

# **Principal Component Analysis (PCA)**

## Principal Component Analysis (PCA): Simplifying High-Dimensional Data

Principal Component Analysis (PCA) is a statistical technique widely used for dimensionality reduction in machine learning and data science. PCA transforms high-dimensional data into a lower-dimensional form while preserving the most important information. It's invaluable in exploratory data analysis, feature extraction, and visualization.

# **Key Features of PCA:**

#### 1. Dimensionality Reduction:

 PCA reduces the number of variables (features) in a dataset by transforming it into a new set of variables called principal components. The goal is to retain the most significant variance in the data, allowing for simpler models and easier visualization.

#### 2. Principal Components:

 Principal components are new, uncorrelated variables formed by linear combinations of the original variables. These components are ranked by the amount of variance they capture, with the first principal component accounting for the maximum possible variance.

## 3. Explained Variance:

 The amount of total variance that is captured by each principal component is referred to as the explained variance. This helps in determining how many components are sufficient to represent the majority of the data's information.

## 4. Eigenvalues and Eigenvectors:

PCA relies on calculating the eigenvalues and eigenvectors of the data's covariance matrix.
 The eigenvectors define the direction of the principal components, and the eigenvalues tell how much variance is captured along each principal component.

#### **How PCA Works:**

#### 1. Standardization:

Standardize the dataset to ensure each feature has a mean of 0 and a variance of 1, which
is important since PCA is sensitive to the scale of the data.

# 2. Covariance Matrix:

 Compute the covariance matrix to measure the relationships between different features in the data.

### 3. Eigenvalue Decomposition:

Calculate the eigenvalues and eigenvectors of the covariance matrix. The eigenvectors point
in the direction of maximum variance, and the corresponding eigenvalues quantify how much
of that variance is captured.

## 4. Projecting the Data:

The original data is projected onto the principal components to form a new dataset with reduced dimensions. This new dataset preserves the most significant variance while simplifying the structure.

## Advantages of PCA:

- **Reduces Complexity**: PCA reduces the number of features without much loss of information, simplifying models and making data easier to visualize.
- **Improves Model Performance**: By eliminating noise and redundant features, PCA can enhance the accuracy and efficiency of machine learning models.
- **Visualization**: PCA is often used to reduce data to 2D or 3D for visualization, making it easier to spot patterns in high-dimensional data.

#### Limitations:

- Loss of Interpretability: The new principal components are combinations of the original variables, making them harder to interpret in context.
- **Linearity**: PCA assumes that the relationships in the data are linear, meaning it may not work well for datasets with non-linear structures.
- **Sensitive to Scaling**: PCA is heavily influenced by the scaling of the input data, which means standardizing or normalizing the dataset is crucial before applying PCA.

## **Applications of PCA:**

- **Image Compression**: PCA is often used to reduce the dimensionality of image data while retaining important features for compression and reconstruction.
- **Feature Engineering**: PCA is widely used to reduce the number of features in machine learning models, especially when dealing with highly correlated data.
- **Genomics**: In bioinformatics, PCA helps in reducing the complexity of gene expression data, enabling researchers to identify patterns and classify samples.

Principal Component Analysis (PCA) is a powerful and versatile tool for simplifying high-dimensional data. Its ability to reduce dimensionality while retaining key information makes it crucial in tasks ranging from exploratory data analysis to machine learning model optimization. However, careful attention to scaling and the linear nature of PCA should be kept in mind for effective application.

## Notebook

#### October 23, 2024

```
[3]: import numpy as np # linear algebra
      import pandas as pd # data processing, CSV file I/O (e.g. pd.read_csv)
      import plotly.offline as py
      py.init_notebook_mode(connected=True)
      import plotly.graph_objs as go
      import plotly.tools as tls
      import seaborn as sns
      import matplotlib.image as mpimg
      import matplotlib.pyplot as plt
      import matplotlib
      %matplotlib inline
      # Import the 3 dimensionality reduction methods
      from sklearn.decomposition import PCA
[43]: # Import the pandas library for data manipulation and analysis
      import pandas as pd
      # Load the training dataset from a CSV file
      # The file path is specified as a raw string to avoid issues with backslashes
      train = pd.read_csv(r"C:\Users\Zahid.Shaikh\100days\61\train.csv")
      # Print the shape of the DataFrame to see the number of rows and columns
      print(train.shape)
      # Display the DataFrame to inspect its contents
      train
     (42000, 785)
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      [42000 rows x 785 columns]
[45]: # Save the labels to a Pandas Series named 'target'
      target = train['label']
      # Drop the label feature from the training dataset as it will not be used for
       \hookrightarrow PCA
      train = train.drop("label", axis=1)
      # Import the StandardScaler class from sklearn for data standardization
      from sklearn.preprocessing import StandardScaler
```

# Convert the training DataFrame to a NumPy array

X = train.values

```
X_std = StandardScaler().fit_transform(X)
      # Calculate the mean vector of the standardized data
      mean_vec = np.mean(X_std, axis=0)
      # Compute the covariance matrix of the standardized data
      cov_mat = np.cov(X_std.T)
      # Calculate the eigenvalues and eigenvectors of the covariance matrix
      eig vals, eig vecs = np.linalg.eig(cov mat)
      # Create a list of (eigenvalue, eigenvector) tuples for further processing
      eig_pairs = [(np.abs(eig_vals[i]), eig_vecs[:, i]) for i in_
       →range(len(eig_vals))]
      # Sort the eigenvalue-eigenvector pairs from high to low based on eigenvalues
      eig_pairs.sort(key=lambda x: x[0], reverse=True)
      # Calculate the total sum of eigenvalues
      tot = sum(eig vals)
      # Calculate individual explained variance for each principal component
      var_exp = [(i / tot) * 100 for i in sorted(eig_vals, reverse=True)]
      \# Calculate cumulative explained variance to see the total variance explained
       \hookrightarrow by the first k components
      cum_var_exp = np.cumsum(var_exp)
[49]: import plotly.graph_objs as go # Import necessary Plotly graphing library
      from plotly.subplots import make_subplots # Import to create subplots
      # Define the first trace for cumulative explained variance
      trace1 = go.Scatter(
          x=list(range(784)), # X-axis values representing feature columns
          y=cum_var_exp, # Y-axis values representing cumulative explained variance
          mode='lines+markers', # Display both lines and markers on the plot
          name="Cumulative Explained Variance", # Name for the legend
          line=dict(
              shape='spline', # Smooth line shape for better visual representation
              color='goldenrod' # Set the line color
          )
      )
      # Define the second trace for individual explained variance
      trace2 = go.Scatter(
         x=list(range(784)), # X-axis values representing feature columns
```

# Standardize the data by removing the mean and scaling to unit variance

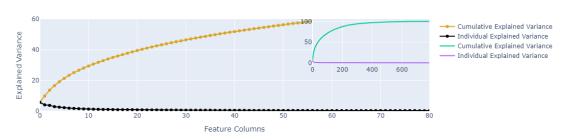
```
y=var_exp, # Y-axis values representing individual explained variance
   mode='lines+markers', # Display both lines and markers on the plot
   name="Individual Explained Variance", # Name for the legend
   line=dict(
        shape='linear', # Straight line shape for clarity
       color='black' # Set the line color
   )
)
# Create the figure using make_subplots to enable subplots and insets
fig = make subplots(
    insets=[{'cell': (1, 1), 'l': 0.7, 'b': 0.5}], # Define inset layout_\( \)
 \rightarrowparameters
   print_grid=True # Print qrid for better visualization
# Add the first trace (cumulative explained variance) to the main plot
fig.add_trace(trace1, 1, 1)
# Add the second trace (individual explained variance) to the main plot
fig.add trace(trace2, 1, 1)
# Configure layout settings for the main plot
fig.update_layout(
   title='Explained Variance Plots - Full and Zoomed-in', # Set the title of L
 → the plot
   xaxis=dict(range=[0, 80], title='Feature Columns'), # Configure X-axis__
 ⇔properties
   yaxis=dict(range=[0, 60], title='Explained Variance'), # Configure Y-axis,
 \hookrightarrowproperties
# Add the cumulative explained variance trace to the inset
fig.add_trace(go.Scatter(
   x=list(range(784)), # X-axis values for the inset
   y=cum_var_exp, # Y-axis values for cumulative explained variance
   xaxis='x2', # Specify which X-axis to use in the inset
   yaxis='y2', # Specify which Y-axis to use in the inset
   name='Cumulative Explained Variance' # Name for the inset trace
))
# Add the individual explained variance trace to the inset
fig.add_trace(go.Scatter(
   x=list(range(784)), # X-axis values for the inset
   y=var_exp, # Y-axis values for individual explained variance
   xaxis='x2', # Specify which X-axis to use in the inset
   yaxis='y2', # Specify which Y-axis to use in the inset
```

```
name='Individual Explained Variance' # Name for the inset trace
))

# Display the plot
fig.show()
```

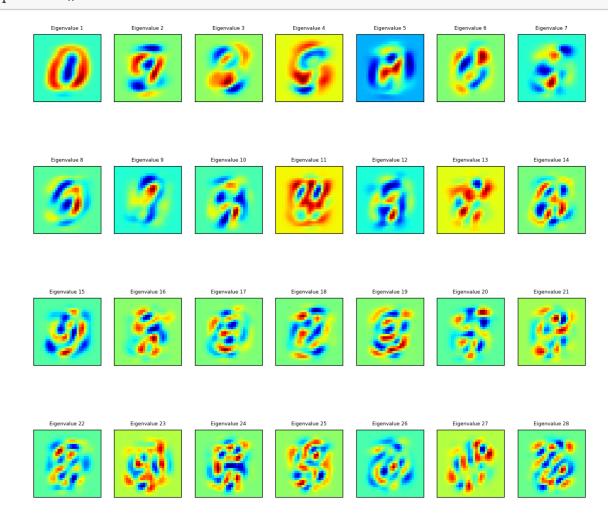
```
This is the format of your plot grid:
[ (1,1) x,y ]
With insets:
[ x2,y2 ] over [ (1,1) x,y ]
```

Explained Variance Plots - Full and Zoomed-in



```
[23]: # Invoke SKlearn's PCA method
      n_{components} = 30
      pca = PCA(n_components=n_components).fit(train.values)
      eigenvalues = pca.components_.reshape(n_components, 28, 28)
      # Extracting the PCA components ( eignevalues )
      eigenvalues = pca.components_
      n_row = 4
      n_{col} = 7
      # Plot the first 8 eignenvalues
      plt.figure(figsize=(13,12))
      for i in list(range(n_row * n_col)):
          offset =0
          plt.subplot(n_row, n_col, i + 1)
          plt.imshow(eigenvalues[i].reshape(28,28), cmap='jet')
          title_text = 'Eigenvalue ' + str(i + 1)
          plt.title(title_text, size=6.5)
          plt.xticks(())
          plt.yticks(())
```

# plt.show()



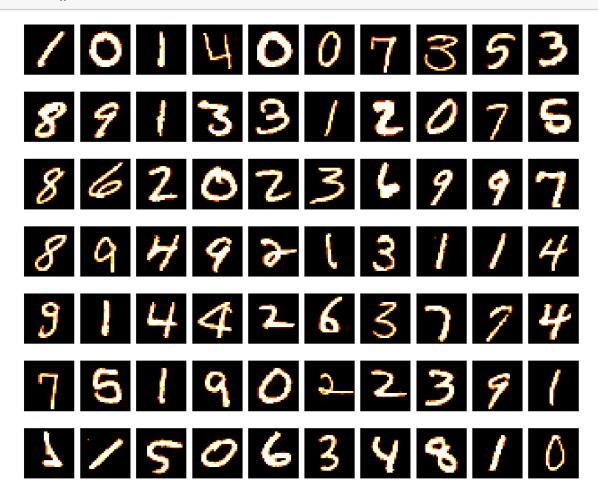
```
[30]: import matplotlib.pyplot as plt

# Plot some of the numbers
plt.figure(figsize=(14,12))
for digit_num in range(0, 70):
    plt.subplot(7, 10, digit_num + 1)

# Convert the row data to a numpy array and reshape it
grid_data = train.iloc[digit_num].to_numpy().reshape(28, 28)

# Plot the reshaped data as an image
plt.imshow(grid_data, interpolation="none", cmap="afmhot")
plt.xticks([])
plt.yticks([])
```

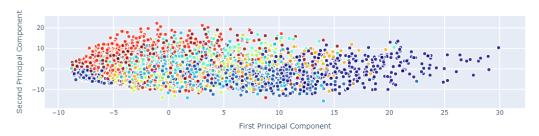
plt.tight\_layout()
plt.show()



```
Target = target[:6000]
```

```
[41]: trace0 = go.Scatter(
          x = X_5d[:,0],
          y = X_5d[:,1],
           name = Target,
          hoveron = Target,
          mode = 'markers',
          text = Target,
          showlegend = False,
          marker = dict(
              size = 8,
              color = Target,
              colorscale ='Jet',
              showscale = False,
              line = dict(
                  width = 2,
                  color = 'rgb(255, 255, 255)'
              ),
              opacity = 0.8
      data = [trace0]
      layout = go.Layout(
          title= 'Principal Component Analysis (PCA)',
          hovermode= 'closest',
          xaxis= dict(
               title= 'First Principal Component',
              ticklen= 5,
              zeroline= False,
              gridwidth= 2,
          ),
          yaxis=dict(
              title= 'Second Principal Component',
              ticklen= 5,
              gridwidth= 2,
          ),
          showlegend= True
      )
      fig = dict(data=data, layout=layout)
      py.iplot(fig, filename='styled-scatter')
```

#### Principal Component Analysis (PCA)



# []:

This notebook was converted with convert.ploomber.io