A Hugoniot-Based Flux for Propagation of Single Shocks without Spurious Wave Production

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So lets look at a right moving shock with speed S, as in the diagram below, We get from a

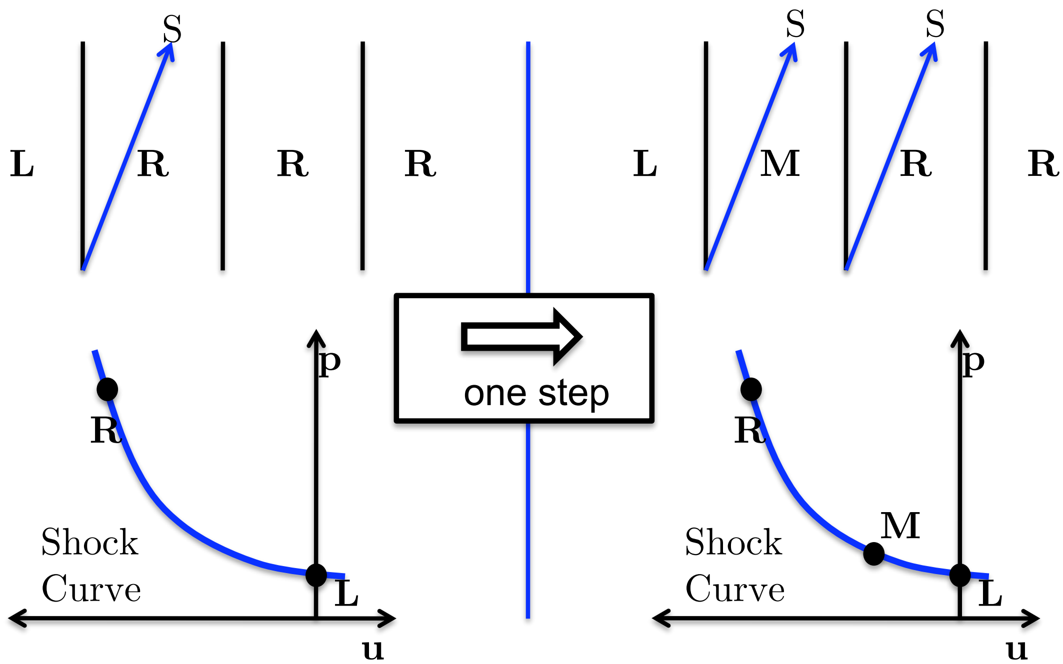

Figure 1: An Idealized Version

first order scheme that

$$\mathbf{u}_M = \mathbf{u}_R - \frac{\Delta t}{\Delta x}(\mathbf{f}_R - \mathbf{f}_L) \tag{1}$$

In order to achieve the goal, let the left flux contain ρ_L, u_L, P, where P is a pressure chosen to ensure the middle state lies on the Hugoniot curve. From previous analysis for shock CFL number $\nu = S\frac{\Delta t}{\Delta x}$, we know that

$$p_M = \nu p_L + (1-\nu)p_R + \frac{\gamma - 1}{2}\left(\frac{\nu(1-\nu)\rho_L\rho_R[u]^2}{(1-\nu)\rho_L + \nu\rho_R}\right) \tag{2}$$

But we really want $p_M = p_H(\mathbf{u}_M)$, where

$$p_H = \tilde{p} + p_L + \sqrt{\tilde{p}^2 + \frac{4\gamma}{\gamma+1}p_L\tilde{p}} \tag{3}$$

and

$$\tilde{p} = \frac{\gamma+1}{4}\left(\frac{\rho_R}{(1-\nu)\rho_L + \nu\rho_R}\right)^2 \nu^2\rho_L[u]^2 \tag{4}$$

but short of putting p_H directly as p_M, we can adjust the Riemann solver to compensate by setting p_L such that $p_M = p_H$. This sets

$$p_L = \frac{1}{\nu}\left(p_H - (1-\nu)p_R - \frac{\gamma-1}{2}\left(\frac{\nu(1-\nu)\rho_L\rho_R[u]^2}{(1-\nu)\rho_L + \nu\rho_R}\right)\right) \tag{5}$$