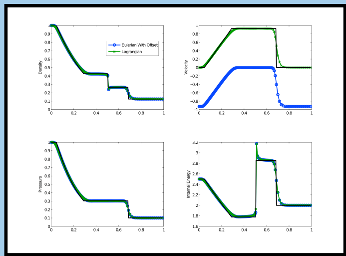
Entropy Traces in Eulerian and Lagrangian Calculations

Daniel W. Zaide, Philip L. Roe
University of Michigan, Department of Aerospace Engineering

CRASH

Introduction

• There exists a common difficulty in the numerical solution of gasdynamic problems involving strong shockwaves.
• At boundaries and near the intersections of shockwaves, regions of anomalously high temperature appear.
• Often thought of as a feature of Lagrangian schemes, but occurs equally in Eulerian schemes if diffusion is sufficiently small.



• Comparing Sod's shocktube problem for both types of methods, offsetting the velocity for the Eulerian method by the speed of the entropy wave, a similar spike in internal energy is observed.

Governing Equations

• The 1D Euler Equations in a fixed frame are shown as

$$\mathbf{u}_t + \mathbf{f}(\mathbf{u})_x = 0$$

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ \rho u \\ E\end{bmatrix} + \frac{\partial}{\partial x}\begin{bmatrix}\rho u \\ \rho u^2 + p \\ u(E+p)\end{bmatrix} = \begin{bmatrix}0 \\ 0 \\ 0\end{bmatrix},$$

with closure from the ideal gas law, $p = (\gamma - 1)\left(E - \frac{1}{2}\rho u^2\right)$

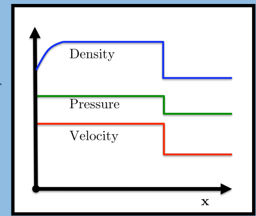
• Discretize in time explicitly with forward Euler time-stepping and in space with a Godunov-type scheme as

$$\mathbf{u}_i^{n+1} = \mathbf{u}_i^n - \frac{\Delta t}{\Delta x}\left(\mathbf{f}^n_{i+\frac{1}{2}}(\mathbf{u}_i^n, \mathbf{u}_{i+1}^n) - \mathbf{f}^n_{i-\frac{1}{2}}(\mathbf{u}_{i-1}^n, \mathbf{u}_i^n)\right)$$

•Second-order (or "high-resolution") methods do not alleviate entropy traces; they instead preserve errors for much longer time than first-order methods.

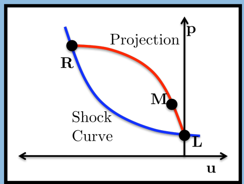
The Noh Problem

• Notorious difficulty encapsulated by William Noh (1986)
• Describes the implosion of a strong shockwave by imposing an inward velocity.
• Exists in three dimensions with radial, cylindrical, and spherical symmetry.
• Sample solutions show a large density defect near the wall; this is a canonical example of an entropy trace.
• Most current remedies use a type of energy diffusion, adjusting the energy equation in some form.



Entropy Traces

• Entropy traces occur for a simple wave when internal states do not lie on the path connecting external states.
• After one step, the internal state lies on the curve shown, here referred to as the Projection curve.
• The solution to future Riemann problems will then contain spurious waves of the other families, which will continue to propagate and result in potential errors.



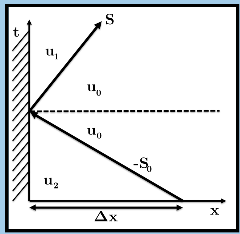
• To further demonstrate that the `start-up' error is a entropy trace, devise a three-wave system whose projections lie exactly on the shock curve.
• One such system is the one shown below,

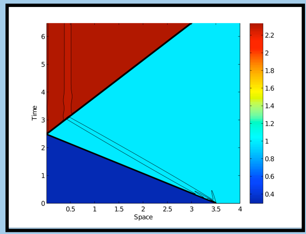
$$\begin{bmatrix}u \\ a \\ b\end{bmatrix}_t + \begin{bmatrix}\frac{1}{2}\left(u^2+a^2+b^2\right) \\ ua \\ ub\end{bmatrix}_x = \mathbf{0}$$

Conference Proceedings

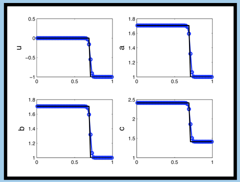
• Roe, P. L., Zaide, D. W., *An Eulerian Look at Lagrangian CFD*. Numerical Methods for Multi-material Fluids and Structures Conference, Sept. 2009.
• Roe, P. L., Zaide, D. W., *Entropy Traces In Lagrangian and Eulerian Calculations*. Sixth International Conference on Computational Fluid Dynamics, July 2010.

Numerical Results





• On the left, a test problem was developed to separate wall effects from entropy traces.
• On the right, looking at density contours in time, two spurious waves are initially produced; the entropy wave carries an error with it, which is picked up by the shock upon reflection.
• To the right, a Noh problem for the the entropy-trace free system is shown
• All internal states lie exactly on the shock curve.
• There is no entropy trace or spurious oscillations.
• Since there is no entropy trace or nonlinear mixing, there is no wall defect observed.



Final Thoughts

• Whenever entropy is created by numerical error, it is propagated along particle paths as though it had been created physically.
• These are referred to as entropy traces.
• The simplest example of this is wall heating, exemplified by the Noh problem.
• These errors derive from insisting on the narrowest possible shock profiles, and so we are currently looking at various shock-broadening mechanisms.

Acknowledgements

Discussions with Professors Krzysztof Fidkowski, Kenneth Powell, and Smadar Karni of UM and Robert Lowrie of LANL were greatly appreciated. The support of Andrew Barlow of the AWE and Nikos Nikoforakis of the University of Cambridge was influential in the foundations of this work.

NNSA National Nuclear Security Administration ASC
SFU