Sur la prédiction numérique du flux de chaleur, la propagation lente d'ondes de choc et la structure du choc à l'intérieur d'une maille

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Sept 6^{th}, 2011

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On Wall Heating, Slowly Moving Shocks, and Sub-cell Shock Structure

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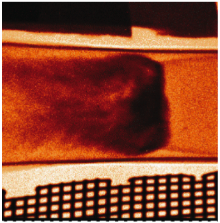
Introduction

- Understanding shockwaves is critical in the prediction and study of many phenomena, where the abrupt changes in material properties due to shockwaves can greatly affect regions of interest such as surfaces and activate other physical mechanisms, such as combustion or ionization.



Drake, 2011

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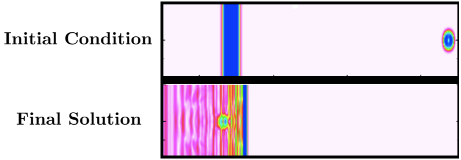
Introduction

- Previous work with shock capturing schemes has classified some pervasive errors as errors in shock position, spurious waves, or unstable shock behavior.
- We will refer to these errors as **Shockwave Anomalies** and focus on three of them; shock position, slowly moving shocks, and wall heating.
- These are numerical artifacts formed due to the presence of shockwaves within the flow solution.



Eric Johnsen, 2011

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Godunov-Type Finite Volume Methods

- In this work, we use the Euler Equations in a fixed frame.
- The explicit Forward Euler in space and time scheme is used,

$$\mathbf{u}_i^{n+1} = \mathbf{u}_i^n - \frac{\Delta t}{\Delta x}\left(\mathbf{f}^n_{i+\frac{1}{2}}(\mathbf{u}_i^n, \mathbf{u}_{i+1}^n) - \mathbf{f}^n_{i-\frac{1}{2}}(\mathbf{u}_{i-1}^n, \mathbf{u}_i^n)\right)$$

  with the flux $\mathbf{f}^n_{i+\frac{1}{2}}(\mathbf{u}_i^n, \mathbf{u}_{i+1}^n)$ coming from a Riemann solver.
- In this work, both an exact and approximate (Roe) Riemann solver are used, with indistinguishable results.
- 2^{nd}-order (or "high-resolution") methods do not alleviate shock anomalies; they instead preserve errors for much longer than 1^{st}-order methods.

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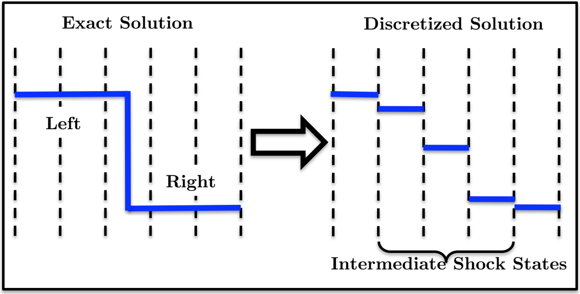
Intermediate Shock States



- For a single captured shock to be located anywhere on a 1D grid, at least one internal value is needed.
- While the number of required intermediate states varies from scheme to scheme, all conservative schemes produce them.

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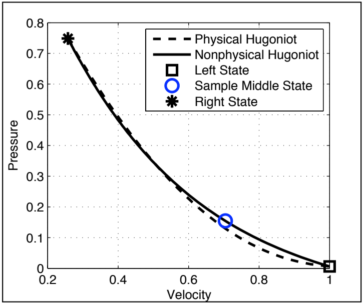
Stationary Shocks

- The simplest example is a stationary shock, for which we have quite a complete theory.
- For the exact or Roe Riemann solvers, this intermediate state lies on the non-physical Hugoniot curve.



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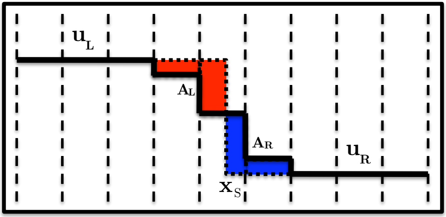
Stationary Shocks

Shock Position

- So what's anomalous about the stationary shock? Lets compute shock position for a scalar conservation law.



- To compute shock position, we can use the equal area rule.
- For the conserved variable, u, the shock position, x_S, divides the discrete solution such that A_L and A_R are equal.

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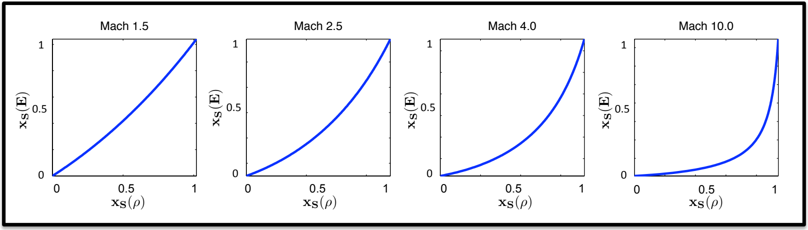
Stationary Shocks

Shock Position - An Ambiguity

- For a system of conservation laws, we would hope that shock positions calculated from each conserved variable would agree.
- **Because the Hugoniot is curved,** they do not; plots of $x_S(E)$ versus $x_S(\rho)$ are shown below for M_0 =1.5, 2.5, 4.0 and 10.0.



- Shock positions from conservation of mass and energy are

$$x_S(\rho) = \frac{\rho_M - \rho_R}{\rho_L - \rho_R}, \quad x_S(E) = \frac{E_M - E_R}{E_L - E_R}$$

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Stationary Shocks

Shock Position - An Ambiguity

- Because of the ambiguity, there is always an order Δx error close to a captured shock.
- It has been shown by Carpenter and Casper (1999) that "high-order" shock-capturing methods are only first-order on fine grids.
- A proposed fix by Kreiss et al (2001) is extremely complicated.

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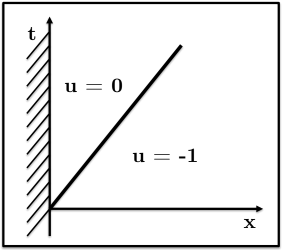
Wall Heating

The Noh Problem

- Notorious difficulty encapsulated by William Noh (1986)
- Describes a uniform gas driven into a wall and the resulting strong shock.
- Exists in three dimensions with radial, cylindrical, and spherical symmetry.
- We look at the 1D problem.



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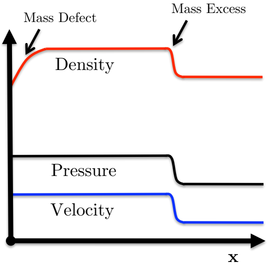
Wall Heating

Representative Solutions

- Virtually all shock-capturing methods provide quite good solutions for pressure and velocity, but predict too small a density in a small region at the origin.
- Thus we have a mass defect at the wall.
- In consequence the temperature there is too high, so that this and related phenomena have been called *wall heating*.



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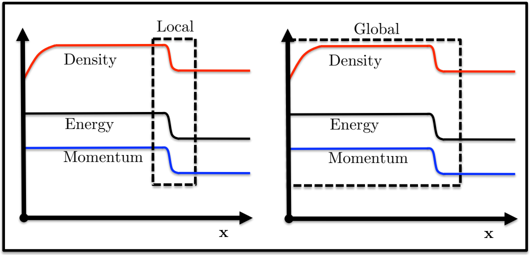
Wall Heating

Shock Position

- To show how shock position plays a role in wall heating, consider two control volumes:
  - **Local** - contains only the region immediately around the shock.
  - **Global** - contains the whole domain.
- **Because the Hugoniot is curved,** there is a mass excess at the shock.



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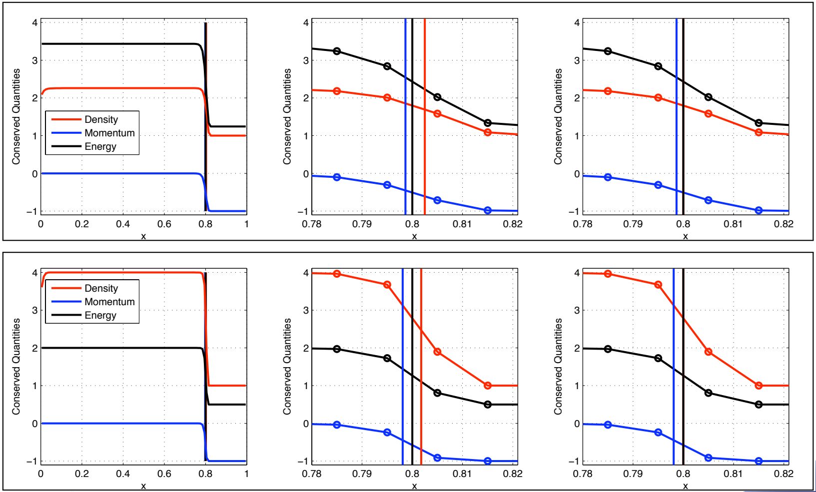
Wall Heating

Conserved quantities for a Mach 1.1 shock (Top) and Mach 10^6 shock (Bottom).



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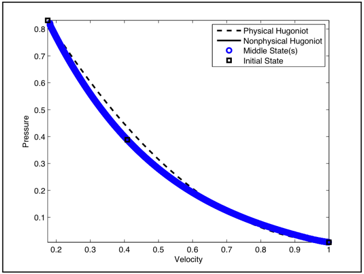
Slowly Moving Shockwave Phenomenon

- Slowly moving shocks generate spurious waves of other families.
- The intermediate states of a slowly moving shock remain close to the equilibrium states of a stationary shock.
- **Because the Hugoniot is curved**, they generate spurious waves.



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What if the Hugoniot were linear?

- The following set of nonlinear conservation laws has straight Hugoniots, and therefore suffers no ambiguity in shock position.

$$\begin{bmatrix} u \\ a \\ b \end{bmatrix}_t + \begin{bmatrix} \frac{1}{2}\left(u^2 + a^2 + b^2\right) \\ ua \\ ub \end{bmatrix}_x = 0$$

- Numerical solutions to the system experience no difficulty with a Noh-like problem or with slowly moving shocks.

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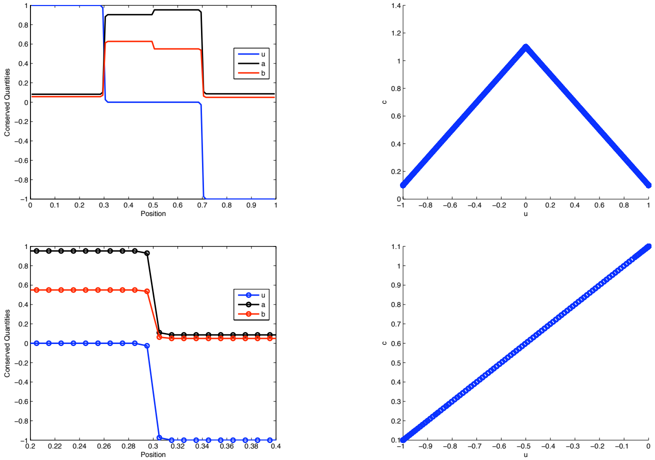
What if the Hugoniot were linear?

(top) Noh-like Problem. (bottom) Slowly moving shock.



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What if the intermediate shock states were collinear in state space?

- This would eliminate the ambiguity in shock position in the stationary shock, and potentially eliminate shockwave anomalies.
- We have designed schemes using penalty terms to enforce this; they are of the form

$$\mathbf{f}^*_{i+\frac{1}{2}} = \mathbf{f}_{i+\frac{1}{2}}(\mathbf{u}_i, \mathbf{u}_{i+1}) + \mathbf{d}_{i+\frac{1}{2}}(\mathbf{u}_{i-1}, \mathbf{u}_i, \mathbf{u}_{i+1}, \mathbf{u}_{i+2})$$

- This leads to intermediate shock states that do not satisfy local thermodynamic equilibrium.
- $\mathbf{d}_{i+\frac{1}{2}}$ is constructed such that in the linear case, $\mathbf{d}_{i+\frac{1}{2}} = 0$ and we are left with the original Riemann solution.

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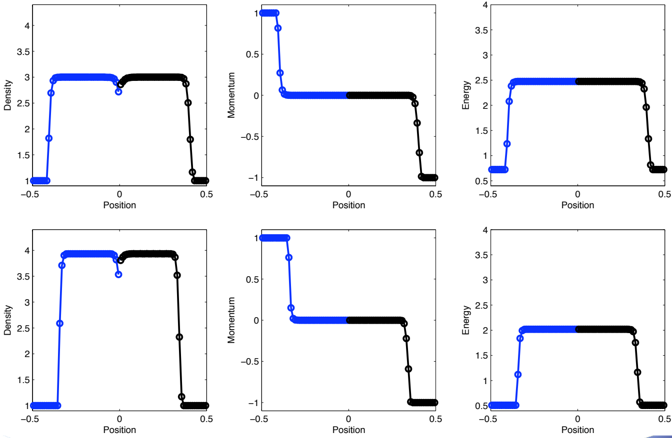
Wall Heating

(top) Mach 2.0. (bottom) Mach 10.0.

Blue - Standard Method, Black - Our Method



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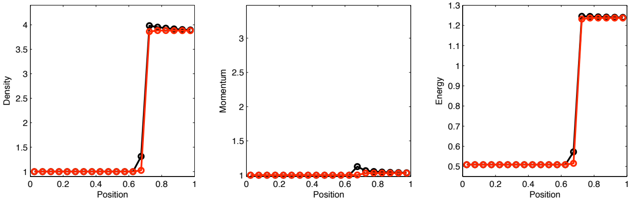
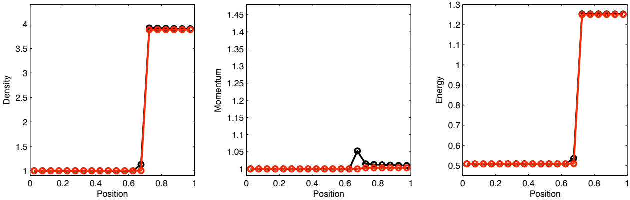
Slowly Moving Shocks

Black - Standard Method, Red - Our Method



$S = 1/100$



$S = 1/1000$

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Concluding Thoughts

- In this work, we have established connections between common defects of shock-capturing methods, by linking each of them to the nonlinearity of the Hugoniot curve.
- Since that nonlinearity is a physical fact, it would seem that an over-reliance on physical derivations may have some drawbacks if they rely on assumptions of thermodynamic equilibrium (LTE).
- Almost all current shock-capturing methods assume LTE for the intermediate states.
- Therefore it seems natural to relax this condition.

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Concluding Thoughts

- The intermediate states are still trusted to contain the correct conserved variables - relaxing LTE only affects the pressure.
- Replacing LTE with a requirement that the intermediate states be linear interpolants of the end states can be thought of as requiring consistency in the captured shock.
- Preliminary numerical results with a penalty-based method show tremendous promise in eliminating the slowly moving shock phenomenon and decreasing wall heating.

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Future Work and Acknowledgements

- Future work will include examining this new penalty method around shock instabilities, such as the carbuncle phenomenon.
- We will also work towards a simpler derivation of the penalty method and an extension to multiple dimensions.

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