Straight lines:

1. Equation of x-axis $y=0$

& equation of y-axis $x=0$

2. Equation of straight line which is parallel to x-axis is $y=b$

and parallel to y-axis is $x=a$.

4. Equation of straight line which passes through the point (x_1, y_1) is;

$$y-y_1 = m(x-x_1)$$

3. Equation of straight line which passes through the origin $(0,0)$ is

$$y=mx$$

5. Equation of straight line which passing through two point (x_1, y_1) and (x_2, y_2) is,

$$\frac{x-x_1}{x_1-x_2} = \frac{y-y_1}{y_1-y_2}$$

6. Angle between two straight lines

$$\theta = \tan^{-1} \frac{m_1-m_2}{1+m_1m_2}$$

7. If two straight lines are perpendicular then $m_1 m_2 = -1$ and parallel then $m_1 = m_2$

8. If $ax + by + c = 0$ is a straight line then equation of parallel line of this line is

$$ax + by + K = 0$$ and

perpendicular of this line is $bx - ay + K = 0$

9. Slop of the line joining the points (x_1, y_1) & (x_2, y_2) is,

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

3. Find the straight lines passing through the point (a) $(-3,-1)$, $(11,13)$

(b) $(11,13)$, $(-1,-3)$

Solution:

We know, the equation of straight line passing through two points are

$$\frac{x-x_1}{x_1-x_2} = \frac{y-y_1}{y_1-y_2} \quad \text{--- (1)}$$

(a) For point $(-3,-1)$, $(11,13)$ we get from (1),

$$\frac{x-(-3)}{-3-11} = \frac{y-(-1)}{-1-13}$$

$$\Rightarrow \frac{x+3}{-14} = \frac{y+1}{-14}$$

$\Rightarrow x - y + 2 = 0$ Ans.

(b) For point $(11, 13), (-1, -3)$ we get from (i),

$$\frac{x-11}{11+1} = \frac{y-13}{13+3}$$

$$\Rightarrow \frac{x-11}{12} = \frac{y-13}{16}$$

$$\Rightarrow \frac{x-11}{3} = \frac{y-13}{4}$$

$\Rightarrow 4x - 3y - 5 = 0$. Ans.

H.W. 4 Find the equation of the straight line passing through $(2,6)$, $(6,-1)$.

5. Find the angle between two straight lines

$$5x + 4y - 6 = 0$$
$$10x - 4y + 45 = 0$$

Solution: We know the angle between two straight line is,

$$\theta = \tan^{-1} \frac{m_1 - m_2}{1 + m_1 m_2} \quad \text{— (i)}$$

Given that,

$$5x + 4y - 6 = 0$$

$$\Rightarrow 4y = -5x + 6$$

$$\therefore y = -\frac{5}{4}x + \frac{3}{2} \quad \text{— (ii)}$$

and $10x - 4y + 45 = 0$

$$\Rightarrow 4y = 10x + 45$$

$$\therefore y = \frac{5}{2}x + \frac{45}{4} \quad \text{——} \text{ (iii)}$$

we know that,

$$y = mx + c \quad \text{——} \text{ (iv)}$$

Now comparing (ii) and (iii) with (iv) we get,

$$m_1 = -\frac{5}{4} \quad \text{and} \quad m_2 = \frac{5}{2}$$

From (i) we get,

$$\theta = \tan^{-1}\left(\frac{-\frac{5}{4} - \frac{5}{2}}{1 - \frac{5}{4} \cdot \frac{5}{2}}\right)$$

$$= \tan^{-1} \frac{\frac{-5-10}{4}}{1 - \frac{25}{8}}$$

$$= \tan^{-1} \frac{\frac{-15}{4}}{\frac{8-25}{8}}$$

$$= \tan^{-1}\left(\frac{-15}{\cancel{4}} \times \frac{\overset{2}{\cancel{8}}}{-17}\right)$$

$$\therefore \theta = \tan^{-1}\left(\frac{30}{17}\right).$$

6. Find the straight line passing through the point (2,5), (5,6) and show that it is perpendicular to the straight line which passing through the point (-4,5) and (-3,2).

(2,6) , (-5,9)

Solution:

Equation of the straight line passing through two points is,

$$\frac{x-x_1}{x_1-x_2} = \frac{y-y_1}{y_1-y_2} \quad \text{—— (1)}$$

The line passing through (2,5) & (5,6) is,

$$\frac{x-2}{2-5} = \frac{y-5}{5-6}$$

$$\Rightarrow \frac{x-2}{-3} = \frac{y-5}{-1}$$

$$\Rightarrow x-2-3y+15=0$$

$$\therefore x-3y+13=0$$

The slope of this straight line is,

$$m_1 = \frac{6-5}{5-2} = \frac{1}{3}$$

Again the slope of the line joining the points (-4,5) & (-3,2) is,

$$m_2 = \frac{2-5}{-3+4} = \frac{-3}{1} = -3$$

We know that two lines are Perpendicular

$m_1 m_2 = -1$

Now L.H.S. $= m_1 m_2$

$= \frac{1}{3}(-3)$

$= -1$

So the lines are Perpendicular.

7. Find the equations of lines passing through $(-5, 6)$ and a) parallel b) Perpendicular to $7x - 8y = 9$.

Solution:

a) Let the equation parallel to $7x - 8y - 9 = 0$ be $7x - 8y - K = 0$ —— (1)

Since it passes through $(-5, 6)$ so from (1) we get,

$7(5) - 8.6 - K = 0$

$\Rightarrow -K = 35 + 48$

$\therefore K = -83$

$\therefore$ The required equation is,

$7x - 8y + 83 = 0$

b) let the equation perpendicular to $7x-8y-9=0$

be, $8x+7y-K=0$. — (ii)

Since, (ii) passes through $(-5,6)$ so from

(ii) we get,

$$8(-5)+7\cdot 6-K=0$$

$$\Rightarrow K=-40+42$$

$$\therefore K=2.$$

$\therefore$ The required equation is $8x+7y-2=0$.

The Circle:

1. Equation of the circle,

$$x^2+y^2+2gx+2fy+c=0.$$

2. Centre $(-g,-f)$ and radius, $r=\sqrt{g^2+f^2-c}$

3. Equation of the circle passing through the point (h,k) is $(x-h)^2+(y-k)^2=r^2$.

1. Find the equation of the circle whose centre is $(1,-2)$ and passing through the point $(9,4)$.

Solution:

Given that,

Centre of the circle is $(9,4)$.

Since the circle passes through $(1,-2)$

$\therefore$ radius of the circle $r=\sqrt{(9-1)^2+(4+2)^2}$

$$=\sqrt{64+36}$$

$$=\sqrt{100}$$

$$=10$$

$\therefore$ The equation of the circle is,

$$(x-1)^2+(y+2)^2=10^2$$

$$\Rightarrow x^2-2x+1+y^2+4y+4-100=0$$

$$\Rightarrow x^2+y^2-2x+4y-95=0. \text{ Ans.}$$

2. Find the equation of the circle whose [centre] is (4, 5) and passing through the point (3, -5).

Ans: $x^2+y^2-8x-10y-60=0$

3. Find the center and radius of the circle $x^2+y^2-4x+5y+9=0$

Solution:

We know,

the equation of the circle is,

$$x^2+y^2+2gx+2fy+c=0 \quad \text{—— (1)}$$

Given that,

$$x^2+y^2-4x+5y+9=0 \quad \text{—} \quad (ii)$$

Comparing (i) and (ii) we get,

$2g=-4$ and $2f=5$, $c=9$

$\Rightarrow g=-2$ $\quad f=5/2$

$\therefore$ Center is $(-g,-f) = (2, -5/2)$ Ans.

Radius is $r=\sqrt{g^2+f^2-c}$

$$=\sqrt{(-2)^2+\left(\frac{5}{2}\right)^2-9}$$

$$=\sqrt{4+\frac{25}{4}-9}$$

$$=\frac{\sqrt{16+25-36}}{2}$$

$$=\frac{\sqrt{5}}{2} \text{ Ans.}$$

4. Find center and radius of $x^2+y^2-8x-10y+1=0$

Ans: $(4, 5)$, $2\sqrt{10}$

Solution: