Do the physical and nonphysical Hugoniot Curves intersect?

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The Euler equations, here shown in one dimension, are

$$\mathbf{u}_t + \mathbf{f}(\mathbf{u})_x = 0. \tag{1}$$

expanded as

$$\frac{\partial}{\partial t}\begin{bmatrix} \rho \\ \rho u \\ E \end{bmatrix} + \frac{\partial}{\partial x}\begin{bmatrix} \rho u \\ \rho u^2 + p \\ u(E+p) \end{bmatrix} = \mathbf{0} \tag{2}$$

with the equation of state $p = p(\rho, i)$. For an ideal gas this is

$$p = (\gamma - 1)\rho i, \qquad \rho i = \left(E - \frac{1}{2}\rho u^2\right). \tag{3}$$

The speed of sound is then $a = \sqrt{\left.\frac{\partial p}{\partial \rho}\right|_s} = \sqrt{\frac{\gamma p}{\rho}}$.

Starting with a preshock left state of ρ_L, u_L, p_L we can define a physical and non physical hugoniot with $p \in [p_L, p_R]$ (since $p_L < p_R$).

$$u_p(p) = u_L - \frac{p - p_L}{\sqrt{\frac{\rho_L}{2}((\gamma-1)p_L + (\gamma+1)p)}} \tag{4}$$

$$u_{np}(p) = u_R - \frac{p - p_R}{\sqrt{\frac{\rho_R}{2}((\gamma-1)p_R + (\gamma+1)p)}} \tag{5}$$

I believe this is correct. I Hope this is correct. Examining slopes at each end, we have that

$$\left.\frac{\partial u_p}{\partial p}\right|_{p_L} = -\frac{1}{\sqrt{\gamma \rho_L p_L}} \tag{6}$$

$$\left.\frac{\partial u_{np}}{\partial p}\right|_{p_L} = -\frac{\rho_R((3\gamma-1)p_R + (\gamma+1)p_L)}{\sqrt{2}(\rho_R((\gamma-1)p_R + (\gamma+1)p_L))^{3/2}} \tag{7}$$

$$\left.\frac{\partial u_p}{\partial p}\right|_{p_R} = -\frac{\rho_L((3\gamma-1)p_L + (\gamma+1)p_R)}{\sqrt{2}(\rho_L((\gamma-1)p_L + (\gamma+1)p_R))^{3/2}} \tag{8}$$

$$\left.\frac{\partial u_{np}}{\partial p}\right|_{p_R} = -\frac{1}{\sqrt{\gamma \rho_R p_R}} \tag{9}$$

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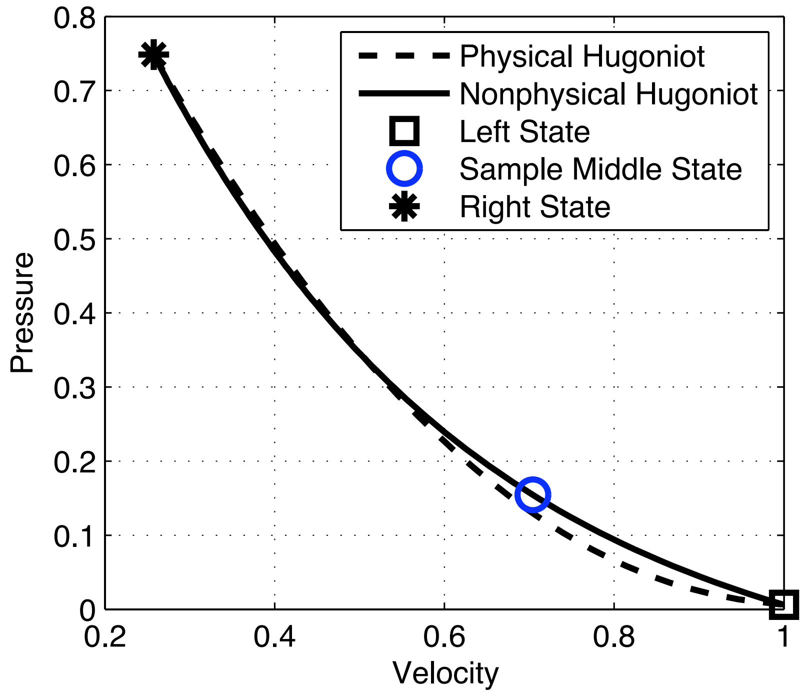

Figure 1. Example of the two curves.

We have that $p_L < p_R$, and $\rho_L < \rho_R$ and that if $\left.\frac{\partial u_p}{\partial p}\right|_{p_L} > \left.\frac{\partial u_{np}}{\partial p}\right|_{p_L}$ and $\left.\frac{\partial u_p}{\partial p}\right|_{p_R} > \left.\frac{\partial u_{np}}{\partial p}\right|_{p_R}$ there is at least one point of intersection. I think I can prove this.