in data-driven optimization and forecasting in order to make a shoe production scheduling model to maximize profit. We utilize an Integer Programming (IP) model to aid WARP with their production schedule for the month of February. To solve the IP model, we develop a robust computer program using AMPL that takes into account the revenue from sales minus all costs incurred. We investigate flexibilities in warehouse space, machine operating time, and budget for the purchase of raw materials. The study concludes that a maximum profit of $4,070,130.42 can be claimed by WARP.

I. Introduction

At the beginning of 2006, a major business opportunity arises after one of WARP's largest competitors goes bankrupt. This market shift has disrupted the competitive landscape between shoe production companies and it has been predicted that the demand for shoes will increase substantially. WARP aims to take advantage of this and reap the total benefits, however, capitalizing on this requires a careful consideration and formulation of WARP's production capabilities, resource allocation, and operational constraints to develop a highly robust and efficient production scheduling plan. Thus, WARP has come to us for aid in creating a production scheduling plan to meet the demands for the month of February, 2006.

The following paper contains sections pertaining to the study of an optimal production sheduling plan for WARP. The sections outline the assumptions made to simplify our IP, the methodology used to create our IP, the results and discussion that go in-depth of the current optimal solution, and an investigation on the sensitivity of certain variables of our model. The aim of this study is to provide an analysis into WARP's capabilities in order to synthesize an actionable production plan that satisfies WARP's immediate costs, and also to lay a framework for operational excellence in the face of fluctuating market demands.

A. Assumptions

The following assumptions were made:

- Demand for February 2006 is double the historical demand from February of previous years.
- Closing inventory for January 2006 is assumed to be zero.
- All sales occur at the end of the month.
- Every shoe produced is sold.
- Raw material budget is capped at $10,000,000.
- Not meeting the demand for a type of shoe translates to loss of current or potential customers, therefore a penalty of $10 per unmet shoe pair has been assigned for not meeting demand.
- The WARP shoe production plant, including each machine, will work up to 12 hours a day, 28 days a month (1,209,600 seconds).
- Machine setup costs are negligible.
- Machine setup times are negligible.
- Workers are paid on an hourly basis at a rate of $25/hour.
- Each machine must be operated by one worker.
- Transportation costs can be ignored.
- Manufacturing sequence can be ignored.

II. Methodology

The production planning model is constructed using decision variables, indices, parameters, and constraints as follows:

A. Sets and Indices

- $J = \{1, 2, \ldots, 557\}$: Set of shoe types.
- $K = \{1, 2, \ldots, 72\}$: Set of machines.
- $I = \{1, 2, \ldots, 165\}$: Set of raw materials.
- $N = \{1, 2, \ldots, 8\}$: Set of warehouses.

B. Decision Variables

Binary Variables:

$w_n \in \{0, 1\}$: 1 if warehouse n is operating, 0 otherwise.

Continuous Variables:

a_j : Number of pairs of shoe j produced.

C. Parameters

- s_j : Sale price per pair of shoe j.
- d_j : Forecasted demand for shoe j.
- rm_{ij} : Quantity of raw material i used for shoe j.
- rmc_i : Cost per unit of raw material i.
- rmq_i : Available quantity of raw material i.
- opm_k : Operating cost per minute for machine k.
- $avgdur_{kj}$: Average time in seconds for machine k to produce shoe j.
- $capw_n$: Capacity of warehouse n.
- $opcw_n$: Operating cost of warehouse n.

D. Objective Function

The objective is to maximize profit Z:

$$
\begin{aligned}
\text{Maximize } Z = \quad & \underbrace{\sum_{j \in J} s_j \cdot a_j}_{\text{Revenue from Sales}} \\
& - \underbrace{\sum_{i \in I} \sum_{j \in J} rmc_i \cdot rm_{ij} \cdot a_j}_{\text{Raw Material Costs}} \\
& - \underbrace{\sum_{k \in K} \sum_{j \in J} \frac{avgdur_{kj} \cdot a_j}{60} \cdot opm_k}_{\text{Machine Operating Costs}} \\
& - \underbrace{25 \sum_{k \in K} \sum_{j \in J} \frac{avgdur_{kj} \cdot a_j}{3600}}_{\text{Labour Costs}} \\
& - \underbrace{\sum_{n \in N} opcw_n \cdot w_n}_{\text{Warehouse Operating Costs}} \\
& - \underbrace{10 \cdot \sum_{j \in J} d_j - a_j}_{\text{Unmet Demand Penalty}}
\end{aligned}
$$

E. Constraints

1. Raw Material Budget:

$$\sum_{i\in I}\sum_{j\in J} rmc_i \cdot rm_{ij} \cdot a_j \leq 10{,}000{,}000.$$

2. Demand Satisfaction:

$$a_j \leq d_j, \quad \forall j \in J.$$

3. Warehouse Capacity:

$$\sum_{j\in J} a_j \leq \sum_{n\in N} capw_n$$

4. Raw Material Availability:

$$\sum_{j\in J} rm_{ij} \cdot a_j \leq rmq_i, \quad \forall i \in I.$$

5. Individual Machine Operating Time:

$$\sum_{j\in J} avgdur_{kj} \cdot a_j \leq 1{,}209{,}600 \quad \forall k \in K.$$

6. Non-Negativity, Integer, and Binary Constraints:

$$a_j, u_j \geq 0, \quad \forall j \in J,$$
$$a_j, u_j \in \mathbb{Z}, \quad \forall j \in J.$$

III. Results and Discussion

We aim to synthesize the results of our study in the following subsections.

A. Results

After our IP formulation, we used AMPL coupled with Gurobi to faciliate the calculation of the optimal profit. We found that the optimal profit is $4,070,130.42 and we sold the exact demand of shoes: 24517.

B. Estimated Demand for February

Among the historical data provided to us by WARP, was the demand for each shoe type j for each month from years 1997 to 2003. This study pertains to the optimization of production scheduling for the month of February alone. To account for this, we extrapolated the demand for each shoe type j in February by averaging the demand for each shoe type j from February in the years 1997 to 2003. We then multiplied each averaged demand by 2, based on WARP analysts' prediction that demand would double.The pseudo-code for this process is show in Table 1.

C. Variables and Constraints

Our IP formulation of the production schedule consists of four decision variables, in which two are binary (m_k, w_n) and two are continuous (a_j, u_j). We formulated five constraints to account for limitations imposed by WARP's raw material budget, demand satisfaction, warehouse capacity, raw material availability, and machine operating time.

TABLE I
PSEUDOCODE FOR WARP PRODUCTION DEMAND

Input: Historical demand for shoe j, Years: 1997–2003 **Output:** Extrapolated demand for each shoe type j
1. Initialize an empty list, *avg_demand* **2.** For each shoe type j: Retrieve demand data for j from 1997 to 2003. Compute the average of the demand values for j. Store the average in *avg_demand[j]*. **3.** Multiply each *avg_demand[j]* by 2 (WARP prediction). **4.** Store the result as the extrapolated demand for j. **5.** Return the extrapolated demand values.

D. AMPL Program

Solutions to the IP model were obtained using both the Gurobi solver along with AMPL for their robust optimization capabilities for operations research. Gurobi was used for its speed in computation and ability to handle large datasets efficiently. AMPL was chosen for its user-friendly syntax and simplicity. The pseudocode for our program is defined in Table. 2, 3, and 4.

TABLE II
PSEUDOCODE FOR .DAT FILE

Input: Data from tables in WARP2011W.mdb **Output:** Parameters
1. Read table from .mdb using table name, ODBC connector and database path. **2.** Set parameters using the sets defined in the .mod file. **3.** Read table to AMPL.

TABLE III
PSEUDOCODE FOR .MOD FILE

Input: Parameters **Output:** Decision Variable (a_j, w_n, m_k)
1. Initialize the decision variables and sets. **2.** Define objective function equation: Maximize profit (z). **3.** For each raw material i in I: Calculate the total raw material usage. Calculate the machine operating time Calculate the total storage requirement. **4.** Update the profit equation. **5.** Return the final decision variable quantities a_j, w_n, m_k).

TABLE IV
PSEUDOCODE FOR .RUN FILE

Input: .mod & .dat Files & Gurobi solver configuration **Output:** Optimized objective value z
1. Reset the environment. **2.** Load the model file into AMPL. **3.** Load the data file into AMPL. **4.** Set the solver options (Gurobi). **5.** Solve the optimization problem. **6.**Display the results.

E. Slack Analysis

The slack analysis obtained by our IP formulation show that the only binding constraints are the demand satisfaction constraints for specific shoe types, such as SH041 and SH329, where the amount produced for each shoe type met the demand exactly. This suggests that the formulated production plan is tightly aligned with the forecasted demand, leaving no room for excess production.

In contrast, the raw material budget constraint is non-binding, as there is significant slack associated with it, indicating that WARP has not fully utilized its $10,000,000$ budget for raw materials. Similarly, the warehouse capacity constraint is non-binding, indicating that the total warehouse capacity has not been fully utilized. Furthermore, there is significant slack associated with the availability of raw materials and operation times for individual machines.These considerations suggests that WARP has many opportunity to scale production further within existing resources and operations provided the demand for shoes increases.

F. Shadow Price Analysis

The shadow price analysis obtained by our IP formulation shows that the binding constraints are mostly the demand satisfaction constraints for specific shoe types, such as SH114 and SH345. For these specific types of shoes, the shadow prices are positive, reflecting the increase of production to meet a surplus of demand could significantly increase overall profit. On the other hand, the raw material budget constraint has a shadow price of zero, solidifying the result of unused resources in the slack analysis and suggesting that increasing the budget by the constraints slack would not improve the objective. Similarly, the warehouse capacity and machine operating time constraints are non-binding,as their shadow prices are zero, further solidifying the unused resources and the under utilization of machines.

IV. Investigation

In this section, we investigate how variations in key operational variables—warehouse space, machine availability, and raw material budget—affect the production schedule and overall profitability, both individually and in combination, through scenario-based analysis.

A. Analysis of Additional Warehouse Space

Suppose that additional warehouse space is available at the price of $10 per box of shoe, is it economical to buy it? What is the optimal amount of space to buy in this situation?

By introducing additional warehouse space at a cost of $10 per shoe, we observe that the objective function worsens. This is primarily because the cost of extra space is too high relative to the benefits it provides, in particular, the existing warehouse capacity already meets all demand as per the previous constraints, meaning the additional space incurs unnecessary costs. As a result, the model ends up purchasing extra space even when it's not needed to meet demand, leading to a decrease in the overall objective value. However, to get the highest possible value with this new constraint in the model, we will require 24799 units of additional space to achieve $4045612.74 profit.

B. Impact of Decreased Machine Availability

Suppose that each machine was available for only 8 hours per day instead of 12, how does our solution change? Which constraints are binding now? And is the new solution realistic?

By introducing a reduced time slot for 8 hours per day, we find that the right-hand side constraint for machine operating time is reduced to 806400 seconds of availability. Additionally, since we assumed that all machines can work in parallel, we find that the work done by every machine individually has to be less than the new constraint. When we solve for the optimal objective function value given these updated constraints, we find that our optimal value remains the same at $4,070,130.42 units of profit; this however makes sense because the reduction in available machine hours does not affect the optimal solution since each machine is only using around 0.5 hours on average, as indicated by the slack. This solution also makes sense because the reduction in available machine hours does not affect the optimal solution, as all machines still require less than 8 hours of work and work together in parallel as stated in the assumptions.

C. Effect of Increased Raw Material Budget

Suppose that there was an increase of $7,000,000 in the budget for raw materials, what should we do?

By introducing an increase in the raw material budget, we find that the right-hand side constraint for the raw material budget is increased to $17,000,000 to produce shoes. Therefore we find that the cost to produce all j shoes must be less than the new constraint. When we solve for the optimal objective function value given these updated constraints, we find that our optimal value remains the same at $4,070,130.42 units of profit. This, however, makes sense because the increase in the right-hand of the budget constraint does not affect the optimal solution due to the slack being $8,599,200 suggesting that the system already has an unused budget, therefore additions to this constraint will not improve the objective function. Knowing that this solution will not improve the optimal value, it wold prove beneficial to instead use this money to find a way to increase demand since that is the primary binding constraint, therefore improving that will result in an increase in total profits. Investment in marketing campaigns can prove to be useful especially when trying to increase demand.

V. Conclusion

The current IP model for the WARP shoe company offers various insights into what decisions the company can make to maximize profits given the market shift following the bankruptcy of a major competitor. The current model operates under various assumptions based on the given data and can be used to predict and model the best decisions to allow the company to maximize profit. From the model, we learned that the profit binding constraint was the demand and that increasing other budgets such as raw material budget and individual machine operating time will not change the optimal profit. This is due to the demand of shoes being low, so there is no need for all resources to be used. However there are limitations to the model due to data and validity of certain assumptions therefore future work can include increased rigour in IP formulation, that is, less raw assumptions and the inclusion of variables that we assumed to be negligible.

References

[1] University of Toronto, "Course Materials: Files and Modules," Available at: https://q.utoronto.ca/courses/364306/files/32615343?module_item_id=5874444, Accessed: Dec.2, 2024.