

## 2 Introduction to Automata Theory, Formal Languages and Computation

- **Substring:** A substring of a string is defined as a string formed by taking any number of symbols of the string.

**Example:** For the string  $w = 012$ , the substrings are  $\lambda, 0, 1, 2, 01, 12$ , and  $012$

### 1.2 Basics of Set Theory

A set is a well-defined collection of objects. The objects used for constructing a set are called elements or the members of the set.

A set has some features as follows:

- A set is a collection of objects. This collection is regarded as a single entity.
- A set is comprised of distinct elements. If an element, say 'a', is in set  $S$ , then it is denoted as  $a \in S$ .
- A set has a well-defined boundary. If  $S$  is a set and 'a' is any element, then depending on the properties of 'a', it can be said whether  $a \in S$  or  $a \notin S$ .
- A set is characterized by its property. In general, if  $p$  is the defined property for the elements of  $S$ , then  $S$  is denoted as  $S = \{a : a \text{ has the property } p\}$

**Example:**

- The set of all integers is denoted as  $S = \{a : a \text{ is an integer}\}$ . Here,  $7 \in S$  but  $1/7 \notin S$
- The set of all odd numbers denoted as  $S = \{a : a \text{ is not divisible by } 2\}$ . Here,  $7 \in S$  but  $8 \notin S$
- The set of prime numbers less than 100 is denoted as  $S = \{a : a \text{ is prime and less than } 100\}$ . Here,  $23 \in S$  but  $98$  or  $101 \notin S$

**1.2.1 Subset** Let there be two sets  $S$  and  $S_1$ .  $S_1$  is said to be a subset of  $S$  if every element  $S_1$  is an element of  $S$ . Symbolically, it is denoted as  $S_1 \subset S$ . The reverse of a subset is the superset. In the previous example,  $S$  is the superset of  $S_1$ .

**Example:**

- Let  $Z$  be the set of all integers.  $E$  is the set of all even numbers. All even numbers are natural numbers. So, it can be denoted as  $E \subset Z$ .
- Let  $S$  be the set of the numbers divisible by 6, where  $T$  is the set of numbers divisible by 2. Property says that if a number is divisible by 6, it must be divisible by 2 and 3. So, it can be denoted as  $S \subset T$ .

**1.2.2 Finite and Infinite Set** A set is said to be finite if it contains no element or a finite number of elements. Otherwise, it is an infinite set.

**Example:**

- Let  $S$  be the set of one digit integers greater than 1,  $S$  is finite as its number of elements is 8.
- Let  $P$  be the set of all prime numbers.  $P$  is infinite, as the number of prime numbers is infinite.