

# ECE 495 Autonomous Vehicles

## Tutorial 7 Overview

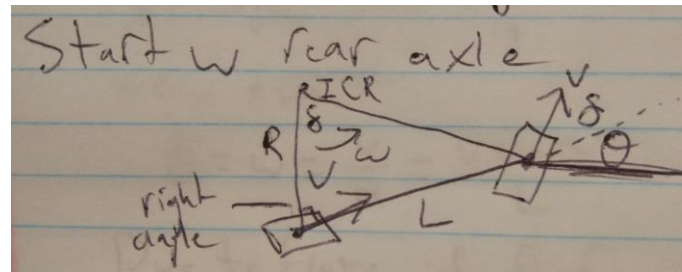
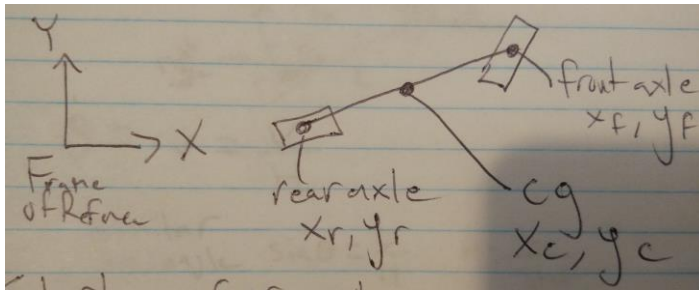
- . Before starting
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  - . Part 2 Output

# Before starting

- We are implementing a kinematic bicycle model
  - Simple compared to dynamic vehicle model
  - Works at low speeds
  - CG (center of gravity) bicycle model
    - w10-1\_vehicle\_modeling
      - slides 25 & 26
  - Let's go through the derivation

# Before starting – Rear bicycle model

- Theta: heading
- L: length between the two wheel axis
- Delta: steering angle measured from theta
- V: velocity pointing from wheel
- ICR: Instantaneous center of rotation



$$\dot{\theta} = \omega = \frac{v}{R}$$

rotation rate

Similar triangle  $\tan \delta = \frac{L}{R}$

new eqn

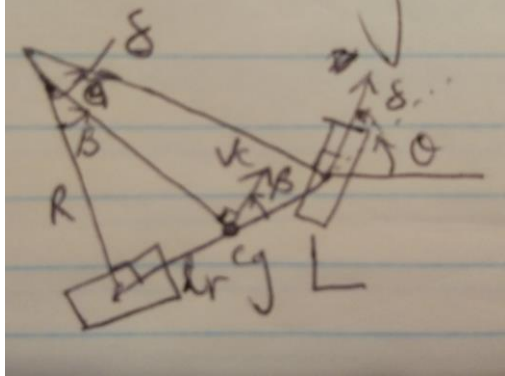
$$\dot{\theta} = \omega = \frac{v}{R} = \frac{v \tan \delta}{L}$$

$$\cos \theta = \frac{\dot{x}_r}{v} \quad \dot{x}_r = v \cos \theta$$

$$\sin \theta = \frac{\dot{y}_r}{v} \quad \dot{y}_r = v \sin \theta$$

# Before starting – CG bicycle model

- $l_r$ : distance from rear axle to cg
- Beta: Vehicle slip angle



for rear

$$\frac{\dot{x}}{V_c} = \cos(\theta + \beta)$$

$$\dot{x} = V_c \cos(\theta + \beta)$$

$$\frac{\dot{y}}{V_c} = \sin(\theta + \beta)$$

$$\dot{y} = V_c \sin(\theta + \beta)$$

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an

for rear

$$\dot{\theta} = \omega = \frac{V_r \tan \delta}{L}$$

$V_r$  angles

$$\cos(\beta) = \frac{V_r}{V_c} \quad V_r = V_c \cos(\beta)$$

for cg

$$\dot{\theta} = \omega = \frac{V_c \cos(\beta) \tan(\delta)}{L}$$

$$\tan \beta = \frac{l_r}{R} \quad \tan \delta = \frac{L}{R}$$

$$\tan \beta = \frac{l_r \tan \delta}{L} \quad \leftarrow R = \frac{L}{\tan \delta}$$

$$\beta = \tan^{-1} \left( \frac{l_r \tan \delta}{L} \right)$$

# A7 Goals

- Implement step (update) function for the CG (center of gravity) bicycle kinematic model
- Find control inputs to the step function in order to steer the bicycle in a figure 8 loop

## A7 – Part 1

- Bicycle class values
  - Max turn rate of 1.22 rad/sec (`self.w_max`)
  - Wheel base length of 2 m (`self.L`)
  - Length from rear axle to center of mass is 1.2 (`self.l_r`)
- Implement the step function within the Bicycle class
  - Arguments
    - $v$  – bicycle velocity
    - $w$  – steering angle rate

## A7 – Part 1

- Calculate values for:
  - $\dot{x}_c$ ,  $\dot{y}_c$ ,  $\dot{\theta}$ ,  $\dot{\delta}$
  - \*\*\*Remember to properly clamp  $w$ \*\*\*

$$\dot{x}_c = v \cos (\theta + \beta)$$

$$\dot{y}_c = v \sin (\theta + \beta)$$

$$\dot{\theta} = \frac{v \cos \beta \tan \delta}{L}$$

$$\dot{\delta} = \omega$$

## A7 – Part 1

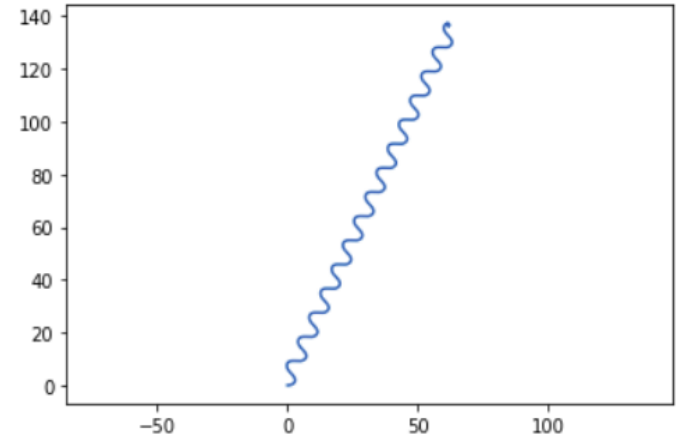
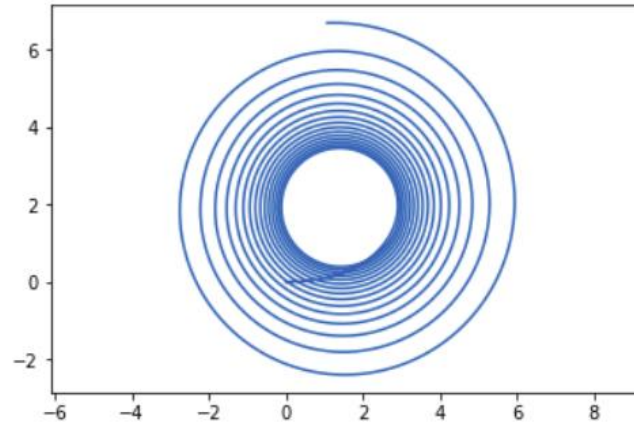
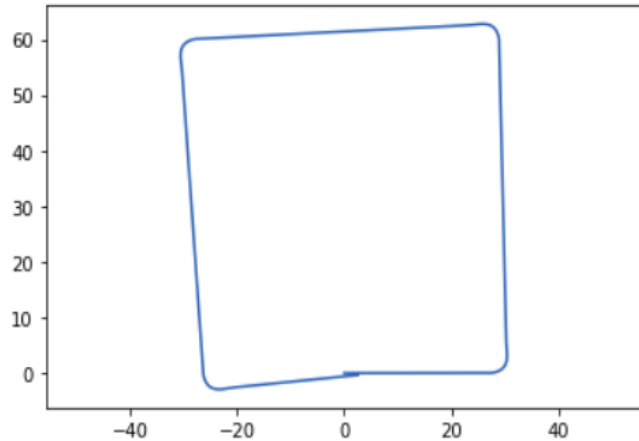
- Update variables
  - Use the calculated values and self.sample\_time
    - self.xc, self.yc, self.delta, self.theta
  - Use Beta equation
    - self.beta

$$\beta = \tan^{-1}\left(\frac{l_r \tan \delta}{L}\right)$$



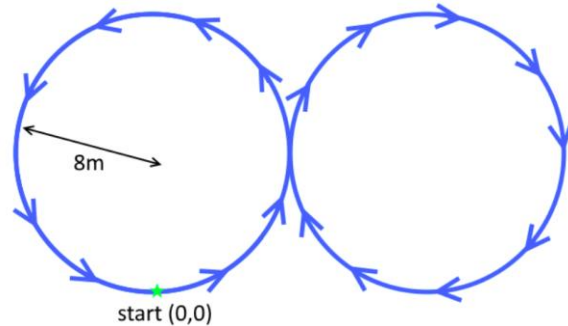
# A7 – Part 1 Output

- Uncomment different code blocks to test output
  - Square, spiral, wave pattern



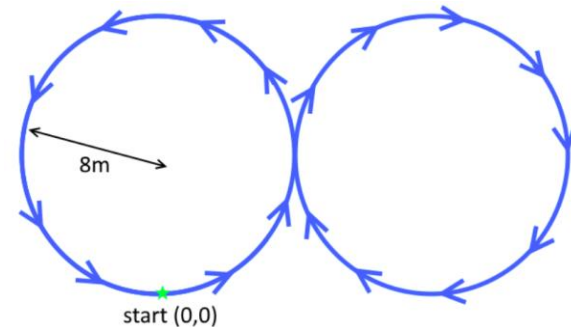
## A7 Part 2

- Program the control inputs to have the bicycle create a figure 8 trajectory
- Requirements:
  - Both circles have a radius of 8 m
  - Finish following the trajectory in 30 s



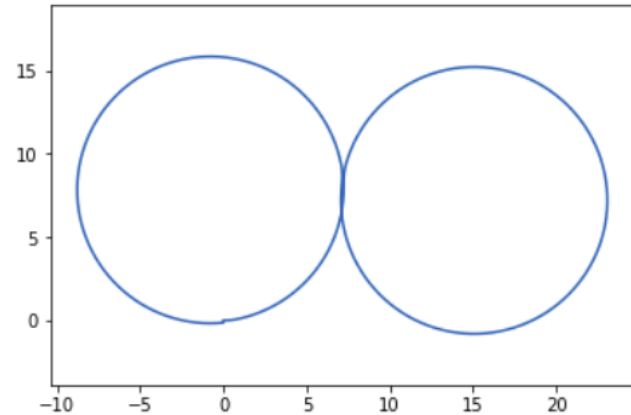
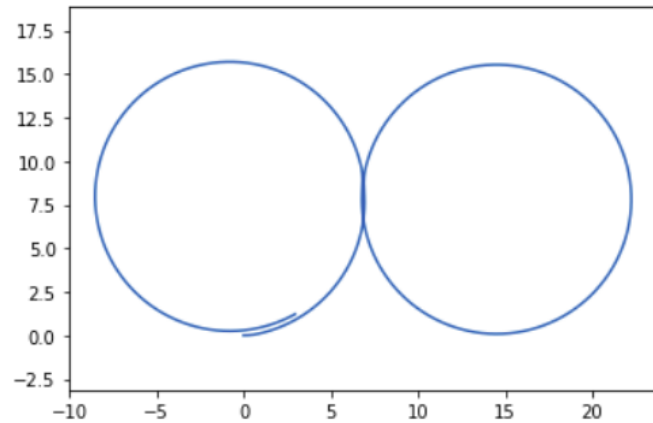
## A7 Part 2 Hints

- I recommend reading over the code and math in the assignment to have the bicycle ride in a circle
- Calculating  $v$ 
  - 2 circles, radius = 8 m, must traverse in 30 seconds
- Calculate desired delta, set  $w$  to steer to it
- Three turns: left, right, left
  - Test angles for when to start next turn



# A7 Part 2 Output

- Example solution outputs



# A7 tutorial

- . Finished