Analysis of Recursive Algorithm

Exercise 03

Exercise 1
Backward Substitution

```
A(n )
{
    if (n > 1)
        return   A(n-1)
}
```

Exercise 2

```
Factorial (n )
{
    if (n == 0)
        return   1;
    else
        return (n * factorial (n-1)
}
```

Exercise 3

$T(n) = n + T(n - 1)$ when $n > 1$,

$T(n) = 1$ when $n = 1$

Exercise 4
Recursive Tree

$T(n) = 2T(n/2) + c, \quad \text{when } n > 1,$

$T(n) = c \qquad\qquad\qquad \text{when } n = 1$

Exercise 5

$T(n) = 2T(n/2) + n$, when $n > 1$,

$T(n) = 1$ when $n = 1$

Tower of Hanoi Puzzle

Tower of Hanoi Puzzle

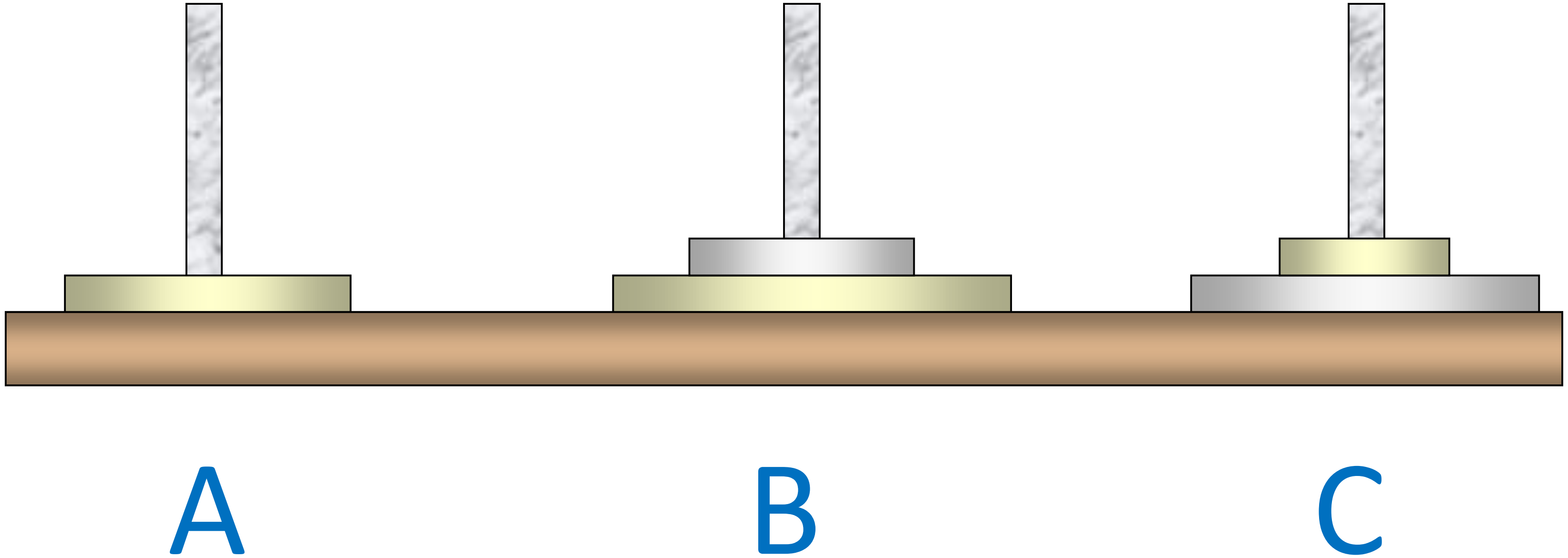
1. There are n **disks of different sizes** that can slide onto any of three pegs/towers.

2. Initially, all disks are on the first peg in order of size, the largest on the bottom and the smallest on the top.

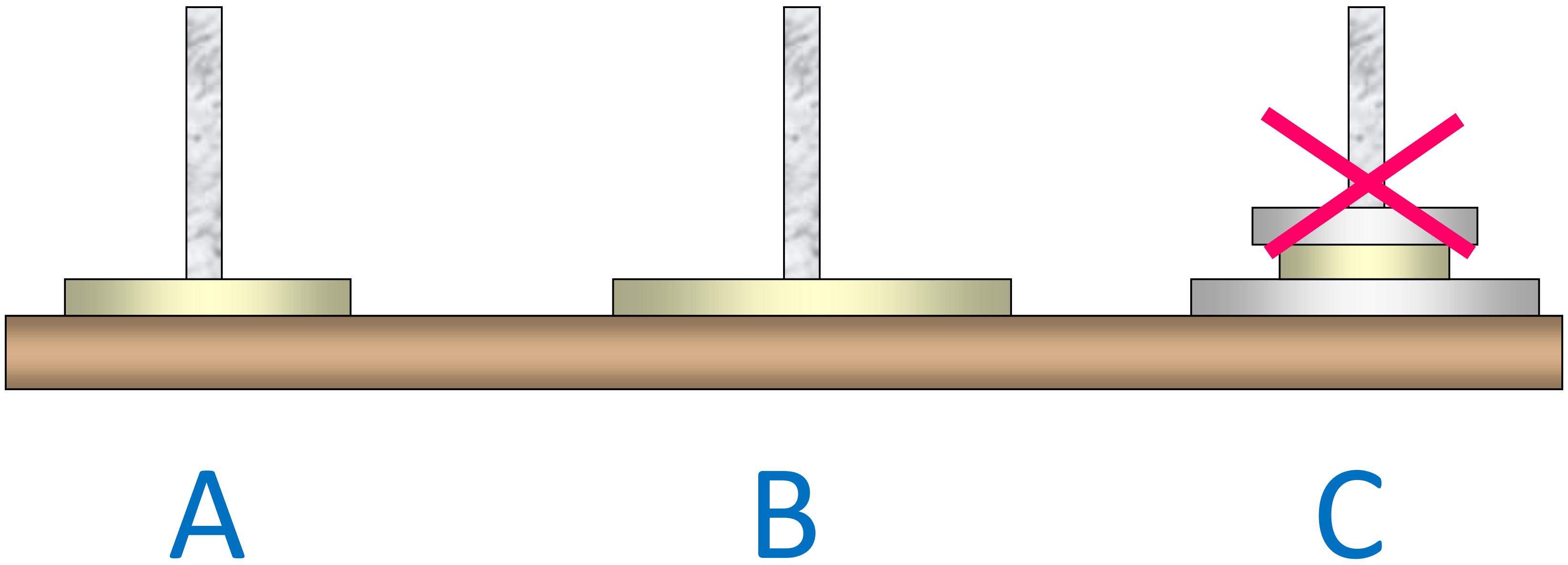
3. The goal is to move all the disks to the third peg, using the second as an auxiliary.

4. At a time, only one disk can be moved and it is forbidden to place a larger disk on the top of a smaller one.

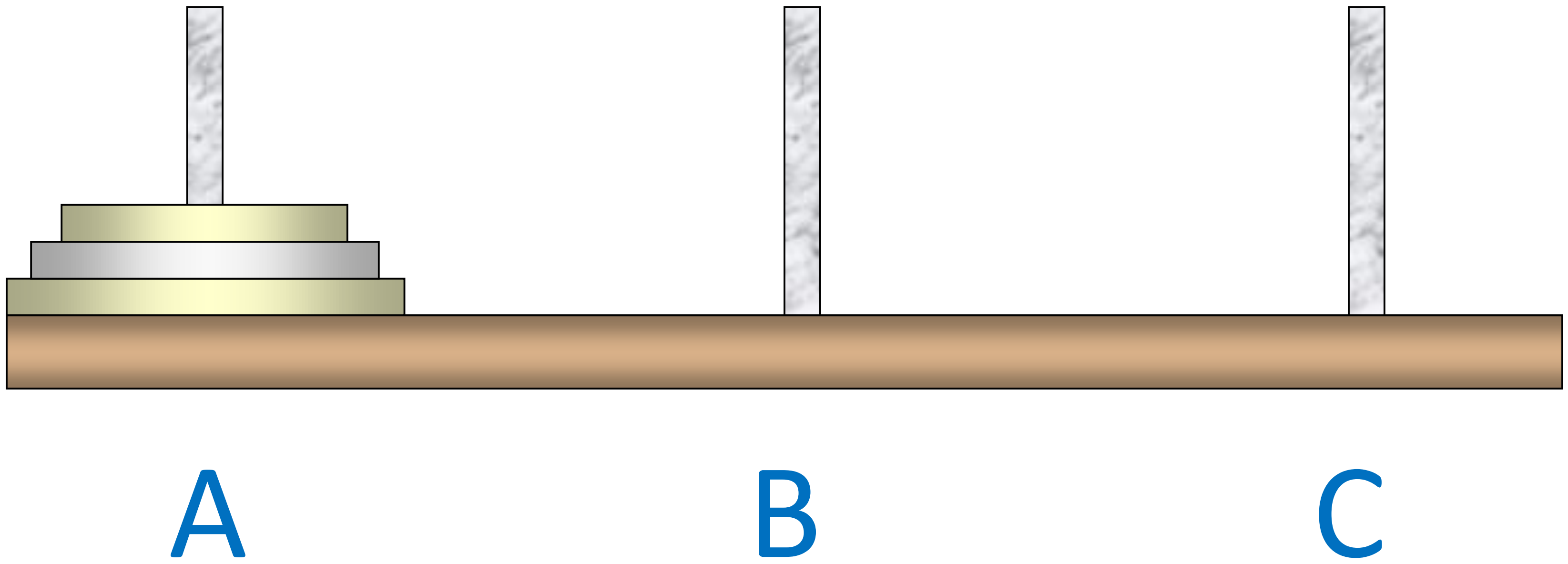
Tower of Hanoi Puzzle



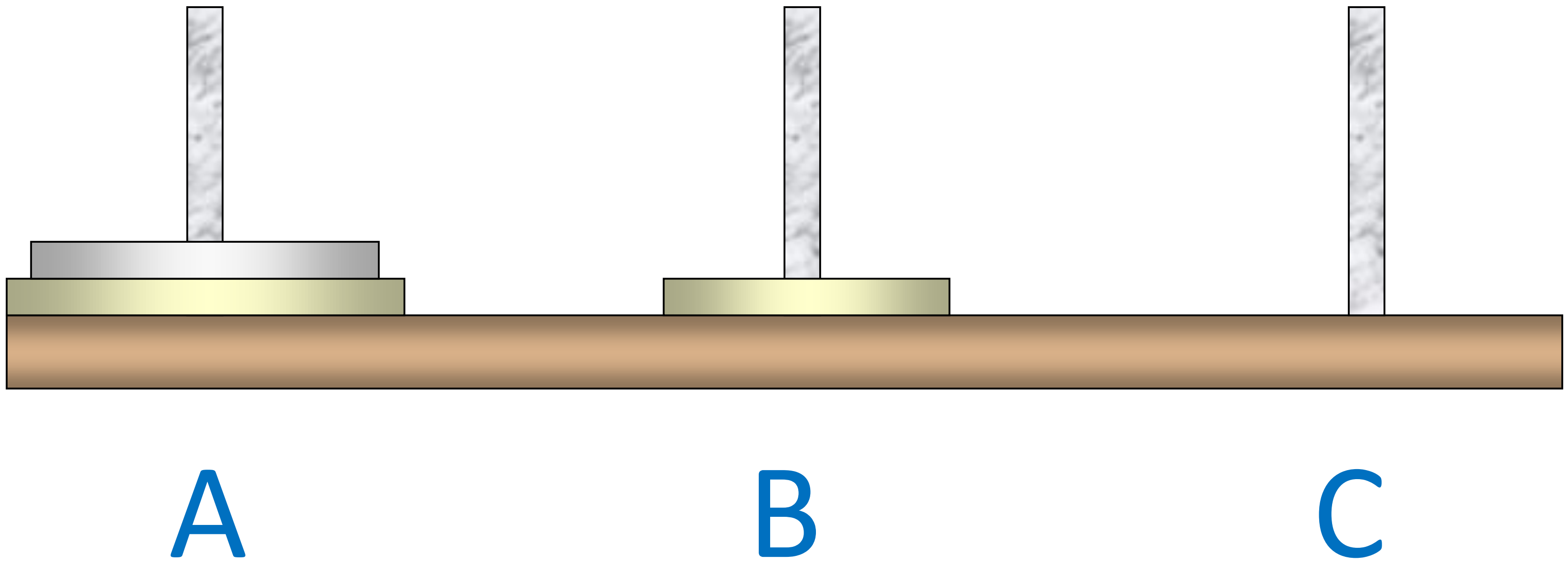
Tower of Hanoi Puzzle



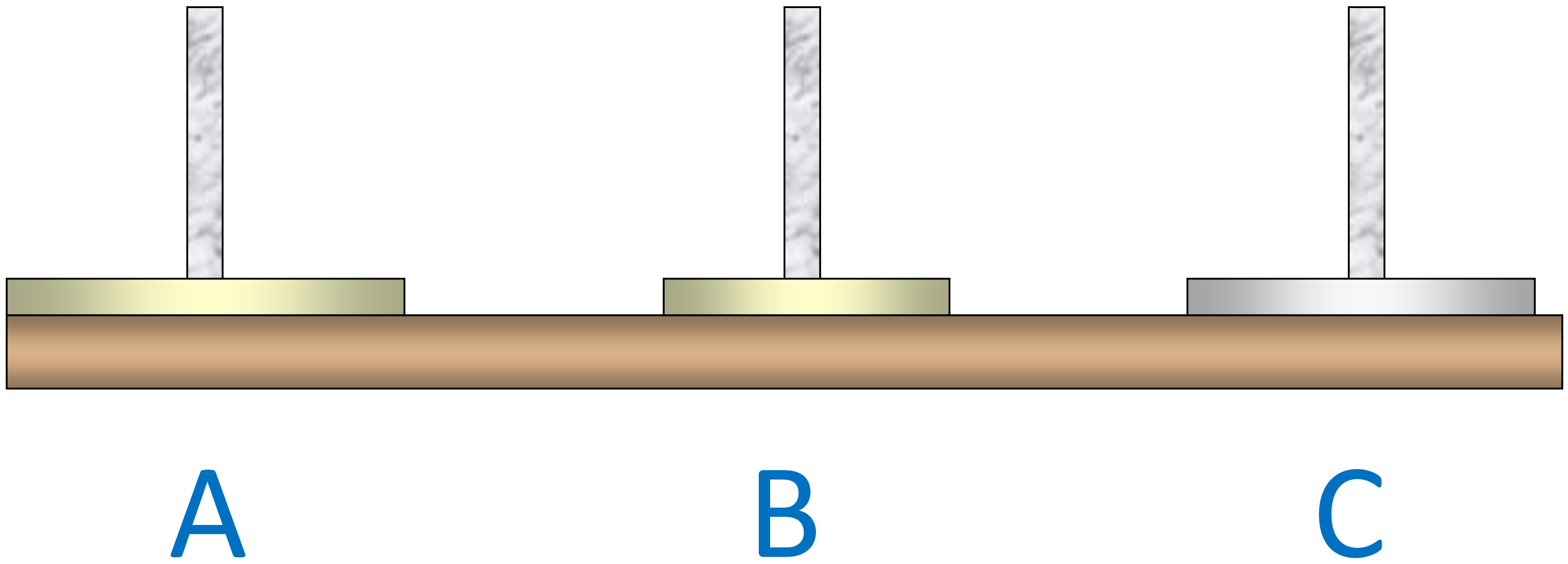
Tower of Hanoi Puzzle



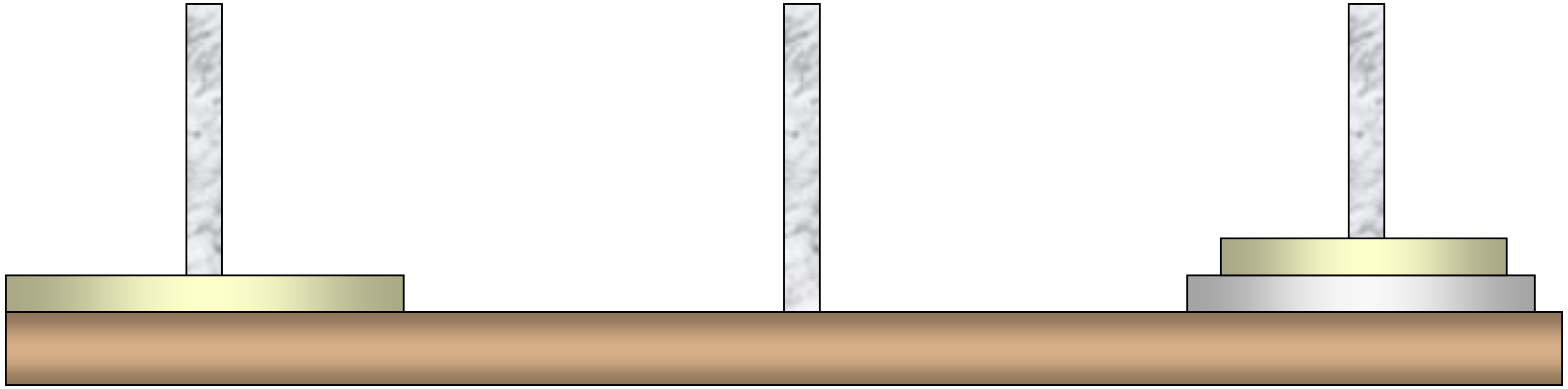
Tower of Hanoi Puzzle



Tower of Hanoi Puzzle

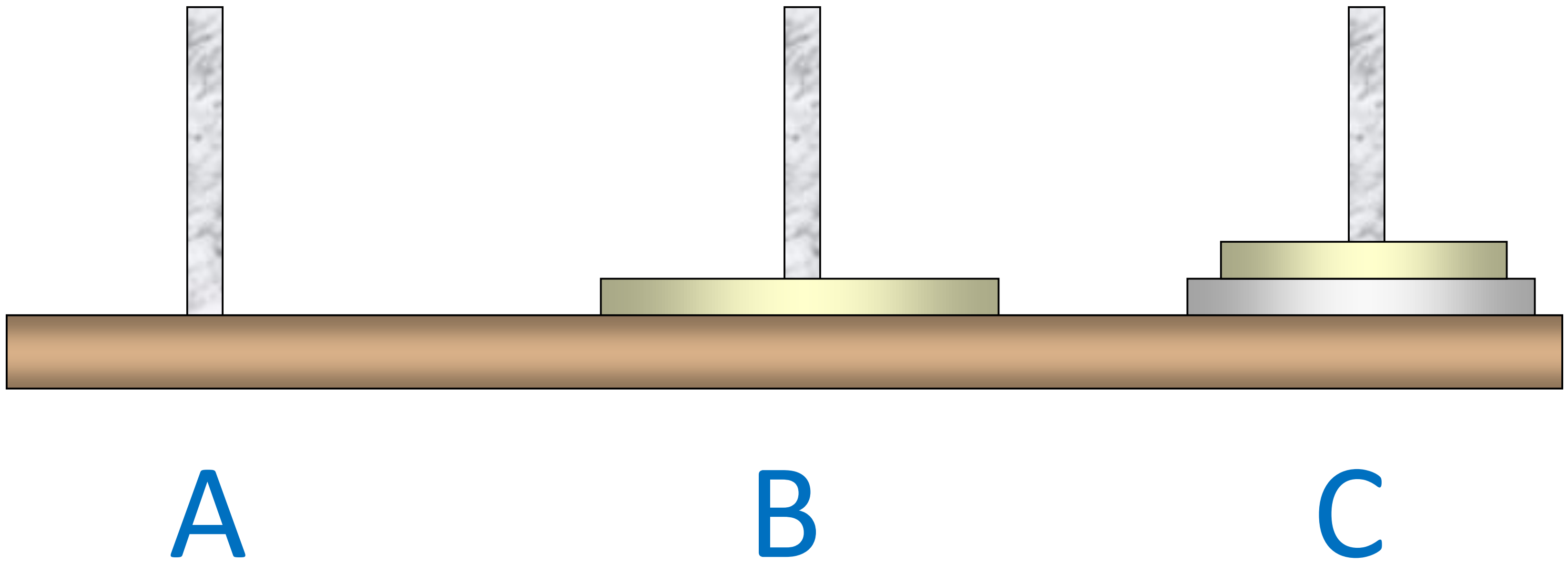


Tower of Hanoi Puzzle

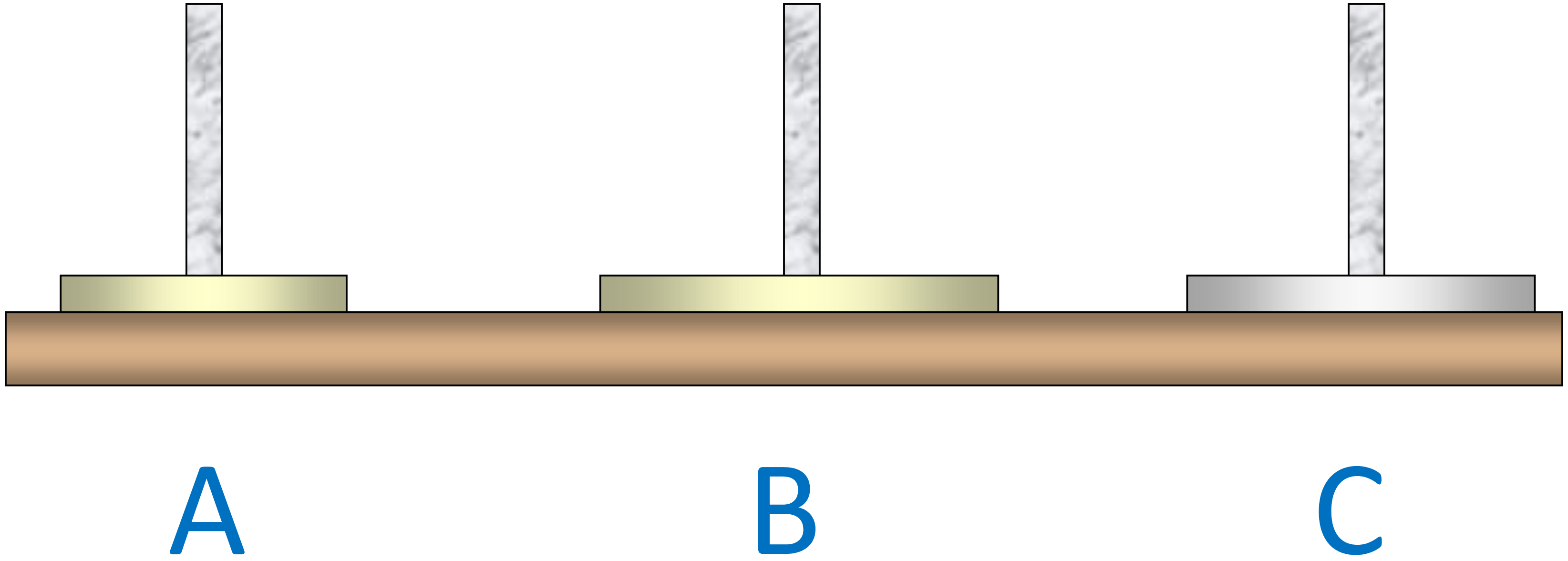


A B C

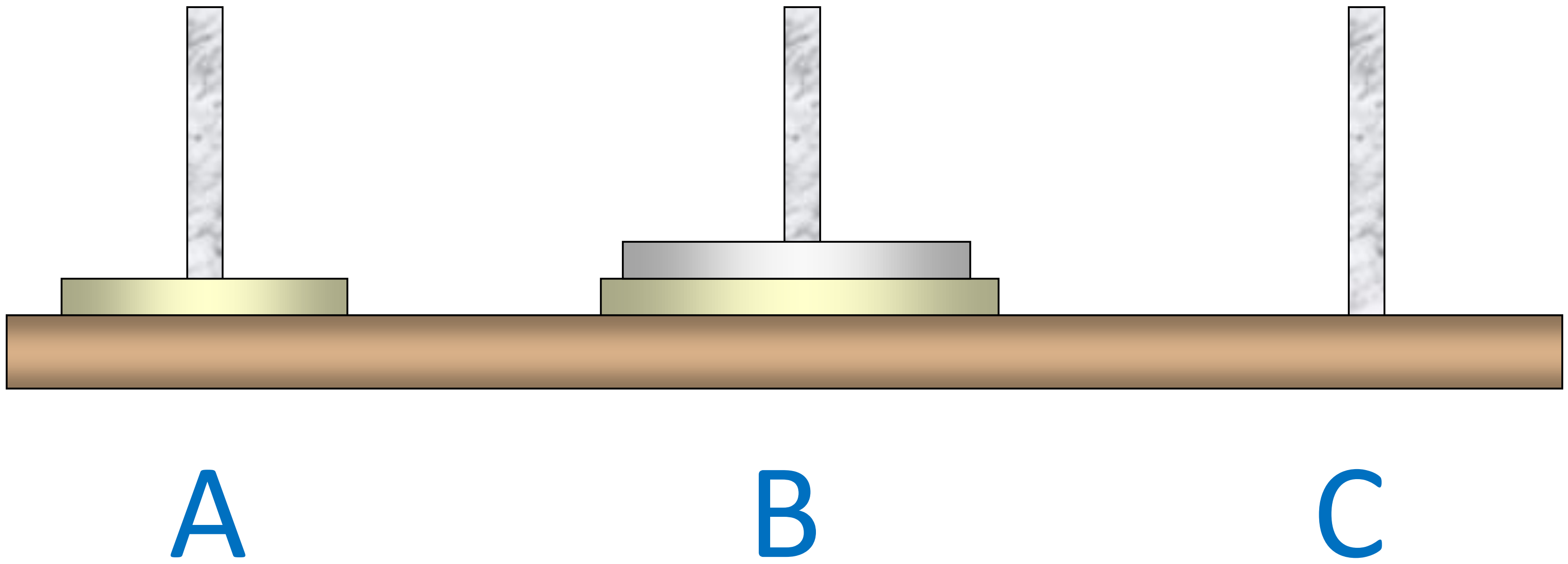
Tower of Hanoi Puzzle



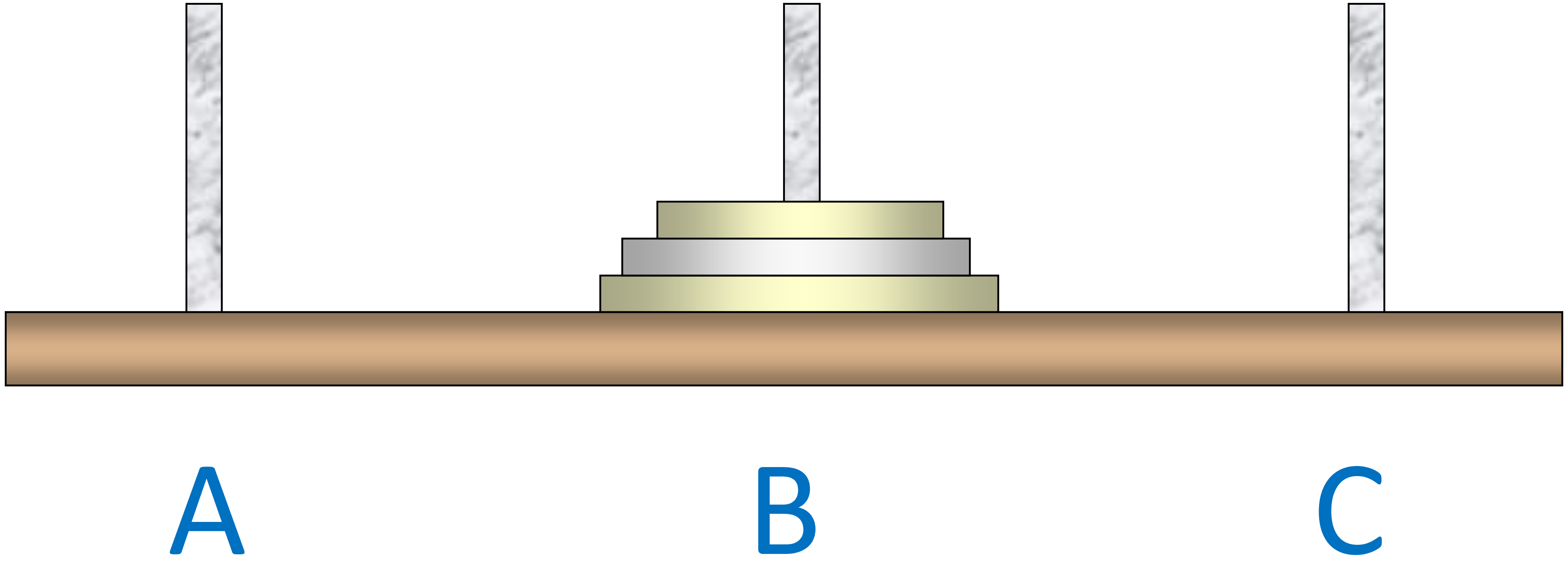
Tower of Hanoi Puzzle



Tower of Hanoi Puzzle



Tower of Hanoi Puzzle



Tower of Hanoi Puzzle

Recursive solution: To move n disks from peg 1 to peg 3 (peg 2 is auxiliary),

- we first move **recursively** $\mathrm{n}-1$ disks from peg 1 to peg 2 (peg 3 is auxiliary),
- then we move the largest disk from peg 1 to peg 3 directly and **recursively** move $\mathrm{n}-1$ disks from peg 2 to peg 3 (peg 1 is auxiliary).
- If $\mathrm{n}=1$, we move the single disk from peg 1 to peg 3.

Try: http://www.mathsisfun.com/games/towerofhanoi.html

Tower of Hanoi Puzzle

Analysis:

- The **input size** indicator is the number of disks n.
- The **basic operation** is moving one disk.
- The total number of moves $M(n)$ depends on n only.
- The **recurrence equation** is

$$M(n) = M(n-1) + 1 + M(n-1)$$

$$M(1) = 1$$

Exercise 6
Tower of Hanoi

$$T(n) = 2T(n-1) + 1\text{, when } n > 0$$

$$T(n) = 1 \qquad\qquad\qquad\text{, when } n = 1$$