Example 1.1) A relation R is defined on set (I) of all integers, by aRb, if and only if ab > 0, for all $\mathrm{a, b} \in \mathrm{I}$. Examine if R is (a) reflexive, (b) symmetric, and (c) transitive.

Solution:

a) Let $a \in I$. then $aa > 0$, holds good if $a \neq 0$ if a $=$ 0,then a.a$=$0. therefor, aRa does not hold for all a in I. So, R is not reflexive.

b) Let $a, b \in I$. If $ab > 0$, then $ba > 0$ also. That is, $aRb \Rightarrow bRa$. So, R is symmetric.

c) Let $a, b, c \in I$. i let aRb and bRc both hold good.It can be said

ab $> 0 and \mathrm{bc} > 0$. if we multiply these two,then(ab)(bc)$> 0, \mathrm{ab}^2\mathrm{c} > 0$. We know b^2 is always $> 0. It can be said$ clearly that $\mathrm{ac} > 0$. Thus, aRb and bRc $\Rightarrow$ aRc. So, R is transitive.

Example 1.2) A relation R is defined on a set of integers (I) by aRb if a $-\mathrm{b}$ is divisible by 3, for $\mathrm{a}, \mathrm{b} \in \mathrm{I}$. Examine if R is (a) reflexive (b) symmetric, and (c) transitive.

Solution:

a) Let a $\in 1$. Then, $\mathrm{a}-\mathrm{a}$ is divisible 3. Therefore, aRa holds good for all a $\in 1$. So the relation R is reflexive.

b) Let $a, b \in I$ and aRb hold good. It means $a-b$ is divisible by 3. If it is true, then $b-a$ is also divisible by 3 .

(Let $a=6, b=3, a-b=3$, divisible by 3, $b-a=-3$, which is also divisible by 3). So, bRa holds good. Therefore, R is symmetric.

c) Let $a, b, c \in I$ and aRb, bRc hold good. It means $a-b$ and $b-c$ both are divisible by 3. Therefore, $(a-c)=(a-b)+(b-c)$ is divisible by 3. (Let $a=18, b=12, c=6, a-b=6, b-c=6$. Both are divisible by 3.$a-c=12$ is also divisible by 3. So, aRc holds good. Therefore, R is transitive.