Rotman School of Management
UNIVERSITY OF TORONTO

Rotman

Group Number: __________N/A________________

Assignment Title: ________RSM2307 Assignment 1___

Course Code: RSM 2307______________________

Section #: 1 2 3 4

Course Name: ____Advanced Derivatives__

Professor Name: __Redouane Elkamhi _

In submitting this **group** work for grading, we confirm:

- That the work is original, and due credit is given to others where appropriate.
- That all members have contributed substantially and proportionally to each group assignment.
- That all members have sufficient familiarity with the entire contents of the group assignment so as to be able to sign off on them as original work.
- Acceptance and acknowledgement that assignments found to be plagiarized in any way will be subject to sanctions under the University's Code of Behaviour on Academic Matters.

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Assignments are to be submitted using student ID numbers only; do not include your name. Please note that assignments that include names or that do not have the box above checked **will not be graded.**

Question 1 ***TD Principal Protected Notes***

Question 1: TD Principal Protected Note - Part A

Detailed steps of our Approach

Step #	Volatility
1	Assumption: Todays date is the 25th April 2013 and we are going to value the note
2	To estimate the volatility, we look at past share price development of each stock in the basket between 1st January 2008 to 24th April 2013
3	We calculate the continuous return for each stock for each day. By averaging the daily returns of each stock, we get the portfolio return for each day. The stocks are equally weighted in the basket
4	The standard deviation of the portfolio return gives us the daily volatility. We annualize it for further calculations

Step #	Dividend Yield
1	The factsheet gives us a dividend yield of 3.86% for the share basket at February 28, 2013. We believe that the historical dividend yield doesn't change much, since Banks use to pay the same amount of dividends
2	To be accurate we calculate the dividend yield ourselves and expect to get a similar number as provided in the factsheet. We use the same time period as we used to calculate the volatiliy
3	The dividend yield is calculated by dividing the sum of dividends in a year by the average share price of that year
4	The 2013 Dividend Yield is annualized. We average the dividend yields of the companies to get the yearly dividend yield of our basket
5	Our calculated dividend yield of 4.15% is close to the dividend yield mentioned on the basket, meaning that our assumption was right

Step #	Pricing Method
1	We know that we are long in a call option if we buy the note. But since we look at the average of 6 years share price we can't use the classical Black-Scholes Formula. So we relied on the VBA code of Redouane Elkamhi to price the option

Pricing the Principal Protected Value			
Information Input for Pricing		**Components**	**Value in $**
Variables	**Value**	**Bond Price**	**91.39**
Index	100	Number of Options	1.00
Ann. Volatility	26.6%	**Avg. Call Option Price**	**9.93**
Div. Yield	4.15%	**Note Value**	**$ 101.32**
Int_Rate	1.50%		
Life	6		
Principal	100		

We find the note value to be $101.32.

Question 1: TD Principal Protected Note - Part B

Step #	Description
1	To calculate the effective yield we need to first calculate the revenue for the Bank
2	The bank sells the Note for 100 dollar
3	However, the information sheet mentions that the selling agent's commision is 3.5%
4	To hedge the options, the issuer needs to enter the contract from the counterpart
5	The Bank wrote call options, which means they need to go long in the same amount of options with the same strike and maturity to make it a zero game
6	If we substract the cost to of the options from the revenue we get the net profit to calculate the yield. This is the price for which we sell the 6-year bond
7	With the things we have we can calculate the effective yield

Calculation Steps			
Income	100		
Selling Agent's Commision	3.50%	**Revenue**	**96.5**
Number of Options	1.00	Option Price	9.932906518 U
Net Profit / 6-year bond sold for	86.57		
Cont. Comp.yield	**2.40%**		

Question 2 **Buffer Notes**

Issued	December 18, 2018	Days:	751.00
Maturity	January 7, 2021	Years:	2.063 years

Part A Note components Based on 3x multiplier

1) Zero-coupon note from Citigroup
2) Three long ATM call
3) Three Short OTM Call
4) One Short OTM Put

Option Type	Maturity	Strike Price	Quantity		
Long Call	January 7, 2021	$ 885.28	1	At the money	K=S
Short Call	January 7, 2021	$ 1,003.32	3	Out of money	K>S
Short Put	January 7, 2021	$ 796.75	3	Out of money	K<S

Bonds	Maturity	Final Payment
Citigroup note	January 7, 2021	$ 10.00

Part B ***Inputs***

SPX Level	885.28
Volatility	45%
Interest Rate	2%
Dividend Yield	1.50%
Time to maturity (years)	2.06

Insturment	*Strike*	*Price*	*Quantity*	*Present Value*
Long Call	$ 885.28	$ 220.69	0.034	$ 7.48
Short Call	$ 1,003.32	$ 181.88	0.034	-$ 6.16
Short Put	$ 796.75	$ 161.80	0.011	-$ 1.83
Bond			$ 1.00	$ 9.61
Total Value of Buffer Note				$ 9.10

Bond	
Final Payment	$ 10.00
Interest Rate	2%
Present Value	$ 9.61

* Assuming continuous compounding

Black Scholes Model	Option 1
Inputs:	
Stock Price (S)	$ 885.28
Strike Price (X)	$ 885.28
Volatility (s)	45%
Risk-free Rate	2%
Time to expiration (T) Years	2.06
Dividend Yield	1.50%
Output:	
D1	0.338883197
D2	-0.306988308
N(D1)	0.632651138
N(D2)	0.379426136
Call Price	$ 220.69
Put Price	$ 211.90

Black Scholes Model	Option 2
Inputs:	
Stock Price (S)	$ 885.28
Strike Price (X)	$ 1,003.32
Volatility (s)	45%
Risk-free Rate	2%
Time to expiration (T) Years	2.06
Dividend Yield	2%
Output:	
D1	0.145093655
D2	-0.50077785
N(D1)	0.557681536
N(D2)	0.308263738
Call Price	$ 181.88
Put Price	$ 286.36

Black Scholes Model	Option 3
Inputs:	
Stock Price (S)	$ 885.28
Strike Price (X)	$ 796.75
Volatility (s)	45%
Risk-free Rate	2%
Time to expiration (T) Years	2.06
Dividend Yield	2%
Output:	
D1	0.502012418
D2	-0.143859086
N(D1)	0.692170607
N(D2)	0.442805872
Call Price	$ 255.55
Put Price	$ 161.80

Question 3 ***Reverse Convertible Notes***

Part A

Components		Position	Quantity
1)	Knock-In Put Options	Short	66.66666667
2)	Debt Insturment	Long	1

Put Option

Maturity		0.02	years
Strike	$	15.00	
Barrier	$	10.00	

Bond

Principal	$	1,000.00

Part B

Know

Maturity	0.25	years
Principal	$1,000	
Stock Price	$15	
Volatility	60%	
Coupon Rate	16%	
Interest Rate	1.20%	
Dividend yield	0%	

Pricing		
Components		Value
Knock-in Puts	-$	70.6051
Bond		$1,036.88
Reverse Convertible Value	$	966.280

Component 1 : Bond

Principal	$1,000
Coupon	$40
Future Value	$1,040
Present Value	$1,036.88

Component 2: Knock-in Put

Value 1.059076 Using BarrierOption VBA code

3 Scenarios

1) At maturity, stock price is less than at time t=0 and get knocked in.
Receive: principal/s0 shares + coupon payment

2) At maturity, stock price >= $15, then get $1000 + interest

3) Stock price stays between barrier and initial price. Therefore, don't get knocked in.
Receive $1000 + coupon.

Question 4 **Morgan Stanely Intel SPARQS**

Issue Date	June 30, 2018
Maturity Date	July 20, 2019
Days	385
Years	1.069444444
Selling Price	$11.18

Pricing		
Components		Value
PV Note Coupons	$	0.92
PV Call Price	$	10.63
Short Call on Forward	-$	1.19
SPARQ Value	$	10.362

Inputs

Intel Closing Price	$	20.77
Volatility		35.40%
Interest Rate		3.13%
Call Price		$11.63
Yield to call		16%
Dividend Yield		2.70%

Selling Price	$11.18
Difference b/w price and value	$0.82
% Margin	7.32%

Assumptions

Used yield to call to calculate PV of call price

Used 360 days per year as done in the prospectus

Note			
Date	COUPONS		PV
October 20, 2018	$	0.2733	$ 0.2707
January 20, 2019	$	0.2236	$ 0.2197
April 20, 2019	$	0.2236	$ 0.2180
July 20, 2019	$	0.2236	$ 0.2162
Total			$ 0.9245

Intel			
Date	Dividends		PV
August 5, 2018	$	0.1400	$ 0.1396
November 5, 2018	$	0.1400	$ 0.1385
February 4, 2019	$	0.1400	$ 0.1374
May 5, 2019	$	0.1400	$ 0.1363
Total			$ 0.5517

Black Scholes Model		
Inputs:		
Stock Price (S)	$	20.82
Strike Price (X)	$	20.82
Volatility (s)		35%
Risk-free Rate		3%
Time to expiration (T) Years		0.566666667
Dividend Yield		0.00%
Output:		
D1		0.199799471
D2		-0.066682048
N(D1)		0.579181293
N(D2)		0.473417413
Call Price	$	2.38
Put Price	$	2.01

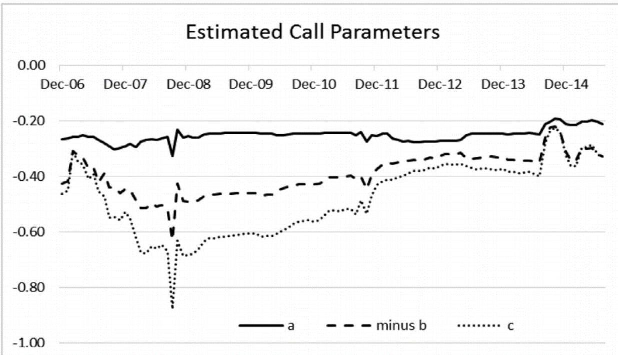
PV of Call Price	
Call Price	$11.63
Interest Rate (YTC)	16.00%
Years to maturity	0.566666667
PV Call Price	$10.63

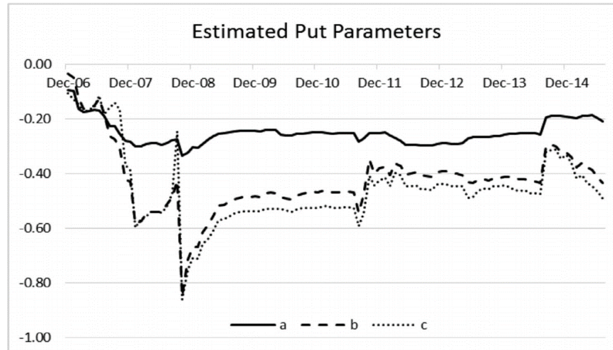
Hull and White Results

Part A *Descriptive Statistics*

Variable	*Observations*	*Mean*	*Standard Deviation*	*Min*	*Max*
S&P 500	1002020	1553.888	391.2582	676.53	2130.82
Strike Prices	1002020	14.83621	41.88139	50	350
Implied Volatility	1002020	0.2087461	0.0862987	0.04156	1.30078

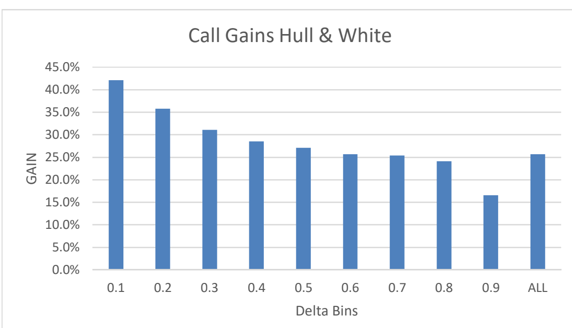
Part B Graphing predicted values of a, b and c over time

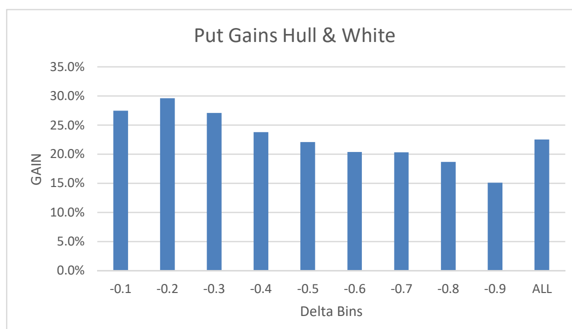




Part C Computing GAIN

BS Delta	Call Gains	BS Delta	Put Gains
0.1	42.1%	-0.1	27.5%
0.2	35.8%	-0.2	29.6%
0.3	31.1%	-0.3	27.1%
0.4	28.5%	-0.4	23.8%
0.5	27.1%	-0.5	22.1%
0.6	25.7%	-0.6	20.4%
0.7	25.4%	-0.7	20.3%
0.8	24.1%	-0.8	18.7%
0.9	16.6%	-0.9	15.1%
ALL	25.7%	ALL	22.5%





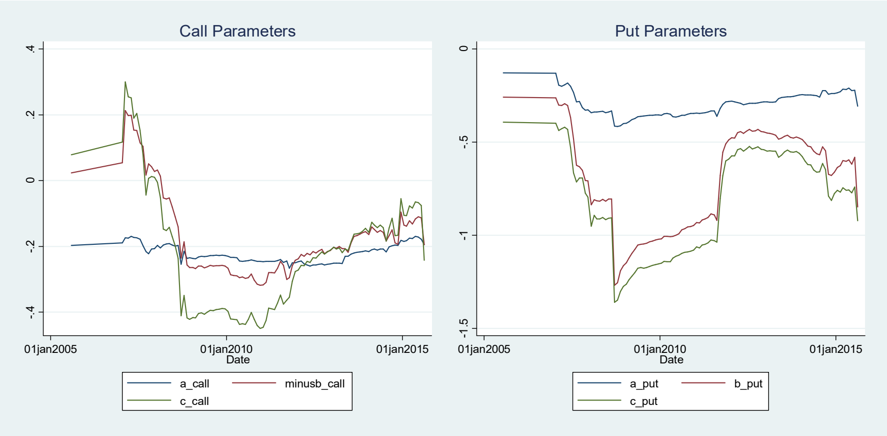
Our Results (Reproducing Hull and White)

Part A *Descriptive Statistics*

Variable	***Observations***	***Mean***	***Standard Deviation***	***Min***	***Max***
S&P 500	1002020	1553.888	391.2582	676.53	2130.82
Strike Prices	1002020	14.83621	41.88139	50	350
Implied Volatility	1002020	0.2087461	0.0862987	0.04156	1.30078

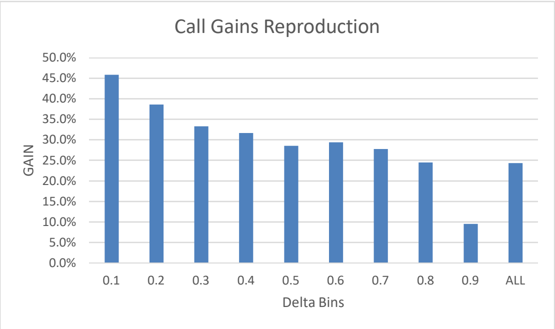
Note: ***Our results did not match exactly with Hull and White's Original results. The call parameters seem to be approximately half of what they should be and the put parameters are approximately double what the paper reports. We believe that this is a scaling problem of some sort. The paper mentiones dividing everything by index level to get sequential quotes. We believe that there is a scaling error (something multiplied or divided) which would equate the results. The regression runs the correct number of times (105) on a rolling 36 month window basis and the multiplier seems to be computed correctly. We are very interested to know what the error is in the code, as we have stuided this problem extensivey. Please see the attched stata do file and dta data file.***

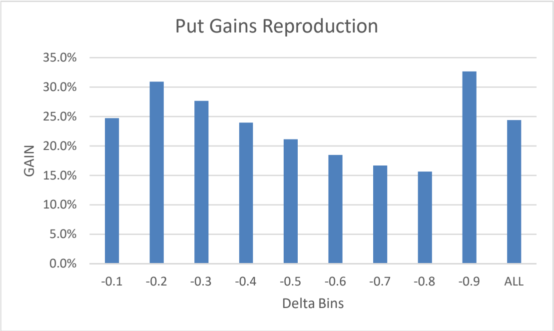
Part B Graphing predicted values of a, b and c over time



Part C Computing GAIN

BS Delta	Call Gains	BS Delta	Put Gains
0.1	45.9%	-0.1	24.7%
0.2	38.7%	-0.2	30.9%
0.3	33.3%	-0.3	27.7%
0.4	31.7%	-0.4	24.0%
0.5	28.6%	-0.5	21.1%
0.6	29.4%	-0.6	18.5%
0.7	27.8%	-0.7	16.7%
0.8	24.5%	-0.8	15.7%
0.9	9.5%	-0.9	32.7%
ALL	24.4%	ALL	24.4%





Improved Model

Part A *Descriptive Statistics*

Variable	*Observations*	*Mean*	*Standard Deviation*	*Min*	*Max*
S&P 500	1002020	1553.888	391.2582	676.53	2130.82
Strike Prices	1002020	14.83621	41.88139	50	350
Implied Volatility	1002020	0.2087461	0.0862987	0.04156	1.30078

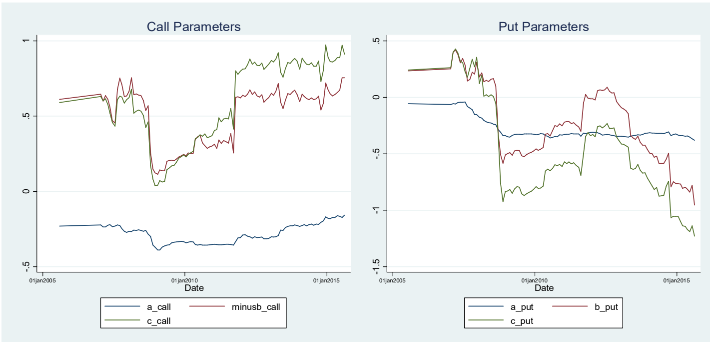
Original Model

$$f \quad \delta_{BS} \quad S = \frac{\vartheta_{BS}}{\sqrt{T}}\frac{S}{S}\left(a + b\delta_{BS} + c{\delta_{BS}}^{2}\right) + \varepsilon$$

Improved Model

$$f \quad \delta_{BS} \quad S = \sqrt{\frac{\vartheta_{BS}}{\sqrt{T}}\frac{S}{S}}\left(a + b\delta_{BS} + c{\delta_{BS}}^{2}\right) + \varepsilon$$

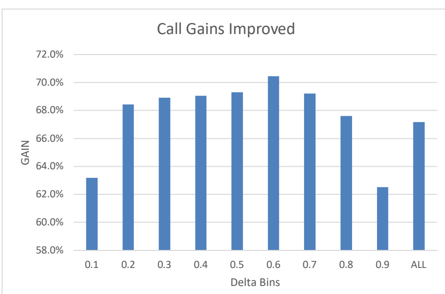
Part B Graphing predicted values of a, b and c over time

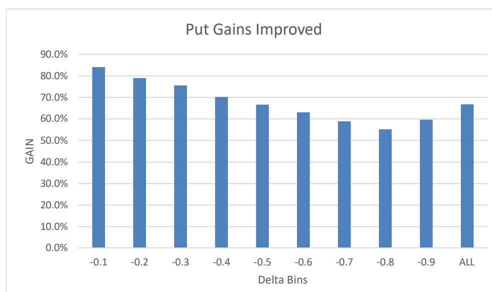


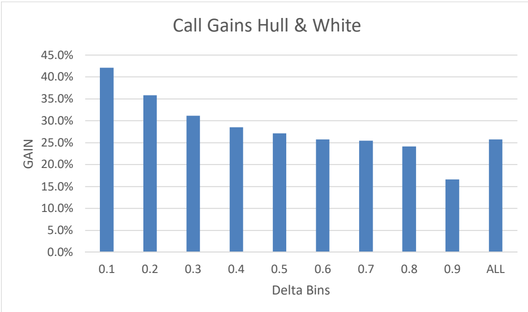
The improved model uses the square root of the multiplier. It was suprising how much higher the calculated gain was when using the square root of the multiplier. It seems that more shape restriction was desired by the model and it therefore fit much better. This result has not beeen tested out of sample and therefore, the addition of the square root would likely decrease the accuracy of prediction and hence out of sample fit. This is a common problem when forcing more shape on the fitted values in a calibration procedure. The average GAIN Increase is approximately 40% which is very interesting. I am hoping that this result holds with the correct code.

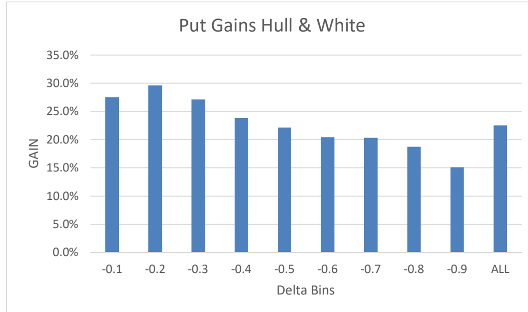
Part C Computing GAIN

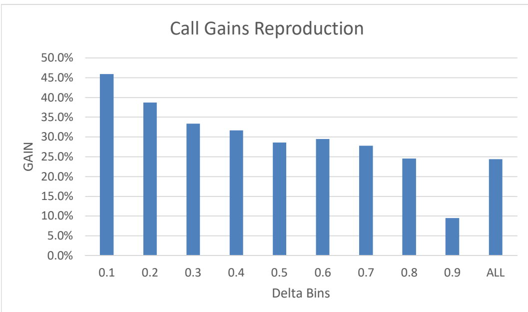
BS Delta	Call Gains	BS Delta	Put Gains
0.1	63.2%	-0.1	84.1%
0.2	68.4%	-0.2	79.0%
0.3	68.9%	-0.3	75.5%
0.4	69.1%	-0.4	70.2%
0.5	69.3%	-0.5	66.6%
0.6	70.4%	-0.6	63.0%
0.7	69.2%	-0.7	58.8%
0.8	67.6%	-0.8	55.1%
0.9	62.5%	-0.9	59.7%
ALL	67.2%	ALL	66.8%

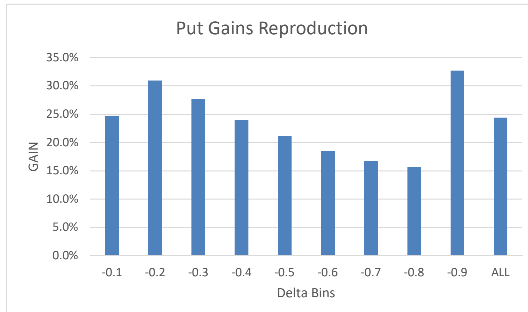


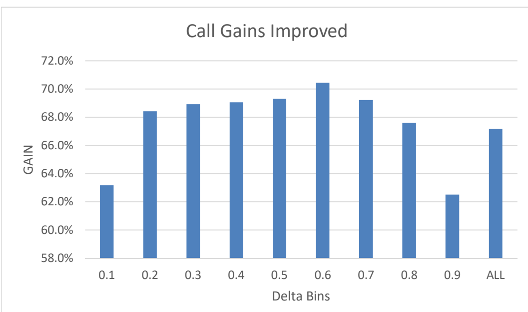


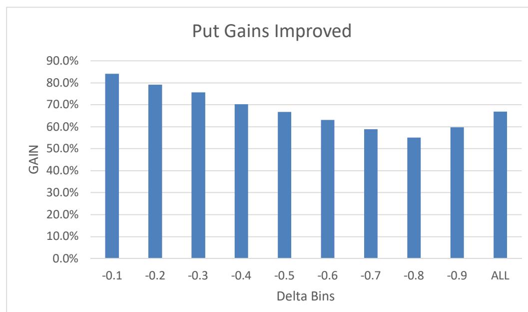


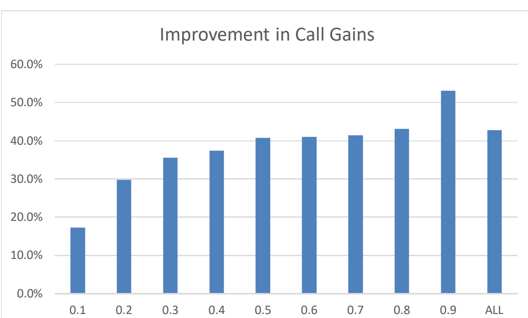


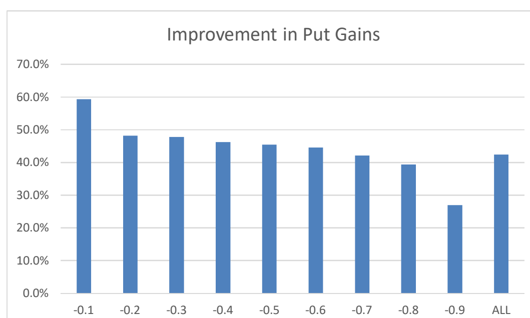










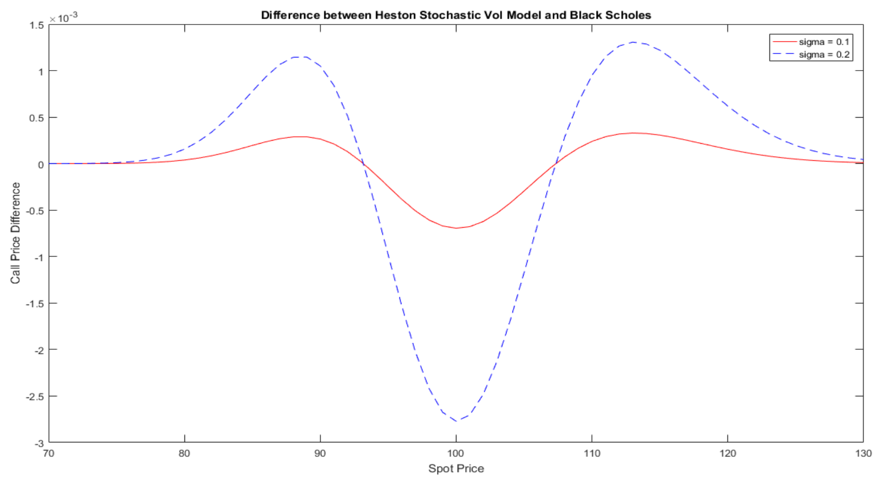


Improved minus Original

BS Delta	Call Gains	BS Delta	Put Gains
0.1	17.3%	-0.1	59.4%
0.2	29.8%	-0.2	48.1%
0.3	35.6%	-0.3	47.8%
0.4	37.4%	-0.4	46.2%
0.5	40.7%	-0.5	45.5%
0.6	41.0%	-0.6	44.6%
0.7	41.4%	-0.7	42.1%
0.8	43.1%	-0.8	39.4%
0.9	53.0%	-0.9	27.0%
ALL	42.8%	ALL	42.4%

Heston Stochastic Volatility Model (1993)

Part 1	The Heston SV Call Option Valuation Model has been coded as a MATLAB function named "StochVol.m" (see MATLAB Files ZIP Folder)
Part 2	The reproduction of Figure 4 in Heston's 1993 paper is given below. To generate in MATLAB, run file named "HestonSV_root.m"



Part 3 The discretized Monte Carlo version of the model can be run by "HestonMC_root.m"
The number of time steps can be changed by changing value of n and the number of simulations can be chnaged by changing value on N
Using the values given below, the results of the Monte Carlo are stated

n =	126	DAILY TIME STEP
N =	100,000	NUMBER OF SIMULATIONS

Call Price sigma = 0.1	2.8231	Absolute Difference with Actual Price	0.0034
Call Price sigma = 0.2	2.8188	Absolute Difference with Actual Price	0.0012

Heston Stochastic Volatility Model (1993)

Part 1	The Heston Nandi GARCH (1,1) Call Option Valuation Model has been coded as a MATLAB function named "NandiGARCH.m" (see MATLAB Files ZIP Folder)

Part 2	The reproduction of prices given on page 620 the 2000 paper is given below. To generate in MATLAB, run file named "NandiGARCH_root.m"

Call Price T = 50 days	1.8178
Call Price T = 100 days	2.4819

Part 3	The discretized Monte Carlo version of the model can be run by "NandiGARCH_MC_root.m"

The number of time steps can be changed by changing value of n and the number of simulations can be chnaged by changing value on N
Using the values given below, the results of the Monte Carlo are stated

n =	10,000	TIME STEP	
N =	100,000	NUMBER OF SIMULATIONS	
Call Price T = 50 days	1.9065	Absolute Difference with Actual Price	0.0887
Call Price T = 100 days	2.5465	Absolute Difference with Actual Price	0.0646

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Final Page

Grade:________