Bounded approximation of log likelihood for plans

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Abstract

The likelihood of a sequence of decisions is very hard to calculate. We suggest a method that allows a bounded approximation of log likelihood.

1 The problem

Most psychophysics tasks assume that pairs of responses and stimuli within a sequence of an experiment are all independent of each other.

$$\mathcal{L}(S) = \prod_{s_i \in S} P(s_i) \tag{1}$$

Therefore it is possible to compare log likelihoods.

$$\log \mathcal{L}(S) = \sum_{s_i \in S} \log(P(s_i))$$

However, when the task involves a sequence the correct way to calculate likelihood is:

$$\mathcal{L}(S) = \prod_{s_i \in S} P(s_i | s_0, s_1, \ldots, s_{i-1})$$

2 Solution

Let $\lambda(s)$ be the approximated distribution over the successors of state s. Let $F(s)$ be the calculated part of the distribution. Let $F^{-1}(s)$ be the approximated part of the distribution.

For a certain model we calculate the partial distribution $\lambda(s)$ as following: For every state in $F^{-1}(s)$ we assign $\epsilon/|F^{-1}(s)|$ for the rest, we assign the true probability.

2.1 Error bounds

The error of a state s on every level is $e(s) = |\lambda(s) - p(s)|$.

For every approximated state $0 < p(s) < \epsilon$ therefore $e(s) < \epsilon(1 - 1/|F^{-1}(s)|)$ or $e(s) < \epsilon/|F^{-1}(s)|$.

3 practical example from rush-hour