Ordinary and Partial Differential Equations and Coordinate Geometry

MAT 103

Differential Equations:

An equation involving derivatives of one or more dependent variables with respect to one or more independent variables is called a differential equation.

Classifications of Differential Equations:

a) Ordinary Differential Equations (ODE)

b) Partial Differential Equations (PDE)

Ordinary Differential Equations (ODE): A differential equation involving ordinary derivatives of one or more dependent variables with respect to a single independent variable is called an ordinary differential equation.

Partial Differential Equations (PDE): A differential equation involving partial derivatives of one or more dependent variables with respect to more than one independent variable is called an ordinary differential equation.

Examples of DE:

i) $\frac{d^2y}{dx^2} + 2y\left(\frac{dy}{dx}\right)^2 = 0$ → y is dependent variable and x is independent variableand this is an **ODE**

ii) $\frac{d^4x}{dt^4} + 5\frac{d^2x}{dt^2} + 3x = \sin t$ → x is dependent variable and t is independent variableand this is an **ODE**

iii) $\frac{\partial u}{\partial s} + \frac{\partial v}{\partial t} = v$ → u *and* v are dependent variables and s&t are independent variableand this is a **PDE**

Order and Degree of DE

Order of DE: The order of the highest ordered derivatives involved in a differential equation is called the order of the differential equation.

Degree of DE: After removing the radicals (square root, cubic root), the power of the highest order derivative is called the degree of a differential equations.

Examples

i) $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^3 - y = 0$ ⇒ Order 2, Degree 1

ii) $\frac{d^2y}{dx^2} = \sqrt{\left(\frac{dy}{dx}\right)^3 + 3}$ ⇒ $\frac{d^2y}{dx^2} = \sqrt{\left(\frac{dy}{dx}\right)^3 + 3}$ ⇒ Order 2, Degree 2

iii) $\left(\frac{d^2y}{dx^2}\right)^4 + 2y\left(\frac{dy}{dx}\right)^2 = 0$ ⇒ Order 2, Degree 4

iv) $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 + 5y = 0$ ⇒ ??????

Classification of DE based on Linearity:

According to linearity, differential equations are two types.

- **Linear differential equations**
- **Nonlinear differential equations**

Linear DE: A linear differential equation of order n, in the dependent variable y and the independent variable x is an equation can be expressed in the form,

$$a_0(x)\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_{n-1}(x)\frac{dy}{dx} + a_n(x)y = b(x)$$

Non- Linear DE: Nonlinear differential equations is an equation which is not linear.

Examples of Linear and Nonlinear DE

- $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$ → 2nd order linear ODE with degree one
- $\frac{d^2y}{dx^2} + y^2 = 0$ → 2nd order nonlinear ODE with degree one
- $\frac{d^3y}{dx^3} + 3\frac{dy}{dx} + 3y = 0$ → 3rd order linear ODE with degree one
- $\left(\frac{dy}{dx}\right)^2 - 2y = 0$ → 1st order nonlinear ODE with degree two
- $\frac{dy}{dx} + \ln y = 0$ → ?????

Exercises: Find the degree, order and linearity of the following differential equations,

i) $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 2y = 8$

ii) $\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^3 + 2y = 0$

iii) $\left(\frac{d^2y}{dx^2}\right)^3 + 2y = \frac{d^3y}{dx^3}$

iv) $\frac{dy}{dx} + \tan y = 0$

v) $\frac{d^2y}{dx^2} + 2y\frac{dy}{dx} + \cos y = 0$

Sample Short Questions:

1. Define DE, PDE, ODE

2. What is the main difference between ODE and PDE?

3. In which DE there will be more than one independent variable?

4. What is degree and order of DE?

5. What is the difference between degree and order of DE?

6. Give an example of linear DE/ nonlinear DE