An IMEX Method for Radiation Hydrodynamics

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Introduction

- Some of the most challenging computational problems are those with coupled physics.
- Coupled physics has two major challenges:
  - Software implementation.
  - Disparities between temporal and spatial scales.
- This work focuses on disparities in temporal scales (stiffness) and the effects of nonlinearity.

Motivation

- Use an explicit scheme on the slower timescale and an implicit scheme on the faster timescale.
- These are generally referred to as **IMEX** schemes (IMplicit-EXplicit).
- These schemes are often computationally less expensive than a purely implicit scheme.
- In this work, a scheme for the general equation

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u_t = E(u) + I(u)$$

  is developed, where $E(u)$ and $I(u)$ are the explicit and implicit components, respectively.
- Conventional IMEX schemes tend to split the implicit operator, for example the second-order IMEX-BDF2 scheme is

$$\frac{3u^{n+1} - 4u^n + u^{n-1}}{2\Delta t} = 2E^n - E^{n-1} + I^{n+1}$$

- This scheme performs well in the implicit limit, but as $I(u) \to 0$, displays poor treatment of the explicit operator.

A BDF2-RK2 IMEX Scheme

- Introduce a scheme such that whenever $I(u) \equiv 0$, it reduces to the second-order, explicit Runge-Kutta (RK-2) method.
- Whenever $E(u) \equiv 0$, it should reduce to the second-order, backward-difference formula (BDF-2) method.
- For a constant time step Δt, we then have

$$v^{n+1} = u^n + \Delta t[E(u^n) + I(v^{n+1})],$$

$$\frac{3}{2}\left(\frac{u^{n+1} - u^n}{\Delta t} - \frac{1}{2}\left[E(u^n) + E(v^{n+1})\right]\right) - \frac{1}{2}\left(\frac{u^n - u^{n-1}}{\Delta t} - \frac{1}{2}\left[E(u^{n-1}) + E(v^n)\right]\right) = I(u^{n+1}).$$

- Since the first stage requires a separate nonlinear solve, an alternative is

$$v^{n+1} = u^n + \Delta t[E(u^n) + I(u^{n+1})]$$

Radiation Hydrodynamics

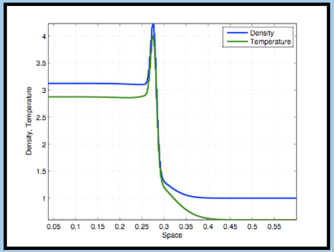
- The radiation hydrodynamics model examined is the non-relativistic Euler equations with a grey P_1 radiation treatment without scattering in non-dimensional form as

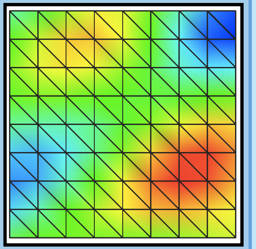
$$\begin{aligned}
\partial_t \rho + \partial_x(\rho v) &= 0,\\
\partial_t(\rho v) + \partial_x(\rho v^2 + p) &= -\mathbb{P}S_F,\\
\partial_t(\rho E) + \partial_x(\rho E + p)v &= -\mathbb{P}S_E,\\
\partial_t E_r + \mathbb{C}\partial_x F_r &= S_E,\\
\partial_t F_r + \frac{1}{3}\mathbb{C}\partial_x E_r &= \mathbb{C}S_F
\end{aligned}$$

- We choose to treat the hydrodynamics explicitly and all radiative effects implicitly, defining

$$\mathbf{E}(\mathbf{u}) = -\begin{pmatrix} \partial_x(\rho v) \\ \partial_x(\rho v^2 + p) \\ \partial_x(\rho E + p)v \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{I}(\mathbf{u}) = -\begin{pmatrix} 0 \\ \mathbb{P}S_F, \\ \mathbb{P}S_E, \\ \mathbb{C}\partial_x F_r - S_E \\ \frac{1}{3}\mathbb{C}\partial_x E_r - \mathbb{C}S_F \end{pmatrix}$$

Preliminary Numerical Results





- Numerical results are computed in a Discontinous Galerkin framework.
- On the left, a 1D, Mach 5 shocktube problem is shown, where the effects of radiation can be seen in both density and temperature.
- On the right, a 2D modified version of Lowrie's manufactured test problem was run.
- For this, temperature results are shown.

Final Thoughts

- Preliminary results show that BDF2-RK2 appears to be giving reasonable solutions to rad-hydro problems.
- While the advantages of BDF2-RK2 are not explicitly demonstrated here, previous results (not shown) do suggest a gain over traditional IMEX methods.
- Future work includes a detailed accuracy and grid convergence study as well as examination of standard test problems.

Acknowledgements

The first author would like to thank Professor Krzysztof Fidkowski for technical support and for the use of his research code. The support of Los Alamos National Lab was influential in the foundations of this research.