Zahid Esad ÇAKICI 150190006

1- Quicksort's asymptotic upper bound for worst-case deterministic pivot selection is θ ($n2$) since the recursion equation is T (n) = T (0) + T (n - 1) + θ (n). This equation is similar to Insertion sort, because one side of the partition is always elementless. In the best case however, partition will divide the matrix evenly and the recursion equation will be T (n) = 2T (n / 2) + θ (n) and this will have a θ ($nlogn$) just like in Merge Sort.

2- Quicksort's asymptotic upper bound for the selection of random pivots may have an effect on probabilistic analysis. Random variables of the indicators have some benefits in facilitating the computation. Each partition will be random in this analysis, so the divisions are random and probability of each division is 1 / n. We choose as each different partition of the random variable of the indicator, so E [Xk] is equal to 1 / n where k = 0, 1 ...., n – 1.

$$T(n) = \sum_{k=0}^{n-1} X_k(T(k) + T(n-k-1) + \theta(n)) \qquad E[T(n)] = E\left[\sum_{k=0}^{n-1} X_k(T(k) + T(n-k-1) + \theta(n))\right]$$

$$E[T(n)] = \sum_{k=0}^{n-1} E[X_k] * E[(T(k) + T(n-k-1) + \theta(n))] \qquad E[T(n)] = \sum_{k=0}^{n-1} E[X_k(T(k) + T(n-k-1) + \theta(n))]$$

$$E[T(n)] = \frac{1}{n}\sum_{k=0}^{n-1} E[T(k)] + \frac{1}{n}\sum_{k=0}^{n-1} E[T(n-k-1)] + \frac{1}{n}\sum_{k=0}^{n-1} \theta(n)$$

$$E[T(n)] = \frac{2}{n}\sum_{k=0}^{n-1} E[T(k)] + \theta(n)$$

Substitution method to prove $E[T(n)] \leq an\log n$:

Fact: $\sum_{k=0}^{n-1} klogk \leq \frac{1}{2}n^2\log n - \frac{1}{8}n^2$

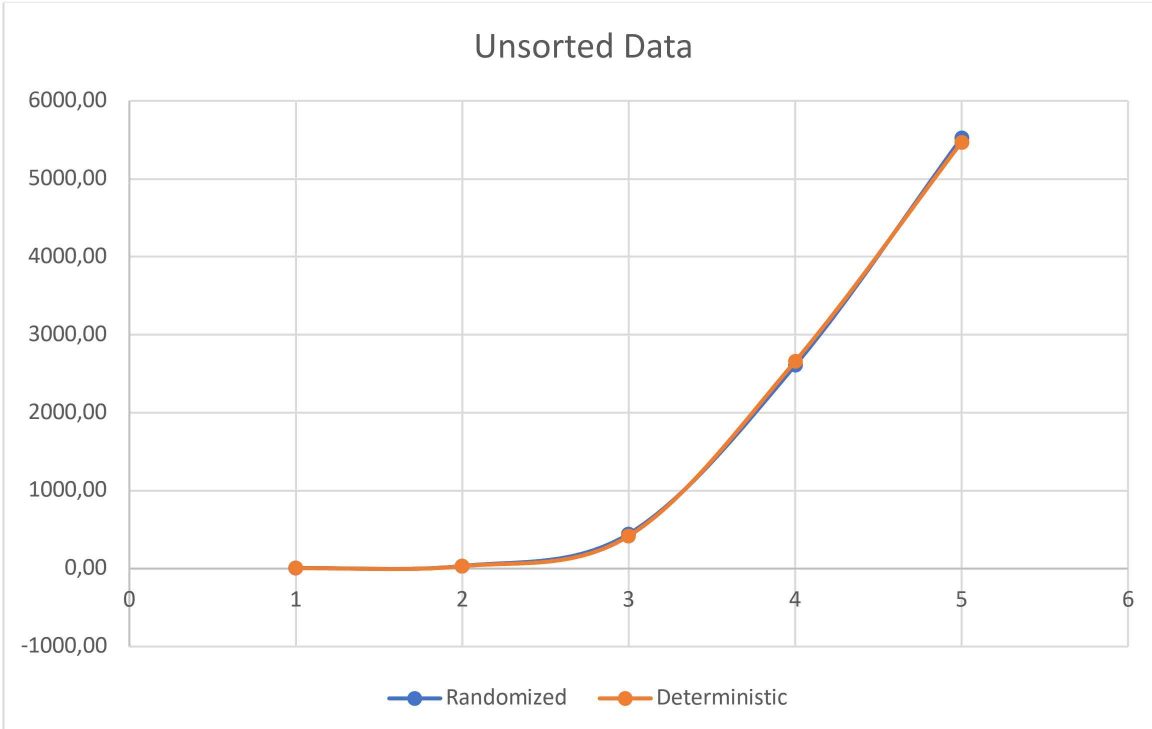
$$E[T(n)] \leq \frac{2}{n}\sum_{k=0}^{n-1} ak\log k + \theta(n)$$

$$E[T(n)] \leq \frac{2a}{n}\left(\frac{1}{2}n^2\log n - \frac{1}{8}n^2\right) + \theta(n)$$

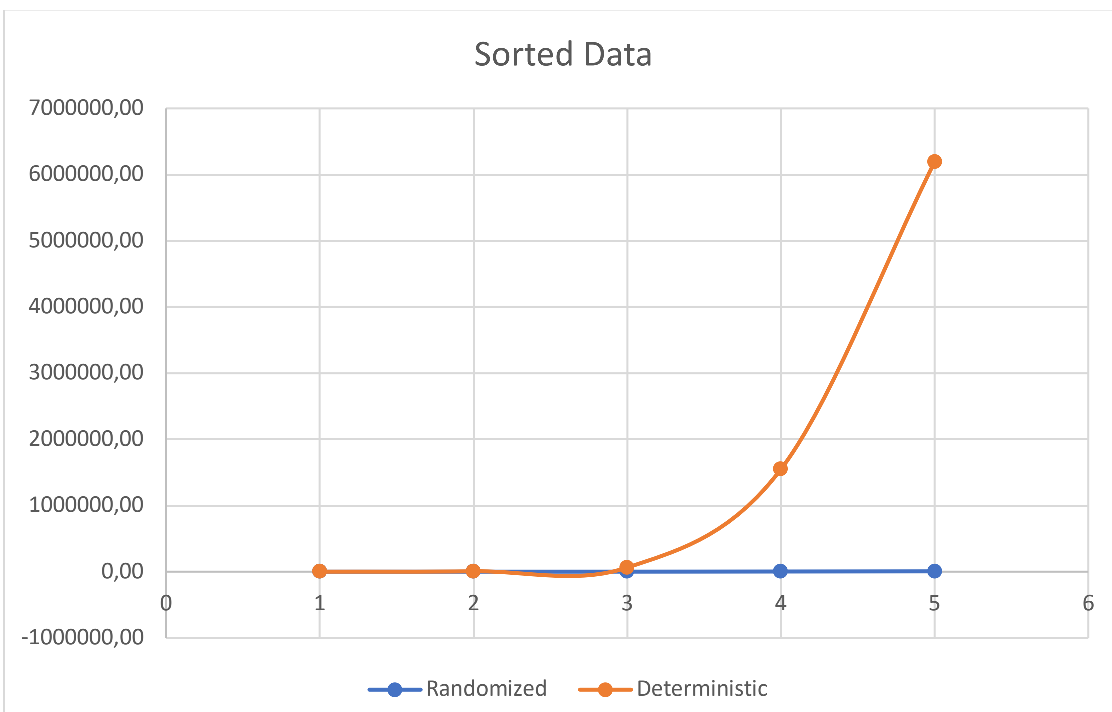
$$E[T(n)] = an\log n - \left(\frac{an}{4} - \theta(n)\right)$$

If a is chosen large enough so that an/4 dominates the Θ(n).

3-



The asymptotic upper limit of the deterministic quicksort is $\theta\ (n2)$ while the asymptotic upper bound of the randomized upper limit is $\theta\ (nlogn)$. So we can clearly see that they have approximately same execution time on unsorted data.



4-

Compared to random quick sort from unsorted data, the execution time of sorted data does not change a lot, however there is a big difference in deterministic sort since its complexity is θ (n2) while in unsorted data complexity is θ(nlogn). Thus, we should clearly see that it makes more sense to choose randomized quicksort as it does not differ in unsorted data and it is faster in sorted data.