Dynamics Modeling and Tracking Control of Robot Manipulators in Random Vibration Environment

Ming-Yue Cui, Xue-Jun Xie, and Zhao-Jing Wu

Abstract—**In this technical brief, the problem of modeling and tracking control for the manipulator with multi-revolute joints in random vibration environment is considered. By analyzing the effect of environment to the mass points and introducing an equivalent stochastic noise process, a stochastic Hamiltonian dynamic model is constructed to describe the motion of the manipulator. Based on the constructed model, a state feedback backstepping controller in vector form is designed such that the unique solution of the closed-loop system is bounded in probability, and the mean square of the tracking error converges to an arbitrarily small neighborhood of zero.**

Index Terms—**Random vibration environment, robot manipulator, stochastic Hamiltonian model, tracking control.**

I. Introduction

Robot manipulators are composed of links connected by joints into a kinematic chain, which are widely applied to many areas where the use of humans is impractical or undesirable [1]. For robot manipulators, kinematics and dynamics modeling is the subject of many monographes such as [2]–[4]. The key to kinematics modeling is to find an appropriate mapping denoted by Jacobian matrix between the joint variables in joint space and the position-orientation of the end effector in task space. A dynamic model can be derived from the Lagrangian equation method or Hamiltonian equation method to describe the relationship between force and motion, which is what we pursue in this technical brief. Based on Lagrangian model, great efforts have been made in developing control schemes to command the end-effector motion to achieve a desired response. In many monographes and references such as [5]–[16], some control techniques such as PID control, computed torque control, adaptive control, passivity-based control, variable structure control and intelligent control are considered. However, most of the existing references mainly focus on the deterministic case.

With the gradual improvement of the robot technology in deterministic case, it is naturally expected that manipulators can work in the random vibration environment such as transportation equipments [17]. Since the traditional deterministic models can't well describe the motion of the system in the random vibration environment, and the control methods based on the deterministic models can't achieve some desired objectives, there are few related results on these problems until now. In [18. Section 8.2.4], for the variable arm-length robot manipulator with one revolute joint and one prismatic joint hung from random vibrating ceiling, a stochastic Hamiltonian model was constructed based on a reasonable explanation to the vibration and an optimal stabilization control was proposed by using the stochastic averaging method. But for the modeling and tracking control of robot manipulators with multi-revolute joints in a random vibration workspace, to the authors' knowledge, there is no any related result.

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The objective of this technical brief is to solve this problem, the main contributions are as follows.

(i) The main difficulty for dynamics modeling is how to transform the random vibration in environment to the mass points along the links between them, which is the main reason why there are few results on the stochastic modeling for robots in the existing references. In this technical brief, by using the robot dynamics and the equivalence principle of mechanics, a stochastic Hamiltonian dynamic equation is established to describe the motion of the robot manipulator with multi-revolute joints in random vibration environment.

(ii) Based on the constructed model, by extending the integral backstepping technique to the vector form, a state feedback controller in vector form is designed such that all of the signals in the closed-loop system is bounded in probability, and the mean square of the tracking error converges to an arbitrarily small neighborhood of zero by choosing design parameters appropriately. The simulation result demonstrates the efficiency of the proposed scheme.

This technical brief is organized as follows. The problem is formulated in Section II. The stochastic dynamic equation is constructed in Section III. In Section IV, controller design and stability analysis are addressed. A simulation result is given in Section V. Section VI concludes the technical brief.

Notations: The following notations are used throughout the technical brief. For a vector x, x^T denotes its transpose and $|x|$ denotes its usual Euclidean norm; for a matrix X, X^{-1} denotes its inverse and $\|X\|_F$ denotes its Frobenius norm defined by $\|X\|_F = (\mathrm{Tr}\{XX^T\})^{1/2}$, where $\mathrm{Tr}(\cdot)$ denotes the matrix trace; $\mathbb{R}^n$ denotes the real n-dimensional space; $\mathbb{R}^{n\times r}$ denotes the real $n \times r$ matrix space; and $\mathcal{C}^i(\mathbb{R}^n)$ denotes the set of all functions with continuous i-th partial derivative on $\mathbb{R}^n$.

II. Problem Formulation

Since there are many random vibration environments which can be found in the internal space of many random moving bodies such as an automobile riding on rough road, a ship on waving sea, a flying airplane, it is expected that a manipulator still can work well in such real environments.

Consider a planar rigid robot manipulator with n revolute joints in a random vibration environment (see Fig. 1). Let us fix some notations as follows: For $i = 1, 2, \ldots, n$, q_i denotes the angle from the horizontal line to i-th link, m_i denotes the mass of i-th link, l_i denotes the length of i-th link, l_{ci} denotes the distance from the previous joint to the center of mass of i-th link, and u_i denotes the torque with respect to i-th joint, which is supplied by the motor, whose units are rad (radian), kg (kilogram), m (meter) and $\mathrm{N}\cdot\mathrm{m}$ $(\mathrm{Newton}\cdot\mathrm{meter})$, respectively. Here, just for the convenience, the end-effector is said as n-th link. g is the acceleration of gravity whose unit is $\mathrm{m/s^2}$. The manipulator is connected to O on the floor by a revolute joint and there is no air resistance acting on it. As in [18, Section 8.2.4], the random vibration considered in this technical brief is described by the random accelerations of the point O and let ξ_1, ξ_2 denote the random accelerations of the point O

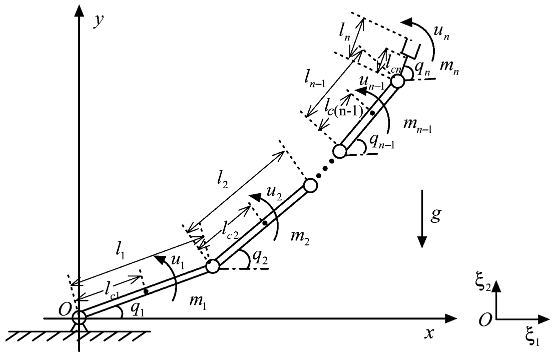

Fig. 1. Planar rigid manipulator with n revolute joints.

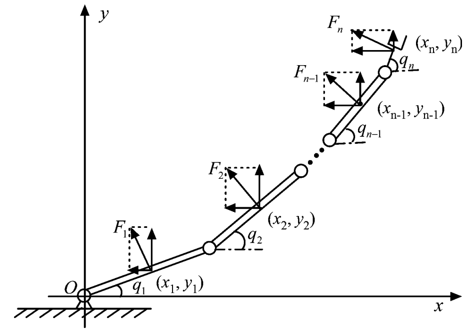

Fig. 2. The decomposition of forces.

in horizontal and vertical directions, which can be seen as independent white noises.

The objective of this technical brief is to design a tracking controller for the manipulator in the random vibration environment. To this end, two efforts will be taken in the following sections.

1) Construct an appropriate mathematical model to describe the motion of the n-joint planar rigid robot manipulator in random vibration environment.
2) Design a controller $u = (u_1, u_2, \ldots, u_n)^T$ such that the configuration $q = (q_1, q_2, \ldots, q_n)^T$ can track a given smooth reference signal $q_r = (q_{r1}, q_{r2}, \ldots, q_{rn})^T$ as close as possible.

III. Stochastic Kinematics Model for Manipulator

By using robot dynamics [5] and the equivalence principle of mechanics [19], the stochastic Hamiltonian equation of the robot manipulato is to be established in this section.

A. Modeling Under the Assumption That the Point O is Stationary

Consider the system of particles consisting of n links which are regarded as the mass points, and select $(q_1, q_2, \ldots, q_n)$ as the generalized coordinate. The base coordinate frame Oxy is shown in Fig. 1, then the natural coordinate of i-th particle is

$$\begin{cases} x_i = \sum_{j=1}^{i-1} l_j \cos q_j + l_{ci} \cos q_i \\ y_i = \sum_{j=1}^{i-1} l_j \sin q_j + l_{ci} \sin q_i. \end{cases} \tag{1}$$

By (1), the total kinetic energy of the system is

$$K(q, \dot{q}) = \sum_{i=1}^{n} \frac{1}{2} m_i (\dot{x}_i^2 + \dot{y}_i^2) = \frac{1}{2} \dot{q}^T M(q) \dot{q} \tag{2}$$

where $q = (q_1, q_2, \ldots, q_n)^T$ and the inertia matrix (generalized mass) is $M(q) = (M_{ij}(q))_{n \times n}$ with $M_{ii}(q) = m_i l_{ci}^2 + \sum_{k=i+1}^{n} m_k l_i^2$ and $M_{ij}(q) = M_{ji}(q) = (m_j l_{cj} + \sum_{k=j+1}^{n} m_k l_j) l_i \cos(q_j - q_i)$ $(i \neq j)$. The total potential energy of the system equals

$$\begin{aligned} P(q) &= \sum_{i=1}^{n} m_i g y_i \\ &= \sum_{i=1}^{n} \left(m_i l_{ci} + \sum_{j=i+1}^{n} m_j l_i \right) g \sin q_i. \end{aligned} \tag{3}$$

Therefore, the Lagrangian function is $L(q, \dot{q}) = K(q, \dot{q}) - P(q) = \dot{q}^T M(q) \dot{q}/2 - P(q)$. Define the generalized momenta by $p = (p_1, p_2, \ldots, p_n)^T = \partial L / \partial \dot{q} = M(q)\dot{q}$ (see P.143 in [5]) whose unit is $\mathrm{kg} \cdot \mathrm{m}^2 \cdot \mathrm{rad/s}$, then the Hamiltonian function is $H(q, p) = K(q, p) + P(q) = p^T M^{-1}(q) p/2 + P(q)$, which denotes the total energy of the system. According to Hamiltonian mechanics [20], the dynamic equation of the system of particles is given by

$$\begin{aligned} \dot{q} &= \frac{\partial H}{\partial p}(p, q), \\ \dot{p} &= -\frac{\partial H}{\partial q}(p, q) + \tau \end{aligned} \tag{4}$$

where $\tau = (\tau_1, \tau_2, \ldots, \tau_n)^T$ is the vector of generalized forces acting on the system.

In order to obtain the generalized forces, suppose that the control u_i $(i = 1, 2, \ldots, n)$ is the torque of force F_i with unit N acting on the i-th particle, i.e., $u_i = F_i l_{ci}$ (see Fig. 2). Since there is no resistance, based on the generalized forces formula[1] (see P.41 in [20]), the generalized force is

$$\begin{aligned} \tau_i &= \sum_{j=1}^{n} \left(-F_j \sin q_j \frac{\partial x_j}{\partial q_i} + F_j \cos q_j \frac{\partial y_j}{\partial q_i} \right) \\ &= F_i l_{ci} + \sum_{j=i+1}^{n} F_j l_i \cos(q_j - q_i) \\ &= u_i + \sum_{j=i+1}^{n} u_j \frac{l_i}{l_{cj}} \cos(q_j - q_i) \end{aligned} \tag{5}$$

whose unit is $\mathrm{N} \cdot \mathrm{m}$. Then $\tau = G(q)u$ with $G(q) = (G_{ij}(q))_{n \times n}$, where $G_{ii}(q) = 1$, $G_{ij}(q) = (l_i / l_{cj}) \cos(q_j - q_i)$ $(i < j)$ and $G_{ij}(q) = 0$ $(i > j)$. By (5) and the definition of Hamiltonian function, (4) can be written as

$$\begin{aligned} \dot{q} &= M^{-1}(q) p \\ \dot{p} &= -\frac{\partial H}{\partial q}(p, q) + G(q) u. \end{aligned} \tag{6}$$

Remark 1: Force F_i $(i = 1, 2, \ldots, n)$ acting on the i-th particle is introduced to obtain the generalized forces. In fact, it is not the actual control. From the view point of engineering, the true control is the torque u_i which is supplied by the motor settled on the i-th joint.

B. The Stochastic Motion of the Particles

In this subsection, based on the equivalence principle of mechanics [19] and the relative motion [21], the effect of the random vibration is considered. From now on, point O is regarded as the reference point.

[1]The generalized force along the j-th generalized coordinates is defined by $Q_j = \sum_{i=1}^{N} (F_{ix} \partial x_i / \partial q_j + F_{iy} \partial y_i / \partial q_j + F_{iz} \partial z_i / \partial q_j)$, $j = 1, 2, \ldots, n$, where (F_{ix}, F_{iy}, F_{iz}) is the force acted on the i-th point. The transformation from the natural coordinates $\mathbf{r}_i = (x_i, y_i, z_i)$ to the generalized coordinates $\{q_1, q_2, \ldots, q_n\}$ is represented as $\mathbf{r}_i = \mathbf{r}_i(q_1, q_2, \ldots, q_n, t)$.

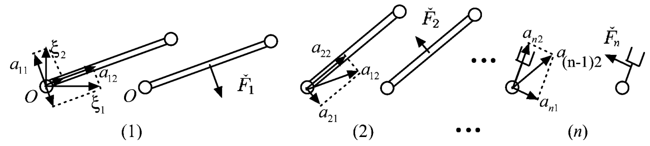

Fig. 3. Stochastic forces.

Step 1: Decomposing ξ_1 and ξ_2 at the point O (see Fig. 3(1)), we have $a_{11} = -\xi_1 \sin q_1 + \xi_2 \cos q_1$ and $a_{12} = \xi_1 \cos q_1 + \xi_2 \sin q_1$. According to the principle of relative motion, a stochastic force $\check{F}_1$ with unit N in the direction of F_1 is introduced such that $\check{F}_1/m_1$ describes the stochastic motion of 1-st particle relative to O, i.e., $\check{F}_1 = -m_1 a_{11} = m_1 \sin q_1 \xi_1 - m_1 \cos q_1 \xi_2$.

Step 2: Decomposing the acceleration a_{12} of the 2nd joint (see Fig. 3(2)), one obtains $a_{21} = -(\xi_1 \cos q_1 + \xi_2 \sin q_1)\sin(q_2 - q_1)$ and $a_{22} = (\xi_1 \cos q_1 + \xi_2 \sin q_1)\cos(q_2 - q_1)$. As in step 1, a stochastic force $\check{F}_2$ with unit N in the direction of F_2 is introduced such that $\check{F}_2/m_2$ describes the stochastic motion of 2-nd particle, i.e., $\check{F}_2 = -m_2 a_{21} = m_2 \cos q_1 \varphi_2(q)\xi_1 + m_2 \sin q_1 \varphi_2(q)\xi_2$ with $\varphi_2(q) = \sin(q_2 - q_1)$.

Step i: Decomposing the acceleration $a_{(i-1)2}$ of the i-th joint, one has

$$\begin{aligned} a_{i1} &= -(\xi_1 \cos q_1 + \xi_2 \sin q_1)\varphi_i(q), \\ a_{i2} &= (\xi_1 \cos q_1 + \xi_2 \sin q_1)\psi_i(q) \end{aligned} \tag{7}$$

where $\varphi_i(q) = \prod_{k=2}^{i-1} \cos(q_k - q_{k-1})\sin(q_i - q_{i-1})$ and $\psi_i(q) = \prod_{k=2}^{i-1} \cos(q_k - q_{k-1})\cos(q_i - q_{i-1})$, $i = 3, \ldots, n$. Then, a stochastic force $\check{F}_i$ with unit N in the direction of F_i is introduced such that $\check{F}_i/m_i$ describes the stochastic motion of i-th particle, i.e.,

$$\begin{aligned} \check{F}_i &= -m_i a_{i1} \\ &= m_i \cos q_1 \varphi_i(q)\xi_1 + m_i \sin q_1 \varphi_i(q)\xi_2. \end{aligned} \tag{8}$$

Based on the generalized forces formula, following the same line as (5), the generalized stochastic force is

$$\check{\tau}_i = \Lambda_{i1}(q)\xi_1 + \Lambda_{i2}(q)\xi_2, \quad i = 1, 2, \ldots, n \tag{9}$$

whose unit is $\mathrm{N} \cdot \mathrm{m}$, where $\Lambda_{11}(q) = m_1 l_{c1} \sin q_1 + \sum_{j=2}^{n} m_j l_1 \cos q_1 \varphi_j(q)\cos(q_j - q_1)$, $\Lambda_{12}(q) = -m_1 l_{c1} \cos q_1 + \sum_{j=2}^{n} m_j l_1 \sin q_1 \varphi_j(q)\cos(q_j - q_1)$, $\Lambda_{i1}(q) = m_i l_{ci} \cos q_1 \varphi_i(q) + \sum_{j=i+1}^{n} m_j l_i \cos q_1 \varphi_j(q)\cos(q_j - q_i)$ and $\Lambda_{i2}(q) = m_i l_{ci} \sin q_1 \varphi_i(q) + \sum_{j=i+1}^{n} m_j l_i \sin q_1 \varphi_j(q)\cos(q_j - q_i)$, $i = 2, \ldots, n$. Then the vector of generalized stochastic forces is $\check{\tau} = \Lambda(q)\xi$ with $\check{\tau} = (\check{\tau}_1, \check{\tau}_2, \ldots, \check{\tau}_n)^T$, $\xi = (\xi_1, \xi_2)^T$ and $\Lambda(q) = (\Lambda_{ij}(q))_{n\times 2}$.

C. A Stochastic Model of the Manipulator

On the basis of the above two subsections, the stochastic dynamic equation is obtained

$$\begin{aligned} \dot{q} &= M^{-1}(q)p, \\ \dot{p} &= -\frac{\partial H}{\partial q}(p, q) + G(q)u + \Lambda(q)\xi. \end{aligned} \tag{10}$$

By replacing ξ with "dB/dt", the Stratonovich stochastic differential equation of (10) can be obtained

$$\begin{aligned} dq &= M^{-1}(q)p\,dt, \\ dp &= \left(-\frac{\partial H}{\partial q}(p, q) + G(q)u\right)dt + \Lambda(q) \circ dB \end{aligned} \tag{11}$$

where B is a 2-D independent Wiener process. Since the diffusion matrix of q subsystem equals 0, and the diffusion matrix $\Lambda(q)$ of p subsystem does not depend on p, the Wong-Zakai correction term (see [22, (6.1.3)]) is zero, i.e.,

$$\frac{1}{2}\begin{pmatrix} \Xi_1 \frac{\partial \Xi_1}{\partial q} + \Xi_2 \frac{\partial \Xi_1}{\partial p} \\ \Xi_1 \frac{\partial \Xi_2}{\partial q} + \Xi_2 \frac{\partial \Xi_2}{\partial p} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{12}$$

where $\Xi_1 = 0$ and $\Xi_2 = \Lambda(q)$. The equivalent Itô stochastic differential equation is written as

$$\begin{aligned} dq &= M^{-1}(q)p\,dt, \\ dp &= \left(-\frac{\partial H}{\partial q}(p, q) + G(q)u\right)dt + \Lambda(q)dB \end{aligned} \tag{13}$$

i.e., the final stochastic Hamiltonian dynamic model of the manipulator is constructed.

IV. Tracking Control via State Feedback

In this section, the objective is to design a smooth or at least locally Lipschitz state feedback controller such that the configuration $q(t)$ can be driven to track a given twice continuously differentiable bounded reference signal $q_r(t) \in \mathbb{R}^n$ as close as possible while maintaining all the signals in the closed-loop system bounded in probability (see the definition in P.15 of [23]).

The configuration q and its velocity $\dot{q}$ can be measured by adding some measuring equipments like encoders to the manipulator. Since the inertia matrix $M(q)$ is known and $p = M(q)\dot{q}$, state p is measurable. It is easy to verify that the functions $M^{-1}(q)$, $-\partial H(p, q)/\partial q$, $G(q)$, and $\Lambda(q)$ are smooth.

A. Tracking Controller Design

Suppose that the power spectral density (PSD) of white noise ξ equals to $\Sigma/(2\pi)$, which is equivalent to the fact $dB = \Sigma dW$, where $\Sigma = (r_{ij})_{2\times 2}$ is a nonnegative definite matrix and W is a 2-D independent standard Wiener process. Then (13) is rewritten as

$$\begin{aligned} dq &= M^{-1}(q)p\,dt \\ dp &= \left(-\frac{\partial H}{\partial q}(p, q) + G(q)u\right)dt + \Lambda(q)\Sigma dW. \end{aligned} \tag{14}$$

Here, the underlying complete probability space is taken to be the quartet $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$ with a filtration $\mathcal{F}_t$ satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all P-null sets). The quasi-lower-triangular form of the system (14) inspires us to extend the backstepping technique [24], [25] to the vector form. In the following, a state feedback tracking controller will be designed by two steps.

Step 1: Introduce the error variables $z_1, z_2 \in \mathbb{R}^n$ as

$$z_1 = q - q_r \tag{15}$$

$$z_2 = p - M(q)\dot{q}_r - \alpha \tag{16}$$

where the smooth function α will be designed later. By (14)–(16), the tracking error z_1 satisfies

$$dz_1 = (M^{-1}(q)p - \dot{q}_r)dt = (M^{-1}(q)z_2 + M^{-1}(q)\alpha)dt. \tag{17}$$

For the first Lyapunov function $V_1 = (z_1^T z_1)^2/4$, by choosing the stabilizing function α as

$$\alpha = -c_1 M(q)z_1 \tag{18}$$

where $c_1 > 0$ is a design parameter, the infinitesimal generator[2] of V_1 along (17) satisfies

$$\begin{aligned}\mathcal{L}V_1 &= z_1^T z_1 z_1^T \left(M^{-1}(q)z_2 + M^{-1}(q)\alpha\right)\\ &= -c_1(z_1^T z_1)^2 + z_1^T z_1 z_1^T M^{-1}(q)z_2. \end{aligned} \tag{19}$$

Step 2: Substituting (18) into (16), one has $z_2 = p - M(q)(-c_1 z_1 + \dot{q}_r)$. From (14) and (17), it follows that

$$\begin{aligned} dz_2 &= dp - dM(q)(-c_1 z_1 + \dot{q}_r)\\ &\quad - M(q)(-c_1 dz_1 + d\dot{q}_r)\\ &= (\psi(q,p,\dot{q}_r,\ddot{q}_r) + G(q)u)dt + \Lambda(q)\Sigma dW \end{aligned} \tag{20}$$

where $\psi(q,p,\dot{q}_r,\ddot{q}_r) = -\partial H(p,q)/\partial q - (dM(q)/dt)\,(-c_1 z_1 + \dot{q}_r) + c_1 p - M(q)(c_1\dot{q}_r + \ddot{q}_r)$, $dM(q)/dt = (dM_{ij}(q)/dt)_{n\times n}$ with $dM_{ii}(q)/dt = 0$ and $dM_{ij}(q)/dt = dM_{ji}(q)/dt = -(m_j l_{cj} + \sum_{k=j+1}^n m_k l_j) l_i (\dot{q}_j - \dot{q}_i)\sin(q_j - q_i)$ $(i \neq j)$. For the second Lyapunov function $V_2 = V_1 + (z_2^T z_2)^2/4$, the infinitesimal generator of V_2 satisfies

$$\begin{aligned}\mathcal{L}V_2 &= -c_1(z_1^T z_1)^2 + z_1^T z_1 z_1^T M^{-1}(q)z_2\\ &\quad + z_2^T z_2 z_2^T(\psi(q,p,\dot{q}_r,\ddot{q}_r) + G(q)u)\\ &\quad + \frac{1}{2}\mathrm{Tr}\left\{\Sigma^T\Lambda^T(q)(2z_2z_2^T + z_2^T z_2 I)\Lambda(q)\Sigma\right\}. \end{aligned} \tag{21}$$

Applying Young's inequality[3] to the second term in (21), one has

$$\begin{aligned} z_1^T z_1 z_1^T M^{-1}(q) z_2 &\le |z_1|^3 \|M^{-1}(q)\|_F |z_2|\\ &\le \frac{c_1}{4}(z_1^T z_1)^2\\ &\quad + \frac{27}{4c_1^3}\|M^{-1}(q)\|_F^4 (z_2^T z_2)^2. \end{aligned} \tag{22}$$

As for the fourth term in (21), we first deal with the function $\Lambda(q)$. According to the definition of Frobenius norm and using the fact $(\cos q_1\cos(q_2 - q_1) - \cos q_{r1}\cos(q_{r2} - q_{r1}))^2 = ((\cos q_1 - \cos q_{r1})\cos(q_2 - q_1) + \cos q_{r1}(\cos(q_2 - q_1) - \cos(q_{r2} - q_{r1})))^2 \le 2(|q_1 - q_{r1}|^2\cos^2(q_2 - q_1) + |(q_2 - q_1) - (q_{r2} - q_{r1})|^2\cos^2 q_{r1}) \le 2(\cos^2(q_2 - q_1) + 2\cos^2 q_{r1})|q - q_r|^2$, it is learned that

$$\begin{aligned} &\|\Lambda(q) - \Lambda(q_r)\|_F^2\\ &= \sum_{i=1}^n \left((\Lambda_{i1}(q) - \Lambda_{i1}(q_r))^2 + (\Lambda_{i2}(q) - \Lambda_{i2}(q_r))^2\right)\\ &\le \sum_{i=1}^n (n-i+1)\Bigg(2im_i^2 l_{ci}^2\phi_i(q,q_r)\\ &\quad + \sum_{j=2}^n 2(j+1)m_j^2 l_1^2\Big(\phi_j(q,q_r)\cos^2(q_j - q_i)\\ &\quad + \prod_{k=2}^{j-1}\cos^2(q_{rk} - q_{r(k-1)})\sin^2(q_{rj} - q_{r(j-1)})\Big)\Bigg)|q - q_r|^2\\ &\triangleq \delta(q,q_r)|q - q_r|^2 \end{aligned} \tag{23}$$

where $\phi_1(q,q_r) = 1$, $\phi_2(q,q_r) = \sin^2(q_2 - q_1) + 1$ and $\phi_j(q,q_r) = \prod_{k=2}^{j-1}\cos^2(q_k - q_{k-1})\sin^2(q_j - q_{j-1}) + \sum_{l=2}^{j-1}\prod_{k=2}^{l-1}\cos^2(q_{rk} - q_{r(k-1)})\prod_{k=l+1}^{j-1}\cos^2(q_k - q_{k-1})\sin^2(q_j - q_{j-1}) + \prod_{k=2}^{j-1}\cos^2(q_{rk} - q_{r(k-1)})$, $j = 3,\ldots,n$. By (23), the Frobenius norm of $\Lambda(q)$ satisfies

$$\begin{aligned}\|\Lambda(q)\|_F^2 &\le 2\|\Lambda(q) - \Lambda(q_r)\|_F^2 + 2\|\Lambda(q_r)\|_F^2\\ &= 2\delta(q,q_r)|q - q_r|^2 + 2\|\Lambda(q_r)\|_F^2 \end{aligned} \tag{24}$$

where $\|\Lambda(q_r)\|_F^2 = m_1^2 l_{c1}^2 + (\sum_{j=2}^n m_j l_1\varphi_j(q_r)\cos(q_{rj} - q_{r1}))^2 + \sum_{i=2}^n (m_i l_{ci}\varphi_i(q_r) + \sum_{j=i+1}^n m_j l_i \varphi_j(q_r)\cos(q_{rj} - q_{ri}))^2$ with $\varphi_2(q_r) = \sin(q_{r2} - q_{r1})$ and $\varphi_j(q_r) = \prod_{k=2}^{j-1}\cos(q_{rk} - q_{r(k-1)})\sin(q_{rj} - q_{r(j-1)})$, $j = 3,\ldots,n$. By the definition of Frobenius norm, the norm compatibility, (24) and Young's inequality, we have

$$\begin{aligned} &\frac{1}{2}\mathrm{Tr}\left\{\Sigma^T\Lambda^T(q)(2z_2z_2^T + z_2^T z_2 I)\Lambda(q)\Sigma\right\}\\ &\le 3|z_2|^2\left(\delta(q,q_r)|z_1|^2 + \|\Lambda(q_r)\|_F^2\right)\|\Sigma\|_F^2\\ &\le \frac{c_1}{4}(z_1^T z_1)^2 + \left(\frac{9}{c_1}\delta^2(q,q_r) + \frac{9}{\epsilon}\|\Lambda(q_r)\|_F^4\right)\\ &\quad \times \|\Sigma\|_F^4 (z_2^T z_2)^2 + \frac{\epsilon}{4} \end{aligned} \tag{25}$$

where $\epsilon > 0$ is a design parameter and $\|\Sigma\|_F^2 = r_{11}^2 + r_{12}^2 + r_{21}^2 + r_{22}^2$. Substituting (22) and (25) into (21), one has

$$\begin{aligned}\mathcal{L}V_2 &\le -\frac{c_1}{2}(z_1^T z_1)^2 + z_2^T z_2 z_2^T\Bigg(\frac{27}{4c_1^3}\|M^{-1}(q)\|_F^4 z_2\\ &\quad + \psi(q,p,\dot{q}_r,\ddot{q}_r) + G(q)u\\ &\quad + \left(\frac{9}{c_1}\delta^2(q,q_r) + \frac{9}{\epsilon}\|\Lambda(q_r)\|_F^4\right)\|\Sigma\|_F^4 z_2\Bigg) + \frac{\epsilon}{4}. \end{aligned} \tag{26}$$

The controller u is designed as

$$\begin{aligned} u = G^{-1}(q)\Bigg(&-\frac{c_2}{2}z_2 - \frac{27}{4c_1^3}\|M^{-1}(q)\|_F^4 z_2\\ &- \psi(q,p,\dot{q}_r,\ddot{q}_r)\\ &- \left(\frac{9}{c_1}\delta^2(q,q_r) + \frac{9}{\epsilon}\|\Lambda(q_r)\|_F^4\right)\|\Sigma\|_F^4 z_2\Bigg) \end{aligned} \tag{27}$$

which we substitute in (26) to obtain

$$\mathcal{L}V \le -\frac{c_1}{2}(z_1^T z_1)^2 - \frac{c_2}{2}(z_2^T z_2)^2 + \frac{\epsilon}{4} \le -cV + \frac{\epsilon}{4} \tag{28}$$

where $c_2 > 0$ is a design parameter, $V = V_2$ and $c = 2\min\{c_1, c_2\}$.

Up to now, the following error system is obtained by (17), (18), (20), and (27):

$$\begin{aligned} dz_1 &= (-c_1 z_1 + M^{-1}(q)z_2)dt,\\ dz_2 &= -\Bigg(\frac{c_2}{2} + \frac{27}{4c_1^3}\|M^{-1}(q)\|_F^4\\ &\quad + \left(\frac{9}{c_1}\delta^2(q,q_r) + \frac{9}{\epsilon}\|\Lambda(q_r)\|_F^4\right)\|\Sigma\|_F^4\Bigg)\\ &\quad \times z_2 dt + \Lambda(q)\Sigma dW. \end{aligned} \tag{29}$$

Just for simplicity, c_1 and c_2 are selected as scalars. In fact, c_1 and c_2 can be chosen as any positive definite constant matrix.

B. Stability Analysis

We give the main result of this technical brief.

[2]For $V(x) \in C^2(\mathbb{R}^n)$, the infinitesimal generator of V along the stochastic nonlinear system $dx(t) = f(x(t),t)dt + g(x(t),t)dW(t)$ is $\mathcal{L}V = V_x f + \mathrm{Tr}\{g^T V_{xx} g\}/2$, where $V_x = (\partial V/\partial x_j)_{1\times n}$ and $V_{xx} = (\partial^2 V/(\partial x_i \partial x_j))_{n\times n}$.

[3]For any vectors $x, y \in \mathbb{R}^n$ and any scalars $\epsilon > 0$, $p > 1$ there holds $x^T y \le (\epsilon^p/p)|x|^p + (1/(q\epsilon^q))|y|^q$, where $q = p/(p-1)$.

Theorem 1: Consider the stochastic Hamiltonian dynamic (14) for the manipulator. By choosing the state-feedback controller (27):

(i) the error system (29) has a unique solution for any initial values $z_1(t_0), z_2(t_0) \in \mathbb{R}^n$ on $[t_0, \infty)$ and all of the signals in the closed-loop system are bounded in probability;

(ii) the mean square of the tracking error satisfies

$$\lim_{t\to\infty} E|q(t)-q_r(t)|^2 \le \left(\frac{2\epsilon}{c}\right)^{1/2} \tag{30}$$

and the right hand side can be made small enough by choosing design parameters appropriately.

Proof: Denoting $z(t) = (z_1^T(t), z_2^T(t))^T$, from the definition of V, it is obvious that $V(z(t)) \in \mathcal{C}^2(\mathbb{R}^{2n})$ and

$$\frac{1}{8}|z(t)|^4 \le V(z(t)) \le \frac{1}{4}|z(t)|^4. \tag{31}$$

Since the functions of the error system (29) satisfy the local Lipschitz condition, in view of (28) and (31), following the same line as the proof of Theorem 4.1 in [26], there exists a unique strong solution $z(t) = z(t; z_0, t_0)$ of system (29) for each $z(t_0) = z_0 \in \mathbb{R}^{2n}$ and it satisfies

$$EV(z(t)) \le e^{-c(t-t_0)}EV(z_0) + \frac{\epsilon}{4c},\ \forall t \ge t_0. \tag{32}$$

According to Jensen's inequality[4], from (31) and (32), one has

$$E|z(t)|^2 \le (E|z(t)|^4)^{1/2} \le 2^{1/2}|z_0|^2 e^{-c/2(t-t_0)} + \left(\frac{2\epsilon}{c}\right)^{1/2} \tag{33}$$

which means that the error system (29) is exponentially practically stable in mean square. According to the Chebyshev's inequality, (33) implies that the solution $z(t)$ of error system (29) is bounded in probability. By (15), (16) and the twice continuously differentiable boundedness of reference signal $q_r(t)$, it can be obtained that states q and p are bounded in probability. From (27), u is also bounded in probability.

Moreover, (30) holds from (33) and the definition of z. Noting $c = 2\min\{c_1, c_2\}$, the right hand side of (30) can be made small enough by choosing c_1 and c_2 large enough and ϵ small enough. □

Remark 2: According to Chebyshev's inequality, for any $\varepsilon > 0$ and $\varepsilon_0 > 0$, there exists a $T > 0$ such that $\forall\, t > T$

$$P\{|q(t)-q_r(t)| > \varepsilon\} \le \frac{1}{\varepsilon^2}\left(\varepsilon_0 + \left(\frac{2\epsilon}{c}\right)^{1/2}\right) \le \varepsilon' \tag{34}$$

where ε' can be made small enough by choosing parameters appropriately, which implies that the asymptotic tracking in probability in some sense can be achieved at the expense of large control effort. In designing controller, the effect between the tracking error and the allowable control effort must be carefully compromised.

Remark 3: Since the control and the reference signal enter in different channels, it is difficult to achieve asymptotic tracking. The parameter ϵ is introduced to deal with the Hessian terms caused by nonvanishing signal $q_r(t)$. For the problem of stabilization, i.e., $q_r(t) = 0$, ϵ can be avoid by using the technique in [24, 3.35], then the stabilization of q can be achieved.

Remark 4: In many papers such as [27]–[29], based on the traditional backstepping technique, the different tracking controllers are designed for different stochastic systems with lower-triangular structure. While system (14) is the quasi-lower-triangular form, which is difficult to be transformed into lower-triangular structure. By extending the integral backstepping technique to the vector form, we design a tracking controller in vector form.

[4]If φ is a concave function and x is a random variable on Ω, then $E\varphi(x) \le \varphi(Ex)$.

V. Simulation Result

In this section, we demonstrate the effectiveness of the control scheme by the planar rigid manipulator with three revolute joints (i.e., $n = 3$). The stochastic Hamiltonian dynamic model is

$$\begin{aligned} dq &= M^{-1}(q)p\,dt, \\ dp &= \left(-\frac{\partial H}{\partial q}(p,q) + G(q)u\right)dt + \Lambda(q)\Sigma dW \end{aligned} \tag{35}$$

where $q = (q_1, q_2, q_3)^T$, $p = (p_1, p_2, p_3)^T$, $u = (u_1, u_2, u_3)^T$, $\Sigma = (r_{ij})_{2\times 2}$, W is a 2-dimensional independent standard Wiener process, $M(q) = (M_{ij}(q))_{3\times 3}$ with $M_{11}(q) = m_1 l_{c1}^2 + m_2 l_1^2 + m_3 l_1^2$, $M_{22}(q) = m_2 l_{c2}^2 + m_3 l_2^2$, $M_{33}(q) = m_3 l_{c3}^2$, $M_{12}(q) = (m_2 l_1 l_{c2} + m_3 l_1 l_2)\cos(q_2 - q_1) = M_{21}(q)$, $M_{13}(q) = m_3 l_1 l_{c3}\cos(q_3 - q_1) = M_{31}(q)$ and $M_{23}(q) = m_3 l_2 l_{c3}\cos(q_3 - q_2) = M_{32}(q)$, the Hamiltonian function $H(q,p) = p^T M^{-1}(q)p/2 - P(q)$ with $P(q) = \sum_{i=1}^{3}(m_i l_{ci} + \sum_{j=i+1}^{3} m_j l_i) g \sin q_i$,

$$G(q) = \begin{bmatrix} 1 & \frac{l_1}{l_{c2}}\cos(q_2-q_1) & \frac{l_1}{l_{c3}}\cos(q_3-q_1) \\ 0 & 1 & \frac{l_2}{l_{c3}}\cos(q_3-q_2) \\ 0 & 0 & 1 \end{bmatrix}$$

and $\Lambda(q) = (\Lambda_{ij}(q))_{3\times 2}$ with $\Lambda_{11}(q) = m_1 l_{c1}\sin q_1 + m_2 l_1 \cos q_1 \sin(q_2 - q_1)\cos(q_2 - q_1) + m_3 l_1 \cos q_1 \cos(q_2 - q_1)\sin(q_3 - q_2)\cos(q_3 - q_1)$, $\Lambda_{12}(q) = -m_1 l_{c1}\cos q_1 + m_2 l_1 \sin q_1 \sin(q_2 - q_1)\cos(q_2 - q_1) + m_3 l_1 \sin q_1 \cos(q_2 - q_1)\sin(q_3 - q_2)\cos(q_3 - q_1)$, $\Lambda_{21}(q) = m_2 l_{c2}\cos q_1 \sin(q_2 - q_1) + m_3 l_2 \cos q_1 \cos(q_2 - q_1)\sin(q_3 - q_2)\cos(q_3 - q_2)$, $\Lambda_{22}(q) = m_2 l_{c2}\sin q_1 \sin(q_2 - q_1) + m_3 l_2 \sin q_1 \cos(q_2 - q_1)\sin(q_3 - q_2)\cos(q_3 - q_2)$, $\Lambda_{31}(q) = m_3 l_{c3}\cos q_1 \cos(q_2 - q_1)\sin(q_3 - q_2)$ and $\Lambda_{32}(q) = m_3 l_{c3}\sin q_1 \cos(q_2 - q_1)\sin(q_3 - q_2)$.

Choose the reference signal $q_r(t) = (0.5 + 0.5\sin t, 0.5 + 0.5\cos t, -0.5\cos t)^T$ whose unit is rad. The state feedback tracking controller is given by (27) with $n = 3$, i.e.,

$$\begin{aligned} u = {} & G^{-1}(q) \\ & \times\Bigg(-\frac{c_2}{2}z_2 - \frac{27}{4c_1^3}\|M^{-1}(q)\|_F^4 z_2 + \frac{\partial H}{\partial q}(p,q) \\ & + \frac{dM(q)}{dt}(-c_1 z_1 + \dot{q}_r) \\ & - c_1 p + M(q)(c_1\dot{q}_r + \ddot{q}_r) \\ & - \left(\frac{9}{c_1}\delta^2(q,q_r) + \frac{9}{\epsilon}\|\Lambda(q_r)\|_F^4\right)\|\Sigma\|_F^4 z_2\Bigg). \end{aligned} \tag{36}$$

In the simulation, we choose PSD of the white noise ξ as $\Sigma = (r_{ij})_{2\times 2}$ with $r_{ii} = 0.1$ and $r_{ij} = 0\ (i \ne j)$; the system parameters $m_1 = 0.5$ kg, $m_2 = 0.5$ kg, $m_3 = 0.5$ kg, $l_1 = 3$ m, $l_2 = 3$ m, $l_3 = 1.5$ m, $l_{c1} = 1.5$ m, $l_{c2} = 1.5$ m, $l_{c3} = 1$ m, and $g = 9.8\ \mathrm{m/s^2}$; the initial values $q_1(0) = 0.55$ rad, $q_2(0) = 1.15$ rad, $q_3(0) = -0.65$ rad, $p_1(0) = 0\ \mathrm{kg\cdot m^2\cdot rad/s}$, $p_2(0) = 0\ \mathrm{kg\cdot m^2\cdot rad/s}$, $p_3(0) = 0\ \mathrm{kg\cdot m^2\cdot rad/s}$; the design parameters $c_1 = 10$, $c_2 = 1$, and $\epsilon = 0.5$. Fig. 4 demonstrates the effectiveness of the control scheme.

VI. Conclusions

In this technical brief, the problem of modeling and tracking control for the manipulator with multi-revolute joints in random vibration environment is considered.

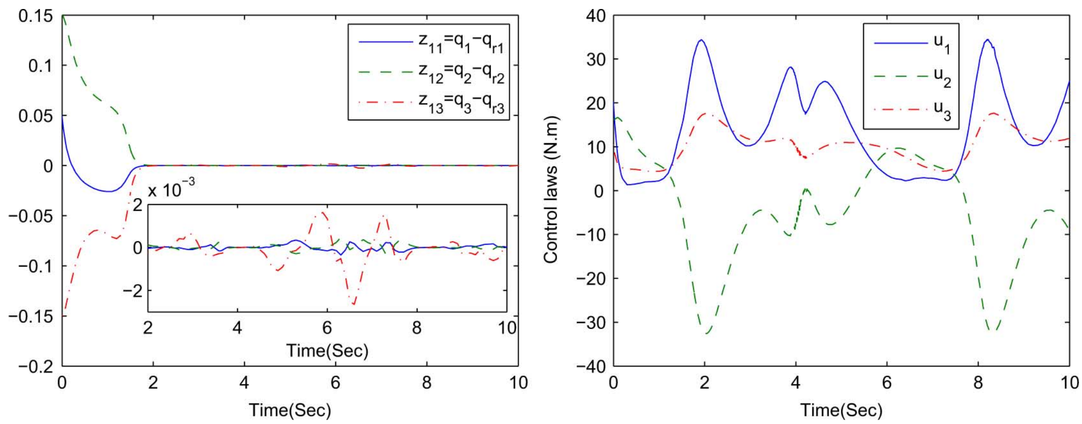

Fig. 4. Responses of closed-loop system.

There are other problems to be considered: 1) an interesting problem is to extend the method to robot manipulators with multi-revolute joints and multi-prismatic joints in a random vibration workspace; 2) the result should be further extended to the general stochastic Hamiltonian control systems; and 3) find other control methods to solve this problem and compare their effectiveness and efficiency.

REFERENCES

[1] M. W. Spong, S. Hutchinson, and M. Vidyasagar, *Robot Modeling and Control*. New York: Wiley, 2006.
[2] C. De Wit, B. Siciliano, and B. Bastin, *Theory of Robot Control*. London, U.K.: Springer–Verlag, 1996.
[3] L. Sciavicco and B. Siciliano, *Modelling and Control of Robot Manipulator*. London, U.K.: Springer-Verlag, 2000.
[4] J. J. Craig, *Introduction to Robotics: Mechanics and Control*, 3rd ed. Upper Saddle River, NJ: Prentice–Hall, 2004.
[5] F. L. Lewis, D. M. Dawson, and C. T. Abdallah, *Robot Manipulator Control Theory and Practice*. New York: Marcel Dekker, 2004.
[6] F. L. Lewis, D. M. Dawson, and C. T. Abdallah, *Control of Robot Manipulators in Joint Space*. London, U.K.: Springer–Verlag, 2005.
[7] A. C. Huang and M. C. Chien, *Adaptive Control of Robot Manipulators: A Unified Regressor-Free Approach*. , Singapore: World Scientific, 2010.
[8] É. Dombre and W. Khalil, *Robot Manipulators: Modeling, Performance Analysis and Control*. London, U.K.: ISTE Ltd., 2007.
[9] S. S. Ge, T. H. Lee, and C. J. Harris, *Adaptive Neural Network Control of Robotic Manipulators*. Singapore: World Scientific, 1998.
[10] L. Villani, C. C. D. Wit, and B. Brogliato, "An exponentially stable adaptive control for force and position tracking of robot manipulators," *IEEE Trans. Autom. Control*, vol. 44, no. 4, pp. 798–802, Apr. 1999.
[11] C. C. Cheah, M. Hirano, S. Kawamura, and S. Arimoto, "Approximate Jacobian control with task-space damping for robot manipulators," *IEEE Trans. Autom. Control*, vol. 49, no. 5, pp. 752–757, May 2004.
[12] J. Kasac, B. Novakovic, D. Majetic, and D. Brezak, "Passive finite-dimensional repetitive control of robot manipulators," *IEEE Trans. Control Syst. Technol.*, vol. 16, no. 3, pp. 570–576, Mar. 2008.
[13] F. L. Lewis, S. Jagannathan, and A. Yeildirek, *Neural Network Control of Robot Manipulators and Nonlinear Systems*. Philadelphia, PA: Taylor & Francis, 1998.
[14] H. D. Patino, R. Carelli, and B. R. Kuchen, "Neural networks for advanced control of robot manipulators," *IEEE Trans. Neural Netw.*, vol. 13, no. 2, pp. 343–354, Feb. 2002.
[15] E. M. Jafarov, M. N. A. Parlakci, and Y. Istefanopulos, "A new variable structure PID-controller design for robot manipulators," *IEEE Trans. Control Syst. Technol.*, vol. 13, no. 1, pp. 122–130, Jan. 2005.
[16] H. Hu and P. Y. Woo, "Fuzzy supervisory sliding-mode and neural-network control for robotic manipulators," *IEEE Trans. Ind. Electron.*, vol. 53, no. 3, pp. 929–940, Mar. 2006.
[17] S. H. Crandall and W. D. Mark, *Random Vibration in Mechanical Systems*. New York: Academic Press, 1963.
[18] W. Q. Zhu, *Nonlinear Stochastic Dynamics and Control: A Framework of Hamiltonian Theory*. Beijing, China: Science Press, 2003.
[19] J. M. Knudsen and P. G. Hjorth, *Elements of Newtonian Mechanics: Including Nonlinear Dynamics*. New York: Springer–Verlag, 2000.
[20] M. G. Calkin, *Lagrangian and Hamiltonian Mechanics*. Singapore: World Scientific, 1996.
[21] R. P. Olenick, T. M. Apostol, and D. L. Goodstein, *The Mechanical Universe: Introduction to Mechanics and Heat*. London, U.K.: Cambridge University Press, 2008.
[22] B. Øksendal, *Stochastic Differential Equations-An Introduction With Applications*, 6th ed. New York: Springer–Verlag, 2003.
[23] R. Z. Khas'minskii, *Stochastic Stability of Differential Equations*. Rockville, MD: S & N International, 1980.
[24] M. Krstic and H. Deng, *Stability of Nonlinear Uncertain Systems*. New York: Springer–Verlag, 1998.
[25] Z. J. Wu, X. J. Xie, P. Shi, and Y. Q. Xia, "Backstepping controller design for a class of stochastic nonlinear systems with markovian switching," *Automatica*, vol. 45, no. 4, pp. 997–1004, 2009.
[26] H. Deng, M. Krstic, and R. Williams, "Stabilization of stochastic nonlinear systems driven by noise of unknown covariance," *IEEE Trans. Autom. Control*, vol. 46, no. 8, pp. 1237–1253, Aug. 2001.
[27] H. B. Ji and H. S. Xi, "Adaptive output-feedback tracking of stochastic nonlinear systems," *IEEE Trans. Automat. Control*, vol. 51, no. 2, pp. 355–360, Feb. 2006.
[28] X. J. Xie and N. Duan, "Output tracking of high-order stochastic nonlinear systems with application to benchmark mechanical system," *IEEE Trans. Automat. Control*, vol. 55, no. 5, pp. 1197–1202, May 2010.
[29] Z. J. Wu, J. Yang, and P. Shi, "Adaptive tracking for stochastic nonlinear systems with markovian switching," *IEEE Trans. Automat. Control*, vol. 55, no. 9, pp. 2135–2141, Sep. 2010.