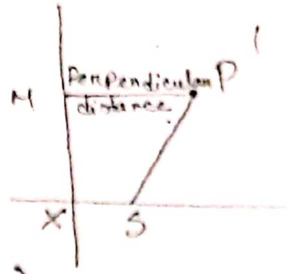
Conic :

$S \rightarrow$ fixed Point

$MX \rightarrow$ fixed line

P is variable point



P moves in such a way that,

$\frac{PS}{PM}$ is a constant $= e$ (let)

Then the Local of P is called a conic.

$PS = ePM$

e is known as eccentricity.

If $e = 1$ then it is a parabola.

$e < 1$ then " " an ellipse

$e > 1$ " " " a hyperbola.

A general equation of 2nd degree

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

will represents a straight line if

(i) a pair of

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

(ii) a pair of parallel lines,

$$\Delta = 0 \text{ and } ab - h^2 = 0$$

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(iii) A pair of perpendicular line if
$\Delta = 0$ and $a+b=0$

(iv) a circle if $a=b$ and $h=0$

(v) a parabola if $\Delta \neq 0$, $ab-h^2=0$

(vi) an ellipse if $\Delta \neq 0$ and $ab-h^2 > 0$

(vii) a hyperbolla if $\Delta \neq 0$ and $ab-h^2 < 0$

(ix) a rectangular hyperbola if $\Delta \neq 0$, $ab-h$
$a+b=0$.

Identify the following conics and hence reduce them to standard form. For an ellipse or hyperbola, find their centres, lengths and equation of major and minor axes. For parabolas find the vertex, length of latus rectum, focus, the equation of the axes, the directrix and the equation of the latus rectum.

1. $25x^2+2xy+25y^2-130x-130y+169=0$

Solution:

Given that,

$$25x^2+2xy+25y^2-130x-130y+169=0 \quad \text{--- (i)}$$

General equation of 2nd degree,

$$ax^2+2hxy+by^2+2gx+2fy+c=0 \quad \text{--- (ii)}$$

comparing (i) & (ii),

$a=25$, $b=25$, $h=1$, $c=169$, $f=-65$,

$g=-65$,

$$\Delta = \begin{vmatrix} 25 & 1 & -65 \\ 1 & 25 & -65 \\ -65 & -65 & 169 \end{vmatrix}$$

$$= 25(4225-4225)-1(169-4225) -65(-65+1625)$$

$$= 4056 - 101400$$

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$$\Rightarrow 25x^2+2xy+25y^2-325+169=0$$

$$\Rightarrow 25x^2+2xy+25y^2=156.$$

Now remove xy term, suppose the reduce form is,

$$a'x'^2+b'y'^2=156 \quad \text{where } h'=0$$

Then $a'+b'=a+b$

$$=25+25$$

$$=50$$

$$a'b'=ab-h^2$$

$$=624$$

$$\therefore a'-b'=\sqrt{(a'+b')^2-4a'b'}$$

$$=\sqrt{2500-2496}$$

$$=\sqrt{4}$$

$$=2$$

$$\therefore a'=26, \quad b'=24$$

Hence the form is,

$$26x'^2+24y'^2=156$$

$$\Rightarrow \frac{x'^2}{6}+\frac{y'^2}{13/2}=1$$

$$\Rightarrow \frac{X^2}{(\sqrt{6})^2}+\frac{Y^2}{(\sqrt{13/2})^2}=1.$$

is the standard form.

Angle of rotation $\theta = \frac{1}{2}\tan^{-1}\frac{2h}{a-b}$

Length of major axis, $= 2b = 2\sqrt{13/2}$

" " minor " $= 2a = 2\sqrt{6}$

$\therefore \tan\theta = 45°, \tan\theta = 135°$

One axis is,

$y - y_1 = m(x$

$y - \frac{5}{2} = 1\left(x - \frac{5}{2}\right)$

$\Rightarrow \frac{2y-5}{2} = \frac{2x-5}{2}$

$\Rightarrow 2y - 5 = 2x - 5$

$\Rightarrow y - x = 0$

$\therefore y = x$.

Another axis is,

$y - \frac{5}{2} = -1\left(x - \frac{5}{2}\right)$

$\Rightarrow y - 5/2 = -x + 5/2$

$\Rightarrow 2y - 5 = -2x + 5$

$\Rightarrow 2y + 2x - 5 = 0$,

Ans 2. $32x^2 + 52xy - 7y^2 - 64x - 52y - 148 = 0$

Solution: Here $a = 32$, $h = 26$, $b = -7$, $g = -3$

$f = -26$, $c = -148$.

$\therefore \Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

$= 32(-7)(-148) + 2(-26)(-32)(26) - 32(-26)^2$
$+ 7(-32)^2 + 148(26)^2$

$=$

$\neq 0$

Now $ab - h^2 = 32(-7) - (26)^2$
$= -224 - 676 = -900 < 0$

So it is hyperbola.

Here, $f(x,y) = 32x^2 + 52xy - 7y^2 - 64x - 52y - 148$

$\frac{\partial f}{\partial x} = 64x + 52y - 64 = 0$

$\frac{\partial f}{\partial y} = 52x - 14y - 52 = 0$

$\Rightarrow 16x + 13y - 16 = 0$ —— (i)

$26x - 7y - 26 = 0$ —— (ii)

Solving (i) and (ii) we get,

$$\frac{x}{-338-112} = \frac{y}{16(-26) + 16 \times 26} = \frac{1}{-112-338}$$

$\therefore x = 1, \quad y = 0$

Here the centre is, $(1, 0)$.

Then the reduced equation is,

$32x^2 + 52xy - 7y^2 + (-32)\cdot 1 + (-26)\cdot 0 - 148 = 0$

$\Rightarrow 32x^2 + 52xy - 7y^2 = 180.$

To remove xy term the angle of rotation

$$\theta = \frac{1}{2}\tan^{-1}\frac{2h}{a-b}$$

$$= \frac{1}{2}\tan^{-1}\frac{52}{32+7}$$

$$= \frac{1}{2}\tan^{-1}\frac{4}{3}$$

Suppose the reduced equⁿ is,

$$a'x'^2 + b'y'^2 = 180$$

then $a'+b' = a+b = 32-7 = 25$

$a'b' = ab - h^2 = -900$

$$a'-b' = \sqrt{(a'+b')^2 - 4a'b'}$$

$$= \sqrt{625+3600}$$

$$= 65$$

Solving $a' = 45$, $b' = -20$

Hence the equation is,

$$45x'^2 - 20y'^2 = 180$$

$$\Rightarrow \frac{x'^2}{4} - \frac{y'^2}{9} = 1$$

Now removing the suffix,

$$\frac{x^2}{2^2} - \frac{y^2}{3^2} = 1$$

which is the standard form.

Length of the major axis $= 2 \cdot 3 = 6$

" " " minor " $= 2 \cdot 2 = 4$

Now, we have,

$$\tan 2\theta = \frac{4}{3}$$

$$\Rightarrow \frac{2\tan\theta}{1-\tan^2\theta} = \frac{4}{3}$$

$$\Rightarrow 6\tan\theta = 4 - 4\tan^2\theta$$

$$\Rightarrow 2\tan^2\theta + 3\tan\theta - 2 = 0$$

$$\Rightarrow (\tan\theta + 2)(2\tan\theta - 1) = 0$$

$\therefore \tan\theta = \frac{1}{2}$ or -2.

Equation of one axis, $y - 0 = \frac{1}{2}(x-1)$

$\Rightarrow 2y = x - 1$ (major)

" " another " $y - 0 = -2(x-1)$

$\therefore y + 2x = 2$ (minor)

3. $16x^2 - 24xy + 9y^2 - 104x - 172y + 44 = 0$

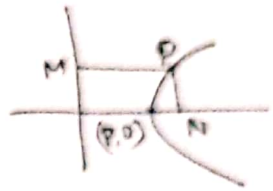
Solution:

Here, $a = 16$, $h = -12$, $b = 9$, $g = -52$, $f = -86$, $c = 44$

$$\therefore \Delta = \begin{vmatrix} 16 & -12 & -52 \\ -12 & 9 & -86 \\ -52 & -86 & 44 \end{vmatrix}$$

$y^2 = 4px$

$PN^2 = 4P\cdot$



$=$

$=$

$\neq 0$

Now $ab - h^2 = 144 - 144 = 0$

So it is a parabola.

Here, $16x^2 - 24xy + 9y^2 = 104x + 172y - 44$

$\Rightarrow (4x - 3y + \lambda)^2 = 104x + 172y - 44 + 2\lambda(4x$

$+\lambda^2$

$\Rightarrow (4x - 3y + \lambda)^2 = (104 + 8\lambda)x + (172 - 6\lambda)y + \lambda^2 - 44$

$\Rightarrow (4x - 3y + \lambda)^2 = 0$ will be perpendicular to the line on R.H.S. if

$$4(104 + 8\lambda) - 3(172 - 6\lambda) = 0$$

$$416 + 32\lambda - 516 + 18\lambda = 0$$

$\Rightarrow 50\lambda = 100$

$\Rightarrow$

$\therefore \lambda = 2$

Hence the above form is,

$$(4x-3y+2)^2 = 120x + 160y - 40$$

$$= 40(3x+4y-1)$$

$$\Rightarrow \left(\frac{4x-3y+2}{\sqrt{4^2+3^2}}\right)^2 \cdot 25 = 40\left(\frac{3x+4y-1}{\sqrt{3^2+4^2}}\right) 5$$

$$\Rightarrow Y^2 \cdot 25 = 200X$$

$$\Rightarrow Y^2 = 8X = 4PX \; [P = 2]$$

which is the standard form.

Here the axes are,

$$3x + 4y - 1 = 0$$

$$\& \; 4x - 3y + 2 = 0$$

$$\therefore \frac{x}{8-3} = \frac{y}{-4-6} = \frac{1}{-9-16}$$

$$\Rightarrow \frac{x}{5} = \frac{y}{-10} = \frac{1}{-25}$$

$$\therefore x = -\frac{1}{5}, \quad y = 2/5$$

Length of Latus rectum $= 4P = 8$

For focus put $X = P$

$$\Rightarrow \frac{3x+4y-1}{5} = 2$$

$\Rightarrow 3x+4y-1=10$

$\Rightarrow 3x+4y-11=0$

which is the equation of latus rectum.

Now,

$4x-3y+2=0$

$3x+4y-11=0$

Solving,

$$\frac{x}{33-8} = \frac{y}{6+44} = \frac{1}{16+9}$$

$$\Rightarrow \frac{x}{25} = \frac{y}{50} = \frac{1}{25}$$

$\therefore x=1, \quad y=2$

$\therefore (1,2)$ is the focus.

Now put, $\frac{3x+4y-1}{5} = -2$

$\Rightarrow 3x+4y-1=-10$

$\Rightarrow 3x+4y+9=0$

This is the equation of directrixes

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4. $9x^2 - 24xy + 16y^2 - 18x - 101y + 19 = 0$

Solution:

$a = 9,\ h = -12,\ b = 16,\ g = -9,\ f = \frac{-101}{2},\ c = 19$

$$\therefore \Delta = \begin{vmatrix} 9 & -12 & -9 \\ -12 & 16 & \frac{-101}{2} \\ -9 & \frac{-101}{2} & 19 \end{vmatrix}$$

$$= 9\left(16\times19 - \frac{101\times101}{4}\right) + 12\left(-12\times19 - \frac{101\times9}{2}\right) - 9(606+144)$$

$=$

$=$

$\neq 0$

Now $ab - h^2 = 144 - 144 = 0$

So it represent a parabola.

Here, $9x^2 - 24xy + 16y^2 = 18x + 101y - 19$

$\Rightarrow (3x-4y)^2 = 18x + 101y - 19$

$\Rightarrow (3x - 4y + \lambda)^2 = 18x + 101y - 19 + 2\lambda(3x - 4y) + \lambda^2$

$= (18 + 6\lambda)x + (101 - 8\lambda)y + \lambda^2 - 19$ —— (1)

Now the lines,

$3x - 4y + \lambda = 0$

& $(18 + 6\lambda)x + (101 - 8\lambda)y + \lambda^2 - 19 = 0$

are perpendicular if,

$$3(18+6\lambda) - 4(101-8\lambda) = 0$$

$$\Rightarrow 54 + 18\lambda - 404 + 32\lambda = 0$$

$$\Rightarrow 50\lambda = 350$$

$$\therefore \lambda = 7$$

Hence from ① we have,

$$(3x-4y+7)^2 = 60x + 45y + 30 = 15(4x+3y+2)$$

$$\Rightarrow \left(\frac{3x-4y+7}{\sqrt{3^2+4^2}}\right)^2 \cdot 25 = 15\left(\frac{4x+3y+2}{\sqrt{4^2+3^2}}\right) \cdot 5$$

$$\Rightarrow \left(\frac{3x-4y+7}{\sqrt{3^2+4^2}}\right)^2 = 3\left(\frac{4x+3y+2}{\sqrt{4^2+3^2}}\right)$$

$$\therefore Y^2 = 3X$$

$= 4PX$ Here, $P = \frac{3}{4}$ → length of the Latus rectum.

Hence the axes are,

$$3x - 4y + 7 = 0$$

and $4x + 3y + 2 = 0$

Solving, $\dfrac{x}{-8-21} = \dfrac{y}{28-6} = \dfrac{1}{9+16}$

$$\Rightarrow \frac{x}{-29} = \frac{y}{22} = \frac{1}{25}$$

$$\therefore x = -\frac{29}{25}, \; y = \frac{22}{25}.$$

∴ Equation of latus rectum.

$x = P$

$\Rightarrow \frac{4x+3y+2}{5} = \frac{3}{4}$

$\Rightarrow 16x+12y+8 = 15$

$\therefore 16x+12y-7=0$

Consider,

$3x-4y+7=0$

$16x+12y-7=0$

Solving,

$\frac{x}{28-84} = \frac{y}{112+21} = \frac{1}{36+64}$

$\Rightarrow \frac{x}{-56} = \frac{y}{133} = \frac{1}{100}$

$\Rightarrow x = -\frac{56}{100}, \quad y = \frac{133}{100}$

∴ Focus is, $\left(-\frac{14}{25}, \frac{133}{100}\right)$

∴ Equation of directrix is,

$x = -P$

$\Rightarrow \frac{4x+3y+2}{5} = \frac{-3}{4}$

$\Rightarrow 16x+12y+8 = -15$

$\therefore 16x+12y+23=0.$

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