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Transient performance improvement in model reference adaptive control using H_∞ optimal method

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Abstract

Poor transient performance is considered as a drawback of model reference adaptive control. To achieve a better transient behavior, the standard model reference adaptive controller is modified by adding a compensator in this paper. Based on the augmented tracking error dynamics, the compensator can be designed as an H_∞ optimal controller to eliminate the adverse effect caused by the parameter estimation error in initial stage, so that the transient performance is improved. The proposed modified adaptive controller will not destroy the asymptotic stability of the closed-loop adaptive system as well as the ideal properties of adaptive law, and a predefined transient performance index is taken into account throughout the controller design. The performance analysis using L_∞ tracking error bound and mean squared tracking error bound in any interval reveals that the amount of the transient performance improvement depends on the performance level of the H_∞ compensator. Simulation results of the designed control scheme application to a flight control system are presented to show the desired transient.

1. Introduction

In the last few decades, model reference adaptive control (MRAC) was used as one of the most appealing technologies to design trustable controllers for various classes of systems [1–4], which can ensure the global stability through online adjustment of control parameters. However, the poor transient response such as chaos, large transient oscillations and bursting may result in

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deterioration of handling qualities and have limited the universality of its application [5,6]. As pointed out in [7,8], the controller parameter estimation error is one of the main reasons that bring about undesired transient. Furthermore, no matter which adaptive algorithm is used, such parameter identification error is inevitable in realistic environment due to the existing of model uncertainties. For example, the limited prior knowledge about the controlled plant model or the dynamic process modeling error in large-scale industrial application based on data-driven [9,10]. Even though the sufficient persistent excitation signals are helpful in parameter convergence, these exogenous signals are unwanted in closed-loop system which may introduce disturbances. Moreover, the beneficial effect of the excitation signals is not so obvious because of slow adaption for stability consideration and "passive" attitude adopted by identification scheme. On the other hand, large reference commands, changes of system state and external disturbances may aggravate the adverse effect of the parameter estimation error on transient performance.

A practical approach for transient performance improvement in MRAC is Luenberger observer-based adaptive control scheme, in which the reference model is modified by adding a state (or output) error feedback item [11,12]. This method gives us a direct insight into transient behavior by increasing convergence rate of the tracking error, but the well designed reference model is replaced and thus the desired reference output to be tracked is changed. Several researchers in [13,7] try to enhance the identification scheme via introducing a high-order parameter estimator, which results in a dynamic certainty equivalence in closed-loop adaptive system so that the transient performance is improved through avoiding direct error normalization. The efforts made in [14,15] are to use a variable structure high gain switching MRAC scheme for obtaining the expected transient and steady-state behavior. In [8], Sun introduces a dynamic compensator in standard adaptive control to counteract errors resulting from the certainty equivalence design. As a result, the improvement of transient performance can be achieved by choosing proper stable transfer function in the compensator. Then Datta gives an alternative method for the construction of dynamic compensator by using Swapping Lemma in [16], which also provides an arbitrarily improved zero-state transient performance. But this desired transient performance improvement is no longer possible in the presence of unmodeled dynamics. Another contribution made by the paper [16] is that it first proposed the mean square tracking error criterion and the L_∞ tracking error bound criterion to assess the transient performance of MRAC system.

In this paper, the widely studied H_∞ optimal control method is used to improve the transient performance of MRAC. Inspired by the approaches of Sun and Datta in [8,16], we also introduce a compensator in the standard model reference adaptive control law to form the modified adaptive scheme. Then based on the resulting augmented tracking error dynamics, in which the parameter estimation error item is considered as a disturbance input, the compensator is designed as an H_∞ optimal controller towards attenuating the adverse effects caused by the parameter estimation in initial stage. To evaluate the effectiveness of the modified adaptive controller, the L_∞ and mean squared error performance bounds are computed, which also reveal that the amount of the transient performance improvement depends on the performance level γ of the H_∞ compensator. In contrast to the existing approaches for transient performance improvement of MRAC, the outstanding properties of the proposed control scheme are as follows:

(i) The asymptotic stability and the ideal properties of adaptive law are maintained by the modified adaptive control scheme.
(ii) Only the output tracking error is adopted as the control signal in the compensator which can be achieved easily in most model reference adaptive systems.

(iii) A predefined transient performance index is taken into account throughout the design of the modified adaptive controller, which also indicates the weight of the compensator in the whole closed-loop control system.

2. Theory preliminary

In this section, we give two lemmas which contain the singular case of state feedback and measurement feedback H_∞ optimal control. The two lemmas play a key role in the compensator design and performance analysis.

Consider a linear time-invariant system

$$\begin{cases} \dot{x} = Ax + B_1 w + B_2 u \ x(0) = 0 \\ z = C_1 x \\ y = C_2 x \end{cases} \tag{1}$$

where $x(t) \in \Re^{n_x}$ is the state, $u(t) \in \Re^{n_u}$ is the control input, $w(t) \in \Re^{n_w}$ is the disturbance input, $z(t) \in \Re^{n_z}$ is the regulated output to be controlled, and $y(t) \in \Re^{n_y}$ is the measured output. A, B_1, B_2, C_1, C_2 are matrices of appropriate dimensions and satisfying the assumptions:

(A1) (A,B_1) is stabilizable.
(A2) (A,C_2) is detectable.
(A3) B_1 is column full rank, and C_2 is row full rank.
(A4) $n_w \le n_y \le n_x$.
(A5) $C_2(sI - A)^{-1}B_1$ is left invertible and strictly minimum phase.

Lemma 1 (*State-feedback H_∞ optimal control*). *If there exists an $\varepsilon > 0$ such that the Riccati equation*

$$A^T P + PA - \varepsilon^{-1} P B_2 R^{-1} B_2^T P + \gamma^{-1} P B_1 B_1^T P + \gamma^{-1} C_1^T C_1 + \varepsilon S = 0 \tag{2}$$

has a positive-definite solution P, where $R \in \Re^{n_u \times n_u}$ and $S \in \Re^{n_x \times n_x}$ are given positive-definite matrices.

Then, there exists a state feedback controller $u = Kx$ with $K = -(1/2\varepsilon)R^{-1}B_2^T P$ such that the system (1) is stabilized, and the transfer function matrix $G_{wz}(s)$ between disturbance w and output z has H_∞ norm less than a prespecified constant $\gamma > 0$. Furthermore, by choosing a sufficiently small ε then γ is arbitrarily close to the H_∞ optimum.

Lemma 2 (*Measurement feedback H_∞ optimal control*). *Suppose K is a suitable state feedback H_∞ optimal controller gain matrix obtained from Lemma 1. Then, give any $\varepsilon^* > 0$, the system (1) can be stabilizable via measurement feedback with guaranteed $\|G_{wz}(s)\|_\infty < \gamma$, if the following Riccati equation*

$$\overline{A}^T Z + Z\overline{A} + \gamma^{-1} P\overline{B}\overline{B}^T Z + \gamma^{-1}\overline{C}\overline{C}^T + \varepsilon^* I = 0, \tag{3}$$

where

$$\overline{A}=\begin{bmatrix} A & B_2K \\ LC_1 & A+B_2K-LC_1 \end{bmatrix}, \quad \overline{B}=[B_1], \overline{C}=[C_2\ 0],$$

has a positive-definite symmetric solution. Where L is the observer gain matrix and can be constructed as follows.

Let $J \in \Re^{n_x \times n_x}$ be a given positive-definite symmetric weighting matrix, given any $\rho > 0$, the Riccati equation

$$AQ+QA^T+J+q^2\rho^{-1}B_1B_1^T-QC_2^TC_2Q+q^{-2}\rho^{-1}QC_1^TC_1Q=0 \tag{4}$$

exists a positive-definite solution Q by choosing $q>0$ be sufficiently large. So that $L=QC_2^T$ and the measurement feedback controller $u=F(s)y$ is obtained and described by the following state equations:

$$\begin{cases} \dot{\hat{x}}=A\hat{x}+B_2K\hat{x}+L(y-C_1\hat{x}) \\ u=K\hat{x} \end{cases} \tag{5}$$

where $\hat{x} \in \Re^{n_x}$ is the observer states.

Remark 1. The system internal stability is included as a property of H_∞ controller and the proofs of above Lemmas can be found in [17,18]. It should be noted that the so-called singular case of H_∞ optimal control problem concerns the regulated and measured outputs with the elimination of the effect of control input and disturbance [19]. As a result, the system (1) has the same form with the augmented tracking error dynamics (Eq. (16) in Section 3.3), so that the compensator can be designed using Lemmas 1 and 2.

3. Modified MRAC for transient performance improvement

The general control plant and reference model along with the same assumptions in standard MRAC are given as follows described by input–output model.

Plant model:

$$y=G_p(s)u=k_p\frac{Z_p(s)}{R_p(s)}u \tag{6}$$

with the assumptions of $G_p(s)$ that

(P1) $Z_p(s)$ is a monic Hurwitz polynomials of degree m_p.
(P2) An upper bound of the degree n_p of $R_p(s)$.
(P3) The relative degree $n^*=n_p-m_p$ of $G_p(s)$.
(P4) The sign of the high frequency gain k_p is known.

Reference model:

$$y_m=W_m(s)r=k_m\frac{Z_m(s)}{R_m(s)}r \tag{7}$$

with the assumptions of $W_m(s)$ that

(M1) $Z_m(s)$ and $R_m(s)$ are monic Hurwitz polynomials of degree q_m, p_m, respectively, where $p_m \le n_p$.
(M2) The relative degree $n_m^* = p_m - q_m$ of $W_m(s)$ is the same as that of $G_p(s)$.

The control objective is to design the control input u so that the output y of the plant tracks the reference output y_m as closely as possible with guaranteed transient performance.

3.1. Modified MRAC

In this subsection, we present the modified MRAC scheme which consists of an adaptive controller augmented by a compensator and a parameter estimator using normalization. We first consider the standard control law

$$u = \theta^T \omega + c_0 r, c_0 = \frac{k_m}{k_p}, \tag{8}$$

where

$$\begin{aligned} &\omega = [\omega_1^T, \omega_2^T, y]^T, \quad \theta = [\theta_1^T, \theta_2^T, \theta_3]^T, \\ &\omega_1 = \frac{a(s)}{\Lambda(s)} u, \quad \omega_2 = \frac{a(s)}{\Lambda(s)} y, \\ &a(s) = [s^{n_p-2}, s^{n_p-3}, \ldots, s, 1], \quad \Lambda(s) = Z_m \lambda(s). \end{aligned}$$

$\lambda(s)$ is a monic Hurwitz polynomial of degree $n_p - 1 - q_m$ and $\theta \in \Re^{2n-1}$ is the controller parameter vector. Then we introduce a compensator u_c as an additional item added in the standard adaptive control law (8) to give the modified control law

$$u = \theta^T \omega + c_0 r + u_c, \tag{9}$$

and define $(\|W_m(s)/c_0\|_\infty)/\gamma$ as the transient performance index, where γ is a design parameter of the compensator u_c. Both of the standard and the modified control law have the same form of the normalized adaptive law:

$$\begin{aligned} &\dot{\theta} = \frac{-\Gamma \epsilon \phi}{m^2}, \\ &\epsilon = \theta^T \phi + c_0 y - W_m(s) u, \\ &\phi = W_m(s) \omega, \end{aligned} \tag{10}$$

where $\Gamma = \Gamma^T > 0$ is the adaptive gain matrix and m is the normalizing signal.

Note that the designs of the standard adaptive controller (8) as well as the normalized adaptive law (10) have been widely studied in the literatures of adaptive control, i.e. [20,23,24], so the details of them are omitted here. To design the compensator u_c, we first present the resulting tracking error equation, based on which the compensator is constructed using H_∞ optimal control discussed in Section 2.

3.2. Tracking error dynamics

Let $\tilde{\theta} = \theta - \theta^*$, where $\theta^* = [\theta_1^{*T}, \theta_2^{*T}, \theta_3^*]^T$ is the desired controller parameter vector, the modified control law (9) can be expressed as

$$u = \theta^{*T} \omega + \tilde{\theta}^T \omega + c_0 r + u_c \tag{11}$$

Then from plant model (6) and reference model (7) together with Eq. (11) we have

$$u = \frac{R_p(s)}{Z_p(s)} \frac{W_m(s)}{c_0} \left[\tilde{\theta}^T \omega + c_0 r + u_c \right], \tag{12}$$

and

$$y = \frac{W_m(s)}{c_0} \left[\tilde{\theta}^T \omega + c_0 r + u_c \right]. \tag{13}$$

So the tracking error $e_1 = y - y_m$ being given by

$$e_1 = \frac{W_m(s)}{c_0} \left[\tilde{\theta}^T \omega + u_c \right], \tag{14}$$

which can be rewritten in the form of state space

$$\begin{cases} \dot{e} = A_c e + B_c \left[\tilde{\theta}^T \omega \right] + B_c u_c \\ e_1 = C_c e \end{cases} \tag{15}$$

where (A_c, B_c, C_c) is a minimum stable realization of $W_m(s)/c_0$.

For the tracking error dynamics (15), one may find that the parameter estimation error item $\tilde{\theta}^T \omega$ can be treated as the disturbance input and has significant effect on system tracking performance. Therefore, we intend to attenuate the disturbance brought by $\tilde{\theta}^T \omega$ on the closed-loop system through designing the compensator u_c as a H_∞ optimal controller. So that the problem of performance improvement in MRAC system can be converted into the problem of H_∞ optimal control. Then we introduce the regulated output $e_z = e_1$ in Eq. (15) to form the following augmented dynamics:

$$\begin{cases} \dot{e} = A_c e + B_c \left[\tilde{\theta}^T \omega \right] + B_c u_c \\ e_z = C_c e \\ e_1 = C_c e \end{cases} \tag{16}$$

which has the same form as Eq. (1) presented in Section 2. And the assumptions (A1)–(A5) can be satisfied by choosing proper reference model $W_m(s)$. So that the systematic design procedures of the H_∞ compensator u_c are given in the following section using Lemmas 1 and 2.

3.3. H_∞ compensator design

Note that not full error state e can be accessed in reality. Then the H_∞ compensator is designed by the following steps based on Eq. (16).

Step 1: Initialize $0 < \gamma < \| W_m(s)/c_0 \|_\infty$ and choose $\varepsilon > 0$ along with $S \in \Re^{p_m \times p_m}$ and $R \in \Re^{q_m \times q_m}$ are given positive-definite matrices, determine whether the following Riccati equation:

$$A_c^T P + P A_c + P B_c^T (\gamma^{-1} I - \varepsilon^{-1} R^{-1}) B_c^T P + \gamma^{-1} C_c^T C_c + \varepsilon S = 0 \tag{17}$$

has a positive-definite solution P. If no solution exists then decrease ε and repeat this step until a suitable solution is got such that the state feedback gain $K = -(1/2\varepsilon) R^{-1} B_c^T P$ is obtained.

Step 2: Initialize q and $\delta > 0$, as well as $J \in \Re^{p_m \times p_m}$ are given positive-definite matrices, determine whether the following Riccati equation:

$$A_c Q + Q A_c^T + J + q^2 \delta^{-1} B_c B_c^T + (q^{-2} \delta^{-1} - 1) Q C_c^T C_c Q = 0 \tag{18}$$

has a positive-definite solution Q. If a solution is obtained then we have the observer gain $L = QC_2^T$. Otherwise, increase q and repeat this step.

Step 3: Initialize $\varepsilon^* > 0$, determine whether the following Riccati equation:

$$\overline{A}^T Z + Z\overline{A} + \gamma^{-1} P\overline{B}\overline{B}^T Z + \gamma^{-1}\overline{C}\overline{C}^T + \varepsilon^* I = 0, \tag{19}$$

where

$$\overline{A} = \begin{bmatrix} A_c & B_c K \\ LC_c & A_c + B_c K - LC_c \end{bmatrix}, \quad \overline{B} = \begin{bmatrix} B_c \\ 0 \end{bmatrix},$$
$$\overline{C} = [C_c \ 0],$$

has a positive-definite solution. If it does then we proceed to step 4, otherwise, decrease ε^* and repeat this step.

Step 4: The H_∞ compensator based on measurement feedback

$$u_c = F(s)e_1 \tag{20}$$

is constructed using state feedback gain K and observer gain L, where $F(s)$ is a stable transfer function described by the state space equations

$$\begin{cases} \dot{\hat{e}} = A_c\hat{e} + B_c K\hat{e} + L(e_1 + C_c\hat{e}) \\ u_c = K\hat{e} \end{cases} \tag{21}$$

which ensures the closed-loop stability of system (16) with $\tilde{\theta}^T\omega$ disturbance attenuation γ.

Step 5: Return to step 1 to revise the transient performance index (i.e. replace γ by $\gamma/2$) until a satisfactory transient is obtained or the optimum (minimized) γ is reached.

Remark 2. To determine the existence of positive-definite solution of a Riccati equation, one can use a standard algorithm in [21]. From the above design steps, the H_∞ optimal compensator with minimized γ always can be obtained by adopting sufficiently small ε and large value of q which may lead to big feedback gain K and L. As that for most control systems, big control gain may decrease the stability margin of the closed-loop system and amplify the high frequency sensor noise. Therefore, trade-offs should be made in practical, actually, the H_∞ suboptimal compensator with a successively prespecified γ is competent to generate satisfactory transient.

4. Stability and performance analysis

In this section, we first demonstrate that the adding of the H_∞ compensator designed in above section will not change the ideal asymptotic stability of the standard MRAC which is assured by the H_∞ optimal control. Then the bounds of L_∞ tracking error as well as the mean squared tracking error in any interval are computed using the property of the H_∞ compensator to show that the amount of transient performance improvement depends on the performance level γ. Finally, the features and benefits of the scheme here are presented by comparing with the approaches proposed in [8,16].

4.1. Stability analysis

To study the stability of the modified MRAC system, we first show that the ideal properties of the normalized adaptive law (10) have not been affected by adding compensator.

Consider the plant (6) with the assumptions (P1)–(P4) and the modified control law (9), there always exists a desired parameter vector $\theta^* = [\theta_1^{*T}, \theta_2^{*T}, \theta_3^*]^T$ such that the following matching condition (22) is satisfied without model uncertainties [8,23]:

$$\frac{c_0 k_p Z_p}{\left(1 - \frac{\alpha^T(s)\theta_1^*}{\Lambda(s)}\right) R_p - k_p Z_p \left(\frac{\alpha^T(s)\theta_2^*}{\Lambda(s)} + \theta_3^*\right)} = W_m(s), \tag{22}$$

multiplying both sides with y and using the equation $R_p(s)y = k_p Z_p(s)u$,we have

$$W_m(s)u - \theta^{*T}\phi = c_0 y. \tag{23}$$

So the estimation error in adaptive law (10) can be expressed as

$$\epsilon = \theta^T\phi + c_0 y - W_m(s)u = \theta^T\phi - \theta^{*T}\phi = \tilde{\theta}^T\phi. \tag{24}$$

Choosing a Lyapunov function $V = (1/2)\tilde{\theta}^T\Gamma^{-1}\tilde{\theta}$, from adaptive law (10) and (24), we can obtain

$$\dot{V} = \tilde{\theta}^T\Gamma^{-1}\frac{\dot{\tilde{\theta}} \le -\frac{\epsilon^2}{m^2} = -(\tilde{\theta}^T\phi)^2}{m^2}, \tag{25}$$

and the following properties hold [24]:

(i) $\tilde{\theta}$ and θ are uniformly bounded,
(ii) $\dot{\theta}$, $\epsilon/m \in L_2$.

This means that the desired properties of the adaptive law have not been destroyed. Actually, the adaptive law and controller have been completely separated by using the error normalization, and only the matching condition (22) and plant equation (1) have been used in deriving estimation error ϵ [8,22]. However, it should be noted that although the modification of control law has nothing to do with the structure of estimation error equation, the modified control input u as well as output y have changed the contents of the estimation error signal ϵ and ϕ, and will have further effects on the identification of θ.

Next, we proof the qualitative stability of the closed-loop system without unmodeled dynamics and disturbances.

From the modified controller (9) and control plant (6),we have the system (15). Then we apply the H_∞ compensator u_c (20) to the system (15), we obtain the resulting tracking error closed-loop system:

$$\begin{cases} \dot{\overline{e}} = \overline{A}\,\overline{e} + \overline{B}[\tilde{\theta}^T\omega] \\ e_1 = \overline{C}\,\overline{e} \end{cases} \tag{26}$$

where

$$\overline{e} = \begin{bmatrix} e \\ \hat{e} \end{bmatrix}, \quad \overline{A} = \begin{bmatrix} A_c & B_c K \\ LC_c & A_c + B_c K - LC_c \end{bmatrix}, \quad \overline{B} = \begin{bmatrix} B_c \\ 0 \end{bmatrix}, \quad \overline{C} = \begin{bmatrix} C_c & 0 \end{bmatrix},$$

which is stable with Lemma 2, and rewrite it in the form of input–output:

$$e_1 = W_c(s)[\tilde{\theta}^T\omega], \tag{27}$$

where the transfer function $W_c(s)=\overline{C}(sI-\overline{A})^{-1}\overline{B}$, for

$$\|W_c(s)\|_\infty<\gamma. \tag{28}$$

From Eqs. (20) and (27) we rewrite the modified control law (9) as

$$u=\theta^{*T}\omega+c_0r+[1+F(s)W_c(s)][\tilde{\theta}^T\omega], \tag{29}$$

thus Eqs. (12) and (13) can be rewritten as

$$u=\frac{R_p(s)}{Z_p(s)}\frac{W_m(s)}{c_0}\left[W_\tau(s)\tilde{\theta}^T\omega+c_0r\right] \tag{30}$$

$$y=\frac{W_m(s)}{c_0}\left[W_\tau(s)\tilde{\theta}^T\omega+c_0r\right], \tag{31}$$

where

$$W_\tau(s)=1+F(s)W_c(s). \tag{32}$$

So that

$$e_1=\frac{1}{c_0}W_m(s)W_\tau(s)\left[\tilde{\theta}^T\omega\right]. \tag{33}$$

Comparing with Eq. (27), we obtain

$$\frac{1}{c_0}W_m(s)W_\tau(s)=W_c(s), \tag{34}$$

as a result, the control input (30) and output (31) are given by

$$u=\frac{R_p(s)}{Z_p(s)}\left[W_c(s)\tilde{\theta}^T\omega+W_m(s)r\right] \tag{35}$$

$$y=W_c(s)\tilde{\theta}^T\omega+W_m(s)r. \tag{36}$$

Choosing $\delta_1>0$ be such that $|sI-\overline{A}|$, $R_m(s)$ and $Z_p(s)$ have all their roots in $Re[s]<-\delta_1/2$. Let $\delta\in(0,\delta_1)$, and since $W_c(s)$ is stable and proper, we have [24]

$$\|W_c(s)\|_\infty^\delta\triangleq\|W_c(s-\delta/2)\|_\infty\in L_\infty. \tag{37}$$

To show the boundedness of all the closed-loop signals, we start with Eqs. (35) and (36) and follow the same procedures as that for the standard MRAC which involve the use of the result (37), properties (i) and (ii) of normalized adaptive scheme, properties of L_2^δ−norm and B–G Lemma. The complete proof is presented in the Appendix, and more details can be found in [16,24].

4.2. Performance analysis

In the section, we are going to evaluate the capability of the H_∞ compensator for transient performance improvement using the criterions of L_∞ bound and mean square tracking error bound in any interval. And the features and benefits of the proposed scheme are presented by comparison with the approaches in [8,16].

From the tracking error (27), we directly have

$$\|e_1(t)\|_\infty\le2n_c\|W_c(s)\|_\infty\|\tilde{\theta}^T\omega\|_\infty, \tag{38}$$

where n_c is the order of $W_c(s)$. Since $\tilde{\theta}^T\omega \in L_\infty$, thus the L_∞ tracking error bound is

$$\|e_1(t)\|_\infty \triangleq \sup_{t \geq 0}|e_1(t)| \leq \gamma n_c c, \tag{39}$$

where c is the generic symbol for a positive constant.

Then we compute the mean value of tracking error in any interval using the following inequality:

$$\int_{t_1}^{t_2} |H(s)x|^2\, dt \leq \|H(s)\|_\infty^2 \int_{t_1}^{t_2} |x|^2\, dt. \tag{40}$$

Squaring and integrating both sides of Eq. (27) and using the above inequality, we get

$$\int_{t_0}^{t_0+t} |e_1|^2\, d\tau \leq \|W_c(s)\|_\infty^2 \int_{t_0}^{t_0+t} |\tilde{\theta}^T\omega|^2\, d\tau < \gamma^2 \int_{t_0}^{t_0+t} |\tilde{\theta}^T\omega|^2\, d\tau. \tag{41}$$

Hence, we obtain the mean squared error bound in any interval

$$\frac{1}{t}\int_{t_0}^{t_0+t} |e_1|^2 d\tau \leq \gamma^2 c. \tag{42}$$

The computations of the two bounds ((39) and (42)) are straight forward by using the property of H_∞ compensator. Apparently, both of the bounds can be reduced by decreasing the value of γ, this means the amount of transient performance improvement depends on the performance level γ of the H_∞ compensator.

In what follows, the approaches proposed in [8,16] are used to compare with the method presented in this paper to show its key features. Although all these schemes concern adding a compensator u_c in the standard adaptive control law to improve transient performance, the design of the compensators is different (see Table 1).

As analyzed in [8], there exists a stable transfer function $F_s(s)$ (i.e. $F_s(s) \approx W_m^{-1}(s)$) such that the bound of $\|W_m(s)(1-F_s(s)W_m(s))\|_\infty$ is close to zero, then the arbitrarily good transient for the modified MRAC is obtained. Similarly, by decreasing the parameter τ of the compensator in [16], the transient performance can be improved as much as desired. In this paper, the expected transient response is obtained by designing the compensator as the optimal H_∞ controller with minimized value of γ. As a result, they share the same capabilities on performance improvement. Moreover, comparing with other approaches, the scheme here has the following features:

(i) The tracking error e_1 is used as control signal in the compensator here which can be obtained conveniently in most control system without using any signals in normalized adaptive law.

Table 1
Comparison of compensator design with other approaches.

Approaches	Compensator structure	Control signals
Proposed in [8]	$u_c = -F_s(s)\epsilon$; $\|W_m(s)(1-F_s(s)W_m(s))\|_\infty < \|W_m(s)\|_\infty$	ϵ
Proposed in [16]	$u_c = -\frac{W_m^{-1}(s)}{(\tau s+1)^{n_m^*}}\left[\epsilon m^2 + w_c(s)\left[\left(w_b(s)\omega^T\right)\dot{\tilde{\theta}}\right]\right]$	ϵ, ω^T, $\dot{\tilde{\theta}}$
Presented in this paper	$u_c = F(s)e_1$	e_1

$w_c(s) = -C_m^T(sI-A_m)^{-1}$, $w_b(s) = (sI-A_m)^{-1}B_m$, (A_m, B_m, C_m) is a minimum stable realization of W_m.

(ii) The construction of the compensator is based on H_∞ optimal control which is more systematic and dose not relay on the experience of designer.

(iii) The weight of the compensator in the whole control system has direct relationship with predefined performance index $(\|W_m(s)/c_0\|_\infty)/\gamma$, so that it can be easily regulated.

Finally, we should note that the adding of compensator has changed the tracking error dynamics in essence so that the transient performance is improved consequently, but it also increases the order of the controller [22].

5. Example and simulation study

To illustrate the effectiveness of the proposed modified MRAC scheme, the following example of longitudinal short period dynamics of the F-16 aircraft derived in [25, Table 5.1] with elevator δ_e input and pitch rate q_d output linearized around a trim point is used

$$\begin{cases} \begin{pmatrix} \dot{\alpha} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} Z_\alpha/V_0 & 1+Z_q/V_0 \\ M_\alpha & M_q \end{pmatrix} \begin{pmatrix} \alpha \\ q \end{pmatrix} + \begin{pmatrix} Z_{\delta e}/V_0 \\ M_{\delta e} \end{pmatrix} \delta_e \\ q_d = (0\ 57.3) \begin{pmatrix} \alpha \\ q \end{pmatrix} \end{cases} \tag{43}$$

where α is the angle of attack as a state in the system (43) and the values of the system parameters are given in Table 2. A second order reference model with relative degree 1 is chosen as

$$q_m = W_m(s) r, W_m(s) = \frac{-(s+2)}{s^2+4s+2}. \tag{44}$$

In what follows, we first give the standard adaptive controller, then the H_∞ compensator u_c with different performance level $\gamma = 3$ and 1 is designed in the modified adaptive controller.

From Eq. (8) and the system dynamics (43) as well as reference model (44), the standard model reference adaptive controller (SMRAC) is given by

$$\begin{aligned} &\delta_e = \theta^T \omega + c_0 r, \quad c_0 = 0.1 \\ &\omega = [\omega_1^T, \omega_2^T, q_d]^T, \quad \theta = [\theta_1^T, \theta_2^T, \theta_3]^T, \\ &\omega_1 = \frac{1}{s+2}\delta_e, \quad \omega_2 = \frac{1}{s+2} q_d. \end{aligned} \tag{45}$$

And the adaptive law together with the normalized signal are chosen as follows:

$$\begin{aligned} &\dot{\theta} = \frac{-\Gamma \epsilon \phi}{m^2} \\ &\epsilon = \theta^T \phi + c_0 q_d - W_m(s) \delta_e, \end{aligned}$$

Table 2
Model parameters.

Parameter	V_0	Z_α	Z_q	Z_{δ_e}	M_α	M_q	M_{δ_e}
Value	502	$-1.01887V_0$	$0.0949V_0$	$-0.00215V_0$	0.82225	-1.07741	-0.17555

$$\phi = W_m(s)\omega,$$
$$m^2 = 1 + \delta_e^2 + q_d^2. \tag{46}$$

Then the modified model reference adaptive controller (MMRAC)

$$\delta_e = \theta^T \omega + c_0 r + u_c \tag{47}$$

is given. And the H_∞ compensator u_c is designed with prespecified performance level $\gamma = 3$ and 1 to show the different capacities on performance improvement.

Rewriting the tracking error equation

$$e_1 = \frac{W_m(s)}{c_0}\left[\tilde{\theta}^T \omega + u_c\right] = \frac{-10(s+2)}{s^2+4s+2}\left[\tilde{\theta}^T \omega + u_c\right]$$

in the form of states space as (16), i.e.

$$\begin{cases} \dot{e} = A_c e + B_c[\tilde{\theta}^T \omega] + B_c u_c \\ e_z = C_c e \\ e_1 = C_c e \end{cases} \tag{48}$$

where

$$A_c = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ -4 \end{bmatrix}, \quad C_c = [2.5\ 2.5].$$

According to steps 1–5 in Section 2, initialize $\varepsilon = 1$ and $0 < \gamma = 3 < \|(-c_0^{-1}(s+2))/(s^2+4s+2)\|_\infty = 10$, the solution P of Riccati equation (17) in step 1 and the state-feedback gain K are obtained by choosing $R = S = I_{2\times 2}$:

$$P = \begin{bmatrix} 1.452 & 0.452 \\ 0.452 & 0.452 \end{bmatrix}, \quad K = [0.904\ 0.904].$$

In order to construct the measurement feedback compensator, we let $q = 2$, $\rho = 1$, so a positive-define solution Q as well as the observer gain L are obtained by solving the Riccati equation (18) in step 2:

$$Q = \begin{bmatrix} 0.6044 & -0.3603 \\ -0.3603 & 3.6612 \end{bmatrix}, \quad L = \begin{bmatrix} 0.6103 \\ 8.2523 \end{bmatrix}.$$

To verify the validity of the designed gains K and L, we point out that there exists a positive-define solution Z of Eq. (19) in step 3:

$$Z = \begin{bmatrix} 0.3900 & 0.3800 & -0.0465 & -0.0475 \\ 0.3800 & 0.3800 & -0.0465 & -0.0475 \\ -0.0465 & -0.0465 & 0.0155 & 0.0062 \\ -0.0475 & -0.0475 & 0.0062 & 0.0068 \end{bmatrix}$$

Then the H_∞ compensator $u_c = F(s)e_1$ with performance level $\gamma = 3$ is constructed and described by the following state equations:

$$\begin{cases} \dot{\hat{e}} = A_c\hat{e} + B_c K\hat{e} + L(e_1 + C_c\hat{e}) \\ u_c = K\hat{e}. \end{cases} \tag{49}$$

Furthermore, let $\gamma=1$ and repeat the same procedures by choosing the parameter $\varepsilon=0.1$, $q=1$ and $\rho=1$, thus we have the larger state feedback gain and observer gain:

$$K'=[4.0633\ 4.0633],\quad L'=\begin{bmatrix}12.5\\10\end{bmatrix},$$

so the compensator $u_c=F'(s)e_1$ with performance level $\gamma=1$ is obtained in the same form

$$\begin{cases}\dot{\hat{e}}=A_c\hat{e}+B_cK'\hat{e}+L'(e_1+C_c\hat{e})\\ u_c=K'\hat{e}.\end{cases}\tag{50}$$

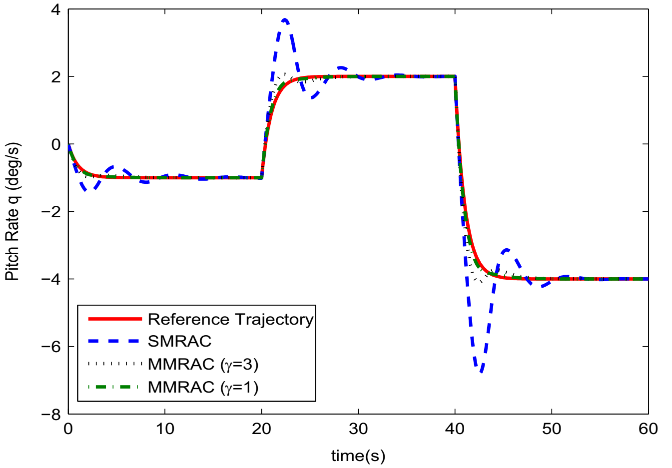

Fig. 1. Responses of pitch rate. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

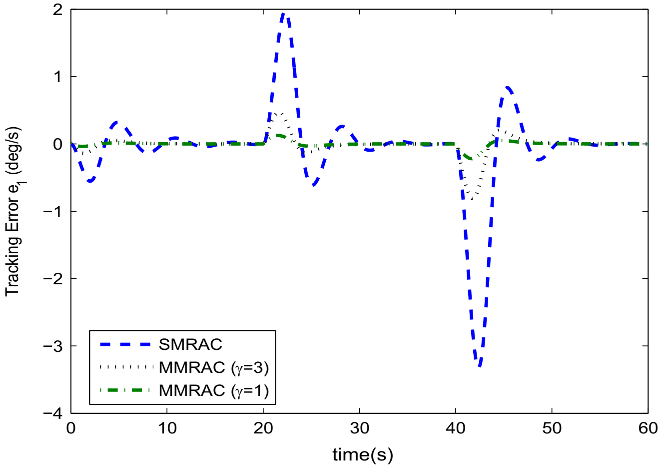

Fig. 2. Tracking error of pitch rate. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

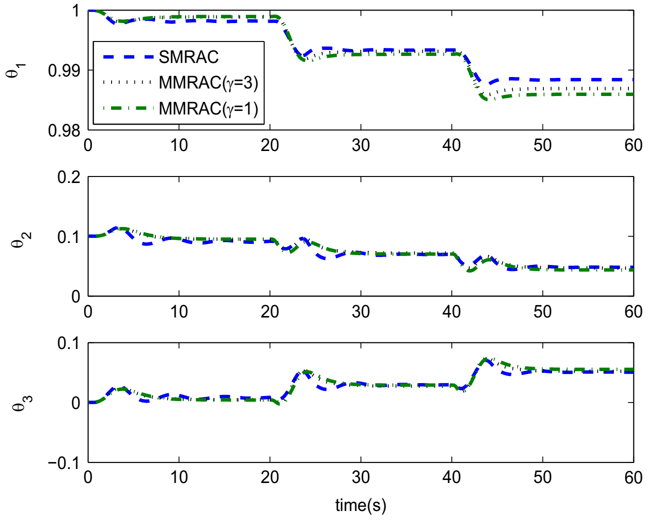

Fig. 3. Parameters estimation.

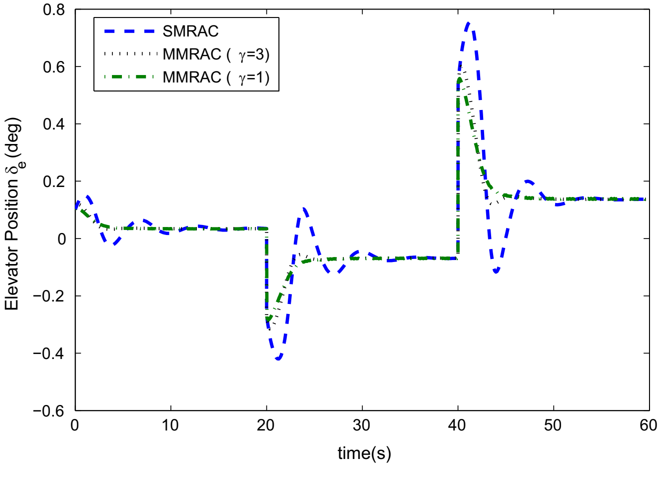

Fig. 4. Deflection of elevator.

To display explicit effects of the parameter estimation error on transient performance, we start with a parameter vector $\theta = [1, 0.1, 0]$ that leads to an initially destabilizing controller with adaptive gain matrix $\Gamma = diag\{1, 1, 1\}$. The standard model reference adaptive controller (SMRAC) is trying to minimize the tracking error $e_1 = q_d - q_m$. However, the poor transient response and tracking error are shown in Figs. 1 and 2 (dash blue line), respectively, for a series of commanded step inputs r. What is more, with the increase of inputs magnitude, the low frequency transient oscillations become serious.

Obviously, the modified model reference adaptive controller (MMRAC) with compensator u_c generates smoother transient with oscillation alleviation (see Fig. 1), and ensures the stability of closed-loop system as well as preserves the desired properties (i) and (ii) of adaptive scheme displayed in Fig. 3. Furthermore, by comparison of MMRAC with $\gamma = 3$ and 1, we can find that the compensator with smaller $\gamma = 1$ has better transient oscillations inhibiting ability and smaller trajectory following error than that with $\gamma = 3$, which is consistent with the performance analysis using L_∞ bound criterion or mean squared value criterion in Section 4. Besides, the compensator ceases to have effect when there is no tracking error shown in Fig. 4.

6. Conclusions

This paper proposes a new scheme to improve transient performance in MRAC, which mainly adopts an H_∞ compensator to attenuate the disturbance caused by the parameter estimation error, rather than seeking ways to drive the control parameter converge to “true” value. The theories of MRAC and H_∞ optimal control are developed separately over the past decades, and both of them have notable properties. The author of this paper tries to synthesize the two control approaches to design a modified adaptive controller with guaranteed transient performance. Systematic construction of the H_∞ compensator is presented in the paper, in which the design parameter γ representing the amount of the transient performance will be improved. Performance analysis and the simulations of the proposed control scheme application to flight control are made to show the desired transient. And our further studies concern the effectiveness of the modified MRAC scheme in the presence of bounded disturbance and unmodeled dynamics.

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Appendix. Stability proof of the modified MRAC

Defining the fictitious normalizing

$$m_f = 1 + \|u_t\|_2^\delta + \|y_t\|_2^\delta. \tag{51}$$

Note that if we establish that $m_f \in L_\infty$ then the boundedness of all signals follows. Using Eq. (37) and the properties of L_2^δ−norm in Eqs. (35) and (36), as well as the fact that $r \in L_\infty$we have

$$m_f \le c + c\|(\tilde{\theta}^T \omega)\|_2^\delta. \tag{52}$$

where c is the generic symbol for a positive constant.

Next, we give the finite L_2 gain of $\tilde{\theta}^T \omega$. Letting $s\Lambda_1(s,\alpha) = 1 - \Lambda_0(s,\alpha)$, $\Lambda_0(s,\alpha) = \alpha^{n_m^*}/(s+\alpha^{n_m^*})$ for $\alpha > \delta/2$ and n_m^* is relative degree of $W_m(s)$, we have [20]

$$\begin{aligned}\tilde{\theta}^T \omega = {} & \Lambda_1(s,\alpha)[\dot{\tilde{\theta}}^T \omega + \tilde{\theta}^T \dot{\omega}] + \Lambda_0(s,\alpha) \\ & \times W_m^{-1}(s)[\tilde{\theta}^T \phi + w_c(s)[w_b(s)[\omega^T]\dot{\tilde{\theta}}]].\end{aligned} \tag{53}$$

Notice that

$$\omega = \begin{bmatrix} \frac{\alpha(s)}{\lambda(s)} & 0 \\ 0 & \frac{\alpha(s)}{\lambda(s)} \\ 0 & 0 \end{bmatrix} [u\ y]^T + [0\ 0\ y]^T. \tag{54}$$

Thus

$$|\omega| \le cm_f + |y|. \tag{55}$$

From Eqs. (36) and (37) and $\tilde{\theta}$ is bounded, it follows that

$$|y| \le c[\|\omega\|_2^\delta + c]. \tag{56}$$

Since $\|\omega\|_2^\delta \le cm_f$, we obtain

$$\begin{aligned} \|\dot{\omega}\|_2^\delta &\le cm_f + \|\dot{y}\|_2^\delta \\ &\le cm_f + c[\|\omega\|_2^\delta + c] \\ &\le cm_f + c. \end{aligned} \tag{57}$$

Now

$$|W_b(s)[\omega^T]| \le c\|\omega\|_2^\delta \le cm_f. \tag{58}$$

Combining Eqs. (53), (56)–(58), we obtain

$$\|(\tilde{\theta}^T \omega)\|_2^\delta \le c + \frac{c}{\alpha} m_f + c\alpha^n \|(\tilde{\theta}^T \phi)\|_2^\delta + \left(\frac{c}{\alpha} + c\alpha^n\right) \|(|\dot{\tilde{\theta}}|m_f)\|_2^\delta.$$

Combining with Eq. (52) and choosing α large enough so that $c/\alpha < 1$, we have

$$m_f \le c + c\alpha^n \left\| \left(\frac{\tilde{\theta}^T \phi}{m} m_f \right) \right\|_2^\delta + \left(\frac{c}{\alpha} + c\alpha^n\right) \|(|\dot{\tilde{\theta}}|m_f)\|_2^\delta.$$

Applying the B–G Lemma we have $m_f \in L_\infty$, it follows that $m \in L_\infty$ and all signals in the closed-loop adaptive system are bounded.

Considering error equation (27), it follows from Swapping Lemma and the related signals which are guaranteed by adaptive law to be in L_2 that $e_1 \in L_2$. In additional, since $\tilde{\theta}^T \omega \in L_\infty$ along with $W_c(s)$ is a stable and proper transfer function, so that $\dot{e}_1 \in L_\infty$. By Barbalat's Lemma, we conclude that $e_1(t) \to 0$ as $t \to 0$.

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