Sharif University Of Technology

Advanced Topics in Neuroscience

Simulation 01 (Neural Encoding)

Instructor: Dr. Ali Ghazizadeh

Zahra Kavian - 98102121

---

1 Integrate and Fire Neuron

a. In this part I generate a spike train using the Poisson process with a rate equal to 100 for $1ms$. I know the instantaneous rate in the Poisson process equals land (frequency rate multiple time steps). In each time bin, I produce a random number in the range $[0, 1]$, from the normal distribution. This number considers a neuron fire probability. If this is less than the instantaneous rate, the neuron will fire and vice versa.

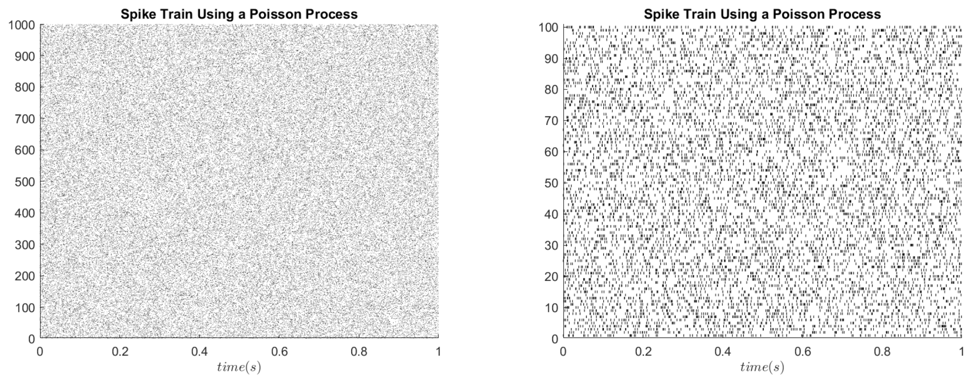


Figure 1: Poisson Spike train. right:1000, left:100 trials.

b. I use the spike train in the previous part and count the number of a spike in every time bin and plot its histogram. Then I use 'histfit' (Matlab function) to fit the best Poisson distribution on it ($\lambda = 100.22$). As I expect, spike count density has a Poisson distribution.

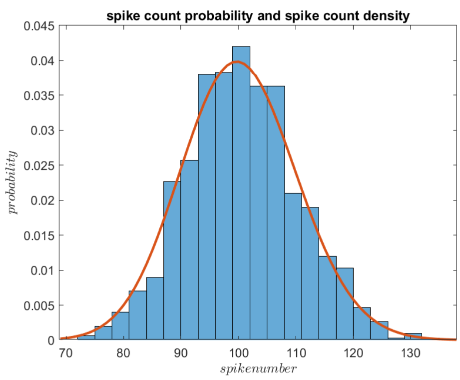


Figure 2: Spike Count Histogram and the related Poisson distribution.

c. Also, I measure the time between every two spike or ISI(Inter Spike Interval), then plot its histogram and fit the best Exponential distribution on it ($\mu = 0.099$).

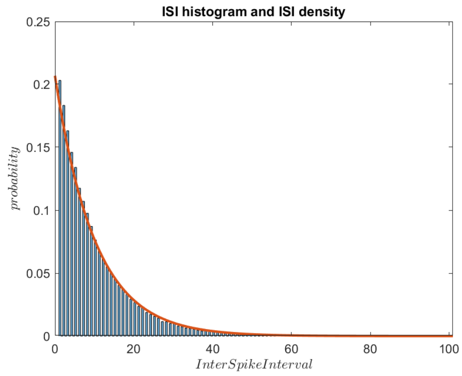


Figure 3: Inter Spike Interval Histogram and the related Exponential distribution.

Renewal Process: The renewal process keeps the memory of the last event (last firing rate) but doesn't care about the earlier events. A way to generate a renewal process spike train is to start with a Poisson spike train and delete all but every kth spike! This procedure is similar to integration over postsynaptic input with Poisson ISI distribution because, for a new spike, it only need the last spike and the spikes in two spike interval are not important. So, if we only keep the kth spike, we can generate the renewal process.

a. spike train using a Poisson process:

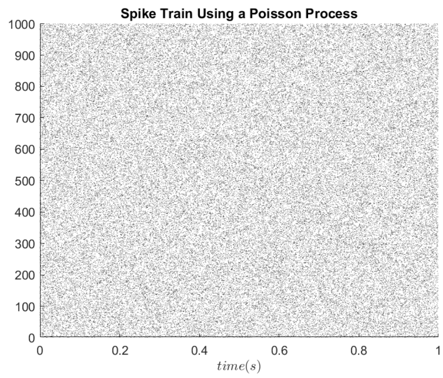


Figure 4: Renewal process spike train.

b. spike count probability histogram: This distribution look like a Poisson distribution ($\lambda = 0.098$)

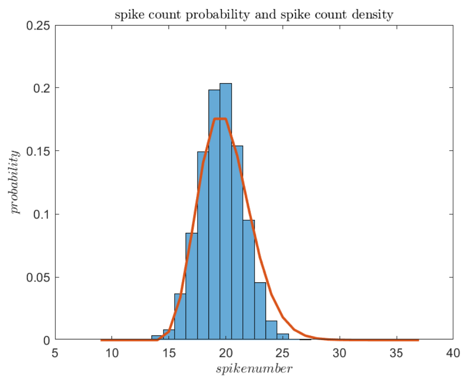


Figure 5: Spike Count Histogram and the related Poisson distribution.

c. Inter-spike interval (ISI) histogram: As I expect, this distribution look like a Erlang (Gamma) distribution ($k = 5.67, \lambda = 8.71$)

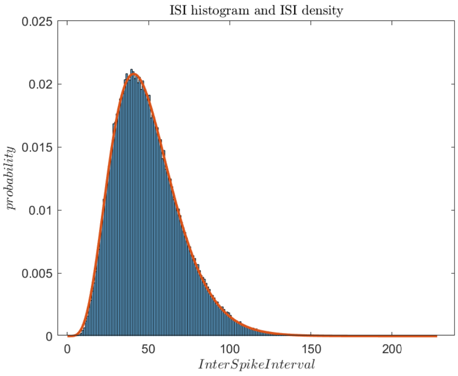


Figure 6: Inter Spike Interval Histogram and the related Exponential distribution.

d. Coefficient Variation (C_v) shows the extent of variability to the mean of it. C_v is zero in the periodic function.

I find the C_v for both Poisson and renewal process:

Poisson process: 0.9536

Renewal process: 0.4233

As expected, the ISI of the Poisson process has exponential distribution, C_v is near one and the ISI of the renewal process has gamma distribution. So C_v is $1/\sqrt{k} = 1/\sqrt{5.67} = 0.42$

e. Gamma Coefficient of Variation proof:

$$E(x) = \int_0^\infty xf(x) = \int_0^\infty x\frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}dx = \frac{\lambda^k}{(k-1)!}\int_0^\infty x^k e^{-\lambda x}dx$$

$$t = \lambda x, \quad z = k$$

$$\int_0^\infty e^{-t}(\tfrac{t}{\lambda})^z \tfrac{1}{\lambda}dt = \frac{1}{\lambda^{z+1}}\int_0^\infty e^{-t}t^z dt = \frac{\Gamma(z+1)}{\lambda^{z+1}}$$

$$E(x) = \frac{\lambda^k k!}{(k-1)!\lambda^{k+1}} = \frac{k}{\lambda}$$

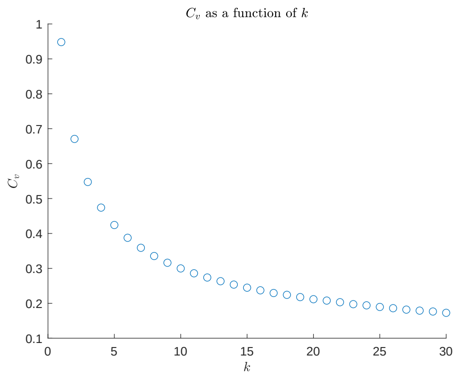
$$var(x) = E(x^2) - E(x)^2$$

$$E(x^2) = \int_0^\infty x^2\frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}dx = \frac{\lambda^k}{(k-1)!}\int_0^\infty x^{k+1}e^{-\lambda x}dx = \frac{\lambda^k}{(k-1)!}\frac{\Gamma(k+2)}{\lambda\lambda^{k+1}} = \frac{k(k+1)}{\lambda^2}$$

$$vear(x) = \frac{k(k+1)}{\lambda^2} - (\tfrac{k}{\lambda})^2 = \frac{k}{\lambda^2}$$

$$C_v = \frac{\sigma(\tau)}{E(\tau)} = \frac{\frac{\sqrt{k}}{\lambda}}{\frac{k}{\lambda}} = \frac{1}{\sqrt{k}}$$

f. To show simulated data variability, We plot the C_v for different k. As we expect, they follow the $1/\sqrt{k}$ function.



g. In this part, I consider the refractory period for the spike train. Everything is the same as the renewal process but the time of two consecutive spikes must be more than the refractory period.

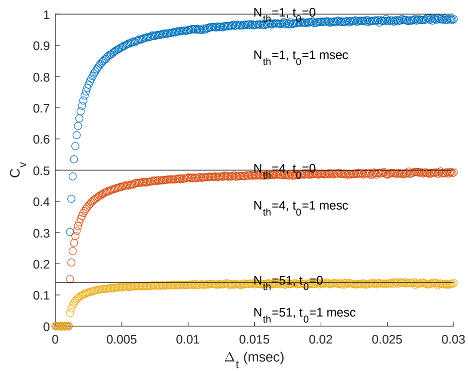


Figure 7: Comparison of C_v. Straight lines represent predictions of C_v, for a neuronal integrator that fires after receiving N_{th} randomly timed input impulses. The curves show C_v modified to account for an absolute refractory period t_0, $= 1$ .O msec. Note that $C_v \leq \frac{1}{N}$ for all models, such that C_v is quite small for large values of N_{th}.

2 Leaky Integrate and Fire Neuron

a. In this part, the leaky integrate-and-fire (LIF) neuron is simulated, probably one of the simplest spiking neuron models. In its simplest form, a neuron is modeled as a "leaky integrator" of its input $I(t)$:

$$\tau_m \frac{dv}{dt} = -v(t) + RI(t) \tag{1}$$

The output of the neuron likes this figure:

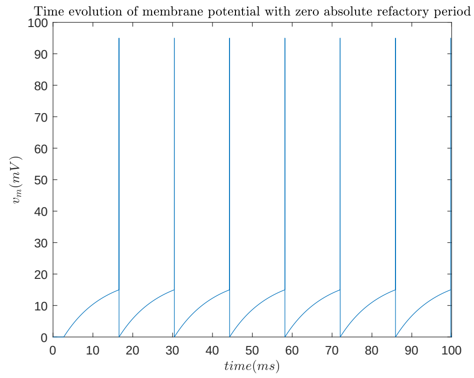


Figure 8: Leaky Integrate and Fire (LIF) neuron, constant input current=20mA, R=1$m\Omega$, resting potential voltage=0mV, and threshold=15mV.

b. The LIF neuron is modeled again but with a considered refractory period. The neuron output is like this figure:

$$\begin{cases} \tau_m \frac{dv}{dt} = -v(t-t_0) + RI(t-t_0) & t \geq t_0 \\ v(t-t0) = v_{test} & t \leq t_0 \end{cases}$$

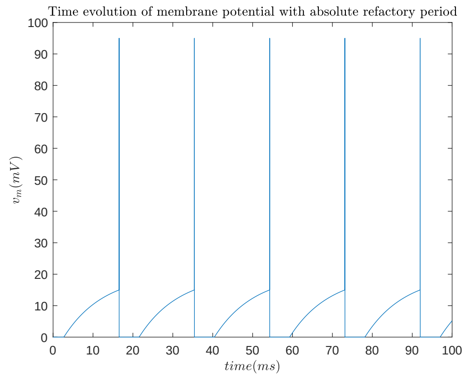


Figure 9: Leaky Integrate and Fire (LIF) neuron, constant input current=20mA, R=1$m\Omega$, resting potential voltage=0mV, threshold=15mV, and refractory period=5$msec$

c. Repeat section a. with stimulating neuron by a time-varying input current $I(t)$. To generate a realistic I(t), first define a K spike train with Poisson distribution and then convolve them in an EPSC kernel, similar to what Softky and Koch did in their paper. Then use this current in the LIF equation 1.

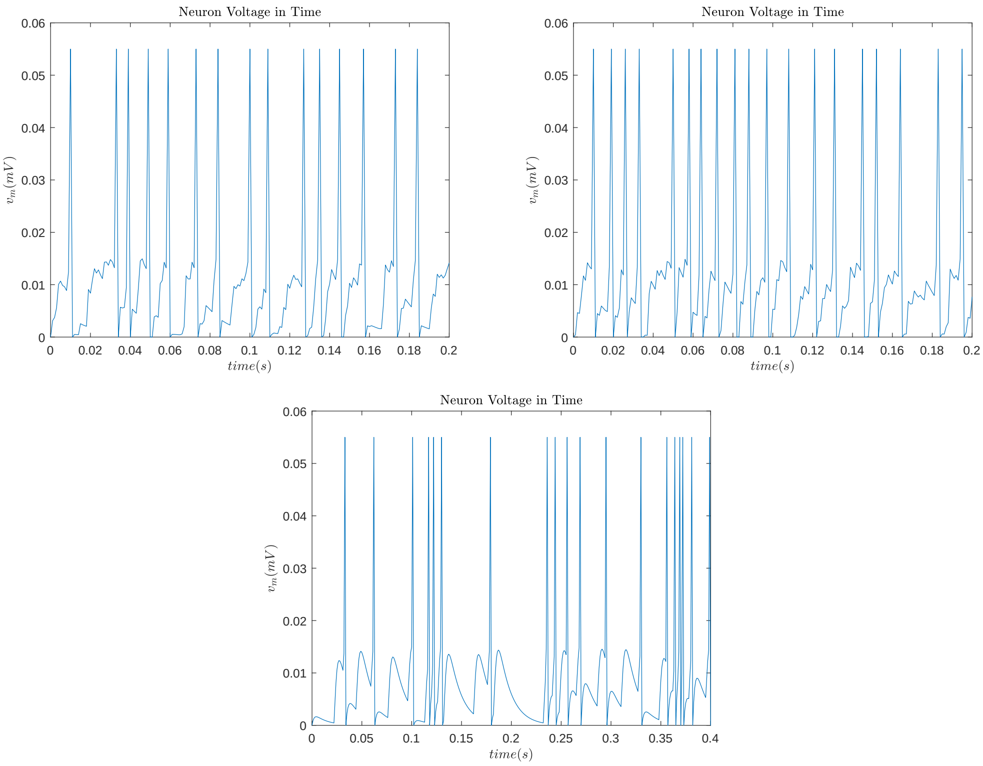
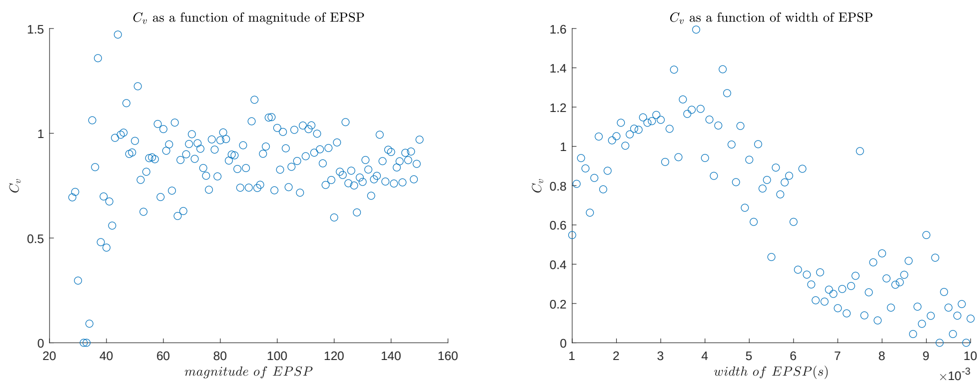


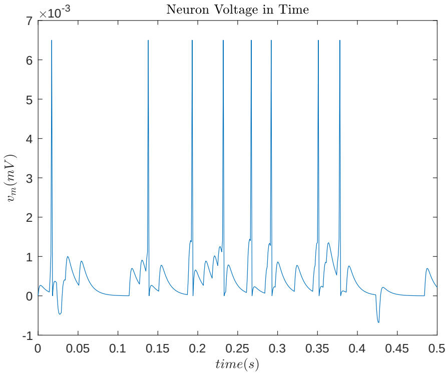
Figure 10: Leaky Integrate and Fire (LIF) neuron with time variable input current and EPSP. The neuron don't fire regularly and the excitatory post synaptic potential sum to reach spike voltage.$Firing\ rate = 100, V_{threshold} = 15mV, Size\ of\ Kernel = 100, left and middle : t_{peak} = 1.5msec, right : t_{peak} = 1.7msec$

The effect of width and magnitude of EPSCs on C_v. When the size of $EPSP$ increase, voltage reach the threshold voltage soon and neuron behave as a LIF model and C_v increase. (or spike variance increase)

Maybe we can describe the result that when the EPSP width increase, we expect the neuron reach threshold soon and C_v increase (like LIF model), but the width limit by refractory period and after some value, the neuron behave regularly and the C_v decrees.



d. To include the IPSP effect in neuron simulations, we select a percentage of synaptic inputs to assign negative kernels. The Integrate model with inhibitory input:



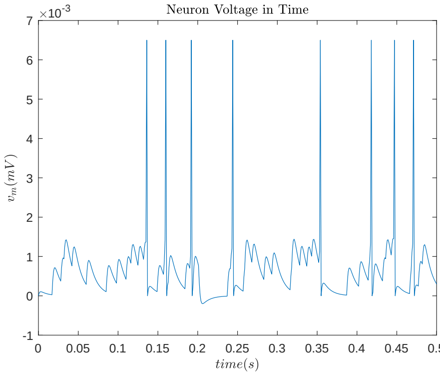


Figure 11: Leaky Integrate and Fire (LIF) neuron with time variable input current, EPSP and IPSP. The neuron don't fire regularly and the excitatory post synaptic potential sum to reach spike voltage.$Firingrate = 100, t_{peak} = 1.7msec, V_{threshold} = 15mV, kernelmagnitude = 100$

Repeat the last part for different percentages of inhibitory inputs. We can see as the inhibitory input increase, the C_v is increase,either. It' because the neuron can not reach the threshold voltage and the ISI increase and neurons spike don't follow Poisson distribution.

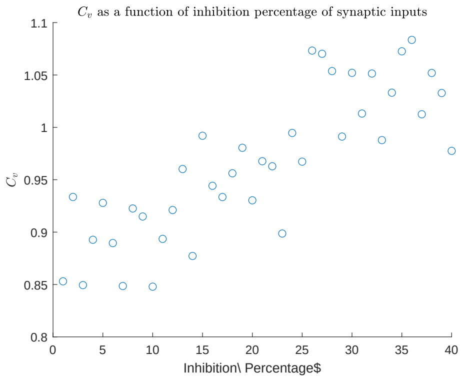


Figure 12: How C_v depends on inhibition percentage of synaptic inputs.

e. Assume the neuron is doing coincidence detection of it excitatory inputs which have a Poisson distribution. The neuron requires N out of M inputs to be active in a short D ms time window. At first we find these windows. Then we compute the ISI and its C_v. We plot the C_v as a function of $\frac{N}{M}$ and D.

As you can see,

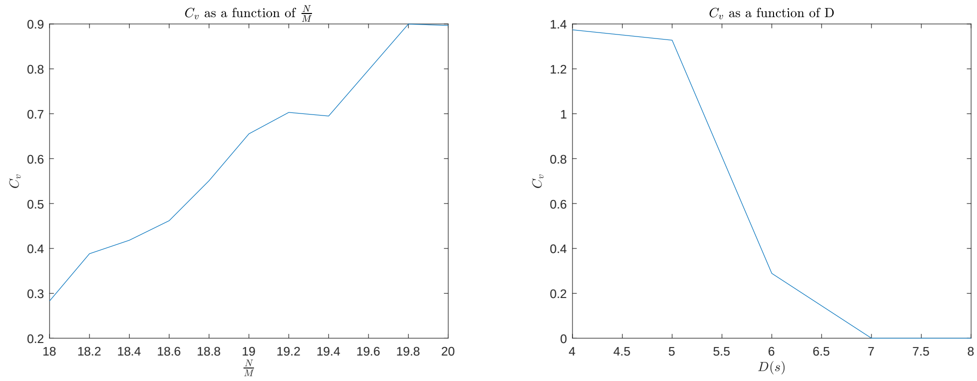


Figure 13: Dependency $\frac{N}{M}$ and D on C_v and coincidence detection of it excitatory inputs

f. Assume the neuron is doing coincidence detection of it excitatory and Inhibitory inputs which have a Poisson distribution. Here the neuron will fire if number of excitatory pulses received minus number of inhibitory pulses ($N_{net} = N_x - N_i$) is bigger than a threshold in D ms time window. At first we find these windows. Then we compute the ISI and its C_v. We plot the C_v as a function of N_x and D.

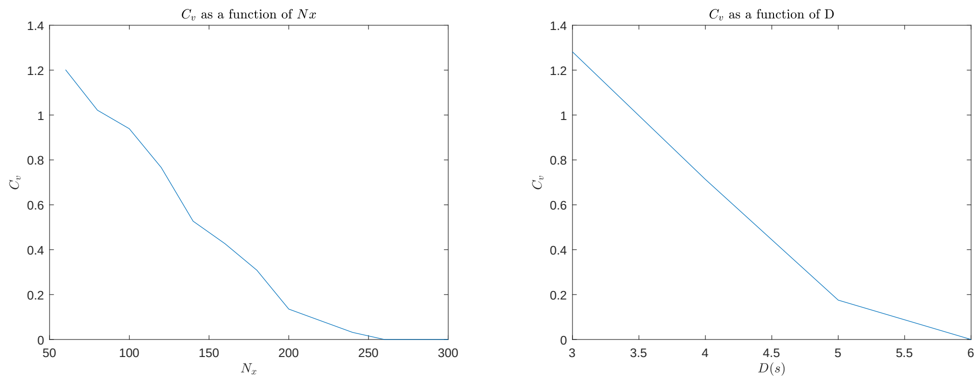


Figure 14: Dependency N_x and D on C_v and coincidence detection of it excitatory inputs