Zaid

INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

End-Autumn Semester Examination 2022-23

Date of Examination: Nov 22, 2022 Session: AN Duration: 3 hrs. Full Marks: 50

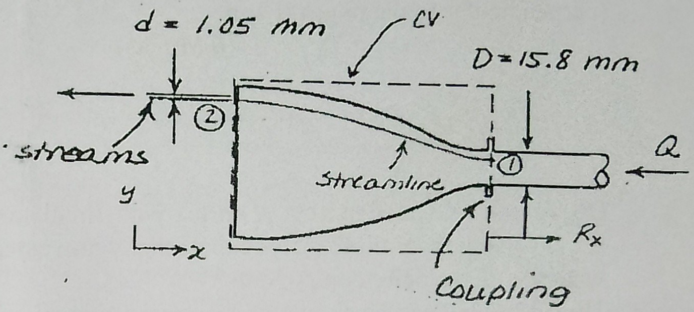
Subject No.: CH21207 Subject: Fluid Mechanics

Department/Center/School: Chemical Engineering

Specific charts, graph paper, log book etc., required: NONE

Special Instructions (if any): Assume any data you feel are missing

Q1. Water at 45 °C enters a shower head through a circular tube with 15.8 mm inside diameter. The water leaves in 24 streams, each of 1.05 mm diameters. The volume flow rate is 5.67 L/min. Estimate

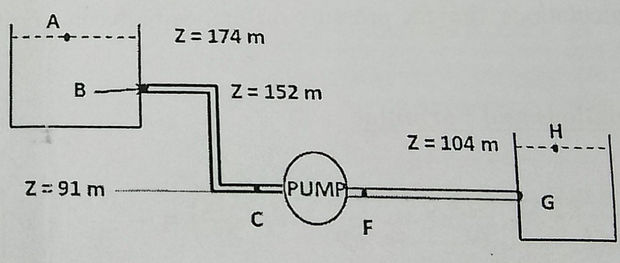


(i) The minimum water pressure needed at the inlet to the shower head.

(ii) Force needed to hold the shower head onto the end of the circular tube. You may use the value of the contraction coefficient, K, to be equal to 0.5, ρ = 990 kg/m³. The frictional losses are generally small for such short lengths associated with the shower head.

(4+4 = 8 Marks)

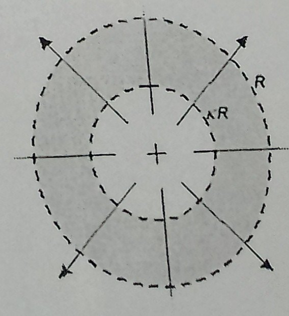
Q2. Water (kinematic viscosity = 1.0×10^{-6} m²/s) is pumped from a reservoir at the rate of 1310 L/s and is being sent to another large tank. The path of water through the pipe is marked as BCFG with the pump being located between C and F. From B to C, the system consists of a square-edged entrance (K = 0.5), 760 m of pipe, three gate valves (Le/D = 8), four 45° elbows (Le/D = 20) and two 90° elbows (Le/D = 30). Gage pressure at C is 197 kPa.



The system between F and G contains 760 m of pipe, two gate valves (Le/D = 8) and four 90° elbows (Le/D = 30) and a loss coefficient equal to 1.0. All the pipes are made of cast iron (ε = 0.26 mm) and of 508 mm diameter. Calculate the average velocity of water in the pipe, the gage pressure at F, the power input to the pump (of efficiency 80%) and the wall shear stress in section FG.

(1+4+2+2=9 Marks)

Q3. An isothermal, incompressible fluid of density ρ flows radially outward owing to a pressure difference between two fixed porous, coaxial cylindrical shells of radii κR and R. Note that the velocity is not zero at the solid surfaces. Assume long cylinders with negligible end effects and steady laminar flow in the region $\kappa R \leq r \leq R$. The top view of the cylinders is provided.



a) Simplify the equation of continuity to obtain a relation between r and v_r.

b) Simplify the equation of motion for a Newtonian fluid.

c) Obtain the pressure profile $P(r)$ in terms of P_R and v_R, the pressure and velocity at the cylinder of radius R respectively.

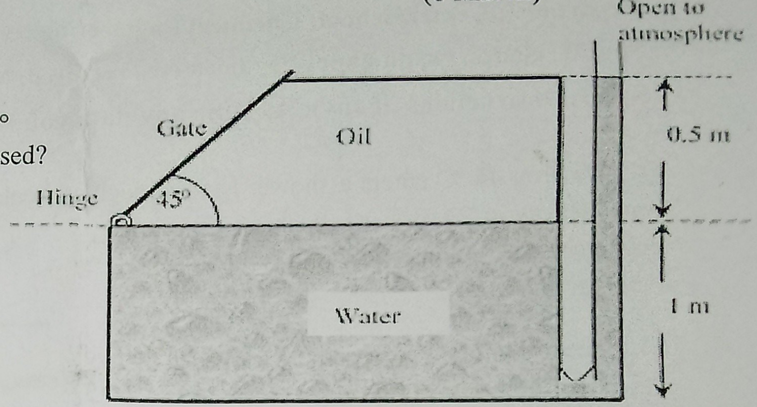
(2+2+4=8 Marks)

<u>Q4.</u> The velocity profile in infinitely wide, and 0.2 m high channel is given by $\vec{V} = (y - \frac{y^2}{0.2})\hat{\imath}$
Determine the stream function for this flow.
Does potential function exist for this flow? What is the vorticity?
Calculate the flowrate by using $\Delta\psi$, and compare with the value obtained by integrating the velocity profile.

(6 Marks)

<u>Q5.</u> What should be the weight of the plate (shown here as gate, making an angle of 45° with the horizontal) such that it remains closed? The density of oil is 800 kg m^{-3}, and width of the plate (perpendicular to the page) is 1 m.

(6 Marks)



<u>Q6.</u> A wall is located at x=0. A vortex with circulation $\Gamma=20\pi$ m/s is placed 1 m above the wall. What is the velocity potential? Do you expect any movement of the centre of the vortex with time? If so, what are the velocity components?
Hint: The wall can be simulated by considering additionally an image vortex, with plane wall regarded as mirror. The image vortex will have a circulation in the reverse direction of the original vortex.

(7 Marks)

<u>Q7.</u> A 2D steady, constant density, inviscid flow is described by the velocity field $u = Ax$, $v = -Ay$, where $A = 1.5$ s^{-1} and the coordinates are measured in meters. Giving clear justifications of your calculations, find the pressure difference between a point at (1,1,0) and a point at (2,2,0).

(6 Marks)

<u>Relations and Formulae</u>

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial P}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = \rho g_z - \frac{\partial p}{\partial z} + \mu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right\}$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\delta\theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r$$

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho\vec{V}.d\vec{A}$$

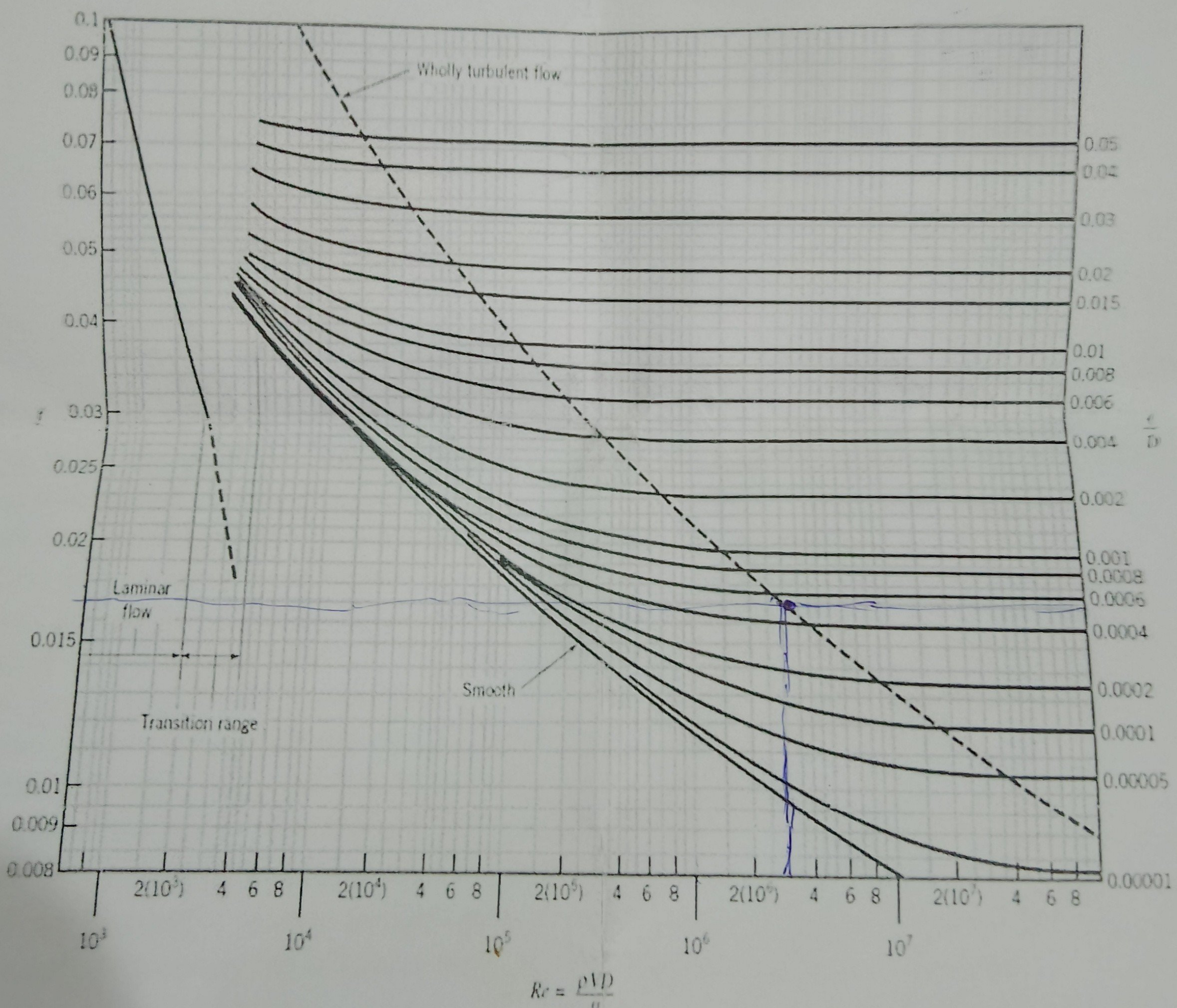
$$F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\,\rho\, d\forall + \int_{CS} u\,\rho\,\vec{V}.\vec{dA}$$

continued

$$\frac{p_o}{\rho}+\frac{V_o^2}{2}+gz_o=\frac{p_1}{\rho}+\frac{V_1^2}{2}+gz_1+f\frac{L}{D}\frac{V_1^2}{2}+h; \quad h_{LM}=K\frac{V^2}{2} \quad or \quad f\frac{L_e}{D}\frac{V^2}{2}; Major\ Loss=f\frac{L}{D}\frac{V^2}{2}$$

$$\dot{W}_{in}=\dot{m}\left[\left(\frac{p_2}{\rho}+\alpha_2\frac{\overline{V_2^2}}{2}+gZ_2\right)+h_{LT}-\left(\frac{p_1}{\rho}+\alpha_1\frac{\overline{V_1^2}}{2}+gZ_1\right)\right]$$

$$Pump\ Head=\frac{\dot{W}_{in}}{\dot{m}}\left(in\ \frac{m^2}{s^2}\right), \quad Power=\rho Q\times Pump\ Head,(W)$$



$$F_{Source}(z)=\frac{m}{2\pi}\ln(z) \qquad F_{Vortex}(z)=-i\frac{\Gamma}{2\pi}\ln(z)$$

$$F_R=\int_A PdA \qquad y'F_R=\int_A yPdA \qquad x'F_R=\int_A xPdA$$