

Intelligent Agents

**PEAS framework** Performance measure, Environment, Actuators, Sensors

| Environment                   | Description   |
|-------------------------------|---|
| Observable                    | access to state of environment at each point in time    |
| Episodic (vs sequential)      | experience is divided in different atomic episodes      |
| Static (vs (semi-) dynamic)   | environment unchan-<br>ged while deliberating           |
| Discrete (vs                  | limit on number of distinct actions                     |
| Single-agent (vs multi-agent) | operates by itself                                      |
| Deterministic (vs stochastic) | state completely determined by current state and action |

| Exploration                     | Exploitation                             |
|---------------------------------|--|
| Learning more about environment | Maximise gain based on current knowledge |

**Representation invariant** Abstract states must have corresponding concrete states

Searching

**Problem solving steps** Goal formulation → problem formulation → search → execute

**Problem formulation** Find states, initial state, goal state/goal test, actions, transition model, action cost function

**Problem vs search tree** Graph structure to model the path (problem) vs path taken to search (search tree)

**Path cost** Cost of path from state to any state

**Optimal path cost** Cost of lowest-cost path cost from any state to any state

**State space** All possible configurations

**Search space** Subset of state space that is to be searched

Evaluation

**Time complexity** Number of nodes generated or expanded

**Space complexity** Maximum number of nodes in memory

**Complete** Algorithm is complete if for every problem instance it will find a solution if one exists

**Optimal** Algorithm is optimal if for every instance

where it produces a solution, the best solution is produced

Uninformed Search

Breadth-first search

|                             |   |
|-----------------------------|---|
| Frontier used: <b>Queue</b> |   |
| Time complexity             | exponential w.r.t depth of optimal solution |
| Space complexity            | exponential w.r.t depth of optimal solution |
| Complete                    | number of nodes finite                      |
| Optimal                     | step cost is the same                       |

Time complexity  $O(b^d)$ , where  $b$  total number of notes,  $d$  depth

Uniform-cost search

|                                      |   |
|--------------------------------------|---|
| Frontier used: <b>Priority queue</b> |   |
| Time complexity                      | exponential w.r.t tier of optimal solution      |
| Space complexity                     | exponential w.r.t tier of optimal solution      |
| Complete                             | step cost always positive and finite total cost |
| Optimal                              | step cost always positive                       |

Space complexity and time complexity  $O(b^{l+\lceil C^* \epsilon \rceil})$ , where  $C^*$  refers to cost of optimal solution,  $\epsilon$  lower bound of cost of each action.

Depth-first search

|                             |   |
|-----------------------------|---|
| Frontier used: <b>Stack</b> |   |
| Time complexity             | exponential w.r.t max. depth of search tree   |
| Space complexity            | polynomial w.r.t to max. depth of search tree |
| Not complete                | when depth of search tree is infinite         |
| Not optimal                 | solution may be in shallower depth            |

Depth-limited search

Limits search to depth  $l$ .

Time complexity, space complexity, completeness, optimality **dependent on search algorithm**.

Iterative-deepening search

Uses DLS, iteratively increasing depth limit from 0 to  $\infty$  until solution found.

Space complexity and time complexity dependent on search algorithm. **Complete and optimal**.

Informed Search

Heuristics

**Admissability** Never overestimates cost

$$h(n) \leq h^*(n)$$

**Consistency** Fulfills triangle inequality

$$h(n) \leq c(n, a, n') + h(n'), h(G) = 0$$

**Dominance** A more dominant admissible heuristic function is better for search.

$$h_1(n) \geq h_2(n) \implies h_1 \text{ dominant}$$

Best-first search

Similar to the other search, with a **priority queue**, with the priority of each node calculated as  $f(n)$  (greedy-best first search:  $f(n) = g(n)$ )

A\*Star Search

Cost:  $f(n) = g(n) + h(n)$

|                  |  |
|------------------|--|
| Time complexity  | exponential                                      |
| Space complexity | exponential w.r.t depth of optimal solution      |
| Complete         | edge costs are positive, branching factor finite |
| Optimal          | depends on heuristics                            |

$h(n)$  admissible  $\implies$  optimal A\* search without visited memory

Non-admissible can still be optimal - if cost-optimal path is where  $h_n$  is admissible on all nodes OR if optimal solution  $C_1$  and second-best optimal solution costs  $C_2$  - if  $h(n)$  overestimates costs, but never by more than  $C_2 - C_1$

$h(n)$  consistent  $\implies$  optimal A\* search with visited memory

Local Search

Used as they use very little memory, and finds reasonable solutions in large or infinite state spaces

Hill-Climbing Search

Idea: Keep climbing until value of neighbour is less than value of current

Prone to local maxima, ridges, plateau - can get stuck as the best move is always picked

Simulated Annealing

Idea: Counter possibility of getting stuck in Hill Climbing

Let  $T$  be the "temperature" - it goes down over time. Picks a random move - if it helps situation it is picked. Otherwise, it is accepted with some probability, which decreases exponentially with the badness of the move, and decreases as  $T$  goes down. Bad moves thus become more unlikely as  $T$  decreases.

Adversarial Search

Used for competitive environments - which are fully observable, deterministic, discrete, have terminal states, zero-sum, and turn-taking.

Minimax

Idea: Assuming players play optimally, the player picks the best move, and the opponent picks the move worst for the player. (Always optimal)

|                   |                                      |
|-------------------|--------------------------------------|
| Terminal State    | $utility(s, max)$                    |
| Max Player moving | Max of utility from possible actions |
| Min Player moving | Min of utility from possible actions |

Alpha-beta pruning

Idea: No point to explore full game tree. Keep track of  $\alpha, \beta$ , the highest-value and lowest-value choices found.

Perfect ordering:  $O(b^{\frac{m}{2}})$ .

Decision Tree

Fully expressive. Size of hypothesis class is generalised to  $2^{2^n}$  for  $n$  boolean attributes.

Entropy

The entropy  $B(q)$  of a Boolean random variable, with probability  $q$

$$B(q) = -(q \log_2 q + (1 - q) \log_2 (1 - q))$$

Thus, the entropy of the output

$$H(output) = B(\frac{p}{p+n})$$

To get the information gain of an attribute  $A$ , find the entropy of everything else except for what is in  $A$ :

$$remainder(A) = \sum_{k=1}^d \frac{p_k + n_k}{p + n} B(\frac{p_k}{p_k + n_k})$$

The information gain is then calculated as the reduction in entropy:

$$IG(A) = H(output) - remainder(A)$$

Methods to generalise more

Pruning: **min-sample-leaf, max-depth**  
Data preprocessing

### Linear Regression

Idea: Best fit linear function for continuous-valued input

#### Loss function

$$J_{MSE}(w) = \frac{1}{2m} \sum_{i=1}^m (h_w(x^i) - y^i)^2$$

In matrix form:

$$\frac{1}{2m} (Xw - Y) \cdot (Xw - Y)$$

where  $X$  are variables,  $w$  are weights,  $Xw = \hat{Y}$  are the predictions, and  $Y$  are the actual values.

#### Minimising loss function

Idea: Differentiate the loss function.

With regard to the weight  $w_i$ , the partial derivative is:

$$\frac{\delta}{\delta w_1} J_{MSE}(w) = \frac{1}{m} \sum_{i=1}^m (w_i x^i - y^i) x^i$$

#### Normal Equation

$$w = (X^T X)^{-1} X^T Y$$

Does not require feature scaling, single iteration, with no need for a learning rate.

$O(n^3)$  time to calculate inverse of  $X^T X$ , and  $X^T X$  needs to be invertible (full column rank)

#### Gradient descent

Idea: Start at a weight and pick a weight that reduces the loss until minimum is found.

With learning rate (hyperparameter),

$$w_j \leftarrow w_j - \frac{\delta}{\delta w_j} J(w_0, \dots)$$

In this scenario, the MSE loss function is convex for linear regression, and thus gradient descent can be done on it.

There are different kinds of gradient descent: Stochastic (randomly selects one at a time), Mini-batch (selects of size m).

As data points decrease, cost of gradient descent is cheaper, and randomness increases, causing possibility of escaping minima (bounce)

#### Logistic regression

Use continuous value output probability for classification

#### Logistic function

Idea: Naive threshold is not differentiable and is completely confident. Soften with sigmoid function.

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

### Loss function

$J_{MSE}$  is not convex due to exponential function. Use: cross-entropy.

$$CE(y, \hat{y}) = \sum_{i=1}^C -y_i \log(\hat{y}_i)$$

The binary cross entropy then can be calculated:

$$BCE(y, \hat{y}) = -y \log(\hat{y}) - (1 - y) \log(1 - \hat{y})$$

#### Minimising the loss

Differentiating, the partial derivative with respect to  $w_j$  is found as

$$\frac{\delta}{\delta w_j} J_{BCE}(w) = \frac{1}{m} \sum_{i=1}^m (h_w(x^i) - y^i) x_j^i$$

#### For many attributes

$$h_{w(x)} = \sigma(w_0 + w_1 f_1 + \dots)$$

#### Multi-class classifiers

|            |   |
|------------|---|
| One vs All | Test probability of all classes, assign highest probability |
| One vs One | Fit classifier for each pair, highest wins assigned         |

#### Performance Measure

#### Confusion Matrix

|             |             |
|-------------|-------------|
| TP          | FP (Type 1) |
| FN (Type 2) | TN          |

$$TPR = \frac{TP}{FP+TN}, FPR = \frac{FP}{TP+TN}$$

#### Correctness (Classification)

Accuracy:

$$accuracy = \frac{1}{N} \sum_{i=1}^N (\hat{y}^i = y^i)$$

Precision  $\mathcal{P}$ . Maximise if false positives (Type I) error costly.

$$\mathcal{P} = \frac{TP}{TP+FP}$$

Recall  $\mathcal{R}$ . Maximise if false negatives (Type II) error costly.

$$\mathcal{R} = \frac{TP}{TP+FN}$$

$$F_1 = \frac{2}{\frac{1}{\mathcal{P}} + \frac{1}{\mathcal{R}}}$$

### Bias and Variance

#### Bias

High bias can cause algorithms to miss relevant relations, resulting in **underfitting**.

(Too less features)

#### Variance

High variance can cause algorithms to model random noise, causing **overfitting**.

(Too much features)

#### Hyperparameter tuning

Grid search (exhaustively try all)

Random search (randomly search)

Successive halving (use all, successively increase with smaller set)

Bayesian optimisation

Evolutionary algorithms