EFFICIENT INVESTMENTS IN PAKISTAN

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Abstract:

The Indian Subcontinent has been a lucrative space for investment by foreign beneficiaries since the dawn of modern history and organized society. From Vasco da Gama's arrival on the sandy shores of Goa, to the years of colonial rule by the British East India Company and then the Crown, the allure of the Subcontinent's resources and immense economic potential brought forth the Portuguese, the Dutch and the British to squabble for blood and land over decades of rule and war in the 18h and 19th centuries. Post-partition and hundreds of years later, the United States, Russia (formerly the Soviet Union), and the People's Republic of China now battle out with projects channeled through institutions like the World Bank, the International Monetary Fund, and the Asian Development Bank; via foreign direct investment; or through large scale infrastructural projects like the One Road One Belt Initiative. All of this points to one thing; Pakistan is a great place to invest with the wealth of resources it has.

The following paper seeks to evaluate why this is so. It seeks to evaluate the efficiency and veracity of investments in listed companies in the Islamic Republic of Pakistan applying a variety of techniques found in modern day financial and investment literature. First; it applies Harry Markowitz's Modern Portfolio Theory to Pakistan along with Data Analytics, Operations Research and Computer Science to devise a code which takes returns of specific indices or portfolios as input and plots efficient portfolios for specific risk profiles in the minimum amount of time as output. Second; it runs a comparative analysis between Pakistani Capital Markets and Capital Markets in significantly important investment havens across the world using statistical techniques found in William Sharpe's Capital Asset Pricing Model (CAPM). Third; it seeks to evaluate whether or not, according to Modigliani and Miller's Propositions in Finance, Pakistani companies are creating home-made leverage at an optimal rate, and whether or not if their Weighted Average

Cost of Capital (WACC) could be lowered. These three tenets seek to prove one key hypothesis; are investments efficient in Pakistan?

Acknowledgements and Memorandum

This paper emerged out of a desire by a group of friends to memorialize our undergraduate experience into a worthwhile senior year project and to serve as a tangible souvenir of our time at university.

This paper would not have been possible without the guidance and support of Dr. Atif Saeed Chaudhry. Thank you, Professor, for being the beacon that guided us during the pits of darkness and confusion when we were lost. Thank you, Professor, for teaching most of us in that one Intermediate Finance class in the fall of 2015 where we fell in love with the course material and decided to spend the remaining majority of our time at university pursuing it. And most importantly, thank you, Professor, for bringing us together.

Part 1 – Efficient Portfolios for Investors

Literature Review

Monte Carlo Simulation:

We have adopted an approach of Monte Carlo Simulation to develop the set of portfolios.

Monte Carlo simulation aims to use randomness in a deterministic manner. The idea of simulation was first proposed in the 18th century by the French scientist Louis LeClerc. This idea of simulation has evolved over time. Early experiments of simulations were to demonstrate the idea of certain theories in practice, or to supplement mathematical intuition particularly in the field of mathematical probability. The traditional applications of simulations aimed to provide an appendage to what were understood deterministic problems. This approach of simulation has changed over time, and increasingly simulation is being used to solve probabilistic analogs of deterministic problems, such as modelling interest rates and options on stock.

The concept of a modern Monte Carlo simulation came into being at the Manhattan Project in the development of Nuclear weapons (Harrison). The scope of these simulations has spread out to different fields that where actual experimentation is either costly or infeasible. The repeatability of these simulations with different conditions has led to its widespread acceptance in modern finance simulation.

The basic idea of a Monte Carlo simulation is split into three parts according to Sawilowsky (2003), which are:

1. Generation of a PDF each detailing the likelihood of an event. In our case, this probability is the likelihood of a particular stock being selected for the portfolio. We have decided two PDFs for this and have used them independently.

a. The first PDF is the uniform distribution. In this case all our stocks had an equal chance of being selected in the portfolio to be constructed. This pdf is fairly simple and assigns a probability of 1/n to each stock where n is the number of total stocks in consideration.

b. The second pdf we have established is that of a modified normal distribution. We established a normal distribution which was centered around μ, the volume weighted average of stock returns, and variance s^2, the variance of a portfolio containing stocks weighted by volume. This stems from our observation that stocks offering higher return to the consumer had an increasingly higher risk.

2. Repeated sampling from the established PDF's. This part is fairly simple and sampling was done iteratively up to a million times. In our case, this sampling established what portfolios were selected.

3. Recording and computing the statistics of interest. In our case, these statistics were the return and variance of the combined portfolio that was formed by a single iteration. We consequently stored these statistics in separate vectors, and plotted them later on.

Our application of the Monte Carlo simulation is more inclined towards the traditional approach. However, it differs from the initial simulations in two distinct ways. Firstly, we have incorporated a growth of some of our most efficient portfolios in the presence of stochastic interest rates mimicking the real-world situation. Secondly, we have treated our objective as something unknown, which makes the Monte Carlo simulations our primary source rather than an addendum to theoretically known solutions.

Quadratic Optimizations:

We have made use of the quadratic optimization algorithm to find either our maximum return or minimum risk for a given level of risk and return respectively. The quadratic optimization involves minimizing the output of a certain equation involving all terms that affect the output either as a linear or a quadratic term subject to certain linear constraints.

The methodology comprises of solving a system of Linear Equation based on the problem at hand. In general, the algorithm aims to minimize the objective function:

$\frac{1}{2}x^TQx + c^Tx$ where x, Q and c all represent vectors.

The constraints are represented by the system $Ex = d$. Then by simple application of basic algebra, the system reduces to the linear system represented below:

$$\begin{bmatrix} Q & E^T \\ E & 0 \end{bmatrix} \begin{bmatrix} x \\ \delta \end{bmatrix} = \begin{bmatrix} -c \\ d \end{bmatrix}$$

In the above set δ represents the set of Lagrange multipliers for the matrix. The solution to this can be obtained by an LU decomposition which has its basis in Linear Algebra.

The LU decomposition aims to split a given matrix into a product of Lower and upper triangular matrices. The methodology was introduced by Tadeusz Banachiewicz in 1938 (Schwarzenberg) and has since been used widely in building algorithms to automate the process of Gaussian elimination. The computer uses a series of numerical techniques in this regard to approximate the solution of the given system.

Solving the system described above yields a vector **x** which fulfills all the given constraints and minimizes our objective function. The above described methodology has been used widely in the theory of operations Research, however its application in finance is relatively new.

We however, did not go through the arduous task of writing down the entire program for this and instead opted to use the "Quad Prog" function found in the optimization toolbox. We have, however attached as reference the entire optimization algorithm that has been used. Our objective function was the minimization of risk for a given particular return. The output, and the only variable was the vector consisting of weights that had been assigned to individual stocks in our portfolio.

Markowitz Frontier:

The Harry Markowitz frontier is a corollary of the concept of Modern Portfolio Theory which was the brainchild of American economist Harry Markowitz and was first published in 1952 in the Journal of Finance (Markowitz). This theory has achieved widespread acceptance since then and still remains one of the cornerstones of investment literature.

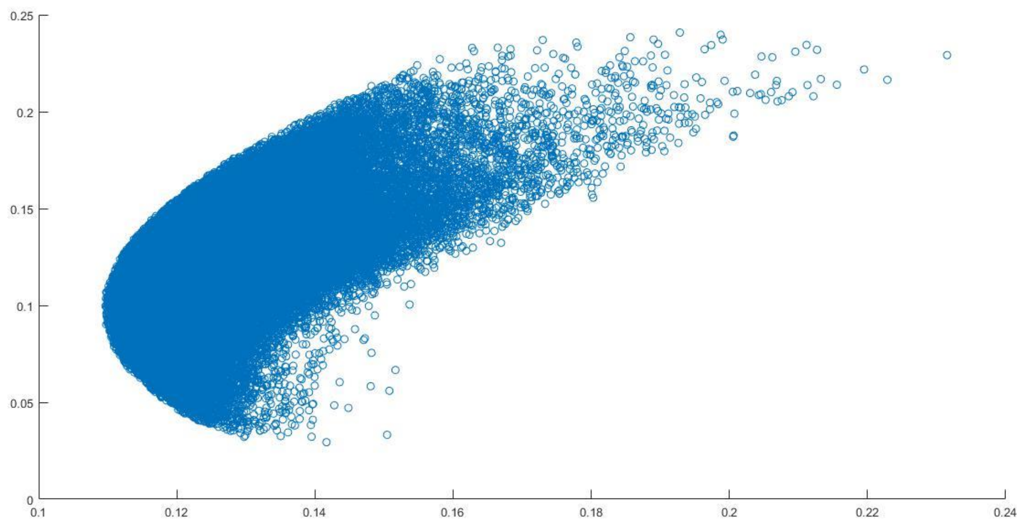
The theory builds itself on the concept of "high risk high return" and addresses how risks associated with individual stocks can be diversified by investing in a set of stocks. Investment in a series of stock that show independence (implied by no correlation in the returns) helps to lower down the risks of investment while continuing to achieve the same level of return.

The Harry Markowitz frontier is simply a collection of points that show the maximum returns associated with each level of risk.

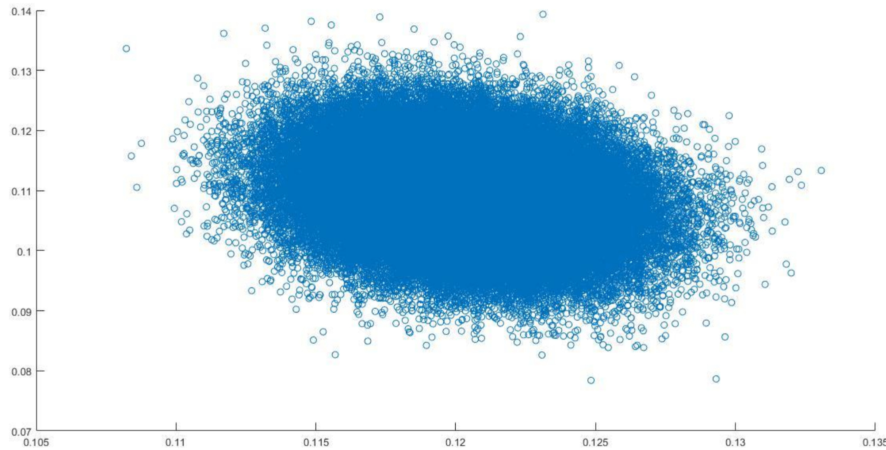
Methodology and Analysis:

We started off with collecting data for all PSX stocks. We developed an algorithm which took the returns of a number of stocks and ran around 100,000 iterations to create 100,000 different random portfolios. The aim behind creating these portfolios and plotting them on a risk-return graph was that the outline of these portfolios would give us the Markowitz Efficient Frontier. Another separate code was written to make the frontier using Quadratic Optimization. The logic behind this was minimizing the risk for a specific return with the weights as decision variables. The code is explained in detail at the end (Exhibit A).

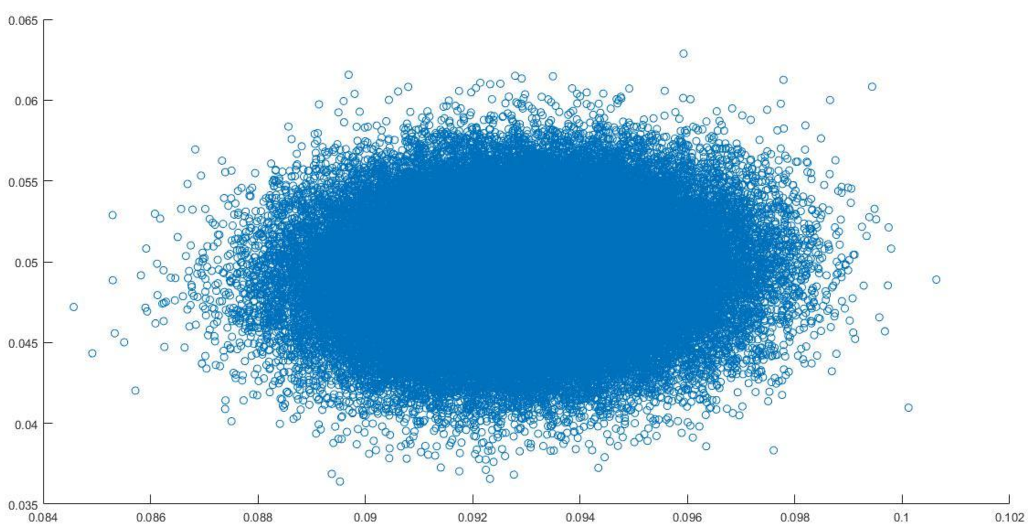
As a test run, we took the returns of 5 high performing stocks (namely HBL, OGDC, UBL, ENGRO and PPL). The code ran successfully and we got a nicely shaped efficient frontier. This success meant that our code was working properly.



As a next step, we collected the returns of all the stocks in the entire PSX. Out of these 543 stocks we took 100 random stocks as input. The reason behind this was to see if the code stays intact and gives an output without any memory or other errors. The code worked but the output was faulty and the shape of the efficient frontier was compromised.

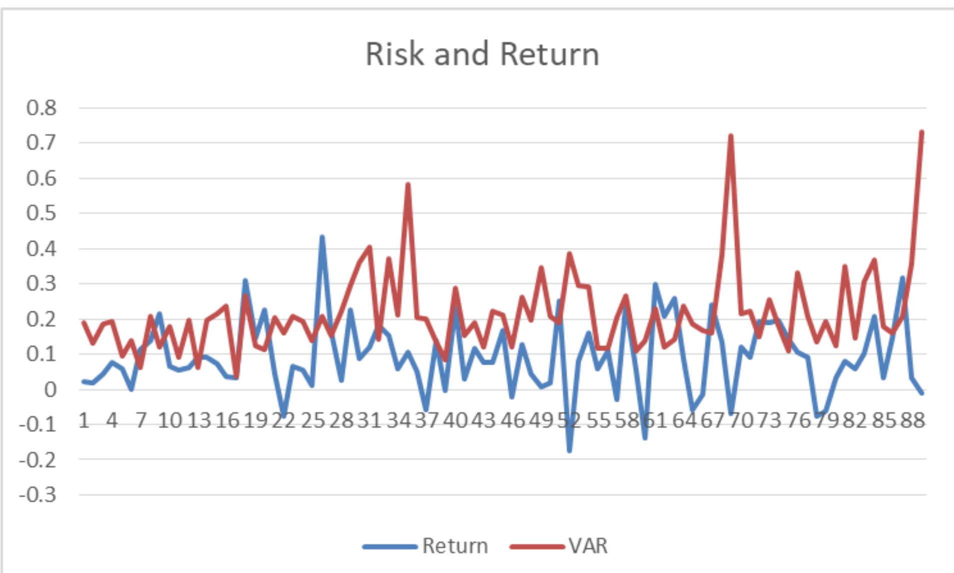


We disregarded this result as we just wanted to check if the code is able to handle the 100 stock returns (a total of 4000 data points). Then to further check the efficiency of the code, we ran the entire 543 stocks and the code still worked with a bit of time delay due to 21,720 data points as input and a total of 54,300,000 outputs.

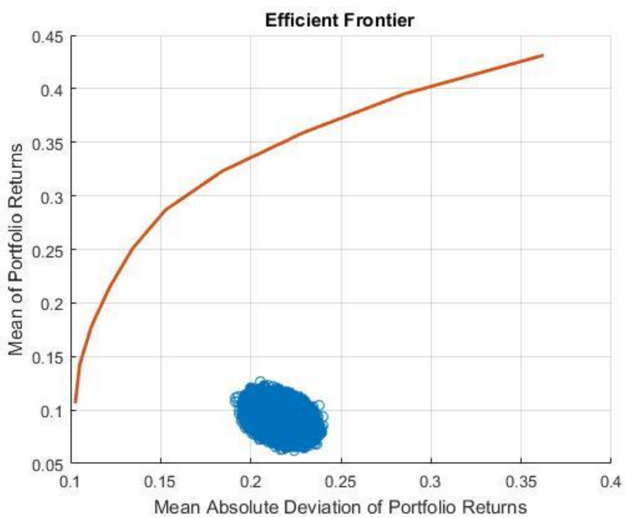


After testing the code extensively, we started to look for the reasons behind the amorphous shaped efficient frontiers. After careful analysis, we came to the conclusion that using random 100 stocks from PSX or the whole PSX itself is the reason behind this bizarre results. This is because many of these stocks are not actively traded and thus have a stock price of 0. We decided, since majority of the investors invest in PSX 100, it would be wise to use the PSX 100 stocks as input. We then reduced the stocks from 100 to 89 as 11 of the 100 portfolios had the same inactive trading due to which their change in price was 0. This reduced our final data to 89 stocks from PSX 100 and thus a total of 3,560 data points.

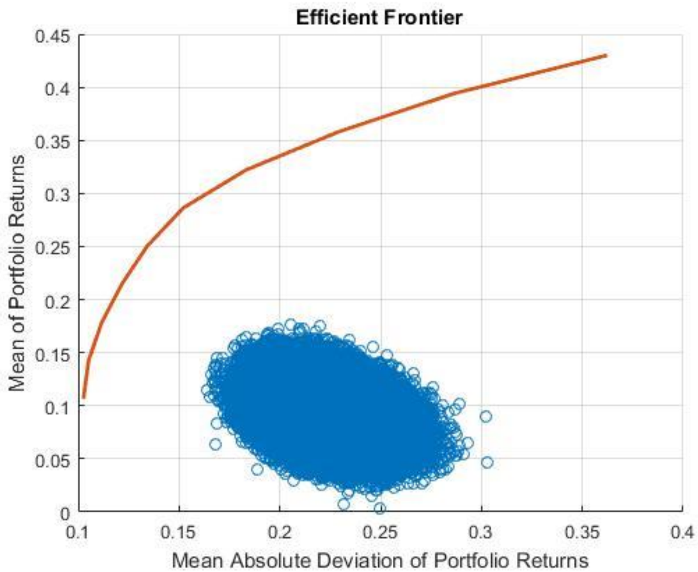
In order to analyze the stock returns, we took average returns and variance and annualized them. We found the correlation between returns (average returns) and risk (variance). The result was surprising as the correlation came out to be -0.005712. Even though this is a very small negative value, this was a surprising and interesting result as we have always studied how increased risk gives increased returns. The graph of returns and risk is given later in the paper.



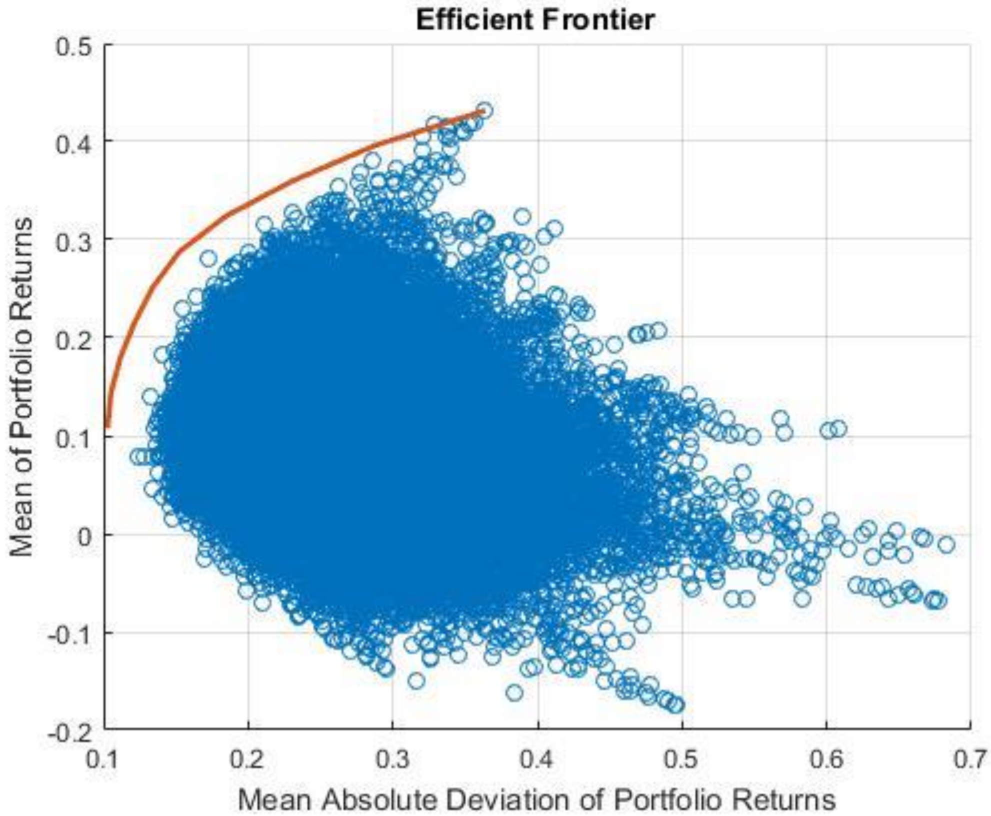
Finally, when we used the 89 stock returns as input the result was faulty. All the portfolios were clustered in a corner with returns just varying over a small range. In order to analyze the issue, we stored all the weights of the 100,000 portfolios to study them for possible problems. There were a total of 8,900,000 different weights calculated using the “randn” function. The maximum weight assigned was just 7.3% and the total variance among the weights was 0.0001. This led us to conclude how using “randn” was an issue and the root of this problem. The reason for this was that “randn” gives normally distributed random variables between -1 and 1. We used absolute values and took ratios to ensure the sum of weights is 1. Due to the normal distribution, all the weights were concentrated towards 0 due to the bell-shaped nature of normal distribution



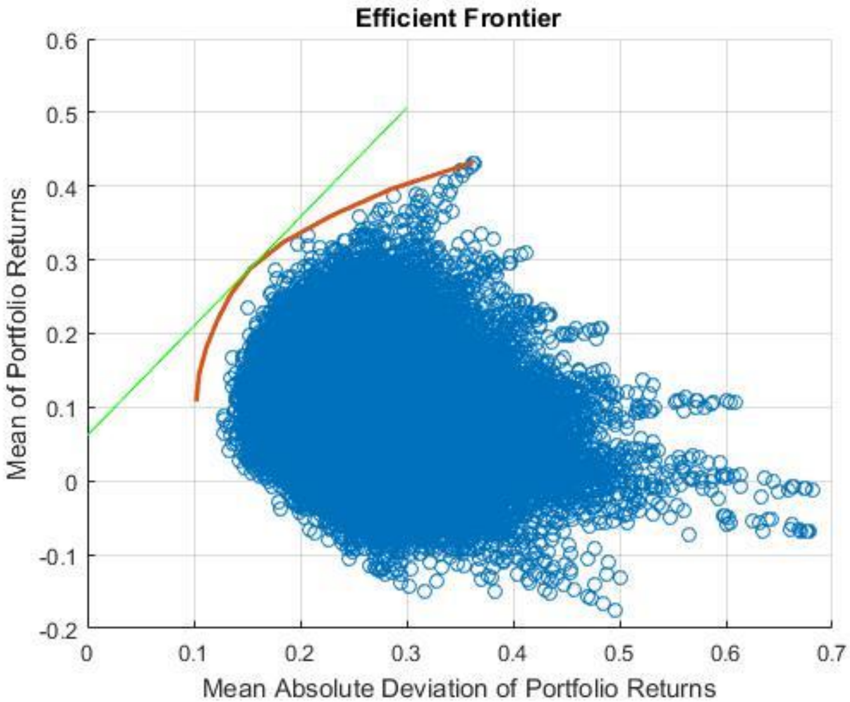
In order to get rid of this issue, we used the “rand” function instead. The rand function gave us uniform random variables between 0 and 1. This handled the issue of concentration towards 0 since any value between 0 and 1 was equally likely to be picked. However, the desired result was still not achieved. The reason behind this, after further analysis, was again the low variance between all the weights. This could be attributed to the fact that since we were assigning weights to all 89 stocks, and 89 random variables between 0 and 1 are a lot, the variation was insignificant.



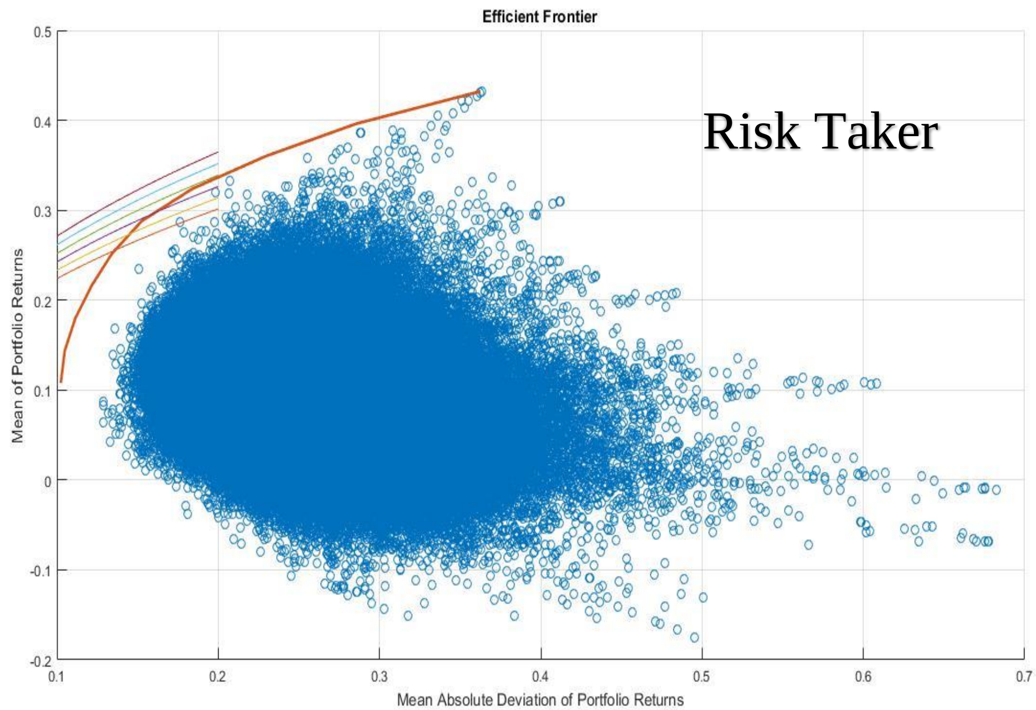
To remove this discrepancy, we decided to randomize the number of stocks that are assigned weights in every iteration. This was achieved by setting the weights above 10% to 0 and adjusting the remaining weights to make sum of weights 100%. By this method, we were able to randomize the number of stocks in every iteration. For example, in one iteration it is possible only 10 stocks are assigned weights greater than 0 and the rest are assigned 0 and in the next iteration 25 stocks are assigned weights greater than 0 and the rest are assigned 0. This proved to be successful and upon running the code a few times we were able to generate the proper efficient portfolio which lined up with the optimization result.

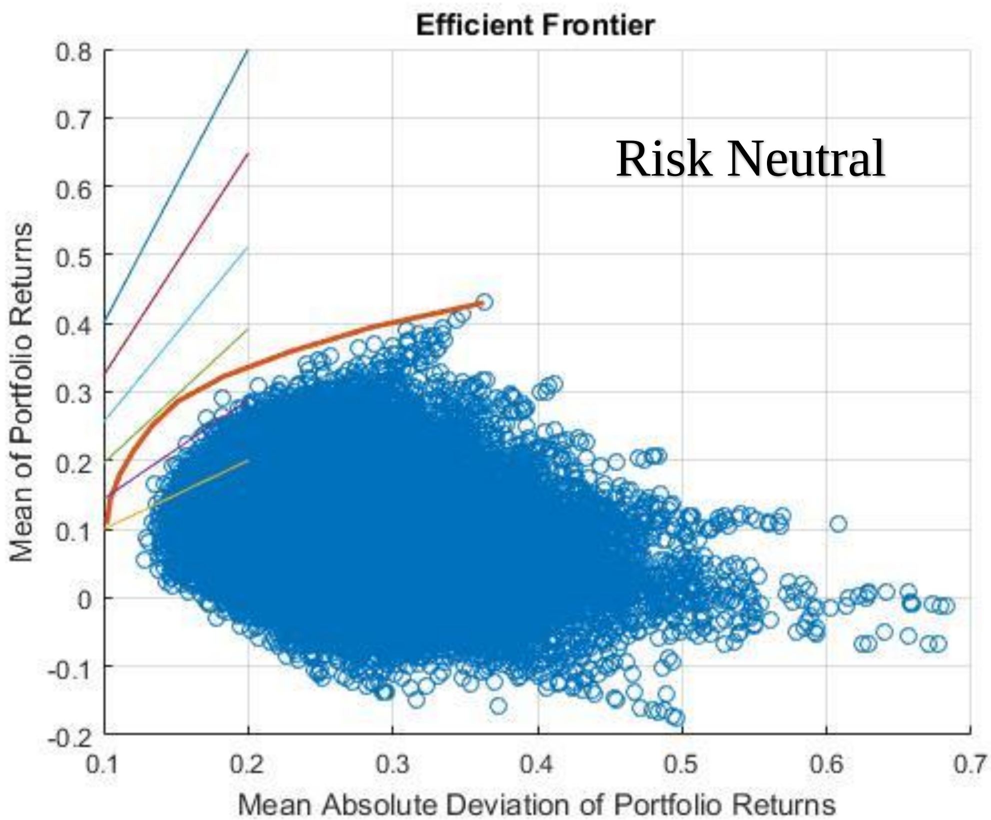


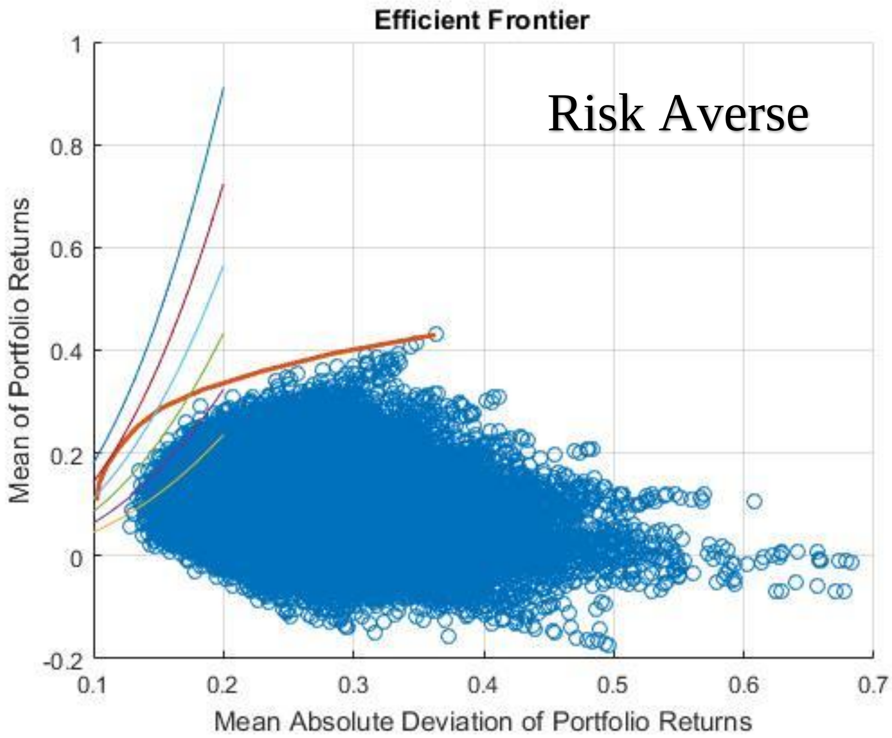
To make the Capital Market Line, we took the risk free rate to be 6-month KIBOR 6.19%. Then we plotted multiple equations with different gradients. The line which was tangent to the efficient frontier gave us the equation of the CML which was **return = 0.0619 + 1.485*risk.** The market efficient portfolio return came out to be 28.8% with corresponding risk of 15.28%. CML plot is given below and the weights of the market efficient portfolio is given in Exhibit B.



To study the behaviors of investors with different risk profiles, we used a specific functional form and varied parameters to incorporate risk neutral, risk averse and risk takers. The functional form that was used was $U(risk, ret) = ret^{\alpha} * risk^{-\beta}$. For the risk taker investor we took alpha as 0.7 and beta as 0.3. The optimum portfolio gave a return of 32.5% with 18.57% risk. For the risk neutral investor we took alpha as 0.5 and beta as 0.5. The optimum portfolio gave a return of 27.5% with 14.51% risk. For the risk averse investor we took alpha as 0.3 and beta as 0.7. The optimum portfolio gave a return of 18.5% with 11.29% risk (Exhibit C)


Efficient Frontier
Risk Taker
Mean of Portfolio Returns
Mean Absolute Deviation of Portfolio Returns
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0.1
0.2
0.3
0.4
0.5
0.6
0.7


Efficient Frontier
Risk Neutral
Mean of Portfolio Returns
Mean Absolute Deviation of Portfolio Returns
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0.1
0.2
0.3
0.4
0.5
0.6
0.7


Efficient Frontier
Risk Averse
Mean of Portfolio Returns
Mean Absolute Deviation of Portfolio Returns
1
0.8
0.6
0.4
0.2
0
-0.2
0.1
0.2
0.3
0.4
0.5
0.6
0.7

Part 2 – The Capital Asset Pricing Model and Public Comparables

Literature Review

This sections adopts two things from established finance practice and literature. First it utilizes the technique of Public Comparables, or 'Public Comps' that are found in modern finance theory (Bruner). This techniques employs a variety of ratios to evaluate the target price or multiples at which a certain security should be trading at a certain point in time. This is used in practice in the vast majority of cases by Investment Banks and Private Equity funds across the world such as Blackstone, Goldman Sachs and Morgan Stanley to price securities that have an active market. They actively prefer this technique over the Discounted Cash Flow (DCF) method and the Divided Discount Model (DDM) for valuation purposes because of the highly discretionary nature of how those two methods operate in terms of valuations in the first place. This means that analysts in terms of carrying out their valuations may tweak their assumptions and base case scenarios to get whatever valuation they desire, and justify their assumptions accordingly.

Why does this make the Public Comps method superior to this? Being quite literally a manifestation of the valuation by multiples approach, it incorporates the biases and imperfections of the market and how active traders view securities. It therefore gives the most reasonable estimate of how much you can expect a security to sell at in terms of a multiple over a given period in time. The ratios and multiples in question over here include, but are not in any way whatsoever limited to, the Price to Earnings Ratio (PE), the EBITDA Multiple (EV/EBITDA), and the Net Debt to Equity Ratio. The ones that we have ended up utilizing for our analysis in this scenario was the use of the Price to Earnings ratio to evaluate whether or not on the comparative are Pakistani companies in certain sectors overvalued compared to their counterparts in other emerging markets.

The second method this section employs is an analysis of Betas. Betas are a measure of market risk for securities that are traded. We did not utilize in our analysis Betas of specific stocks though however. We utilized the methodology of Betas as measures of risk in terms valuing one market versus another. This technique was employed to measure how risky an investment in one stock exchange is versus another, and calculating the Beta of a certain stock exchange whilst assuming a stable market as the base for it. The base that we assumed in the vast majority of our analyses was the NASDAQ and the NYSE and compared it to various regional markets – including Pakistan in the first place.

Why is the Beta so important? In its calculation over here in the first place, it incorporate the covariance of the variable being analyzed with the base and the variance of the base to give a final value. This covariance is an excellent measure of to what degree an exchange's returns are pegged to another. To develop a working criteria of what is best on these terms, we must understand the interpretation of the Beta itself. A negative Beta stock is an excellent addition to a portfolio as its returns are going in the opposite direction of what the market is going in. Basically in terms of risk, the lower the Beta, the better.

Methodology, Results and Interpretations

Our approach towards public comparables was structured to evaluate the Price to Earnings ratios of various countries sectors in Oil and Gas, Chemicals, Pharmaceuticals, and Power Generation.

The objective of this was to evaluate if Pakistani firms operating in the same sector relative to other emerging markets were overvalued or undervalued. A higher average P/E ratio of the emerging markets would mean that Pakistani markets are undervalued and would therefore be a good investment.

The sample size we chose included India, Indonesia, Malaysia, and Thailand to compare to Pakistan. While all of these countries are experiencing on average much greater economic prosperity compared to Pakistan, they are still very similar in key areas such as demographics, resources, the means and concentration of production, income levels and urbanization rates. This means that even though they maybe much further ahead of Pakistan in the game of the development, Pakistan will probably end up following a similar trajectory and therefore a comparison is apt to begin with. The results of our analysis are given as follows:

P/E Average Multiples
Country and Industry Specific Comparables

Industry	India	Indonesia	Malaysia	Thailand	Pakistan	Emerging Market Average
Oil and Gas	17.49x	14.76x	30.84x	18.27x	10.74x	20.34x
Chemical	36.60x	23.76x	16.31x	17.03x	16.28x	23.43x
Pharma	35.69x	24.15x	15.17x	15.59x	14.07x	22.65x
Power	27.26x	20.69x	15.12x	12.90x	6.88x	18.99x

This further confirmed what we felt was going to be an almost certain outcome, that it is worthwhile to invest in Pakistan because it is relatively cheaper for them compared to other markets. This is particularly true for the international investor that has a choice between investing in multiple markets.

For our segment on Betas, our data that was extracted was from Standard and Poor's (S&P) Capital IQ proprietary software and the Bloomberg Terminal available in the LUMS Trading Lab. Our prices for American, Indian, Pakistani, Chinese, London, and Hong Kong were extracted from S&P's platform for the past 10 years, while the second part of our data which was 5 year spot prices for the US Dollar in terms of the currencies of these respective regions was extracted from the Bloomberg Terminal.

Why did we need foreign currency spot rates? The return which was not adjusted for foreign currency fluctuations was insufficient, as it did not account for the appreciation/depreciation of the currency in question. The return of the index including the increase in value of the currency would be the best measurement of this. The following output was given as a consequence of our calculations:

		Pakistan	India	Hong Kong	London	China	NYSE	NASDAQ
WITHOUT FX ADJ	Variance	0.00008	0.00014	0.00010	0.00007	0.00025	0.00005	0.00008
	Covariance with NYSE	0.00000	0.00003	0.00002	0.00003	0.00002	0.00005	0.00005
	Beta (NYSE)	0.07538	0.47147	0.37531	0.50403	0.30583	1.00000	1.01106
	Covariance with NASDAQ	0.00000	0.00003	0.00002	0.00003	0.00002	0.00005	0.00008
	Beta (NASDAQ)	0.04657	0.34581	0.29319	0.39691	0.25943	0.66715	1.00000

		Pakistan	India	Hong Kong	London	China	NYSE	NASDAQ
WITH FX ADJ	Variance	0.00009	0.00009	0.00010	0.00010	0.00025	0.00005	0.00008
	Covariance with NYSE	0.00000	0.00002	0.00002	0.00002	0.00002	0.00005	0.00005
	Beta (NYSE)	0.06852	0.38303	0.37239	0.38988	0.30288	1.00000	1.01106
	Covariance with NASDAQ	0.00000	0.00002	0.00002	0.00003	0.00002	0.00005	0.00008
	Beta (NASDAQ)	0.04292	0.28218	0.28985	0.31929	0.25599	0.66715	1.00000

Figure X:

These results confirm our suspicions that relative to other markets Pakistan is relatively less risky to invest in as we can see that the value of a currency adjusted Beta as well as a non-currency adjusted Beta is very low which means that it is a low risk investment to begin with.

Part 3 – Zero Debt Capital Structures

Literature Review

The Modigliani-Miller Theorem seeks to define the irrelevance of the capital structure of a company to its WACC. The theory comprises of a number of prepositions. The first preposition states that under a number of assumptions, the firm's market value is not affected by debt-equity ratio. The second preposition seeks to prove that a firm's leverage does not affect its WACC. The third preposition states that a firm's dividend policy does not affect its market value. And lastly, the fourth proposition states that the shareholders of the firm are indifferent to the firm's financial policy. (MM 1958)

Further analysis on this theory analyses the effect of debt when the assumptions are relaxed. The MM trade off theory assumes that there exists an optimal structure where the WACC will be lowest. This happens because of the benefits of taxation which tends to reduce the WACC of the company up to a particular point after which the WACC starts increasing again due to bankruptcy costs. (Kendy, et al.)

Analysis

MM Theory initially proposed that the choice of a capital structure is irrelevant for the form since it will have no effect on WACC. However, the theory has been further evaluated to account for the effect of corporate taxation. The tax benefits under debt reduce the WACC of a firm as debt is added. This implies that a 99.9% debt financed capital structure is optimum. This begs the question that why companies are still equity financed. The explanation given by MM is that the bankruptcy costs eventually offset the advantages of using debt in the capital structure. This paper uses two companies from Pakistan to study the effects of debt of WACC.

This paper also seeks to evaluate the phenomena of zero debt companies in Pakistan. During the analysis of various companies and their returns, it was discovered that a few of the major listed companies have a zero debt capital structure. This goes against the famous MM theory which proposes that using debt in a company's capital structure reduces the WACC. Hence, the further analysis was undertaken to discover the reasons for this.

Two companies have been chosen as a sample. Pakistan Petroleum Limited, one of the major players in the oil and gas sector of Pakistan, follows the zero debt capital structure. The calculation of their WACC came out to be 10.9% through the CAPM, assuming Market return as 11% (historic returns over 5 years) and risk free rate as 6.19% (taken from 10 year treasury bonds). The forward looking beta has been extracted from Bloomberg. The following scenario analysis has been done for the WACC of PPL assuming they introduce debt into their structure. The cost of debt used is the cost that is available to PPL's competitor, Mari Petroleum. They have attained long term loans at the cost of KIBOR + 0.15.

% Debt	0%	5%	10%	15%
WACC	10.9%	10.6%	10.4%	10.2%

As it can be seen from the table above, the WACC falls as debt increases in the capital structure of PPL. So why is it that these companies prefer to stay debt free? Various explanations could be put forward.

Firstly, most of the zero debt companies in Pakistan are government owned. Hence, financing does not seem to be of utmost importance to them. PPL has sufficient cash on hand in all years which shows that the liquidity ratios are above par. The following table shows the Cash flow situation of PPL.

	2012	**2013**	**2014**	**2015**	**2016**	**2017**
Cash flow per share	22.48	17.51	10.97	11.52	11.30	12.68
CFO/Total Liabilities	81.14	104.43	67.43	76.03	69.33	38.36

Secondly, although Mari Petroleum uses debt financing, it might not be safe to assume that PPL and other firms will be able to achieve financing on the same level at the same rates. One significant factor that was identified during research was the limited portfolio of financial instruments available to companies in Pakistan. The only debt that is traded is government treasury bills and PIBs (Pakistan Investment Bonds). Due to a lack of efficient pricing mechanisms of debt securities, investors are hesitant in making investments in corporate bonds which is why companies are unable to make use of corporate debt to finance their capital structures. Historically, Packages ltd was the first company to offer corporate bonds in 1995 in Pakistan but the issue was not successful because of the above mentioned reasons. In the current market, the available corporate bonds are in the form of Term Finance Certificates.

In the current scenario, Term Finance Certificates have the potential to offer better returns to investors and cheaper sources of borrowing to companies. If Pakistan is able to overcome the hurdles to the trade of such financial instruments, then it can be said that the companies will be able to achieve a lower WACC and will be able to fully utilise the advantages of debt in their corporate structures.

Exhibits

Exhibit A (MATLAB Codes)

portfoliogeneration.m

```
hold on
stockcov = cov(data);
%calculates a nxn covariance matrix of all the n stocks present in data
stockreturns = mean(data);
%calculates a 1xn matrix of the expected returns of each of the n stocks
stockcov = 4.*stockcov;
%annualizes the expected returns
stockreturns = 4.*stockreturns;
%annualizes the expected returns
iterations = 1000000;
%n = count(stockreturns);
W = zeros(iterations,89);

%start of a loop that will give 100,000 different portfolios
%records portfolio returns and portfolio variances
for i = 1:iterations
    weights = (rand([1 89]));
    for j = 1:89
        if (weights(1,j) > 0.1)
            weights(1,j) = 0;
        end
    end
    weights = weights./(sum(weights));
    weights = round(weights,3);
    %above set of codes generate a 1*n vector of weights summing to 1
    portret(1,i) = stockreturns*(weights.');
    %stores portfolio return in a vector
    portvar(1,i) = (weights)*(stockcov)*(weights.');
    %stores portfolio variance in a vector
end
portsd = portvar.^0.5;
portsd = portsd./(sqrt(pi/2));
%calculated portfolio sd from portfolio variances
scatter(portsd,portret)
%plots sd against return

AssetScenarios = mvnrnd(stockreturns, stockcov, 100000);
p = PortfolioMAD;
p = setScenarios(p, AssetScenarios);
p = setDefaultConstraints(p);
plotFrontier(p);
%logic behind above code in minmeanvar.m
```

minmeanvar.m

```
load('datamatlab.mat')
stockcov = cov(data);
%calculates a nxn covariance matrix of all the n stocks present in data
stockreturns = mean(data);
%calculates a 1xn matrix of the expected returns of each of the n stocks
stockcov = 4.*stockcov;
%annualizes the expected returns
stockreturns = 4.*stockreturns;
%annualizes the expected returns

for i = 1:38
nAssets = numel(stockreturns);
rate = 0.05+(i/100);
Aeq = ones(1,nAssets);
beq = 1;
Aineq= -stockreturns;
bineq = -rate;
lb = zeros(nAssets,1);
ub = ones(nAssets,1);
c = zeros(nAssets,1);

options = optimoptions('quadprog','Algorithm','interior-point-convex');

tic;
weightsforrate = quadprog(stockcov,c,Aineq,bineq,Aeq,beq,lb,ub,[],options);
toc;

optimumret(i) = stockreturns*weightsforrate;
minimumvar = (weightsforrate.')*stockcov*weightsforrate;
minimumsd = minimumvar.^0.5;
minimummad(i) = minimumsd./(sqrt(pi/2));
end

plot(minimummad,optimumret);
```

rootforoptimization.m

```
%input rate for which you want minimum mean variance portfolio
rate = 0.275;

[weights ret mad] = optimumportfolio(rate);
```

optimumportfolio.m

```
function [weightsforrate, optimumret, minimummad] = optimumportfolio(rate)

load('datamatlab.mat');
stockcov = cov(data);
%calculates a nxn covariance matrix of all the n stocks present in data
stockreturns = mean(data);
%calculates a 1xn matrix of the expected returns of each of the n stocks
stockcov = 4.*stockcov;
%annualizes the expected returns
stockreturns = 4.*stockreturns;
%annualizes the expected returns

%optimization function
nAssets = numel(stockreturns);
Aeq = ones(1,nAssets);
beq = 1;
Aineq= -stockreturns;
bineq = -rate;
lb = zeros(nAssets,1);
ub = ones(nAssets,1);
c = zeros(nAssets,1);

options = optimoptions('quadprog','Algorithm','interior-point-convex');

weightsforrate = quadprog(stockcov,c,Aineq,bineq,Aeq,beq,lb,ub,[],options);

%calculation of return to check and the minimum variance
optimumret = stockreturns*weightsforrate;
minimumvar = (weightsforrate.')*stockcov*weightsforrate;
minimumsd = minimumvar.^0.5;
minimummad = minimumsd./(sqrt(pi/2));


end
```

sml.m

```
openfig('figures.fig');
hold on
rfr = 0.0619;
x_axis = 0:0.1:0.3;
for i = 1.485
    sml_i = rfr + (i.*x_axis);
    plot(x_axis,sml_i,'g-')
end

%plots the sml for the efficient frontier
```

curves.m

```
openfig('figures.fig');
hold on
%alpha and beta vary with the risk profile
alpha = 0.5;
beta = 0.5;
risk = 0.1:0.0005:0.2;

for i = 1:0.2:2
ret = ((i)./(risk.^(-beta))).^(1/alpha);
plot(risk,ret)
end

%manual inputs allow to make ICs for different investor behaviours
```

Exhibit B (Weights of the Market Efficient Portfolio)

Stocks	Efficient Portfolio
HBL PA Equity	0.00%
OGDC PA Equity	0.00%
UBL PA Equity	0.00%
ENGRO PA Equity	0.00%
PPL PA Equity	0.00%
MCB PA Equity	0.00%
HUBC PA Equity	0.00%
LUCK PA Equity	0.00%
NESTLE PA Equity	3.18%
POL PA Equity	0.00%
FFC PA Equity	0.00%
PSO PA Equity	0.00%
BAHL PA Equity	0.00%
DAWH PA Equity	0.00%
SNGP PA Equity	0.00%
DGKC PA Equity	0.00%
KAPCO PA Equity	0.00%
MARI PA Equity	0.00%
MEBL PA Equity	0.00%
MTL PA Equity	20.54%
NML PA Equity	0.00%
NBP PA Equity	0.00%
PSEL PA Equity	0.00%

FCCL PA Equity	0.00%
BAFL PA Equity	0.00%
SEARL PA Equity	18.01%
INDU PA Equity	0.00%
KEL PA Equity	0.00%
PAKT PA Equity	0.00%
PAEL PA Equity	0.00%
MLCF PA Equity	0.00%
THALL PA Equity	0.00%
KTML PA Equity	0.00%
PKGS PA Equity	0.00%
TRG PA Equity	0.00%
NRL PA Equity	0.00%
AICL PA Equity	0.00%
ABOT PA Equity	0.00%
HMB PA Equity	0.00%
HCAR PA Equity	0.00%
ABL PA Equity	0.00%
INIL PA Equity	0.00%
APL PA Equity	0.00%
ATRL PA Equity	0.00%
ICI PA Equity	0.00%
FFBL PA Equity	0.00%
CHCC PA Equity	0.00%
SSGC PA Equity	0.00%
EFUG PA Equity	0.00%
PSMC PA Equity	0.00%
COLG PA Equity	8.72%
BOP PA Equity	0.00%
PIOC PA Equity	0.00%
PMPK PA Equity	0.00%
GLAXO PA Equity	0.00%
IGIIL PA Equity	0.00%
FABL PA Equity	0.00%
MUREB PA Equity	0.00%
SHEL PA Equity	0.00%
PTC PA Equity	0.00%
NATF PA Equity	0.67%
JDWS PA Equity	0.00%
JLICL PA Equity	16.04%
NCL PA Equity	0.00%

AKBL PA Equity	0.00%
PGF PA Equity	0.00%
SHFA PA Equity	7.66%
KOHC PA Equity	0.00%
JSCL PA Equity	0.00%
HUMNL PA Equity	0.00%
CSAP PA Equity	0.00%
ATLH PA Equity	0.00%
PICT PA Equity	0.00%
BATA PA Equity	0.00%
GHGL PA Equity	0.00%
BWCL PA Equity	0.00%
ACPL PA Equity	0.00%
SCBPL PA Equity	0.00%
SNBL PA Equity	0.00%
JGICL PA Equity	0.00%
CPPL PA Equity	0.00%
OLPL PA Equity	0.00%
GADT PA Equity	0.00%
FML PA Equity	4.88%
IBFL PA Equity	0.00%
ARM PA Equity	0.53%
PUNO PA Equity	19.76%
BNWM PA Equity	0.00%
CJPL PA Equity	0.00%
Return	28.80%
MAD	15.28%

Exhibit C (Weights of Risk Profile Portfolios)

Stocks	Risk Taker	Risk Averse	Risk Neutral
HBL PA Equity	0.00%	0.00%	0.00%
OGDC PA Equity	0.00%	0.00%	0.00%
UBL PA Equity	0.00%	0.00%	0.00%
ENGRO PA Equity	0.00%	0.00%	0.00%
PPL PA Equity	0.00%	0.00%	0.00%
MCB PA Equity	0.00%	0.00%	0.00%
HUBC PA Equity	0.00%	0.01%	0.00%
LUCK PA Equity	0.00%	0.00%	0.00%
NESTLE PA Equity	0.02%	2.14%	4.01%
POL PA Equity	0.00%	0.00%	0.00%
FFC PA Equity	0.00%	0.15%	0.00%
PSO PA Equity	0.00%	0.00%	0.00%
BAHL PA Equity	0.00%	0.74%	0.00%
DAWH PA Equity	0.00%	0.00%	0.00%
SNGP PA Equity	0.00%	0.00%	0.00%
DGKC PA Equity	0.00%	0.00%	0.00%
KAPCO PA Equity	0.00%	22.23%	0.00%
MARI PA Equity	0.00%	0.00%	0.00%
MEBL PA Equity	0.00%	0.00%	0.00%
MTL PA Equity	15.86%	14.88%	21.57%
NML PA Equity	0.00%	0.00%	0.00%
NBP PA Equity	0.00%	0.00%	0.00%
PSEL PA Equity	0.00%	0.00%	0.00%
FCCL PA Equity	0.00%	0.00%	0.00%
BAFL PA Equity	0.00%	0.00%	0.00%
SEARL PA Equity	34.48%	0.95%	13.90%
INDU PA Equity	0.00%	0.00%	0.00%
KEL PA Equity	0.00%	0.00%	0.00%
PAKT PA Equity	0.00%	0.00%	0.00%
PAEL PA Equity	0.00%	0.00%	0.00%
MLCF PA Equity	0.00%	0.00%	0.00%
THALL PA Equity	0.00%	0.00%	0.00%
KTML PA Equity	0.00%	0.00%	0.00%
PKGS PA Equity	0.00%	0.00%	0.00%
TRG PA Equity	0.00%	0.00%	0.00%

NRL PA Equity	0.00%	0.00%	0.00%
AICL PA Equity	0.00%	0.00%	0.00%
ABOT PA Equity	0.00%	0.00%	0.00%
HMB PA Equity	0.00%	0.00%	0.00%
HCAR PA Equity	0.00%	0.00%	0.00%
ABL PA Equity	0.00%	0.00%	0.00%
INIL PA Equity	0.00%	0.00%	0.00%
APL PA Equity	0.00%	0.00%	0.00%
ATRL PA Equity	0.00%	0.00%	0.00%
ICI PA Equity	0.00%	0.00%	0.00%
FFBL PA Equity	0.00%	0.00%	0.00%
CHCC PA Equity	0.00%	0.00%	0.00%
SSGC PA Equity	0.00%	0.00%	0.00%
EFUG PA Equity	0.00%	0.00%	0.00%
PSMC PA Equity	0.00%	0.00%	0.00%
COLG PA Equity	5.99%	6.55%	8.01%
BOP PA Equity	0.00%	0.00%	0.00%
PIOC PA Equity	0.00%	0.00%	0.00%
PMPK PA Equity	0.00%	5.19%	0.53%
GLAXO PA Equity	0.00%	0.00%	0.00%
IGIIL PA Equity	0.00%	0.00%	0.00%
FABL PA Equity	0.00%	0.00%	0.00%
MUREB PA Equity	0.00%	0.00%	0.00%
SHEL PA Equity	0.00%	0.00%	0.00%
PTC PA Equity	0.00%	0.00%	0.00%
NATF PA Equity	4.06%	0.00%	0.01%
JDWS PA Equity	0.00%	0.00%	0.00%
JLICL PA Equity	16.30%	5.81%	14.80%
NCL PA Equity	0.00%	0.00%	0.00%
AKBL PA Equity	0.00%	0.00%	0.00%
PGF PA Equity	0.00%	0.00%	0.00%
SHFA PA Equity	1.23%	6.33%	8.31%
KOHC PA Equity	0.00%	0.00%	0.00%
JSCL PA Equity	0.00%	0.00%	0.00%
HUMNL PA Equity	0.00%	0.00%	0.00%
CSAP PA Equity	0.00%	0.00%	0.00%
ATLH PA Equity	0.00%	0.79%	0.00%
PICT PA Equity	0.00%	0.00%	0.00%
BATA PA Equity	0.00%	0.03%	0.00%
GHGL PA Equity	0.00%	4.43%	0.00%

BWCL PA Equity	0.00%	0.00%	0.00%
ACPL PA Equity	0.00%	0.00%	0.00%
SCBPL PA Equity	0.00%	0.00%	0.00%
SNBL PA Equity	0.00%	0.00%	0.00%
JGICL PA Equity	0.00%	0.00%	0.00%
CPPL PA Equity	0.00%	0.00%	0.00%
OLPL PA Equity	0.00%	0.00%	0.00%
GADT PA Equity	0.00%	0.00%	0.00%
FML PA Equity	1.39%	5.11%	5.34%
IBFL PA Equity	0.00%	0.00%	0.00%
ARM PA Equity	0.00%	9.84%	4.01%
PUNO PA Equity	20.64%	14.81%	19.50%
BNWM PA Equity	0.00%	0.00%	0.00%
CJPL PA Equity	0.00%	0.00%	0.00%
Return	32.50%	18.50%	27.50%
MAD	18.57%	11.29%	14.51%

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