**Course:** Discrete Mathematics **Assignment Type:** Problem-Solving / Logic
**Due Date:** March 18, 2026 **Total Points:** 100

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## Instructions

Solve all problems showing complete work. Partial credit will be awarded for correct methodology even if the final answer is incorrect. All proofs must be rigorous and clearly stated.

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## Problem 1: Set Theory (20 points)

Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} be the universal set. Let A = {1, 2, 3, 4, 5} Let B = {4, 5, 6, 7, 8} Let C = {2, 4, 6, 8, 10}

Calculate the following:

a) A (B C) (5 points)

b) (A - B) C (5 points)

c) A' B' where A' denotes the complement of A (5 points)

d) Draw a Venn diagram representing A, B, and C (5 points)

**Show all work:**

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## Problem 2: Logic and Propositional Calculus (25 points)

### Part A (10 points)

Construct a truth table for the following proposition: **(p → q) (¬q r)**

Include columns for all intermediate steps.

### Part B (10 points)

Prove or disprove the following logical equivalence using truth tables or logical laws: **(p → q) (q → r) p → r**

### Part C (5 points)

Convert the following English statement into propositional logic: "If it is raining, then the ground is wet, and if the ground is not wet, then it is not raining."

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## Problem 3: Graph Theory (20 points)

Consider the following graph G with vertices V = {A, B, C, D, E, F} and edges: E = {(A,B), (A,C), (B,C), (B,D), (C,D), (C,E), (D,E), (D,F), (E,F)}

a) Draw the graph (5 points)

b) Determine if the graph is Eulerian, Semi-Eulerian, or neither. Justify your answer. (7 points)

c) Find a Hamiltonian path in the graph, or prove that none exists. (8 points)

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## Problem 4: Combinatorics (20 points)

### Part A (8 points)

A committee of 5 people must be formed from a group of 8 men and 6 women. - How many different committees can be formed if there must be exactly 3 women? - Show your calculation using combinations.

### Part B (7 points)

How many different 4-digit PIN codes can be created if: - The first digit must be even - No digit can be repeated - The last digit must be odd

### Part C (5 points)

In how many ways can the letters in the word "MATHEMATICS" be arranged?

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## Problem 5: Proof by Induction (15 points)

Prove by mathematical induction that for all integers n   1:

**1 + 2 + 3 + ... + n = n(n+1)/2**

Your proof must include: - Base case - Inductive hypothesis - Inductive step - Clear conclusion

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## Problem 6: Relations and Functions (20 points)

### Part A (10 points)

Let R be a relation on the set A = {1, 2, 3, 4} defined by: R = {(1,1), (1,2), (2,1), (2,2), (3,3), (3,4), (4,3), (4,4)}

Determine whether R is: - Reflexive - Symmetric - Transitive - An equivalence relation

Justify each answer.

**Part B (10 points)**

Determine if the following function is injective, surjective, both (bijective), or neither: **f:** $\rightarrow$ **defined by f(x) = 3x - 5**

Prove your answer rigorously.

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## Submission Guidelines

- Show ALL work and reasoning
- Use proper mathematical notation
- Submit as a PDF with clearly labeled problems
- Scanned handwritten solutions are acceptable if legible
- Points will be deducted for unclear or disorganized work