Euler equations with entropy equation.

Start with

$$\mathbf{u}_t + \mathbf{f}_x = 0$$

or expanded as

$$\begin{bmatrix} \rho \\ \rho u \\ \rho s \end{bmatrix}_t + \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho u s \end{bmatrix}_x = 0$$

with $p = s\rho^\gamma$.This can be decomposed as

$$\frac{\partial \mathbf{f}}{\partial \mathbf{u}} = \begin{bmatrix} 0 & 1 & 0 \\ (\gamma-1)\rho^{\gamma-1}s - u^2 & 2u & \rho^{\gamma-1} \\ -us & s & u \end{bmatrix} = \mathbf{A}$$

where further decomposition leads to $\mathbf{A} = \mathbf{R}\Lambda\mathbf{L}$. Define a propagation speed

$$a^2 = \frac{\gamma p}{\rho}$$

and write the eigenvalue matrix as

$$\Lambda = \begin{bmatrix} u-a & 0 & 0 \\ 0 & u & 0 \\ 0 & 0 & u+a \end{bmatrix}$$

and right and left eigenvector matrices as

$$\mathbf{R} = \begin{bmatrix} 1 & 1 & 1 \\ u-a & u & u+a \\ s & -(\gamma-1)s & s \end{bmatrix}$$

$$\mathbf{L} = \frac{1}{2a}\begin{bmatrix} \frac{\gamma-1}{\gamma}a + u & -1 & \frac{a}{\gamma s} \\ \frac{2a}{\gamma} & 0 & -\frac{2a}{\gamma s} \\ \frac{\gamma-1}{\gamma}a - u & 1 & \frac{a}{\gamma s} \end{bmatrix}$$