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Simpson's 3/8 rule 1

Simpson's 3/8 rule is another method for numerical integration proposed by Thomas Simpson. It is based upon a cubic interpolation rather than a quadratic interpolation. It is shown in figure 1. Simpson's 3/8 rule is as follows:

$$\int_{a}^{b} f(x)dx \approx \frac{3h}{8} [f(a) + 3f(\frac{2a+b}{3}) + 3f(\frac{a+2b}{3}) + f(b)] = \frac{(b-a)}{8} [f(a) + 3f(\frac{2a+b}{3}) + 3f(\frac{a+2b}{3}) + f(b)]$$

where ba=3h. The error of this method is: -(b-a)^5 $_{\overline{6480f(4)(\xi)}}$

$$-(b-a)^5 \frac{1}{6480 f(4)(\xi)}$$

where ξ is some number between a and b. Thus, the 3/8 rule is about twice as accurate as the standard method, but it uses one more function value. A composite 3/8 rule also exists, similarly as above.[6]

A further generalization of this concept for interpolation with arbitrarydegree polynomials are the Newton-Cotes formulas.

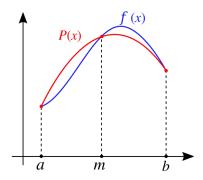


Figure 1: Simpson's 3/8 rule