

Example 1.1) A relation R is defined on set I of all integers, by aRb , if and only if $ab > 0$, for all $a, b \in I$. Examine if R is (a) reflexive, (b) symmetric, and (c) transitive.

Solution:

a) Let $a, b \in I$. then $aa > 0$, holds good if $ba > 0$ if $a = 0$, then $a.a = 0$. therefor, aRa does not hold for all a in I . So, R is not reflexive.

b) Let $a, b \in I$. If $ab > 0$, then $ba > 0$ also. That is, $aRb \Rightarrow bRa$. So, R is symmetric.

c) Let $a, b \in I$. i let aRb and bRc both hold good. It can be said

$ab > 0$ and $bc > 0$. if we multiply these two, then $(ab)(bc) > 0$, $ab^2c > 0$. We know b^2 is always > 0 . It can be said clearly that $ac > 0$. Thus, aRb and $bRc \Rightarrow aRc$. So, R is transitive.

Example 1.2) A relation R is defined on a set of integers (I) by aRb if $a - b$ is divisible by 3, for $a, b \in I$. Examine if R is (a) reflexive (b) symmetric, and (c) transitive.

Solution:

a) Let $a \in I$. Then, $a - a$ is divisible by 3. Therefore, aRa holds good for all $a \in I$. So the relation R is reflexive.

b) Let $a, b \in I$ and aRb hold good. It means $a - b$ is divisible by 3. If it is true, then $b - a$ is also divisible by 3.

(Let $a = 6, b = 3, a - b = 3$, divisible by 3, $b - a = -3$, which is also divisible by 3). So, bRa holds good. Therefore, R is symmetric.

c) Let $a, b, c \in I$ and aRb, bRc hold good. It means $a - b$ and $b - c$ both are divisible by 3. Therefore, $(a - c) = (a - b) + (b - c)$ is divisible by 3. (Let $a = 18, b = 12, c = 6, a - b = 6, b - c = 6$. Both are divisible by 3. $a - c = 12$ is also divisible by 3. So, aRc holds good. Therefore, R is transitive.