Neuroscience, Learning, Memory, Cognition Course

HW3 - theory section

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Question 1

Q1. Suppose the statistical data of an experiment are extracted from the normal distribution $N(\mu, \sigma^2)$. We assume that the variance of this population is known and equal to σ^2, but its mean, μ, is unknown, and we want to estimate from N independent samples $x_1, \ldots, x_N$.

- **1. Obtain the ML estimate for the population mean.**
- **2. Assume that the prior distribution of the parameter μ is the normal distribution $N(\mu, \sigma^2)$ Get the MAP estimate for the population mean. What effect do you think choosing such a prior distribution has on the posterior distribution?**
- **3. Explain how the increase in the number of community samples creates a relationship between these two estimates.**

1.1

> To obtain the Maximum Likelihood (ML) estimate for the population mean, we first need to write down the likelihood function. Given that the samples are independent and identically distributed (i.i.d.), the likelihood function is the product of the probability density functions (PDFs) of the normal distribution for each sample:

$$L(\mu) = P(x_1, \ldots, x_N|\mu) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}} = (\frac{1}{\sqrt{2\pi\sigma^2}})^N e^{-\frac{\sum_{i=1}^{N}(x_i-\mu)^2}{2\sigma^2}}$$

Taking logarithm of the likelihood function, which gives us the log-likelihood function:

$$\ell(\mu) = \ln L(\mu) = -\frac{N}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{N}(x_i-\mu)^2$$

To find the ML estimate, we need to maximize the log-likelihood function with respect to μ. We can do this by taking the derivative of the log-likelihood function with respect to μ and setting it to zero:

$$\frac{d\ell(\mu)}{d\mu} = \frac{1}{\sigma^2}\sum_{i=1}^{N}(x_i-\mu) = 0$$

Solving for μ, we get the ML estimate for the population mean:

$$\hat{\mu}_{ML} = \frac{1}{N}\sum_{i=1}^{N} x_i = \bar{x}$$

1.2

To find the MAP estimate for the population mean, we need to consider the prior distribution of the parameter μ. Given that the prior distribution is also a normal distribution $N(\mu_0, \sigma_0^2)$, we can write the posterior distribution as:

$$P(\mu|x_1, \ldots, x_N) \propto P(x_1, \ldots, x_N|\mu)P(\mu) \propto L(\mu)P(\mu)$$

Since $L(\mu), P(\mu)$ are both normal distributions, the posterior distribution will also be a normal distribution (based on the conjugacy property of the normal distribution). The MAP estimate is the mode of the posterior distribution, which, for a normal distribution, is the same as the mean. (we need to maximize the posterior distribution with respect to μ. Since the posterior distribution is a normal distribution, the maximum value will occur at the mean of the distribution). To find the MAP estimate, we can equate the log of the posterior distribution to a constant and differentiate with respect to μ:

$$\frac{d}{d\mu}(\ell(\mu) + \ln P(\mu)) = 0 \Rightarrow \frac{d\ell(\mu)}{d\mu} + \frac{d\ln P(\mu)}{d\mu} = 0$$

Solving for μ, we get the MAP estimate for the population mean:

$$\hat{\mu}_{MAP} = \frac{\sigma^2\mu + \sigma^2\sum_{i=1}^{N} x_i}{N\sigma^2 + \sigma^2} = \frac{\mu + N\bar{x}}{N+1}$$

where $\bar{x}$ is the sample mean. Choosing a normal prior distribution for the parameter μ has the effect of making the posterior distribution a normal distribution as well. This simplifies the calculations and allows us to obtain a closed-form solution for the

> MAP estimate. Additionally, the choice of prior distribution can influence the final estimate, especially when the sample size is small. As the sample size increases, the influence of the prior distribution diminishes, and the estimate converges to the Maximum Likelihood Estimate (MLE).

1.3

> As the number of samples (N) increases, the ML and MAP estimates converge. This can be seen by examining the MAP estimate formula:
> $$\hat{\mu}_{MAP} = \frac{\sigma^2 \mu + \sigma^2 \sum_{i=1}^{N} x_i}{N\sigma^2 + \sigma^2} = \frac{\mu + N\bar{x}}{N+1}$$
> As N becomes large, the term $N\sigma^2$ dominates the denominator, and the MAP estimate approaches the ML estimate:
> $$\lim_{N\to\infty} \hat{\mu}_{MAP} = \lim_{N\to\infty} \frac{\sigma^2 \mu + \sigma^2 \sum_{i=1}^{N} x_i}{N\sigma^2 + \sigma^2} = \lim_{N\to\infty} \frac{\mu + N\bar{x}}{N+1} = \bar{x} = \hat{\mu}_{ML}$$
> This relationship shows that as more data is collected, the influence of the prior distribution on the estimate diminishes, and the ML and MAP estimates become more similar. This is consistent with the idea that as more data is available, the data itself becomes more informative, and the prior distribution becomes less important in determining the parameter estimate.

Question 2

Stimulus-response law

The figure in pdf shows the relationship of spike rates in response to light intensity in a lobster with a logarithmic scale This diagram shows that the relative power or the amount of activity of neurons has increased with the increase of light intensity.

- **Research the Power law for skill acquisition.**
- **What will be the effect of practicing and learning better on the power and activity of neurons?**
- **If a skill is acquired once and forgotten, does it follow the previous rule again? How? (In this regard, an experiment has been designed to learn to read reversed text, if possible, describe the experiment and answer the questions in its format.**

2.1

> The power law of skill acquisition describes the relationship between the amount of practice and the improvement in performance. It states that the time it takes to perform a task decreases as a power function of the number of times the task has been practiced. This law suggests that the more you practice a skill, the better you become at it, but the rate of improvement decreases over time.

> The power law of practice, or power law of skill acquisition, is a psychological principle that describes the relationship between the amount of practice and the improvement in performance. It's an empirical observation that states that the time it takes to perform a task decreases as a power function of the number of times the task has been practiced. The power law can be expressed mathematically as:
> $$T(n) = T(1) * n^{-a}$$
> Here, T(n) is the time it takes to perform the task on the nth trial, T(1) is the time it took to perform the task on the first trial, n is the number of trials, and a is a constant that depends on the specific task and individual. This law suggests that the more you practice a skill, the better you become at it, but the rate of improvement decreases over time. In other words, you'll see rapid improvement when you first start practicing, but as you become more skilled, the gains from additional practice become smaller. It's important to note that the power law of practice is an empirical observation, and it may not hold true for all tasks or individuals. However, it has been observed in a wide range of skill acquisition scenarios, from typing to playing musical instruments, and even in learning complex tasks like chess or solving mathematical problems.

2.2

> When it comes to the effect of practicing and learning on the power and activity of neurons, it's important to understand that the brain is a highly adaptable organ. As you practice and learn a skill, the neural connections related to that skill become stronger and more efficient. This process, known as synaptic plasticity, allows the brain to adapt to new experiences and improve performance. As a result, the power and activity of neurons involved in the skill will likely increase as you practice and learn, leading to better performance.

> When you practice and learn a skill, the power and activity of the neurons involved in that skill are affected in several ways:
>
> - Strengthening of synaptic connections: As you practice and learn, the connections between neurons (synapses) related to the skill become stronger. This process, known as synaptic plasticity or long-term potentiation (LTP), allows the brain to adapt to new experiences and improve performance. Stronger synaptic connections lead to more efficient communication between neurons, which in turn enhances your ability to perform the skill.
> - Increased neural efficiency: With practice, the brain becomes more efficient at processing information related to the skill. This means that fewer neurons may be required to perform the task, or the same neurons may fire more efficiently. This increased efficiency can lead to better performance and faster reaction times.
> - Formation of new neural connections: As you learn and practice a skill, your brain may form new connections between neurons, creating additional pathways for information processing. This can help improve your ability to perform the skill and adapt to new situations.
> - Changes in neural representation: The way the skill is represented in the brain may change as you practice and learn. For example, the brain areas involved in the skill may become more specialized, or the neural patterns associated with the skill may become more distinct. These changes in neural representation can lead to improved performance and a better understanding of the skill.

> Practicing and learning a skill can lead to increased power and activity of neurons involved in the skill, as well as changes in the brain's structure and function. These changes can result in improved performance, faster reaction times, and a better understanding of the skill being learned.

2.3

> If a skill is acquired once and then forgotten, it's possible that relearning the skill may follow a similar trajectory as the initial learning, but the process might be faster. This phenomenon, known as "savings" or "latent learning," suggests that some aspects of the previously learned skill are retained, even if the skill appears to be forgotten. (That's because some of the neural pathways we formed when we first learned the skill are still hanging around, even if we've forgotten the skill itself)

> Regarding the experiment, Imagine that it involves participants learning to read reversed text (i.e., text that is mirrored or flipped horizontally). If participants initially learn this skill and then forget it, they might still show some savings when relearning the skill, meaning that they could relearn it more quickly than they did the first time. This would suggest that some aspects of the neural connections formed during the initial learning are still present, even if the skill appears to be forgotten.

> The idea is that relearning the skill would be quicker and easier than learning it from scratch, thanks to the concept of savings. This would show that even when we forget a skill, some of the neural pathways are still there, making it easier to pick it back up again.

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