CS2040S Cheatsheet

for midterms AY23-24, Sem 2

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Asymptotic Analysis

$T(n) = O(f(n))$ | **Upper Bound**

There exists constants $c > 0$, $n_0 > 0$ such that $\forall n > n_0$, $T(n) < cf(n)$.

$T(n) = \Omega(f(n))$ | **Lower Bound**

There exists constants $c > 0$, $n_0 > 0$ such that $\forall n > n_0$, $T(n) \geq cf(n)$.

$T(n) = \Theta(f(n))$

if and only if $T(n) = O(f(n))$ & $T(n) = \Omega(f(n))$

Generally, $n^n \succ n! \succ a^n \succ n^a \succ log_a(n)$

$log_a(b) = log_c(a) \div log_c(b)$

$a^{log(x)} = x^{log(a)}$

Binary Search

Reduce and Conquer

$T(n) = T(n/2) + O(1)$

Reduce the size of search by half every single iteration.

Sorting

Sort	**Worst-Case**	**Avg-Case**	**Stable?**
Bubble	$O(n^2)$	$O(n^2)$	Yes
Selection	$O(n^2)$	$O(n^2)$	No
Insertion	$O(n^2)$	$O(n^2)$	Yes
Merge	$O(n^2)$	$O(nlogn)$	Yes
Quick	$O(nlogn)$	$O(nlogn)$	No

Sorts of $T(n) = T(n-1) + O(n) = O(n^2)$

BubbleSort $O(n^2)$, Stable, In-Place

Invariant: At the end of iteration j:
the **biggest** j items are correctly sorted in the **final** j positions of the array.

SelectionSort $O(n^2)$, Not Stable, In-Place

Invariant: At the end of iteration j:
the **smallest** j items are correctly sorted in the **first** j positions of the array.

InsertionSort $O(n^2)$, Stable, In-Place

Invariant: At the end of iteration j:
the **first** j items are in sorted order.

Note: Complexity goes closer to $O(n)$ as the array becomes increasingly closer to sorted.

Sorts of $T(n) = 2T(n/2) + O(n) = O(nlogn)$

MergeSort $O(nlogn)$, Stable, Not In-Place

Divide & Conquer
1. Split array into two halves
2. Sort two halves
3. Combine

Iterative
1. Start with pairs of 2 elements, and sort.
2. Merge two pairs...
3. Continue

QuickSort $O(nlogn)$, Not Stable, In-Place

Divide & Conquer
1. Partition array into two sub-arrays around a pivot
Elements in lower subarray $< x <$ elements in upper subarray
2. Recursively sort sub-arrays

Handling Duplicates
3-Way Partition: Pack duplicates together as a partition

Order Statistics
QuickSelect finds k smallest element, $O(logn)$
1. Choose random pivot.
2. If current index of pivot is bigger than k, recurse right. Otherwise, recurse left.

Binary Tree

Definition: A binary tree is either empty or a node pointing to two binary trees.

Traversals (generally)
In-order: Left > *SELF* > Right
Pre-order: *SELF* > Left > Right
Post-order: Left > Right > *SELF*
Level-order: BFS

Operations

delete(v) $O(height)$
1. No children (just remove)
2. 1 child (remove v, connect child(v) to parent(v))
3. 2 children (find x = successor(v), remove it temporarily, remove v, then connect it back to left(v), right(v) and parent(v))

insert(v) $O(height)$
searchMin() $O(height)$ keep recursing left
searchMax() $O(height)$ keep recursing right
successor() $O(height)$
node has right subtree: searchMin(rightSubtree)
else: first parent with key in left subtree.

AVL Trees

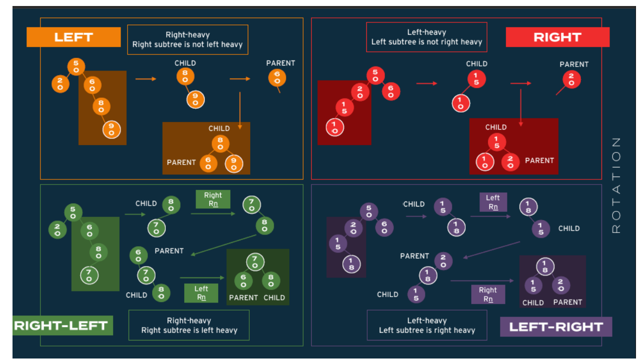
Invariants:
1. All nodes have 0-2 children.
2. All keys in left subtree < value in node < All keys in right subtree
3. **Height-balanced** $|height(v.left) - height(v.right)| \leq 1$
If every node is height-balanced, the tree is height-balanced.

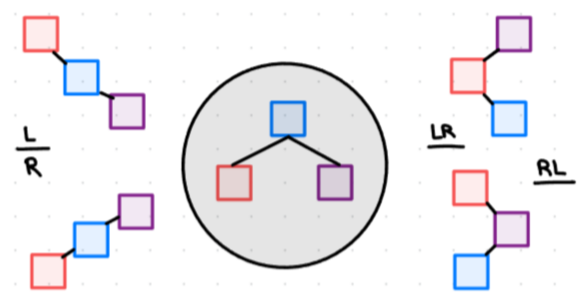
A height balanced tree is balanced $(h = O(logn))$
Min number of nodes n: $n > 2^{h/2}$ $(h < 2logn)$
Max number of nodes n: $n \leq 2^{h+1} - 1$

Rotations

Insertion max 2
left-hvy, not right-sub hvy:
right-rotate(v)

left-hvy, right-sub hvy:
left-rotate(v.left), right-rotate(v)
(symmetric for the other side)





Deletion $O(log(n))$
1. If node has 2 children, swap with successor.
2. Delete node from tree, reconnect children
3. Check all ancestors are height-balanced.

Tries

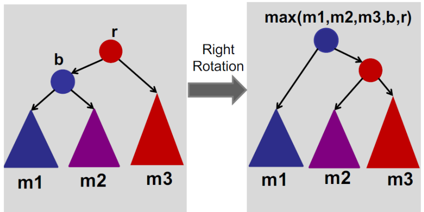
Time Complexity $O(L)$
Space Complexity more nodes & overhead

Node fixed amount of children for strings

Interval Trees

Sorted: Left endpoint
Possible augmentation: **max endpoint in subtree**

Rotations (and maintaining max)



All-Overlaps (with k overlapping intervals): $O(klogn)$
Fastest range query implementation: $O(k + logn)$

(a,b)-Trees

Invariants/Rules:
1: Child-Policy:

	Key		Child	
Node Type	Min	Max	Min	Max
Root	1	$b-1$	2	b
Internal	$a-1$	$b-1$	a	b
Leaf	$a-1$	$b-1$	0	0

2: Key-Range: For every non-leaf node, child = keys + 1.
3: Leaf-Depth: All leafs are at same depth from root.

Insertion
1, Put element in correct order
2. If overflow, find median key in the node.
3. Put median key in parent, split current node.

Deletion
1, Delete key at present node
2. Check for orphan children/downflow
3. If orphan children, replace deleted key with successor or predecessor.
4. If downflow, check for two cases.
5. If sibling has enough keys, then share keys with the other sibling.
6. Else, merge siblings and parent.

Maximum Height $log_a(n)$
Minimum branches a.
Each non-root node $a-1$ keys.
Root node has 1 key, 2 children.

Minimum Height $log_b(n)$
Maximum branches b.
Each non-root node $b-1$ keys.

KD Trees

Generally: split by different dimensions:
$x \to y \to x \to y$ for x, y 2D example

Construction Utilise *QuickSelect*:
$T(n) = 2T(n/2) + O(n) = O(nlogn)$