## Exact Differential Equation:

A differential equation of type

$$M(x,y)dx + N(x,y)dy = 0$$

is called exact differential equation if there exists a function of two variables $F(x,y)$ with continuous partial derivatives such that

$$dF(x,y) = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy$$

$$= M(x,y)dx + N(x,y)dy$$

The general solution of an exact equation is given by

$$\boldsymbol{F(x,y) = c}$$

## Test for Exactness:

Let the function $M(x,y)$ and $N(x,y)$ have continuous partial derivatives in a certain domain D. The differential equation

$$M(x,y)dx + N(x,y)dy = 0$$

is an exact equation if and only if

$$\boldsymbol{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

## Algorithm for solving an Exact Differential Equation:

Step1: Test for exactness

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Step 2: If exact, we write the system of two differential equations that define the function $F(x,y)$

$\frac{\partial F}{\partial x} = M(x,y)$ -------- (1)

$\frac{\partial F}{\partial y} = N(x,y)$-----------(2)

Step 3:Integrate the first equation over the variable x. Instead of the constant C, we write an unknown function of y.

$F(x,y) = \int M(x,y)dx + \emptyset(y)$--------------(3)

Step 4:Differentiating with respect to y, we substitute the value of $\frac{\partial F}{\partial y}$ into the second equation

$\frac{\partial F}{\partial y} = \frac{\partial}{\partial y}[\int M(x,y)dx + \emptyset(y)]$---------(4)

$$=> N(x,y) = \frac{\partial}{\partial y}[\int M(x,y)dx + \emptyset(y)]$$

$$=> \emptyset(y) = N(x,y) - \frac{\partial}{\partial y}[\int M(x,y)dx]$$

Step 5: The general solution of the exact differential equation is given by

$$F(x,y) = c$$

1: $(3x^2y+2)dx + (x^3+y)dy = 0$ (Class work)

2: $(x+y)dy + (y-x)dx = 0$ (Class work)

3: $(y^2 - 2xy + 6x)dx - (x^2 - 2xy + 2)dy = 0$ (Class Work)

4: $ysin2x\ dx - (y^2 + cos^2x\ dy) = 0$ (HW)

(ans.$3y\ cos2x - 2y^3 + c = 0$)

5:$(2x - y + 1)dx + (2y - x - 1)dy = 0$ (HW)

(ans.$x^2 - xy + x + y^2 - y = 0$)

6: $(x^2y - 2xy^2)dx - (x^3 - 3xy^2) = 0$

7: $(1-xy)ydx - (1+xy)xdy = 0$