The University of Jordan

School of Engineering

Department of Mechatronics Engineering

Prof. Zaer Salem Abu-Hammour

Robotic systems

0918474

Comprehensive Programming Project

part I of the project

Project weight

30% of course marks

Submission deadline

22-12-2024

1. Spatial Descriptions and Transformations

1) **Performing a rotation of a point in space with respect to one reference frame: -**
   i. **Two-dimensional rotation of a point in the xy plane about the positive z-axis.**
   ii. **Two-dimensional rotation of a point in the yz plane about the positive x-axis.**
   iii. **Two-dimensional rotation of a point in the xz plane about the positive y-axis.**
2) **Performing a rotation of two frames with respect to each other.**
3) **Calculation of the 24 Euler and Fixed Angles Conventions based on: -**
   i. **Forward calculations.**
   ii. **Inverse calculations including the special cases.**
4) **Transformation of a point in space with respect to one reference frame.**
5) **Changing the reference frame of a point due to transformation.**
6) **Calculation of the inverse of the transformation matrix.**

2. Forward Kinematics of Serial Manipulators

Given the D-H Parameters (geometry) for serial manipulator with certain number of joints (degrees of mobility), it is required perform the following: -

1) **Draw the manipulator in 3D using any one of the following 3D CAD tools: -**
   i. **SolidWorks with Motion and Simulation Add-Ons.**
   ii. **Autodesk Fusion 360.**
   iii. **PTC Creo.**
   iv. **Autodesk Inventor.**
   v. **RoboDK (Robot Simulation Software).**
   vi. **MATLAB with Simulink and Robotics System Toolbox.**
   vii. **CATIA.**
   viii. **FreeCAD with Robotics Workbench.**

   The drawing should be scaled and show the following assignments of the manipulator: -
   i. **Link Assignment from link 0 (base) to link N (the free end of the arm).**
   ii. **joint Assignment from joint 1 to joint N.**
   iii. **Frame Assignment from frame [0] to frame [N+1] (if required).**
2) **Find the individual homogenous transform matrix ${}^{i-1}_{i}T$ for all frames staring from ${}^{0}_{1}T$ until ${}^{N-1}_{N}T$.**
3) **Find the forward kinematics equations of the manipulator**

$$ {}^{0}_{N}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T \ldots {}^{N-1}_{N}T = \begin{bmatrix} r_{11} & r_{12} & r_{14} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

The forward kinematics equations should cover the following possibilities: -

i. **Symbolic fixed parameters. The three fixed parameters, $(a_{i-1}, \alpha_{i-1}, d_i)$ for a revolute joint or $(a_{i-1}, \alpha_{i-1}, d_i)$ for a prismatic joint, are symbolic (not numbers).**

ii. **Numerical fixed parameters. The three fixed parameters, $(a_{i-1}, \alpha_{i-1}, d_i)$ for a revolute joint or $(a_{i-1}, \alpha_{i-1}, d_i)$ for a prismatic joint, are numbers.**

iii. **Symbolic joint variables. The joint variables, (θ_i) for a revolute joint or d_i for a prismatic joint are variables.**

iv. **Numerical joint variables. The joint variables, (θ_i) for a revolute joint or d_i for a prismatic joint are numbers.**

4) **Generation of the desired configuration matrix $\{{}^{0}_{N}T_{des}\}$:-**
   a) **Set numerical values for all fixed parameters of the manipulator for one time.**
   b) **Set numerical values for five sets of all joint variables of the manipulator.**
   c) **Based on the previous five sets of numerical values, find the five desired configuration (position and orientation) of the end effector (wrist) with respect to the base of the manipulator $\{{}^{0}_{N}T_{des}\}$ based on the forward kinematics model of the manipulator.**

3. Inverse Kinematics of Serial Manipulators

1) Multi-dimensional Newton-Raphson method

Given the D-H Parameters indirectly or the forward kinematics model of the manipulator directly as

$$ {}^{0}_{N}T_{FK}(q_1, q_2, \dots, q_N) = \begin{bmatrix} r_{11} & r_{12} & r_{14} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_Z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

It is required to solve the inverse kinematics problem of the manipulator using the multi-dimensional Newton-Raphson method according to the following steps: -

a) **Determination of the desired configuration matrix $\{{}^{0}_{N}T_{des}\}$:-**
   d) **Set numerical values for all fixed parameters of the manipulator.**
   e) **Set numerical values for all joint variables of the manipulator.**
   f) **Based on the previously assumed numerical values, find the desired configuration (position and orientation) of the end effector (wrist) with respect to the base of the manipulator $\{{}^{0}_{N}T_{des}\}$ based on the forward kinematics model of the manipulator.**

b) **Based on $\{{}^{0}_{N}T_{des}\}$, Find all solutions of the joints' variables (q_i) using different initial guess vectors**

$$ \begin{Bmatrix} q_i = \theta_i & \quad \textit{for revolute joints} \\ q_i = d_i & \quad \textit{for prismatic joints} \end{Bmatrix}, \quad \text{where } i = 1, 2, \dots, N $$

Using the following two scenarios: -

i. Square Jacobian matrix.

ii. Rectangular Jacobian matrix.

c) For each solution obtained in section (b), substitute the solution in the forward kinematics model of the manipulator and find the resultant desired configuration matrix $\{{}^{0}_{N}T_{des}\}$.

d) Print all solutions of the joints' variables (q_i) based on the given $\{{}^{0}_{N}T_{des}\}$.

2) Pieper's Closed-Form Method

Given the D-H parameters of the manipulator indirectly, or its the forward kinematics model directly such that it should satisfy one of the three cases of Pieper's method

$$ {}^{0}_{N}T_{FK}(q_1, q_2, \dots, q_N) = \begin{bmatrix} r_{11} & r_{12} & r_{14} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_Z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

It is required to solve the inverse kinematics problem of the manipulator using Pieper's method according to the following steps: -

a) Determination of the desired configuration matrix $\{{}^{0}_{N}T_{des}\}$:-

i. Set numerical values for all fixed parameters of the manipulator.

ii. Set numerical values for all joint variables of the manipulator.

iii. Based on the previously assumed numerical values, find the desired configuration (position and orientation) of the end effector (wrist) with respect to the base of the manipulator $\{{}^{0}_{N}T_{des}\}$ based on the forward kinematics model of the manipulator.

b) Based on $\{{}^{0}_{N}T_{des}\}$, Find all solutions of the joints' variables (q_i)

$$ \begin{Bmatrix} q_i = \theta_i & \quad \textit{for revolute joints} \\ q_i = d_i & \quad \textit{for prismatic joints} \end{Bmatrix}, \quad \text{where } i = 1, 2, \dots, N $$

c) For each solution obtained in section (b), substitute the solution in the forward kinematics model of the manipulator then find the resultant desired configuration matrix $\{{}^{0}_{N}T_{des}\}$.

d) Print all solutions of the joints' variables (q_i) based on the given $\{{}^{0}_{N}T_{des}\}$.

4. Motion Planning of Serial Manipulators

Given the D-H parameters of the manipulator indirectly, or its the forward kinematics model directly such that it should satisfy one of the three cases of Pieper's method

$$ {}^{0}_{N}T_{FK}(q_1, q_2, \dots, q_N) = \begin{bmatrix} r_{11} & r_{12} & r_{14} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_Z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

It is required to generate a path (spatial without time) and a motion (spatial and temporal) using the following phases: -

a) Path planning phase.

1. **Assume a shape of the motion (for example, straight line, sine motion, etc.) of the end effector such that it will be within the workspace of the manipulator.**
2. **Select one of the two planning approaches: -**
   a. **For cartesian motion planning, sample this motion using sampling points within the range $N_S \in [20, 100]$ points such that they will be very close to each other.**
   b. **For point-to-point motion planning, sample this motion using sampling points within the range $N_S \in [2, 5]$ points such that they will be away from each other.**
3. **Solve for the inverse kinematics of all sampling points such that all solutions for every point are obtained.**
4. **Assume that there are motion limits for all joints of the manipulator $(\theta_{i,min}, \theta_{i,max})$.**
5. **Eliminate any mathematical solution obtained from the inverse kinematics that is outside the joints' limits (infeasible solutions).**
6. **For the remaining set of feasible solutions, select the solution in joint space that result in the minimum displacement between any two successive sampling points within all sampling points for all joints out of the multiple solutions obtained using the inverse kinematics.**
7. **For the feasible solution with minimum displacement, print the number of the sampling point along with the values of all joint variables in a form of a table. After this step, the path planning is completed and the spatial data points $(i, q_i^j), i \in [1, N_S],\ j \in [1, N_{DOM}]$ are obtained.**

b) Motion planning phase 1 (temporal data points preparation).

Given the spatial data points (i, q_i^j), it is required to assume a time frame for the generated path$(0, t_f)$, where the manipulator will start from zero time and end its motion at t_f.

1. **Find the numerical derivatives for the first derivative (speed) and the second derivative (acceleration) for all sampling points.**
2. **Check whether the speed or acceleration at any sampling point violates the maximum speed limits for all joints $(\dot{\theta}_{i,max})$, or the maximum acceleration limits for all joints $(\ddot{\theta}_{i,max})$.**
3. **In case of any violation, then increase the final time according to the equation**

$$t_{fnew} = t_{fold} \times 1.1$$

4. **In case of no violations, then decrease the final time according to the equation**

$$t_{fnew} = t_{fold} \times 0.9$$

5. **Repeat steps 2 to 5 until achieving the minimum time (t_{fnew}) that don't have any violation for all joints and all data points.**
6. **Print the temporal data points $(t_i, q_i^j), i \in [1, N_S],\ j \in [1, N_{DOM}]$ of all joints in a form of a table. After this step, phase 1 of the motion planning is completed and the temporal data points $(t_i, q_i^j), i \in [1, N_S],\ j \in [1, N_{DOM}]$ are ready for phase 2.**

c) Spline motion planning (phase 2).

Given the temporal data points (t_i, q_i^j), it is required to generate a set of splines of order n $(generally\, n \geq 3)$ that connects a pair of two successive temporal data points (t_i, q_i^j) for $i \in [1, N_S],\ j \in [1, N_{DOM}]$.

1. **Assume the order of the splines to be used for the given data points.**
2. **Generate the splines for all data points of every joint.**
3. **Check the continuity conditions for the derivatives at the via points for all derivatives from the first till the $(n-1)$ derivative for all joints.**
4. **Find the analytical first and second derivatives for all splines of every joint over the time frame $(0, t_f)$.**
5. **Check whether the speed or acceleration over the given time frame violates the maximum speed limits for all joints $(\dot{\theta}_{i,max})$, or the maximum acceleration limits for all joints $(\ddot{\theta}_{i,max})$.**
6. **In case of any violation, then increase the final time according to the equation**

$$t_{fnew} = t_{fold} \times 1.1$$

7. **In case of no violations, then decrease the final time according to the equation**

$$t_{fnew} = t_{fold} \times 0.9$$

8. **In case of modification on t_{fnew} according to steps 6-7, then regenerate the original temporal data points (t_i, q_i^j) where only the time will be modified.**
9. **Repeat steps 2 to 8 until achieving the minimum time (t_{fnew}) that don't have any violation for all joints and all data points.**
10. **Print the generated splines for all data points of all joints in a form of a table. the motion planning is completed and ready to be executed in the manipulator.**

End of part I of the project