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Section 1: LFP Analysis

1.1 Data Observation

We have LFP signal of 48 electrodes in 823 trials. I use 'Intersect_Clean_Trials.mat' file and pre-process signals (after that, only 490 trial select for extra work).

At first, I plot the LFP signal all channels in specific trial (figure 1).

Also In figure 2,I use 'pwelch' to show the power spectrum of each trial and their average for the first and 40th channels (no reason to select 40th channel).

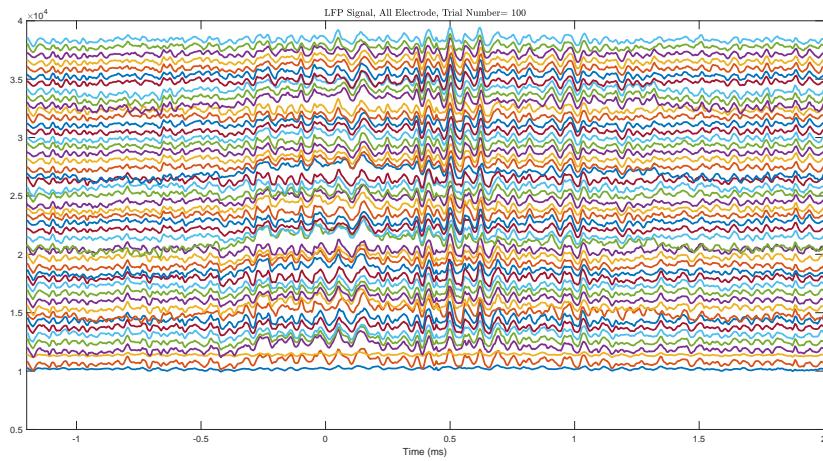


Figure 1: LFP Signal all electrodes, 100th trial.

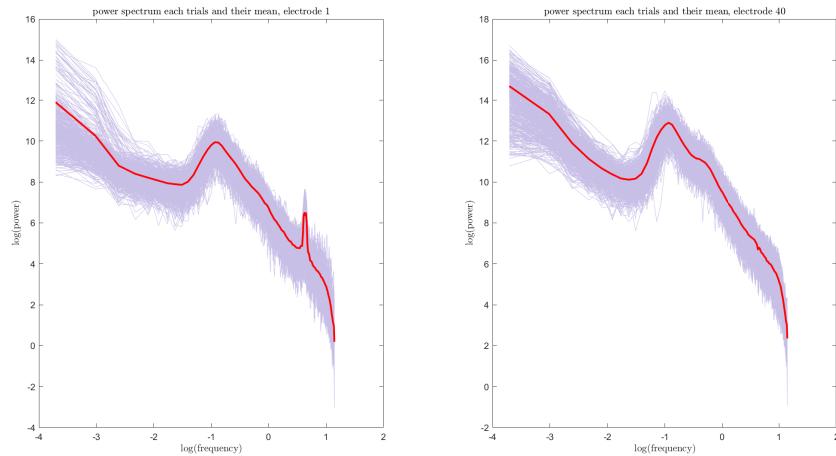


Figure 2: Power spectrum of all trials and their average.

1.2 Most Dominant Frequency Oscillation

In this part, I want to find the most dominant frequency oscillation in each channels. In this procedure, I use two method ('pspectrum' and 'Morlet wavelets') to compute power of oscillation at frequency 0 to 100 (In 'pspectrum' method I use two type of data, first, I concatenate all trials and second, I average signal. There is not specific different between these results). I fit a line on power spectrum using linear regression ('fitlm' matlab) as a pink noise effect. Then, I subtract the power spectrum from the linear regression line. This normalize signal power that remove the $\frac{1}{f}$ noise. The frequency which I have the local maximum in this normalized power, is the dominant frequency (Notice, in wavelet method, I use 'zscore' to normalize signal and get the result more similar with the article).

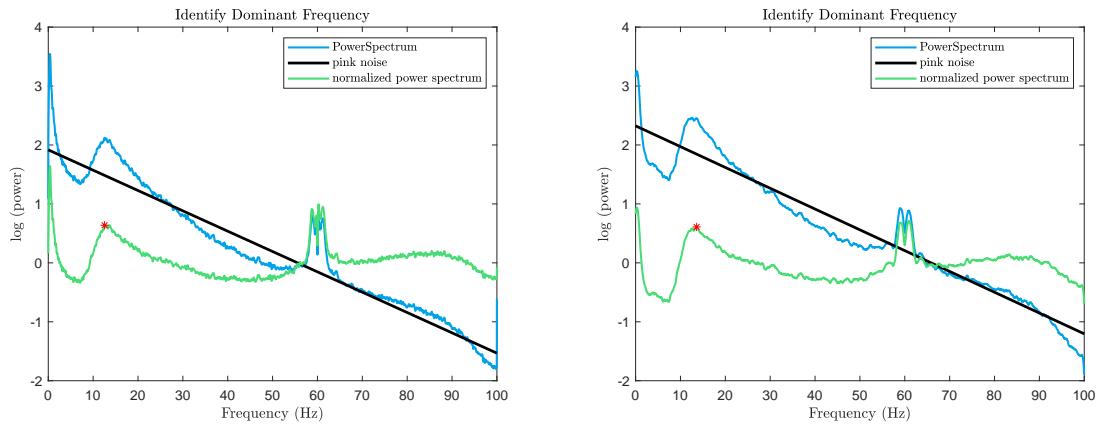


Figure 3: Identifying dominant frequency oscillation use 'pspectrum' method. right: concatenation all trials, left: average all trials. Red asterisk shows the local maximum

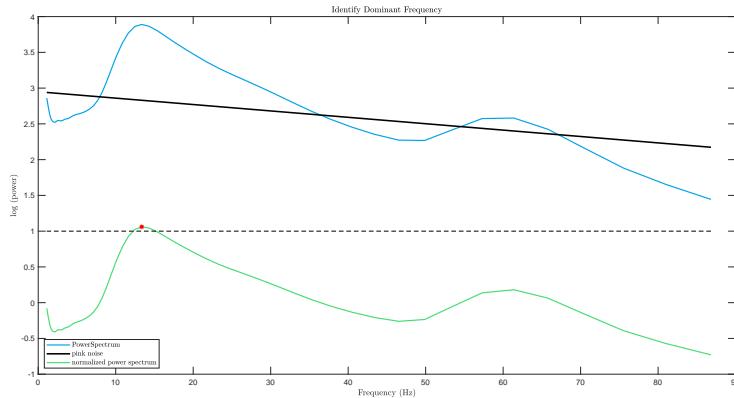


Figure 4: Identifying dominant frequency oscillation use 'Morlet wavelet' method. Red asterisk shows the local maximum

I continue other next parts with whitened signal with 'pspectrum' method (average over trials).

1.3 Clustering Electrodes

In this part, I want to cluster electrode group based on their dominant oscillation frequency. In figure 5, I plot histogram of electrodes dominant frequencies and group them base on histogram frequency bins. I show these group by different color (figure 5 ,right). The number is average of upper and lower frequency bound. Approximately, the neighbor electrodes have a similar dominant frequency.Maybe because near electrode record more similar neuron activity and they have common noise.So their dominant frequencies are closer. In figure , you can see the true value of dominant frequency.

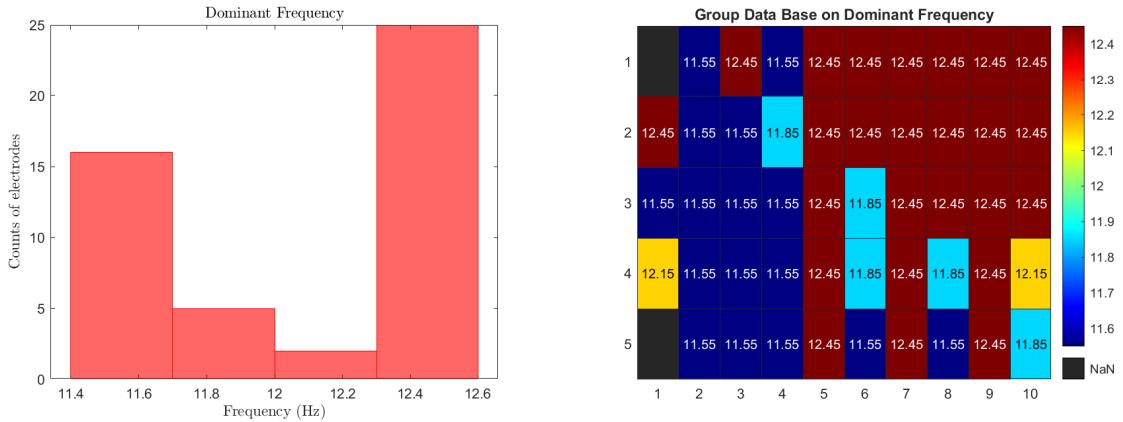


Figure 5: Identifying dominant frequency oscillation use 'pspectrum' method.right: concatenation all trials, left: average all trials.

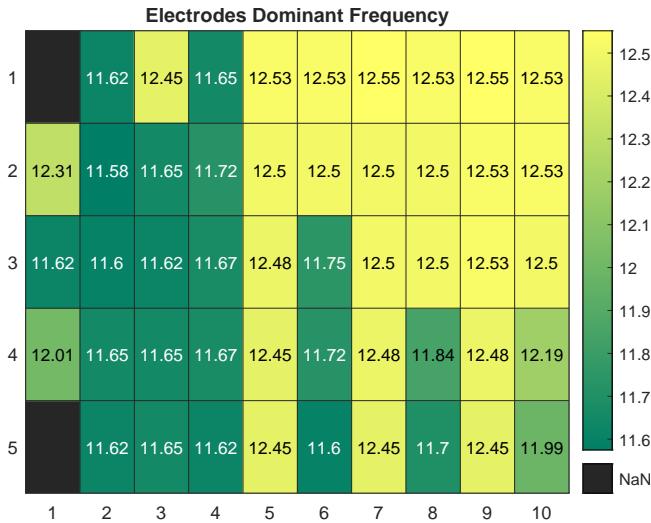


Figure 6: Dominant frequency oscillation use 'pspectrumt' method

I do this part again by whitened signal with 'Wavelet' method, calculate in previous section. All electrodes have a unique dominant frequency (13.36). I think 'Wavelet' compute the power spectrum with better precision.

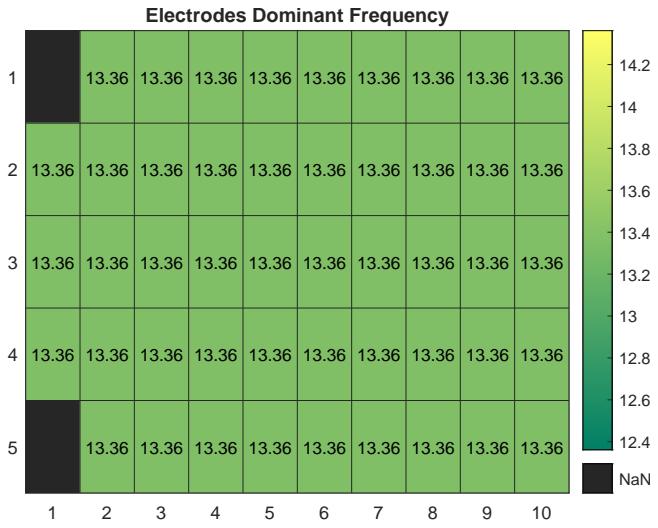


Figure 7: Dominant frequency oscillation use 'pspectrum' method

1.4 Time-Frequency Analysis

In this part, I want to show power spectrum in time. I use Short Fourier transform ('STFT' matlab) and average over all trials and then over all electrodes. Also, I remove the pink noise (by fitting the linear regression and subtraction the noise effect on power signal) to whiten signal. In figure 8, you can compare the results.

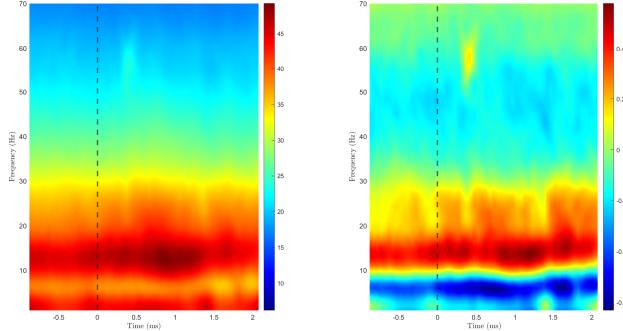


Figure 8: Short Time Fourier Transform. left: raw signal, right: whitened signal. Black dashed line show stimulus onset.

I repeat this part again with 'Wavelet' analysis. In figure 9, we also can see power increases almost 5ms before stimulus onset.

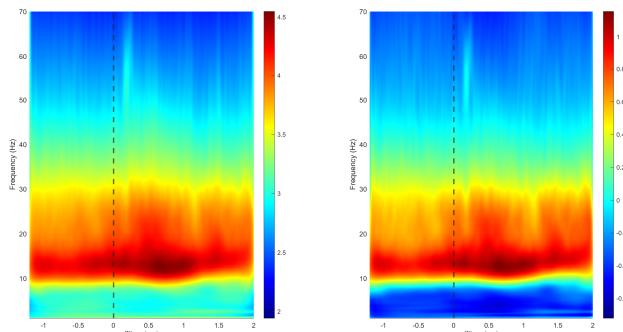


Figure 9: Wavelet time-frequency analysis. left: raw signal, right: whitened signal. Black dashed line show stimulus onset.

1.5 Part d

After removing pink-noise, we have more power in $10 - 20\text{Hz}$ frequencies (beta bounds). "Power spectra revealed a dominant beta frequency band during the instruction period". The power increase after onset and again decrease. My result is almost similar Hatsopoulos et.al 2006 (figure 10).

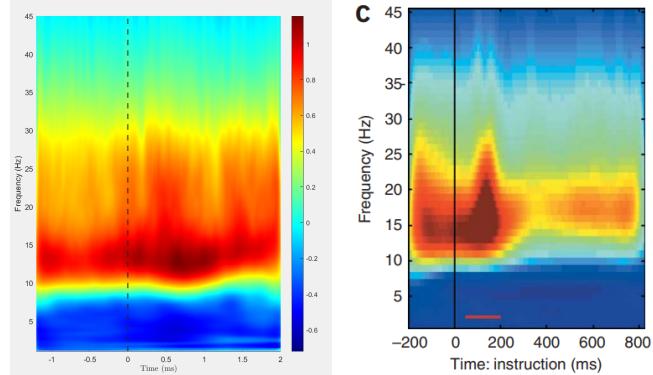


Figure 10: Averaged wavelet spectrogram. right: my result, left: Hatsopoulos result.

Section 2: Phase Propagation (Traveling Waves)

2.1 Band-Pass Butterworth Filter

In this part, I design filter to measured the instantaneous phases of the signals. I use 'butter' Matlab function, with lower frequency at 11Hz , higher at 13Hz and second order. You can see the filter frequency response in figure 11. I want to use Fourier transform next steps. I consider dominant frequency of each electrode (which are found in previous section) as a filter center frequency, with 2Hz bandwidth. I filter signal with it.

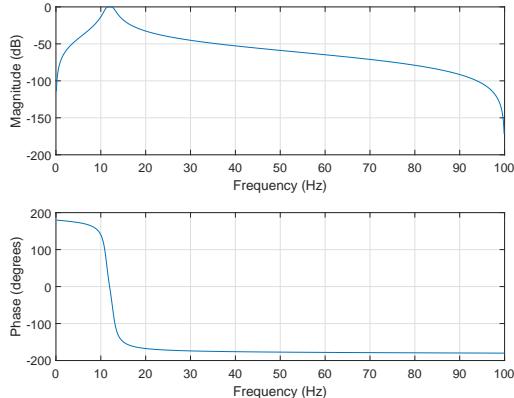


Figure 11: Band-Pass Butterworth Filter.

In figure 12 , you can compare signal before and after filtering (pwelch analysis).

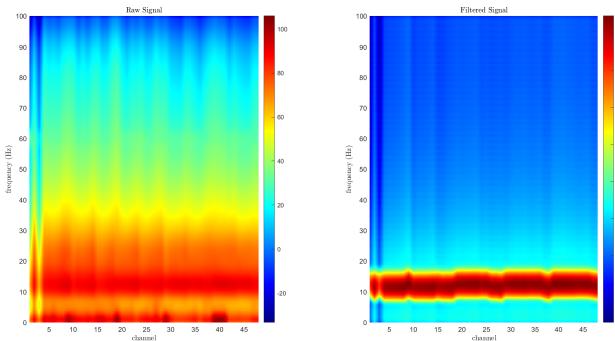


Figure 12: Band-Pass Butterworth Filter.

2.2 Hilbert transform

I use Hilbert transform to calculate instantaneous phase of filtered signals and store its result in $(5 \times 10 \times \text{number-of-trails} \times \text{number-of-time-points})$ matrix named 'electrode_phase'.

$$\begin{aligned}
 S_a(f) &= S(f) + \text{sgn}(f)S(f), \\
 s_a(t) &= F^{-1}[S_a(f)] = F^{-1}\{S(f)\} + F^{-1}\{\text{sgn}(f)\} * F^{-1}\{S(f)\} \\
 &s(t) + j \left[\frac{1}{\pi t} * s(t) \right] = s(t) + js(\hat{t}) \\
 \text{Envelope: } s_m(t) &= |s_a(t)| \\
 \text{Phase angle: } \phi(t) &= \arg[s_a(t)]
 \end{aligned}$$

2.3 Cos (phase) Demo

As we read in Hatsopoulos article: 'the oscillatory peaks within one cycle of the beta oscillation occurred at different times,suggesting wave propagation', I try to show this in figure 13.

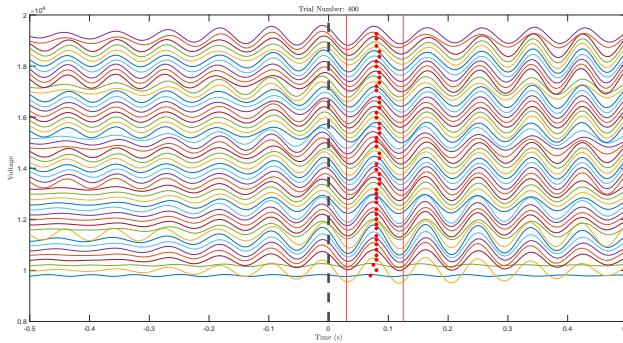


Figure 13: Local field potentials in the dominant frequency band (2 Hz bandwidth) along the electrodes of the array. Red stars mark the local maxima of one characteristic cycle, and the red lines mark the beginning and end of the wave presented in figure 14

By computing cos instantaneous phase of each electrode in specific trial (trial number = 100), figure 14 show this changes in some selected time (I save demo of this part named 'plane wave'. Also shade color between channels and save this demo name 'plane wave_shading'. You also can find them in this [link](#)). "The averaged phase revealed that the phase of the beta oscillations varied systematically across the array, which is evidence of a propagating wave". We can see the wave begin in right button and propagate to top left. In my opinion, the wave is more evidence after stimulus.

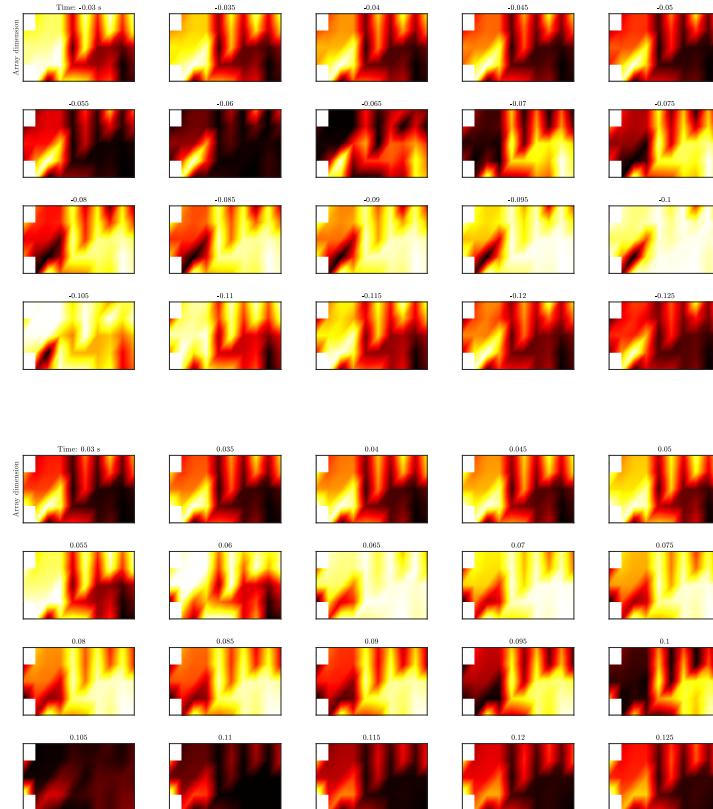


Figure 14: Cos Instantaneous Phase Over some Time, in trial number = 259. Top: before stimulus onset, Bottom: after stimulus onset

I also show instantaneous phase of wave respect to the multi-electrode array dimensions. Phase is presented by contour lines in black-white false color.



Figure 15: Cos Instantaneous Phase Over some Time, in trial number = 259. Top: before stimulus onset, Bottom: after stimulus onset

I also average all trial and repeat two previous step. Now, we can not see the wave in electrode array (save this demo as 'plane wave_shading_average_all_trials'. You can find it [here](#)).

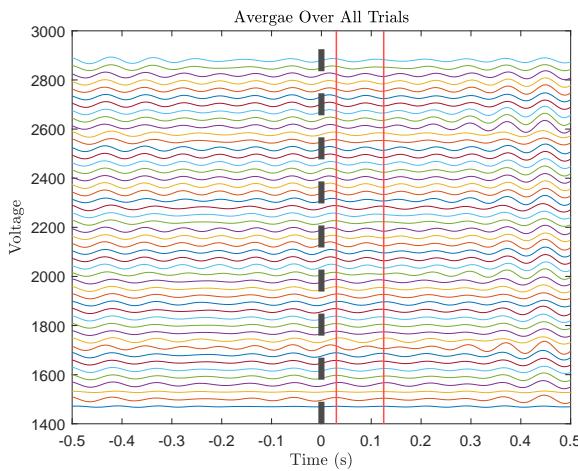


Figure 16: Local field potentials in the dominant frequency band (2 Hz bandwidth) along the electrodes of the array. The red lines mark the beginning and end of the wave presented in figure 17

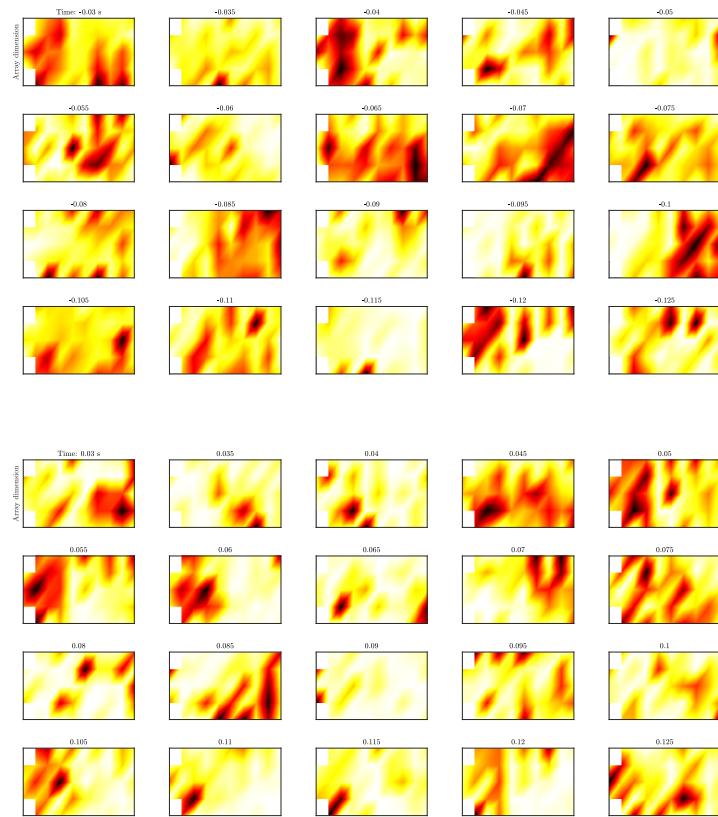


Figure 17: Cos Instantaneous Phase Over some Time, in trial number = 259. Top: before stimulus onset, Bottom: after stimulus onset

2.4 Part 4

2.4.1 Phase Gradient Directionality (PGD)

I find the 'PGD' for each trial in all times base on Hatsopoulos article, 'We define the phase gradient directionality, PGD(t), to measure how well phase gradients align across the array as a function of time:'

$$PGD(t) = \|\bar{\nabla}\phi\| / \|\nabla\phi\|$$

The bar denotes the spatial average at a fixed time. If the phase gradients at all spatial points on the array align at time t, $PGD(t)$ will be 1. On the other hand, if the phase gradients are randomly distributed, PGD will be close to 0.

In figure 18 , I plot PGD averaged across trials as a function of time. It shows the average PGD increase after stimulus onset.

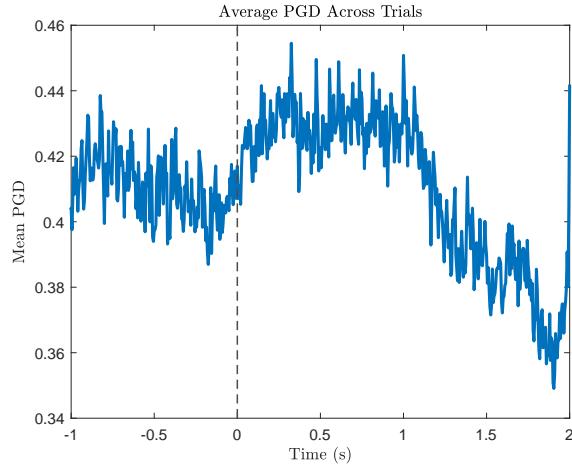


Figure 18: Phase gradient directionality (PGD) averaged across trials as a function of time. Dashed line is a stimulus onset.

In figure 19 , I average PGD across time for each trials and plot its polar histogram. It shows most of trials have the same average PGD.

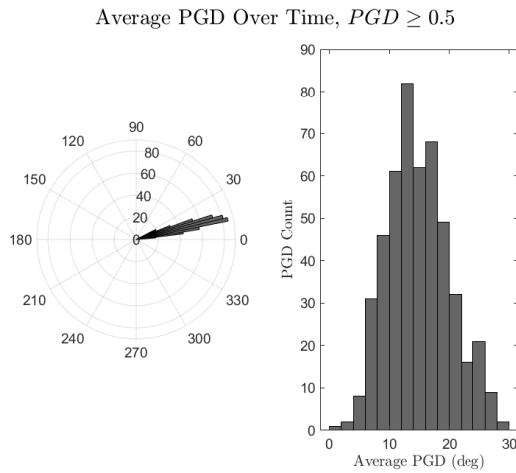


Figure 19: Polar-histogram of PGD averaged across time for each trials.

Also, I select a trial and show the PGD over time ($PGD \geq 0.5$). I think PGD increase almost $0.5ms$ before stimulus onset and again after onset, PGD increase (this demo is saved as '*PGD a trial over time*' in this [link](#).Color bar change after stimulus onset.If $PGD < 0.5$,the bar is red.).)

2.4.2 Direction of Propagation

The velocity direction, which is perpendicular to the phase contours, is $-\nabla\phi$.



Figure 20: Direction of Propagation in some time point, trial number= 100

You can see this demo in this [link](#)., which is saved as '*Velocity Direction Over Time*'

2.4.3 Speed

The velocity magnitude or speed is:

$$speed(t) = \overline{|\partial\phi/\partial t|} / \overline{\|\nabla\phi\|}$$

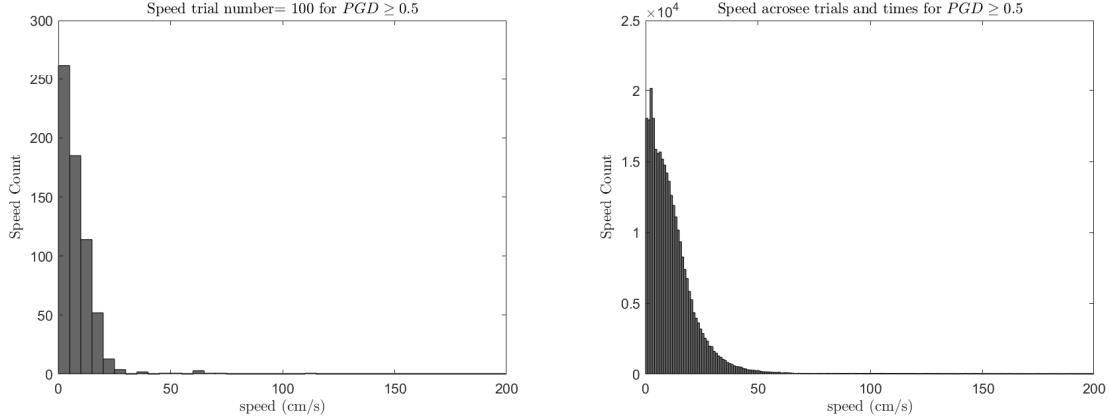


Figure 21: Histograms of the wave propagation speeds $PGD > 0.5$. right: in 100^{th} trial, left: all trials

You can see the speed over time for a selected trial in demo which is saved as '*Speed over time*' or get it [here](#) (Color bar change after stimulus onset. The sample time with $PGD \leq 0.5$ is red).

2.5 Plane Wave

Now, I want to find PGD by fitting a plane on phase in a specific time. As we learn in *Honghui Zhang* article:

"We used circular statistics to identify planar waves of phase progression across each oscillation cluster. For each spatial phase distribution, we used a two-dimensional circular-linear regression to assess whether the observed phase pattern varied linearly with the electrode's coordinates in $2 - D$. In this regression, for electrode i , x_i and y_i represent the $2 - D$ coordinates and θ_i is the instantaneous phase. x and y are determined by projecting the $3 - D$ Talairach coordinates for each cluster into the best-fitting $2 - D$ plane. Circular-linear models do not have an analytical solution and must be fitted iteratively. A $2 - D$ circular-linear model has three parameters to be fit: the phase slopes a and b , which each correspond to the rate of phase change (or spatial frequencies) in each dimension, and the phase offset θ . We converted this model to polar coordinates to simplify fitting. This polar model has two parameters: The angle of wave propagation α , defined as $\alpha = \text{atan}2(b, a)$, and the spatial frequency ζ , defined as $\zeta = \sqrt{a^2 + b^2}$."

I follow two approach: First, I use regression and fit a plane on spatial phase. Then use coefficient and find α and ζ . Second, use greedy algorithm explain in *Honghui Zhang* article."Change alpha in range $[0, 360]$ and ζ in range $[0, 50]$ in increments of 5° and $0.5^\circ/mm$. For each value of α and ζ , we compute the estimated phase $\hat{\theta}_i$ at each electrode i as:"

$$\begin{aligned}\hat{\theta}_i &= (ax_i + by_i + \nu) \bmod 360^\circ \\ a &= \zeta \cos(\alpha), \quad b = \zeta \sin(\alpha)\end{aligned}$$

Then we compute the goodness of fit for these parameters. The selected values of α and ζ are chosen to maximize \hat{r} .

$$\hat{r} = \sqrt{\left[\frac{1}{n} \sum_{i=1}^n \cos(\theta_i - \hat{\theta}_i) \right]^2 + \left[\frac{1}{n} \sum_{i=1}^n \sin(\theta_i - \hat{\theta}_i) \right]^2}$$

At the end, I find the *PGD* as:

$$\rho_{cc} = \frac{\sum_{i=1}^n \sin(\theta_i - \bar{\theta}) \sin(\hat{\theta}_i - \bar{\hat{\theta}}_i)}{\sum_{i=1}^n \sin(\theta_i - \bar{\theta})^2 \sin(\hat{\theta}_i - \bar{\hat{\theta}}_i)^2}$$

$$\rho_{adj}^2 = 1 - \frac{(1 - \rho_{cc}^2)(n - 1)}{n - k - 1}$$

"where n is the number of electrodes, and k is number of independent regressors ($k = 3$). We refer to ρ_{adj}^2 as phase gradient directionality (*PGD*)."

My result:

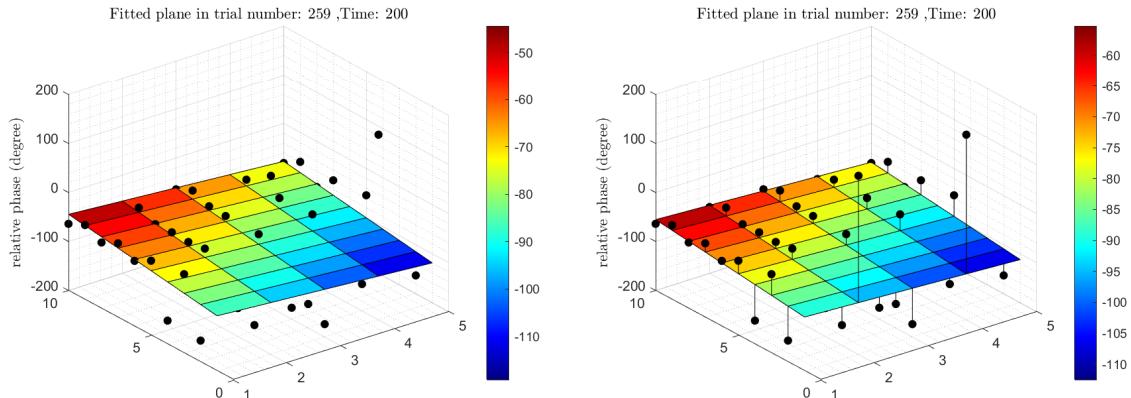


Figure 22: "Illustration of the fitted circular-linear model for quantifying spatial phase gradients and identifying traveling waves. Black dots indicate the average relative phase for each electrode; colored surface indicates the fitted phase plane; black lines indicate residuals."

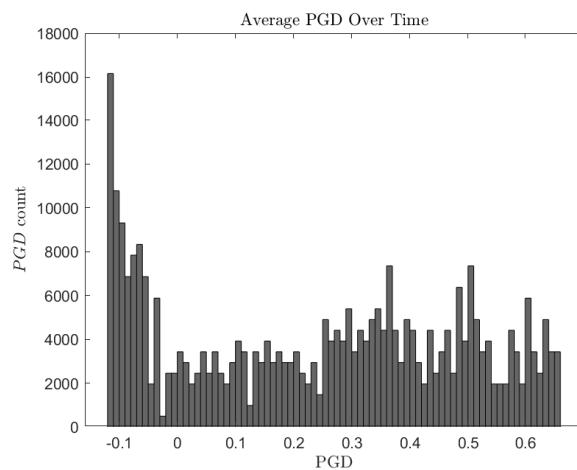


Figure 23: Histogram of PGD all sample times.

I also save demo of plane wave over time as 'fitted Plane Wave over time'.

2.6 Part e

I add speed and PGD in each sample time in 'plane wave_shading', which shows the phase cos, and save as a 'plane 'wave_shading with info'.

2.7 Preferred Direction Propagation

As we see in previous demos, the direction of wave is from left button to right top (if we consider array in horizontal). I plot the preferred angle in all trials and time samples in figure 24 to prove this evidence.

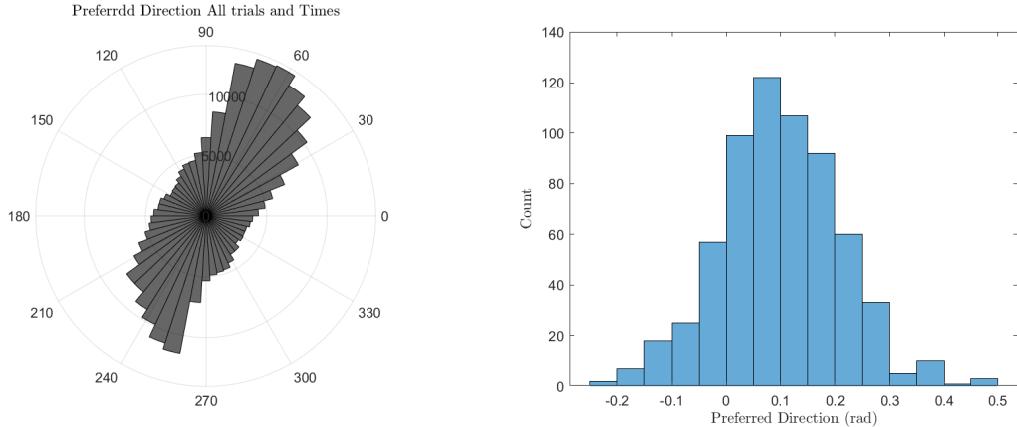


Figure 24: Preferred direction propagation in all trials and times with $PGD \geq 0.5$

To validate this result, I shuffle the signal in time to remove the systematic LFP peak time. The direction is calculated again and you can see in figure 25 we don't have a unique preferred propagation.

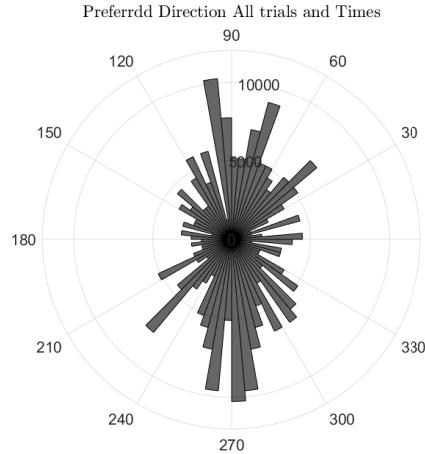


Figure 25: Preferred direction propagation in all trials and times after shuffling

2.8 part g

I calculated phase propagation speed for all trials and times. You can see the speed range is between similar with Sejnowski et.al 2018.

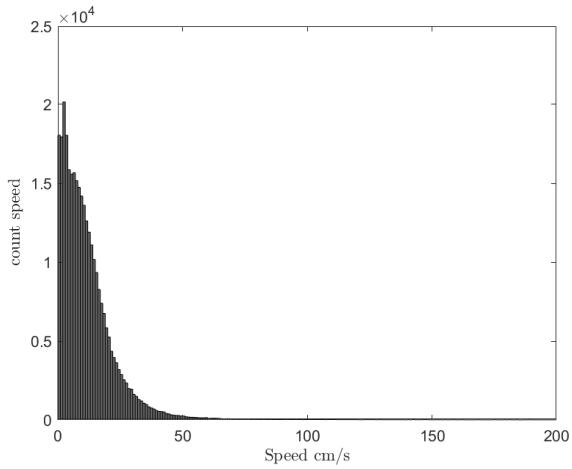


Figure 26: The wave speed in all trials and times

2.9 Extra simulation

2.9.1 PGD spectrum

PGD shows how the phase gradient systematically change over array. I filter data with band-pass filter (10 Hz bandwidth) from 0 to 100. PGD is found and averaged over all trials per times. The peak of PGD spectrum is on beta band. It means the beta is wavelike frequency band. We see similar result in 'Hatsopoulos' (figure 3.b).

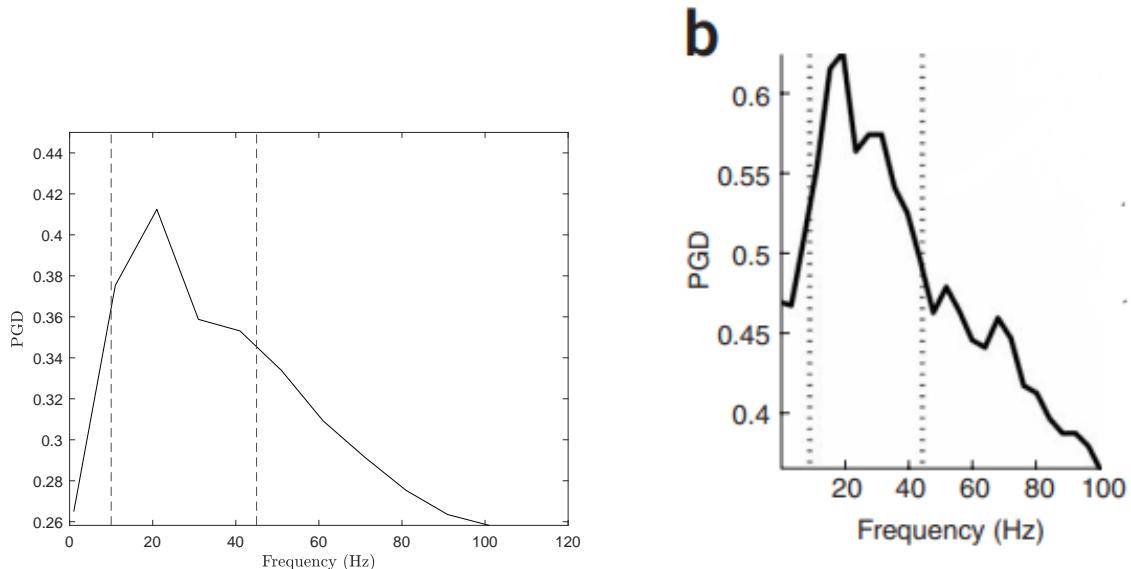


Figure 27: Right: PGD as a function of frequency. Dashed line show beta range. Left: article result.

2.9.2 Percent Phase Locking (PPL)

As we read in 'Hatsopoulos': "To assess the degree of stimulus-related phase-locking, we computed the percent phase-locking (PPL) across trials", I use its method and found the PPL for all electrodes in time. You can see the result in figure . I think the LFP signal show greater phase-locking a short period after stimulus and more specifically, after stimulus onset.

$$PPL(x, y, t) = 100[1 - H(\phi(x, y, t))/H_{max}]$$

$$H(x, y, t) = - \sum_{k=1}^N p_k \log_2(p_k)$$

where N is the number of trials and p_k is the fraction of values of $j(x, y, t)$ at fixed x , y , and t that lie within the k th bin. $H_{max} = \log_2(N)$. We used 5 bins for the calculation. $PPL(x, y, t)$ conveys the degree of certainty in the phase angles at a single location in space and time.

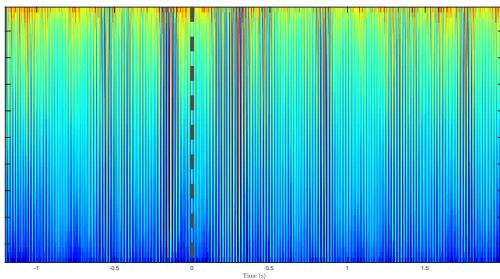


Figure 28: "PPL computed across all channels during the instruction. Time is presented on the horizontal axis, and channels are presented on the vertical axis, sorted by maximum PPL". Dashed line is stimulus onset.