

Linear Differential Equation

Linear Differential Equations:

The equation of the form $A(x) \frac{dy}{dx} + B(x)y = C(x)$ is called a Linear equation of order one.

$$\begin{aligned} A(x) \frac{dy}{dx} + b(x)y &= C(x) \\ \Rightarrow \frac{dy}{dx} + \frac{B(x)}{A(x)}y &= \frac{C(x)}{A(x)} \\ \Rightarrow \frac{dy}{dx} + P(x)y &= Q(x) \end{aligned}$$

Which is the standard form of **Linear DE of order one**.

Working Rules:

1. Put the equation into standard form $\frac{dy}{dx} + P(x)y = Q(x)$
2. Obtain the integrating factor $e^{\int p dx}$
3. Apply the integrating factor to the equation.
4. Solve this equation.

Questions:

Solve the following linear differential equations:

1. $(1+x^2) \frac{dy}{dx} + y = \tan^{-1} x$
2. $x \frac{dy}{dx} - 3y = x^2$ **H.W** (Ans: $\frac{y}{x^3} = -\frac{1}{x} + c$)
3. $\frac{dy}{dx} + y \tan x = \sec x$

4. $(x^2 - 1)\frac{dy}{dx} + 2xy - 1 = 0$

5. $x \log x \frac{dy}{dx} + y = 2 \log x$ **H.W (Ans: $y \log x = (\log x)^2 + c$)**

6. $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$

7. $\frac{dy}{dx} + \frac{x}{1-x^2} = \frac{y^2}{x^2}$