ELEC6141 – Wireless Communications

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Summer 2018, Project Part 2

Project Report on

Effect of Interleaver on BER performance of Coded QPSK over Rayleigh fading channels

We certify that this submission is our original work and meets the Faculty's Expectations of Originality.

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Effect of Interleaver on BER performance of Coded QPSK over Rayleigh fading channels

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Abstract — **In this work, it is investigated the impact of interleaving on bit-error-rate (BER) performance of linear error-correcting codes over Rayleigh fading channels. For this analysis, a Quadrature Phase Shift Keying (QPSK) communication system with single-error-correcting (15, 11) Hamming code is considered. The numerical simulations are implemented on MATLAB and validated by the expected theoretical results.**

Keywords — QPSK, Rayleigh, Hamming Code, Interleaver

I. INTRODUCTION

Digital communication is a process which makes possible to transmit the required signal in a wired or wireless medium with measurable errors. This is possible by representing as digital symbols the analog signal before its transmission. In this process a carrier signal of much higher frequency is modulated by a baseband signal. There are many elemental schemes of modulation such as Amplitude Shift Keying (ASK), Frequency Shift Keying (FSK) and Phase Shift Keying (PSK).

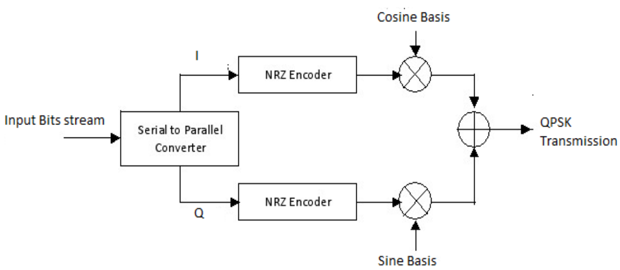

Fig. 1. QPSK Transmission Source [1]

In our project, it is considered that the data is modulated using QPSK scheme, a modulation technique where data bits are transmitted two orthogonal waveforms as shown in Fig. 1. This permits the bits to be grouped as 00, 01, 10 and 11 instead of being imitated as 0 or 1 only. In order to achieve maximum phase separation, constellation diagrams are put in use. To assure minimum error from one symbol to the other, Gray coding is employed. In QPSK, the states are separated by 90 degrees. The phases of the QPSK symbols are 45, 135, 225 and 315 degrees, respectively (see Fig. 2).

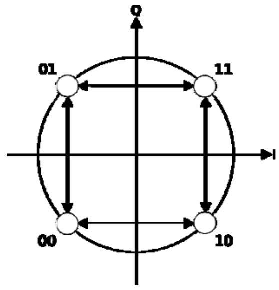

Fig 2. Constellation diagram of QPSK mapping [2]

Since QPSK transmits two bits per symbol instead of one as in basic modulation schemes as ASK and BPSK without degrading the system performance, it is quite used in wireless transmission, notably in the cellular system.

In the present work, the performance of QPSK system in terms of BER is evaluated in deep fading environments is presented and compared with the AWGN channel case.

II. CHANNEL MODELING

Channel is one of the main concerns of any communication system since it is their major source of loss. In general, a channel is classified according to the mathematical model describing how its physical conditions will affect the signals propagating through it. In wireless system, seeing that those conditions are random and time-variant, they can only be estimated statistically. This is to say, the channel model can only assures with a certain probability how the transmitted signal will be affected by the intervening medium. The accuracy of the model's prediction will depend on the degree of simplification of the physical phenomena.

a) AWGN channel

The most simple of those models is the Additive White Gaussian Noise (AWGN) channel. It only assumes linear addition of wideband noise with a constant spectral density and a Gaussian distribution of the amplitude, which accounts for thermal activities such as the vibrations of atoms, black body radiation, solar radiation, etc. The model does not account for frequency selectivity, interference, shadowing and other phenomena. However, it produces simple and easy to implement mathematical equations that give an insight into the underlying behavior of a system before other phenomena are considered.

b) Log-normal fading channel

In order to build a more realistic model, one must also consider the decaying of the signal strength with distance and the interference of obstacles along the way. This random attenuation of the signal throughout its propagation is called fading. Since in the majority of the applications (particularly in large cities) there is no direct line-of-sight path from a transmitter to a receiver, BER performance is most of the time limited by fading. The main fading mechanisms can be divided in basically two groups: the macroscopic and the microscopic ones.

Macroscopic or large-scale fading accounts for the attenuation over long propagation paths, i.e., due to atmospheric absorption and the obstruction of the direct line-of-sight path by buildings and mountains. In cellular applications, atmospheric effects are normally negligible. In this case, the path loss due to macroscopic fading in decibels can be modeled as a random variable with log-normal distribution, whose probability density function is given by:

$$f_{path}(x) = \frac{1}{\sqrt{2\pi\sigma_{path}^2}} e^{-\frac{x^2}{2\sigma_{path}^2}} \quad (1)$$

The variance σ_{path}^2 of Eq. (1) is calculated based on measurements taken over different locations while the median path loss accounting for different sort of environments (from rural to large city areas) is estimated by empirical models such as Okumura and Hata ones.

c) Rayleigh fading channel

When information is transmitted in a medium with limited line-of-sight (LOS), more than one transmission paths will result due to reflection, diffraction and scattering of the signal in the obstacles. It means the received signal will be actually a superposition of several copies of the transmitted signal with different propagation delays. Assuming that the number M of transmission paths is sufficiently large, those paths may be modeled as statistically independent. Thus, according to the central limit theorem the channel will be statistically characterized as a Rayleigh distribution whose probability density function is given by:

$$f_{Rayleigh}(\alpha) = \alpha \frac{e^{\frac{-\alpha^2}{2\sigma^2}}}{\sigma^2} \quad (2)$$

Because of multipath interference, there are frequencies at which the received signal will add constructively and other at which it will add destructively. The maximum bandwidth over which the loss will remain constant is estimated by the coherence bandwidth:

$$B_{coh} = \frac{1}{5\sigma_d} \quad (3)$$

where σ_d is the RMS delay spread of the received signals. If the signal bandwidth B_{sig} is much smaller than B_{coh}, then the fading does not vary with frequency (flat fading).

In small-scale fading, however, not only multipath accounts for fading but also the Doppler spread effect. The relative motion of the transmitter and the receiver implies in a frequency shift of the M signals. This means that the channel conditions is time-variant, which limit the maximum bit period (or bit rate) at which information can be sent with approximately constant path loss. The maximum period of constant loss is estimated by the coherence time:

$$T_{coh} = \frac{9}{16\pi f_d} \quad (4)$$

where f_d is the Doppler shift defined as ratio between the relative velocity v between the transmitter and the receiver and the signal wavelength:

$$f_d \triangleq \frac{v}{\lambda} \quad (5)$$

If the signal time (symbol period) T_{sig} is much smaller than T_{coh}, then the fading is said to be slow.

For simultaneously slow and flat fading channels (Rayleigh fading), the BER of QPSK signal is given by:

$$P_b(QPSK) = \frac{1}{2}\left(1 - \sqrt{\frac{E_b/N_0}{1 + E_b/N_0}}\right) \quad (6)$$

III. Channel Coding

As discussed in our previous work [3], channel coding can be used in order to make some errors detectable. This assures that they can be later corrected by signal processing. Considering the (15, 11) Hamming code from the class of linear block coding (LBC) techniques, every 11-bit message block (MB) will be rewritten into a 15-bit codeword (CW). This creates a redundancy that permits to correct one error per CW improving the BER. This improvement of the system performance is normally expressed in terms of coding gain:

$$\text{Coding gain} \triangleq \left(\frac{\mathrm{E_b}}{\mathrm{N_0}}\bigg|_{\text{uncoded}}\right)_{\text{dB}} - \left(\frac{\mathrm{E_b}}{\mathrm{N_0}}\bigg|_{\text{coded}}\right)_{\text{dB}} \quad (7)$$

The advantage of LBC is that it creates CWs through linear combination of MBs. Hence, data can be coded and decoded using easy-to-implement linear operators such as the generator matrix $\boldsymbol{G}$ and the parity-check matrix $\boldsymbol{H}$ [4].

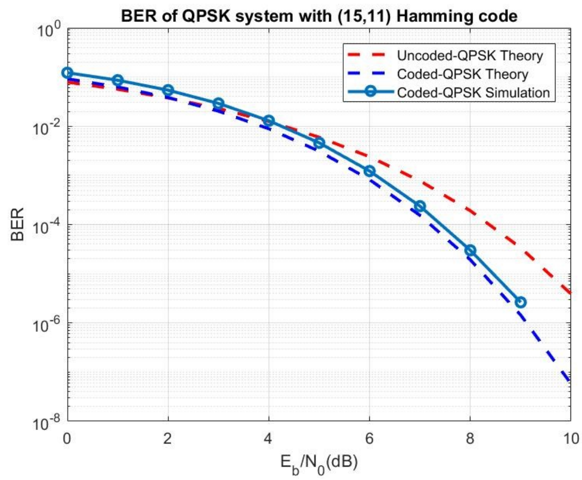

Fig. 3. BER of coded QPSK system over AWGN [3]

Since the redundancy introduced by LBC is quite limited, they only detect and correct information when the BER becomes small. As shown in Fig. 3, for a QPSK signal over AWGN channel with required BER of 10^{-5} a gain of 1 dB is expected after a (15, 11) Hamming code block is applied to the system. However, for higher BER ($> 10^{-2}$) coding gain is negligible or negative.

IV. INTERLEAVING

Due to the effect of microscopic fading, it is often found that error control coding such as the Hamming code is not enough to improve BER. This is because in deeply faded channels errors tend to occur in many consecutive bits rather than in independent bits. If the duration of this burst of errors is too long, interleaver/de-interleaver blocks must be added at the transmitter and the receiver, respectively. By interleaving the data, the bit rate is kept constant but the burst of errors is spread between many different CWs as random errors (see Fig. 4). This allows the error-correcting code to deal with the errors effectively.

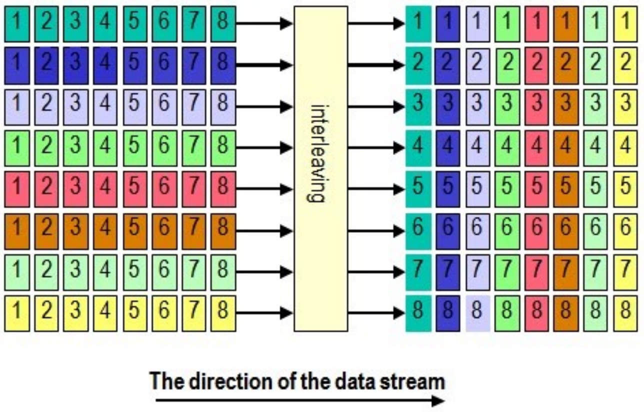

Fig. 4. Interleaving permits to distribute the burst of errors between many codewords by separation of adjacent bits [5]

To guarantee that the design of the interleaver/de-interleaver will scatter the errors sufficiently far away from each other the interleaving depth d (the separation between adjacent bits) should be equal or greater than the ratio of T_{coh} to CW period T_c:

$$d \geq \frac{T_{coh}}{T_c} \quad (8)$$

Therefore, the size of the interleaver for the considered (15,11) Hamming code will be 15 by d bits while the size of the de-interleaver will be d by 15 bits.

V. PROPOSED SYSTEM

In order to evaluate the impact of the interleaver block on BER performance when the system suffers from deep fading, let's consider a QPSK system with single-error-correcting (15, 11) Hamming code with an information rate $R_b = 1\, Mb/s$ and a carrier frequency $f_c = 10\, GHz$. Also, let's suppose that the receiver is moving in speeds of up to $60\, Km/Hour$.

For the proposed system, we obtain:

$$f_d = 555.55\, Hz \quad (9)$$

$$T_{coh} = 0.32\, ms \quad (10)$$

$$T_c = 733\, ns \quad (11)$$

$$d \cong 440\, bits \quad (12)$$

Since $T_c \ll T_{coh}$, the system has slow fading. For simplicity, receiver and transmitter are considered to be close enough in order to make large-scale fading negligible. Also, it will be assumed that the system presents flat fading. Thus, the small-scale fading can be said to be a Rayleigh one.

VI. NUMERICAL RESULTS

According to communication theory, base band and pass band systems are equivalent [6]. So, the numerical simulations of BER performance of the proposed system were done in base band since its implementation is much simpler on MATLAB.

First of all, the considered system was evaluated without coding and compared with an ideal AWGN channel. As it can be seen in Fig. 5, when multipath and Doppler spread are taking into consideration, the BER performance becomes much worse. In Rayleigh fading channels, BER decreases only linearly with the normalized SNR. At $\frac{E_b}{N_0} = 10$, for example, the BER difference is already of five orders of magnitude.

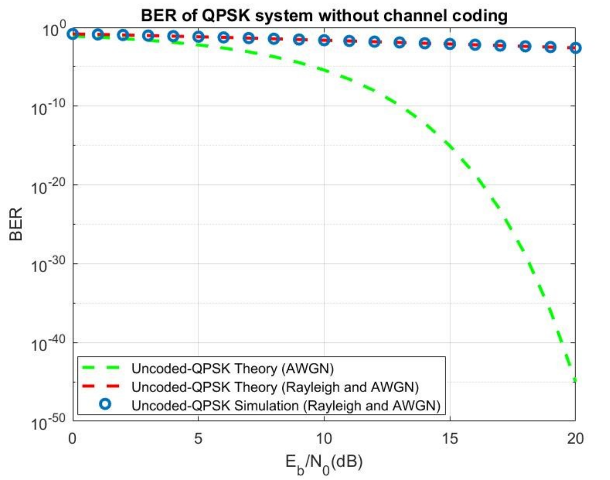

Fig. 5. Comparison in terms of BER of uncoded QPSK system over AWGN and Rayleigh fading channels

Subsequently, the coding block was introduced in the simulation. As expected, in deep fading the addition of the Hamming code only contributed to degrade even more the system performance (see Fig. 6) since it cannot correct burst error and coding reduces the energy per bit due to frequency spreading:

$$\frac{E_c}{N_0} = \frac{11}{15}\frac{E_b}{N_0} \cong 0.733\frac{E_b}{N_0} \quad (13)$$

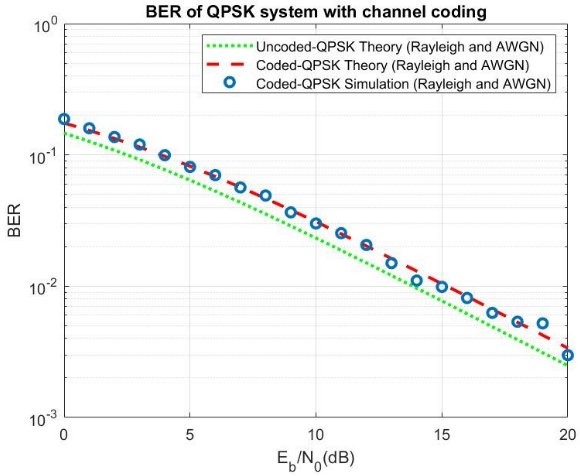

Fig. 6. BER of coded QPSK system over Rayleigh fading channel

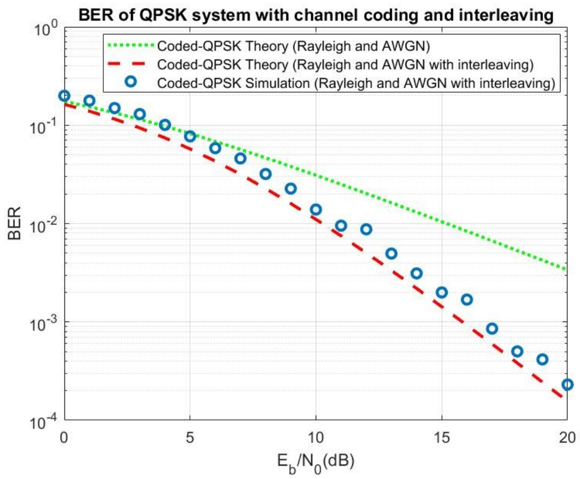

Fig. 7. Comparison in terms of BER of coded QPSK system over Rayleigh fading channel with and without an interleaver

Finally, in order to solve the burst of errors problem, the interleaver block was added to the system. Seeing that $d = 440$, the size of the interleaver block used on the simulation was 15 by 440 bits. Consequently, the size of the de-interleaver was 400 by 15 bits. As it is shown in Fig. 7, when channel coding is assisted by the interleaver, BER performance starts to get better after a certain threshold (a SNR above 4 dB). Nonetheless, despite of all improvements, the system performance is still much poor than the expected performance in an AWGN channel.

VII. Conclusion

An evaluation of the impact of the interleaver block on BER of a communication system suffering of deep fading has been presented. As demonstrated by analytical and numerical means, small-scale fading has a huge impact on system performance and cannot be improved by channel coding only since it introduces a burst of errors to the transmitted data. In order to make channel coding effective, errors must be spread out by interleaving the CWs.

References

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APPENDIX

a. Matlab code for (15,11) Hamming coded QPSK modulation scheme

```
clc
close all;
clear all;

%Simulation of a coded QPSK system in
base band simulation
%Channel coding: single error correcting
code Hamming (15,11)

n=15; %length of the codeword
k=11; %length of the message
t=1; %Number of corrected bits
N=2285800; %Total number of bits of the
data stream
EBN0dB=0:1:10; %Normalized SNR per bit
in dB
EBN0=10.^(EBN0dB/10); %Normalized SNR
per bit
ECN0dB = EBN0dB + 10*log10(k/n);
%Normalized SNR per coded bit in dB
ECN0=10.^(ECN0dB/10); %Normalized SNR
per coded bit
N0=10.^(-ECN0dB/10); %Noise spectral
density

I=eye(k);
P1=[1 0 1 1 1 0 0 0 1 1 1]';
P2=[1 1 0 1 1 0 1 1 0 0 1]';
P3=[1 1 1 0 1 1 0 1 1 0 0]';
P4=[1 1 1 1 0 1 1 0 0 1 0]';
P=[P1 P2 P3 P4];
G=[P I];
H=[eye(n-k) transpose(P)];

theory_uncoded_BER=(1/2)*erfc(sqrt(EBN0)
); %Theoretical BER for uncoded QPSK

Pc=(1/2)*erfc(sqrt(ECN0)); %Theoretical
BER for coded QPSK
aux=0; %Auxiliar variable
theory_coded_BER=0;

for p=(t+1):1:n
aux=(1/n)*(p*(factorial(n)/(factorial(p)
*factorial(n-p)))*(Pc.^p).*(1-Pc).^(n-
p));
theory_coded_BER=theory_coded_BER+aux;
end

for x=1:length(EBN0dB)
    i=round(rand(1,N));
%In-phase random bit stream
    q=round(rand(1,N));
%Quadrature random bit stream
    ui=reshape(i,k,N/k)';
%in-phase 11-bits message
    uq=reshape(q,k,N/k)';
%Quadrature 11-bits message

    ci=mod(ui*G,2);
%In-phase 15-bits codeword
    cq=mod(uq*G,2);
%Quadrature 15-bits codeword

    ci_s=reshape(ci',1,N*n/k);
%In-phase coded bits stream
    ci_s(ci_s==0)=-1;
%In QPSK 0 = -1
    cq_s=reshape(cq',1,N*n/k);
%Quadrature coded bits stream
    cq_s(cq_s==0)=-1;
%In QPSK 0 = -1

    s=ci_s+j*cq_s;
%Transmitted signal in base band


noise=(sqrt(N0(x)/2))*(randn(1,N*n/k)+j*
randn(1,N*n/k));    %AWGN channel

    r=s+noise;
%Received signal

    di=reshape(sign(real(r)),n,N/k)';
%In-phase hard decision decoding
    di(di<0)=0;
    dq=reshape(sign(imag(r)),n,N/k)';
%Quadrature hard decision decoding
    dq(dq<0)=0;

    syi=mod(di*H',2);
%In-phase syndrome calculation
    syq=mod(dq*H',2);
%Quadrature syndrome calculation

    e=[zeros(1,n) ; fliplr(eye(n))];
%Error pattern matrix

    s_est=mod(e*H',2);
%Syndrome matrix to build the look-up
table

    ci_est=zeros((N/k),n); %initializing
in-phase codeword estimation matrix
    cq_est=zeros((N/k),n); %initializing
quadrature codeword estimation matrix
    mi=zeros(N/k,k); %initializing in-
phase message estimation matrix
    mq=zeros(N/k,k); %initializing
quadrature codeword estimation matrix

        for p=1:(N/k)
            for t=1:(n+1)
                if syi(p,:)==s_est(t,:)
```

```
ci_est(p,:)=mod(di(p,:)+e(t,:),2); %In-
phase estimation of the codeword
                   mi(p,:)=ci_est(p,(n-
k+1):n); %In-phase estimation of the
message block
               end
               if syq(p,:)==s_est(t,:)

cq_est(p,:)=mod(dq(p,:)+e(t,:),2);
%Quadrature estimation of the codeword
                   mq(p,:)=cq_est(p,(n-
k+1):n); %Quadrature estimation of the
message block
               end
           end
       end

    ii=reshape(mi',1,N);
%Received in-phase data stream
    qq=reshape(mq',1,N);
%Received quadrature data stream

    BER1=(N-sum(i==ii))/N;          %In-
phase signal BER calculation
    BER2=(N-sum(q==qq))/N;
%Quadrature signal BER calculation
    BER(x)=mean([BER1 BER2]);
%Total BER
end

semilogy(EBN0dB,theory_uncoded_BER,'r--
',EBN0dB,theory_coded_BER,'b--',EBN0dB,
BER,'o-')
xlabel('E_b/N_0(dB)')
ylabel('BER')
title('BER of QPSK system with (15,11)
Hamming code')
legend('Uncoded-QPSK Theory','Coded-QPSK
Theory','Coded-QPSK Simulation');
grid on
```

b. Matlab code for uncoded QPSK modulation scheme with Rayleigh fading

```
clc
close all;
clear all;

%Simulation of uncoded QPSK system in
base band (AWGN + Rayleigh fading
channel)

N=1e6; %Total number of bits of the data
stream
EBN0dB=0:1:20; %Normalized SNR per bit
in dB
EBN0=10.^(EBN0dB/10); %Normalized SNR
per bit
N0=10.^(-EBN0dB/10); %Noise spectral
density

theory_BER_AWGN=(1/2)*erfc(sqrt(EBN0));
%Theoretical uncoded QPSK BER (AWGN)
theory_BER_Rayleigh=(1/2)*(1-
sqrt(EBN0./(1+EBN0))); %Theoretical
uncoded QPSK BER with Rayleign fading

for x=1:length(EBN0dB)
    data=round(rand(1,N)); %Random data
stream

    i=data(1:2:end); %In-phase bits
    q=data(2:2:end); %Quadrature bits

    Ac=sqrt(2); %signal amplitude

s=Ac*((q==0).*(i==0)*(exp(j*(5*pi/4)))+(
q==0).*(i==1)...

*(exp(j*(7*pi/4)))+(q==1).*(i==1)*(exp(j
*(9*pi/4)))...

+(q==1).*(i==0)*(exp(j*(11*pi/4))));
%Transmitted signal with Gray Coding


noise=Ac*sqrt(N0(x)/2)*(randn(1,N/2)+j*r
andn(1,N/2)); %AWGN channel
    ray_var=1; %slow fading coeff.
variance

alpha=sqrt(ray_var*((randn(1,N/2)).^2+(r
andn(1,N/2)).^2)); %Slow fading coeff.
(Rayleigh PDF approximation)

    r=alpha.*s+noise; %Received signal
(Multipath and AWGN over the signal)

    ddata=zeros(1,N); %Decoded data
vector initialization
    ddata(1:2:end)=sign(real(r)); %In-
phase hard decision decoding
    ddata(2:2:end)=sign(imag(r));
%Quadrature hard decision decoding

    ddata(ddata==-1)=0; %Mapping -1s to
0s again

    BER(x)=(N-sum(data==ddata))/N;
%Calculated BER vector
end

semilogy(EBN0dB,theory_BER_AWGN,'g:',EBN
0dB,theory_BER_Rayleigh,'r--',EBN0dB,
BER,'o','LineWidth',2)
xlabel('E_b/N_0(dB)')
ylabel('BER')
title('BER of QPSK system without
channel coding')
```

```
legend('Uncoded-QPSK Theory (AWGN)',...
    'Uncoded-QPSK Theory (Rayleigh and
AWGN)',...
    'Uncoded-QPSK Simulation (Rayleigh
and AWGN)');
grid on
```

c. Matlab code for (15,11) Hamming coded QPSK modulation scheme with Rayleigh fading

```
clc
close all;
clear all;

%Simulation of coded QPSK system in base
band (AWGN + Rayleigh fading channel)
%Channel coding: single error correcting
code Hamming (15,11)

%Coherence and signal time calculation
n=15; %length of the codeword
k=11; %length of the message
t=1; %Number of corrected bits
fc=1e10; %operating frequency (Hz)
c=3e8; %light speed
lambda=c/fc; %wavelength of the carrier
v=6e4/3600; %Speed (m/s) == 60 km/h
fm=v/lambda; %Maximum Doppler frequency
Tcoh=(9/16/pi)*(1/fm); %Coherence time
Rb=1e6; %Bit rate (bit/s)
Tb=1/Rb; %Bit duration (s/bit)
Ts=2*Tb; %Signal time (QPSK)
d=round(Tcoh/(Tb*(k/n)))+1; %Depth of
the interleaver (greater than Tcoh/Tb-
coded)

N=1936000; %2285800; %Total number of
bits of the data stream
EBN0dB=0:1:20; %Normalized SNR per bit
in dB
EBN0=10.^(EBN0dB/10); %Normalized SNR
per bit
ECN0dB = EBN0dB + 10*log10(k/n);
%Normalized SNR per coded bit in dB
ECN0=10.^(ECN0dB/10); %Normalized SNR
per coded bit
N0=10.^(-ECN0dB/10); %Noise spectral
density

I=eye(k);
P1=[1 0 1 1 1 0 0 0 1 1 1]';
P2=[1 1 0 1 1 0 1 1 0 0 1]';
P3=[1 1 1 0 1 1 0 1 1 0 0]';
P4=[1 1 1 1 0 1 1 0 0 1 0]';
P=[P1 P2 P3 P4];
G=[P I];
H=[eye(n-k) transpose(P)];

theory_uncoded_BER=(1/2)*(1-
sqrt(EBN0./(1+EBN0))); %Theoretical
uncoded QPSK BER with Rayleign fading

theory_coded_BER=(1/2)*(1-
sqrt(ECN0./(1+ECN0))); %Theoretical
coded QPSK BER with Rayleign fading

for x=1:length(EBN0dB)
    data=round(rand(1,N)); %Random data
stream

    i=data(1:2:end); %In-phase bits
    q=data(2:2:end); %Quadrature bits

    ui=reshape(i,k,N/k/2)';        %in-
phase 11-bits message
    uq=reshape(q,k,N/k/2)';
%Quadrature 11-bits message

    ci=mod(ui*G,2);              %In-
phase 15-bits codeword
    cq=mod(uq*G,2);
%Quadrature 15-bits codeword

    ci_s=reshape(ci',1,N*n/k/2);   %In-
phase coded bits stream
    cq_s=reshape(cq',1,N*n/k/2);
%Quadrature coded bits stream

    Ac=sqrt(2); %signal amplitude

c=Ac*((cq_s==0).*(ci_s==0)*(exp(j*(5*pi/
4)))+(cq_s==0).*(ci_s==1)...

*(exp(j*(7*pi/4)))+(cq_s==1).*(ci_s==1)*
(exp(j*(9*pi/4)))...

+(cq_s==1).*(ci_s==0)*(exp(j*(11*pi/4)))
); %Transmitted signal with Gray Coding


noise=Ac*sqrt((N0(x)/2))*(randn(1,N*n/k/
2)+j*randn(1,N*n/k/2));    %AWGN channel
    ray_var=1; %slow fading coeff.
variance

alpha=sqrt(ray_var*((randn(1,N*n/k/2)).^
2+(randn(1,N*n/k/2)).^2)); %Slow fading
coeff. (Rayleigh PDF approximation)

    % Variation of fading coeff. every
coherence time
    r1 = [ ];
    for p = 1:length(c)/d
        raux = alpha(p)*c(((p-
1)*d)+1:(p*d));
        r1 = [r1,raux];
    end

    r=r1+noise; %Received signal
(Multipath and AWGN over the signal)

    % Seperating bits with same fading
    sr = [];
```

```
    for p = 1:length(c)/d
    sraux = r((((p-
1)*d)+1:(p*d)))/alpha(p);
    sr = [sr,sraux];
    end

    di=reshape(sign(real(sr)),n,N/k/2)';
%In-phase hard decision decoding
    di(di<0)=0; %In-phase mapping -1s to
0s again
    dq=reshape(sign(imag(sr)),n,N/k/2)';
%Quadrature hard decision decoding
    dq(dq<0)=0; %Quadrature mapping -1s
to 0s again

    syi=mod(di*H',2); %in-phase syndrome
calculation
    syq=mod(dq*H',2); %Quadrature
syndrome calculation

    e=[zeros(1,n) ; fliplr(eye(n))];
%Error pattern matrix

    s_est=mod(e*H',2); %Syndrome matrix
to build the look-up table

    ci_est=zeros((N/k/2),n);
%initializing in-phase codeword
estimation matrix
    cq_est=zeros((N/k/2),n);
%initializing quadrature codeword
estimation matrix
    mi=zeros(N/k/2,k); %initializing in-
phase message estimation matrix
    mq=zeros(N/k/2,k); %initializing
quadrature codeword estimation matrix

       for p=1:(N/k/2)
           for t=1:(n+1)
               if syi(p,:)==s_est(t,:)

ci_est(p,:)=mod(di(p,:)+e(t,:),2); %In-
phase estimation of the codeword
                   mi(p,:)=ci_est(p,(n-
k+1):n); %In-phase estimation of the
message block
               end
               if syq(p,:)==s_est(t,:)

cq_est(p,:)=mod(dq(p,:)+e(t,:),2);
%Quadrature estimation of the codeword
                   mq(p,:)=cq_est(p,(n-
k+1):n); %Quadrature estimation of the
message block
               end
           end
       end

    ii=reshape(mi',1,N/2);
%Received in-phase data stream
    qq=reshape(mq',1,N/2);
%Received quadrature data stream

    ddata=zeros(1,N); %Decoded data
vector initialization
    ddata(1:2:end)=ii; %In-phase hard
decision decoding
    ddata(2:2:end)=qq; %Quadrature hard
decision decoding

    BER(x)=(N-sum(data==ddata))/N;
%Calculated BER vector
end

semilogy(EBN0dB,theory_uncoded_BER,'g:',
EBN0dB,theory_coded_BER,'r--
',EBN0dB,BER,'o','LineWidth',2)
xlabel('E_b/N_0(dB)')
ylabel('BER')
title('BER of QPSK system with channel
coding')
legend('Uncoded-QPSK Theory (Rayleigh
and AWGN)',...
    'Coded-QPSK Theory (Rayleigh and
AWGN)',...
    'Coded-QPSK Simulation (Rayleigh and
AWGN)');
grid on
```

d. Matlab code for (15,11) Hamming coded QPSK modulation scheme over Rayleigh-fading channel with interleaving

```
clc
close all;
clear all;

%Simulation of coded QPSK system in base
band (AWGN + Rayleigh fading channel)
%Channel coding: single error correcting
code Hamming (15,11)
%With interleaver

%Coherence and signal time calculation
n=15; %length of the codeword
k=11; %length of the message
t=1; %Number of corrected bits
fc=1e10; %operating frequency (Hz)
c=3e8; %light speed
lambda=c/fc; %wavelength of the carrier
v=6e4/3600; %Speed (m/s) == 60 km/h
fm=v/lambda; %Maximum Doppler frequency
Tcoh=(9/16/pi)*(1/fm); %Coherence time
Rb=1e6; %Bit rate (bit/s)
Tb=1/Rb; %Bit duration (s/bit)
Ts=2*Tb; %Signal time (QPSK)
d=round(Tcoh/(Tb*(k/n)))+1; %Depth of
the interleaver (greater than Tcoh/Tb-
coded)
```

```
N=1936000; %Total number of bits of the
data stream
EBN0dB=0:1:20; %Normalized SNR per bit
in dB
EBN0=10.^(EBN0dB/10); %Normalized SNR
per bit
ECN0dB = EBN0dB + 10*log10(k/n);
%Normalized SNR per coded bit in dB
ECN0=10.^(ECN0dB/10); %Normalized SNR
per coded bit
N0=10.^(-ECN0dB/10); %Noise spectral
density

I=eye(k);
P1=[1 0 1 1 1 0 0 0 1 1 1]';
P2=[1 1 0 1 1 0 1 1 0 0 1]';
P3=[1 1 1 0 1 1 0 1 1 0 0]';
P4=[1 1 1 1 0 1 1 0 0 1 0]';
P=[P1 P2 P3 P4];
G=[P I];
H=[eye(n-k) transpose(P)];

%Theoretical coded QPSK BER with
Rayleign fading and interleaving
Pc=(1/2)*(1-sqrt(ECN0./(1+ECN0)));
aux=0; %Auxiliar variable
theory_coded_interleaved_BER=0;

for p=(t+1):1:n
aux=(1/n)*(p*(factorial(n)/(factorial(p)
*factorial(n-p)))*(Pc.^p).*(1-Pc).^(n-
p));
theory_coded_interleaved_BER=theory_code
d_interleaved_BER+aux;
end

theory_coded_BER=(1/2)*(1-
sqrt(ECN0./(1+ECN0))); %Theoretical
coded QPSK BER with Rayleign fading

for x=1:length(EBN0dB)
    data=round(rand(1,N)); %Random data
stream

    i=data(1:2:end); %In-phase bits
    q=data(2:2:end); %Quadrature bits

    ui=reshape(i,k,N/k/2)';         %in-
phase 11-bits message
    uq=reshape(q,k,N/k/2)';
%Quadrature 11-bits message

    ci=mod(ui*G,2);              %In-
phase 15-bits codeword
    cq=mod(uq*G,2);
%Quadrature 15-bits codeword

    Nbk=(N/k)/2/d; %Number of blocks
    ci_s=[];
    cq_s=[];
    aux=[];
    for p=1:Nbk
        for t=1:n
        aux=ci(((p-1)*d+1):p*d,t)';
        ci_s=[ci_s aux];
        aux=cq(((p-1)*d+1):p*d,t)';
        cq_s=[cq_s aux];
        end
    end

    Ac=sqrt(2); %signal amplitude

c=Ac*((cq_s==0).*(ci_s==0)*(exp(j*(5*pi/
4)))+(cq_s==0).*(ci_s==1)...

*(exp(j*(7*pi/4)))+(cq_s==1).*(ci_s==1)*
(exp(j*(9*pi/4)))...

+(cq_s==1).*(ci_s==0)*(exp(j*(11*pi/4)))
); %Transmitted signal with Gray Coding


noise=Ac*sqrt((N0(x)/2))*(randn(1,length
(c))+j*randn(1,length(c)));    %AWGN
channel
    ray_var=1; %Slow fading coeff.
variance

alpha=sqrt(ray_var*(randn(1,length(c)).^
2+(randn(1,length(c)).^2))); %Slow
fading coeff. (Rayleigh PDF
approximation)


    % Variation of fading coeff. every
coherence time
    r1 = [ ];
    for p = 1:length(c)/d
        raux = alpha(p)*c(((p-
1)*d)+1:(p*d));
        r1 = [r1,raux];
    end

    r=r1+noise; %Received signal
(Multipath and AWGN over the signal)

    %Seperating bits with same fading
    sr = [];
    for p = 1:length(c)/d
    sraux = r((((p-
1)*d)+1:(p*d)))/alpha(p);
    sr = [sr,sraux];
    end

    dii=sign(real(sr)); %In-phase hard
decision decoding
    dii(dii<0)=0; %In-phase mapping -1s
to 0s again
    dqq=sign(imag(sr)); %Quadrature hard
decision decoding
    dqq(dqq<0)=0; %Quadrature mapping -
1s to 0s again
```

```
    di=[];
    dq=[];
    for p=1:Nbk
        for t=1:n
        di((p-1)*d+1:p*d,t)=dii(1,d*(t-1)+1+6600*(p-1):d*t+6600*(p-1))';
        dq((p-1)*d+1:p*d,t)=dqq(1,d*(t-1)+1+6600*(p-1):d*t+6600*(p-1))';
        end
    end

    syi=mod(di*H',2); %in-phase syndrome calculation
    syq=mod(dq*H',2); %Quadrature syndrome calculation

    e=[zeros(1,n) ; fliplr(eye(n))]; %Error pattern matrix

    s_est=mod(e*H',2); %Syndrome matrix to build the look-up table

    ci_est=zeros((N/k/2),n); %Initializing in-phase codeword estimation matrix
    cq_est=zeros((N/k/2),n); %Initializing quadrature codeword estimation matrix
    mi=zeros(N/k/2,k); %Initializing in-phase message estimation matrix
    mq=zeros(N/k/2,k); %Initializing quadrature codeword estimation matrix

        for p=1:(N/k/2)
            for t=1:(n+1)
                if syi(p,:)==s_est(t,:)
ci_est(p,:)=mod(di(p,:)+e(t,:),2); %In-phase estimation of the codeword
                    mi(p,:)=ci_est(p,(n-k+1):n); %In-phase estimation of the message block
                end
                if syq(p,:)==s_est(t,:)
cq_est(p,:)=mod(dq(p,:)+e(t,:),2); %Quadrature estimation of the codeword
                    mq(p,:)=cq_est(p,(n-k+1):n); %Quadrature estimation of the message block
                end
            end
        end

    ii=reshape(mi',1,N/2); %Received in-phase data stream
    qq=reshape(mq',1,N/2); %Received quadrature data stream
    ddata=zeros(1,N); %Decoded data vector initialization
    ddata(1:2:end)=ii; %In-phase hard decision decoding
    ddata(2:2:end)=qq; %Quadrature hard decision decoding

    BER(x)=(N-sum(data==ddata))/N; %Calculated BER vector
end

semilogy(EBN0dB,theory_coded_BER,'g:',EBN0dB,theory_coded_interleaved_BER,'r--',EBN0dB,BER,'o','LineWidth',2)
xlabel('E_b/N_0(dB)')
ylabel('BER')
title('BER of QPSK system with channel coding and interleaving')
legend('Coded-QPSK Theory (Rayleigh and AWGN)',...
    'Coded-QPSK Theory (Rayleigh and AWGN with interleaving)',...
    'Coded-QPSK Simulation (Rayleigh and AWGN with interleaving)');
grid on
```