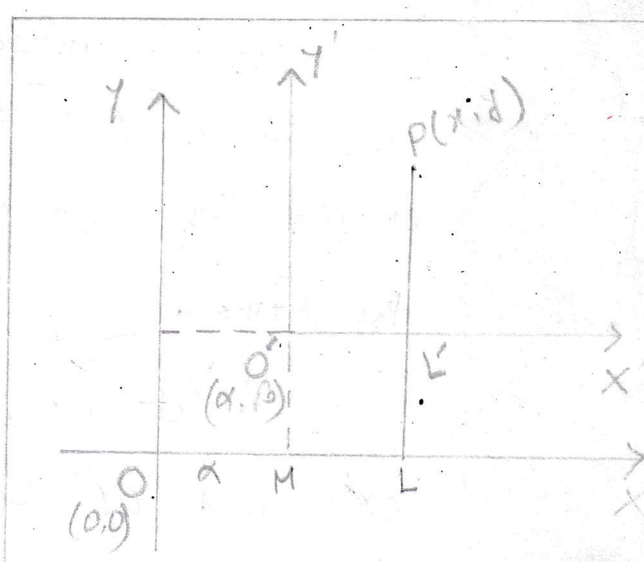
Transformation of co-ordinates:

The co-ordinates of a point or the equation of a curve are always given with reference to a fixed origin and a set of axes of co-ordinates.

The above co-ordinates or the equation of the curve changes when the origin is changed or the direction of axes is changed or both. The process of changing the co-ordinate of a point or the equation of a curve is called transformation of co-ordinates.

Translation of axes (shift of the origin):

To find the change in the co-ordinates of a point when the origin is shifted to another point (α, β) where the direction of axes remains unaltered.



O is the original origin with respect to the axis OX, OY. Shift the origin O to $O'(\alpha, \beta)$

Let $P(x, y)$ be the point with respect to axis

OX, OY. Suppose the new axes are $O'X'$ and $O'Y'$ and the point P' w.r.to new axes $O'X'$ and $O'Y'$ is (x', y').

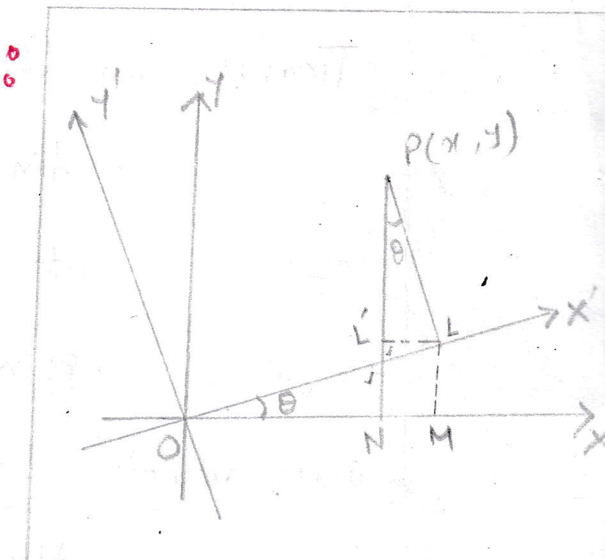
$$\therefore x = OL = OM + ML$$
$$= \alpha + O'L'$$
$$= \alpha + x'$$

$$y = PL = PL' + LL'$$
$$= y' + \beta$$

$\therefore$ The transformed co-ordinate is,

$$\therefore x' = x - \alpha$$
$$y' = y - \beta$$

Rotation of axes (Origin fixed):



To find the change in the co-ordinates of a point when the direction of axes is turned through an angle θ where as the origin of co-ordinates remains the same.

Let OX and OY be the old axes and OX' and OY' be the new axis. O is the common origin for the two sets of axes.

Let the angle $\angle XOX'$ through which the axes have rotated be represented by θ.

Let P be any point in the plane and let its co-ordinates referred to the old axes be (x, y) and referred to the new axes be (x', y'). Draw PN perpendicular to OX, PL perpendicular to OX' and LM perpendicular to OX, ~~and~~

Then, $x = ON$

$= OM - MN$

$= OM - LL'$

$= OL\cos\theta - PL\sin\theta$

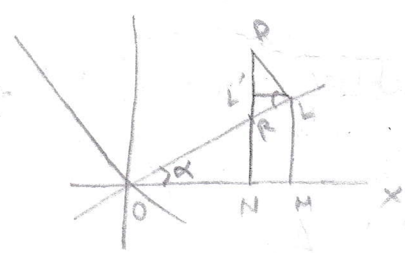
$= x'\cos\theta - y'\sin\theta$.

$y = PN = NL' + PL'$

$= ML + PL'$

$= OL\sin\theta + PL\cos\theta$

$= x'\sin\theta + y'\cos\theta$



1. Transform to Parallel axes through the origin of the equations:

(1) origin $(1,-2)$, $2x^2+y^2-4x+4y=0$

Solution:

Given that,

$$2x^2+y^2-4x+4y=0 \quad \text{--- (1)}$$

The new origin is, $(1,-2)$

$\therefore x = x'+1$

$y = y'-2$

Now the equation (1) is,

$$2(x'+1)^2+(y'-2)^2-4(x'+1)+4(y'-2)=0$$

$$\Rightarrow 2(x'^2+2x'+1)+(y'^2-4y'+4)-4x'-4+4y'-8=0$$

$$\Rightarrow 2x'^2+4x'+2+y'^2-4y'+4-4x'-4+4y'-8=0$$

$$\Rightarrow 2x'^2+y'^2-6=0$$

$$\Rightarrow 2x'^2+y'^2=6$$

Removing the suffix we have,

$$2x^2+y^2=6. \quad \text{Ans.}$$

(ii) Origin $(3,1)$, $x^2-6x+2y^2+7=0$

Ans: $x^2+2y^2+4y=0$

Solution:

2. Transform the following equations into the equations after rotating the axes through an angle 45°.

(i) $x^2-y^2=a^2$

Solution: Given that,

$$x^2-y^2=a^2 \quad \text{--- (1)}$$

Here $\theta = 45^\circ$

$$\therefore x = x' \cos 45^\circ - y' \sin 45^\circ$$

$$= x' \frac{1}{\sqrt{2}} - y' \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} (x' - y')$$

$$y = x' \sin 45^\circ + y' \cos 45^\circ$$

$$= x' \frac{1}{\sqrt{2}} + y' \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} (x' + y')$$

Then the corresponding equation,

$$\left\{\frac{1}{\sqrt{2}} (x' - y')\right\}^2 - \left\{\frac{1}{\sqrt{2}} (x' + y')\right\}^2 = a^2$$

$$\Rightarrow \frac{1}{2} (x'^2 - 2x'y' + y'^2) - \frac{1}{2} (x'^2 + 2x'y' + y'^2) = a^2$$

$$\Rightarrow \frac{1}{2} (x'^2 - 2x'y' + y'^2 - x'^2 - 2x'y' - y'^2) = a^2$$

$$\Rightarrow \frac{1}{2} (-4x'y') = a^2$$

$$\Rightarrow -2x'y' = a^2$$

Removing suffix we get

$$-2xy = a^2.$$

(vi) $x^2 - y^2 - 2\sqrt{2}x - 10\sqrt{2}y + 2 = 0$ Ans: $xy + 6x + 4y = 1$

3. Remove the first degree terms in each of the following equations and find the new origin

(i) $3x^2 - 4y^2 - 6x - 8y - 10 = 0$

Solution:

Let (α, β) be the new origin.

Then $x = x' + \alpha$

$y = y' + \beta$

Now, $3x^2 - 4y^2 - 6x - 8y - 10 = 0$

$\Rightarrow 3(x'+\alpha)^2 - 4(y'+\beta)^2 - 6(x'+\alpha) - 8(y'+\beta) - 10 = 0$

$\Rightarrow 3(x'^2 + 2x'\alpha + \alpha^2) - 4(y'^2 + 2y'\beta + \beta^2) - 6x' - 6\alpha - 8y'$

$- 8\beta - 10 = 0$

$\Rightarrow 3x'^2 + 6x'\alpha + 3\alpha^2 - 4y'^2 - 8y'\beta - 4\beta^2 - 6x' - 6\alpha - 8y'$

$- 8\beta - 10 = 0$

$\Rightarrow 3x'^2 - 4y'^2 + (6\alpha - 6)x' + (-8\beta - 8)y' + 3\alpha^2$

$- 4\beta^2 - 6\alpha - 8\beta - 10 = 0$

To remove the 1st degree term,

$6\alpha - 6 = 0$, $-8\beta - 8 = 0$

$\Rightarrow \alpha = 1$ $\Rightarrow \beta = -1$

$\therefore$ New origin $(1, -1)$

New equation,

$\therefore$ $3x'^2 - 4y'^2 + 3 - 4 - 6 + 8 - 10 = 0$

$\Rightarrow 3x'^2 - 4y'^2 = 9$

Removing suffix we have,

$$3x^2 - 4y^2 = 9 .$$

(ii) $2x^2 + 5y^2 - 12x + 10y - 7 = 0$ 2(x

Ans: $2x^2 + 5y^2 = 30$

Solution:

(iii) $12x^2-10xy+2y^2+11x-5y+2=0$

Solution:

Let (α, β) be the new origin.

$$\therefore \quad x = x'+\alpha$$
$$y = y'+\beta$$

Then given eqn become,

$$12(x'+\alpha)^2-10(x'+\alpha)(y'+\beta)+2(y'+\beta)^2+11(x'+\alpha)-5(y'+\beta)+2=0$$

$$\Rightarrow 12(x'^2+2x'\alpha+\alpha^2)-10(x'y'+y'\alpha+x'\beta+\alpha\beta)+2(y'^2+2y'\beta+\beta^2)+11(x'+\alpha)-5(y'+\beta)+2=0$$

$$\Rightarrow 12x'^2+24x'\alpha+12\alpha^2-10x'y'-10y'\alpha-10x'\beta-10\alpha\beta+2y'^2+4y'\beta+2\beta^2+11x'+11\alpha-5y'-5\beta+2=0$$

$$\Rightarrow 12x'^2+x'(24\alpha-10\beta+11)+y'(-10\alpha+4\beta-5)+2y'^2+12\alpha^2-10x'y'-10\alpha\beta+2\beta^2+11\alpha-5\beta+2=0 \quad \text{——} (1)$$

Removing 1st degree terms,

$$24\alpha-10\beta+11=0$$

and $$-10\alpha+4\beta-5=0$$

Solving these we get,

$\alpha = -3/2$, $\beta = -5/2$

$12x'^2 + 2y'^2 + 12 \cdot \frac{9}{4} - 10x'y' - 10\left(-\frac{3}{2}\right)\left(-5/2\right) + 2 \cdot \frac{25}{4}$

$= \quad + 11\left(-\frac{3}{2}\right) - 5\left(-\frac{5}{2}\right) + 2 = 0$

$\Rightarrow 12x'^2 - 10x'y' + 2y'^2 + 27 - \frac{75}{2} + \frac{25}{2} - \frac{33}{2} + \frac{25}{2} + 2$
$= 0$

$\Rightarrow 12x'^2 - 10x'y' + 2y'^2 + \frac{108 - 108}{2} = 0$

$\Rightarrow 12x'^2 - 10x'y' + 2y'^2 = 0$

Removing suffix we get,

$12x^2 - 10xy + 2y^2 = 0.$ Ans.

H.W. 4. Determine the equation of the parabola $x^2 - 2xy + y^2 + 2x - 4y + 3 = 0$ after rotating the angle through 45°.

Ans: $2y^2 - \sqrt{2}x - 3\sqrt{2}y + 3 = 0$