First order

Ordinary Differential Equation

Standard form: (Order will be one)

$$\boldsymbol{\frac{dy}{dx} = f(x, y)}$$

Example:

$$\frac{dy}{dx} = \frac{x^2 + 2y}{2x - 3y^2}$$

$$=> (x^2 + 2y)dx - (2x - 3y^2)dy = 0$$

$$=> (x^2 + 2y)dx + (3y^2 - 2x)dy = 0$$

We can generalize as,

$$M(x, y)dx + N(x, y)dy = 0$$

First order differential equations can be solved by the following methods:

1) Separable equation / Separable of variable.
2) Homogeneous differential equation.
3) Exact differential equation.
4) Linear differential equation &Bernoullidifferential equation.

<u>Method 1:</u>

Variable Separable Equation: Separable equations are the class of differential equations that can be solved using this method.

General form of Separable Equation:

$$F(x)G(y)dx + f(x)g(y)dy = 0$$

Solution way:

Divide the equation by $f(x)G(y)$

$$=> \frac{F(x)G(y)}{f(x)g(y)}dx + \frac{f(x)g(y)}{f(x)G(y)}dy = 0$$

$$=> \frac{F(x)}{f(x)}dx + \frac{g(y)}{G(y)}dy = 0$$

Then integrating both side we will get the solution.

Example1: Solve the following differential equation using separation of variable method.

$$(xy - x)dx + (xy + y)dy = 0$$

Solution: $(xy - x)dx + (xy + y)dy = 0$

$=> x \quad (y - 1)dx + (x + 1) \; y \quad dy = 0$

$F(x)G(y)f(x)g(y)$

$$=> \frac{x(y-1)}{(x+1)(y-1)}dx + \frac{(x+1)y}{(x+1)(y-1)}dy = 0$$

$$=> \frac{x}{x+1}dx + \frac{y}{y-1}dy = 0$$

$$=> \int \frac{x}{x+1}dx + \int \frac{y}{y-1}dy = \int 0$$

$$=> \int \frac{x+1-1}{x+1} dx + \int \frac{y+1-1}{y-1} dy = c_1$$

$$=> \int \frac{x+1}{x+1} dx - \int \frac{1}{x+1} dx + \int \frac{y-1}{y-1} dx + \int \frac{1}{y-1} dy = c_1$$

$$=> \int dx - \ln(\mathrm{x}+1) + \int \mathrm{dy} + \ln(y-1) = c_1$$

$$=> x + y + \ln(y-1) - \ln(x+1) = c_1$$

(Answer)

Example 2: Solve the following differential equation using separation of variable method.

$$(1+y^2)dx + (1+x^2)dy = 0$$

Solution: Given,

$$(1+y^2)dx + (1+x^2)dy = 0$$

$$=> \frac{(1+y^2)}{(1+y^2)(1+x^2)} dx + \frac{(1+x^2)}{(1+y^2)(1+x^2)} dy = 0$$

$$=> \frac{1}{1+x^2} dx + \frac{1}{1+y^2} dy = 0$$

$$=> \int \frac{1}{1+x^2} dx + \int \frac{1}{1+y^2} dy = \int 0$$

$$=> \tan^{-1} x + \tan^{-1} y = c$$

(Answer)

Exercises:

1) $(1+y^2)dx+\sqrt{1-x^2dy}=0$

2) $e^{-y}dx+(1+x^2)dy=0$

3) $(1+y^2)dx=(1+x^2)dy$

4) $\frac{dy}{dx}=e^{x-y}+x^2e^{-y}$

5) $\frac{dy}{dx}=x^2(e^{3y}-1)$

6) $\frac{dy}{dx}=x^2(e^{3y})$

7) $(2+x)dy=y^4$ **H.W**

8) $(1+y^2)dx-\sqrt{1-x^2dy}=0$ **H.W**

9) $2x(1+y)dx-ydy=0$

10) $e^x(y-1)dx+2(e^x+4)dy=0$