Plane

A plane is a surface such that if any two points are taken on it, the straight line joining them lies wholly on the surface that is every point of the line joining the two points will be on the plane.

Example: Surface of a wall, floor, a piece of paper etc.

The general equation of first degree in x, y, z i.e $ax+by+cz+d=0$ represents a plane.

Formulas:

(i) Equation of the plane passing through the point (x_1, y_1, z_1) is

$$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$$

(ii) Equation of a plane through the intersection of two planes is

$$(ax+by+cz+d)+k(a_1x+b_1y+c_1z+d)=0$$

(iii) perpendicular distance of the point (x', y', z') from the plane: $ax+by+cz+d=0$ is

$$\frac{ax'+by'+cz'+d}{\sqrt{a^2+b^2+c^2}}$$

(iv) Angle between two planes $ax+by+cz+d=0$ and $a_1x+b_1y+c_1z+d=0$ is

$$\cos\theta = \frac{aa_1+bb_1+cc_1}{\sqrt{a^2+b^2+c^2}\sqrt{a_1^2+b_1^2+c_1^2}}$$

(v) Two planes are perpendicular if $a_1a_2+b_1b_2+c_1c_2=0$ and parallel if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$.

(vi) Plane passing through two given points (x_1,y_1,z_1), (x_2,y_2,z_2) and perpendicular to a given plane $ax+by+cz+d=0$ is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_1-x_2 & y_1-y_2 & z_1-z_2 \\ a & b & c \end{vmatrix} = 0.$$

problem: Find the equation of the plane through the points (2,3,1), (1,1,3) and (2,2,3). Find also the perpendicular distance from the point (5,6,7) to this plane.

Soln: The equation of the plane passing through (2,3,1) is

$$a(x-2)+b(y-3)+c(z-1)=0 \quad \cdots (1)$$

Since it passes through (1,1,3) and (2,2,3) we have

$$a(1-2)+b(1-3)+c(3-1)=0$$

$$\Rightarrow -a-2b+2c=0$$

$$\Rightarrow a+2b-2c=0 \quad \cdots (2)$$

and $a(2-2)+b(2-3)+c(3-1)=0$

$$\Rightarrow -b+2c=0$$

$$\therefore b-2c=0 \quad \cdots (3)$$

Solving (2) and (3) we get,

$$a=-2,\ b=2,\ c=1.$$

putting these values into (1) we get,

$$-2(x-2)+2(y-3)+1(z-1)=0$$

$$\Rightarrow -2x+4+2y-6+z-1=0$$

$$\Rightarrow -2x+2y+z-3=0 \quad \cdots (4)$$

Cross Multiplication

$$a+2b-2c=0$$

$$0+b-2c=0$$

$$\Rightarrow \frac{a}{-4+2}=\frac{b}{0+2}=\frac{c}{1-0}$$

$$\therefore \frac{a}{-2}=\frac{b}{2}=\frac{c}{1}$$

The perpendicular distance p from (5, 6, 7) to (4) is

$$p = \frac{(-2)\cdot 5 + 2\cdot 6 + 7 - 3}{\sqrt{(-2)^2 + 2^2 + 1^2}}$$

$$= \frac{-10 + 12 + 7 - 3}{\sqrt{9}}$$

$$= \frac{6}{3} = 2 \text{ (Ans)}$$

Problem:

Find the equation of the plane passing through the intersection of the planes $x + 2y + 3z + 4 = 0$ and $4x + 3y + 2z + 1 = 0$ and the point $(1, 2, 3)$.

Soln: Any plane through the intersection of the two planes is

$$x + 2y + 3z + 4 + k(4x + 3y + 2z + 1) = 0 \quad \cdots (1)$$

Since it passes through (1, 2, 3) we get,

$$1 + 2\cdot 2 + 3\cdot 3 + 4 + k(4\cdot 1 + 3\cdot 2 + 2\cdot 3 + 1) = 0$$

$$\Rightarrow 18 + 17k = 0$$

$$\Rightarrow k = -\frac{18}{17}$$

putting the values of k in (i) we get,

$$x+2y+3z+4+\left(-\frac{18}{17}\right)(4x+3y+2z+1)=0$$

$$\Rightarrow 17x+34y+51z+68-72x-54y-36z-18=0$$

$$\Rightarrow -55x-20y+15z+50=0$$

$$\Rightarrow 55x+20y-15z-50=0$$

$$\Rightarrow 11x+4y-3z=10.$$

Problem:

Find the angle between the planes $2x-y+z=6$ and $x+y+2z=7$.

Soln: Let θ be the angle between the planes then

$$\cos\theta=\frac{2\cdot 1+(-1)\cdot 1+1\cdot 2}{\sqrt{2^2+(-1)^2+1^2}\sqrt{1^2+1^2+2^2}}$$

$$=\frac{3}{\sqrt{6}\sqrt{6}}$$

$$=\frac{3}{6}=\frac{1}{2} \text{ (Ans)}$$

Problem: Find the equation of the plane passing through the point $(2,-1,-4)$ and perpendicular to the planes $3x+4y-5z+6=$ and $x-2y+2z+1=0$

Soln: Since the plane passes through the point $(2,-1,-4)$, so the equation of the plane becomes

$$a(x-2)+b(y+1)+c(z+4)=0 \quad \cdots (i)$$

Since (i) is perpendicular to each planes

$$3a+4b-5c=0 \quad \cdots (ii)$$

$$a-2b+2c=0 \quad \cdots (iii)$$

By (ii) and (iii) using cross multiplication we get,

$$\frac{a}{8-10}=\frac{b}{-5-6}=\frac{c}{-6-4}$$

$$\Rightarrow \frac{a}{-2}=\frac{b}{-11}=\frac{c}{-10}$$

putting the values of a, b, c into (i)

$$-2(x-2)-11(y+1)-10(z+4)=0$$

$$\Rightarrow -2x+4-11y-11-10z-40=0$$

$$\Rightarrow 2x+11y+10z+47=0$$

(Ans)

Problem: Find the equation of the plane through the points (2,2,1) and (9,3,6) and perpendicular to the plane $2x+6y+6z-9=0$.

Soln: We know the plane passing through the given points (2,2,1) and (9,3,6) and perpendicular to the plane $2x+6y+6z-9=0$ is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_1-x_2 & y_1-y_2 & z_1-z_2 \\ a & b & c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-2 & z-1 \\ 2-9 & 2-3 & 1-6 \\ 2 & 6 & 6 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-2 & z-1 \\ -7 & -1 & -5 \\ 2 & 6 & 6 \end{vmatrix} = 0$$

$\Rightarrow (x-2)\ [(-6+30) + (y-2)(-10+42) + (z-1)(-42+2) = 0$

$\Rightarrow (x-2)\cdot 24 + (y-2)\cdot 32 - 40(z-1) = 0$

$\Rightarrow (x-2)\cdot 3 + (y-2)\cdot 4 - 5(z-1) = 0$

$\Rightarrow 3x - 6 + 4y - 8 - 5z + 5 = 0$

$\Rightarrow 3x + 4y - 5z - 9 = 0$ (Ans)

<u>H.W</u> (i) Find the equation of the plane passing through the points $(1, 0, -1)$ and $(2, 1, 3)$ and perpendicular to the plane $2x + y + z = 1$

Ans: $3x - 7y + z = 2$.

(ii) Show that the equation of the plane through the points $(-1, 3, 2)$ and perpendicular to the planes $x + 2y + 2z = 5$ and $3x + 3y + 2z = 8$ is

$2x - 4y + 3z + 8 = 0$

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