Logistic map and finding Feigenbaum constants

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Theory

The logistic map is a polynomial mapping (equivalently, recurrence relation) of degree 2, often referred to as an archetypal example of how complex, chaotic behaviour can arise from very simple nonlinear dynamical equations

$$x_n = rx_n(1 - x_{n-1})$$

The map is implemented in a julia function with r and x_n argument passing the array consisting the x_n. The Initval specifies a random set of 100 initial values.

Plotting

The r value goes from 0.001 to 1 (1000 iterations). The computation is repeated for 1000 runs. finally x_n is plotted over r.

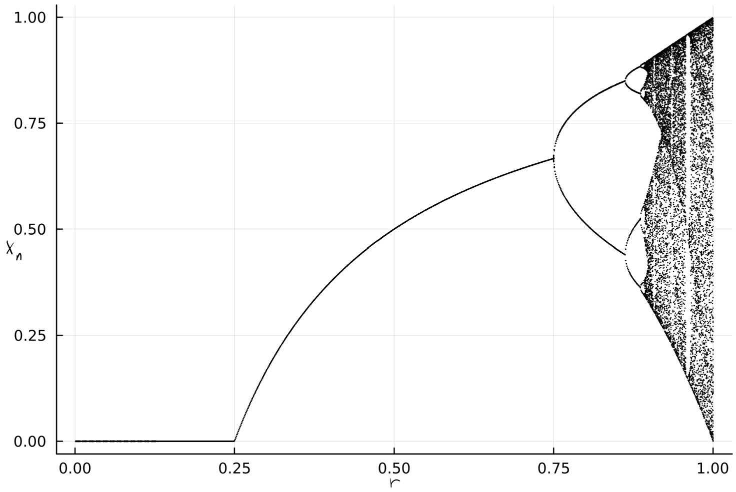

Figure 1: logistic map with 1000 iterations.

1 Finding Feigenbaum constans

For finding bifurcation points, we need to check for the points in our graph where the number of points are powers of 2. For doing so, points with a too small distance need to be unified(repetitive points appear becuase of the noise involved.) Feigenbaum constans are finally calculated using the bifurcation points. Bifurcation points are:

Branches	2	4	8	16	32
r	0.249	0.746	0.864	0.887	0.892

According to wikipedia, The first Feigenbaum constant is the limiting ratio of each bifurcation interval to the next between every period doubling:

$$\delta = 4.60$$

The second Feigenbaum constant is is the ratio between the width of a tine and the width of one of its two subtines (except the tine closest to the fold):

$$\alpha = 2.78 \tag{1}$$