Entropy Distance

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1 Entropy Norm?

Define the entropy distance $z = \Delta\mathbf{v}^T\Delta\mathbf{u}$. Proof of positivity comes from considering the path $\mathbf{v} = \mathbf{v}_1 + \theta(\mathbf{v}_2 - \mathbf{v}_1)$ for $\theta \in [0, 1]$ and writing

$$z = \Delta\mathbf{v}^T\Delta\mathbf{u} = \Delta\mathbf{v}^T \int_{\mathbf{u}_1}^{\mathbf{u}_2} \mathbf{u}_{\mathbf{v}} \mathrm{d}\mathbf{v} = \Delta\mathbf{v}^t \int_0^1 \mathbf{u}_{\mathbf{v}} \mathrm{d}\theta \Delta\mathbf{v} \tag{1}$$

I believe that choosing $\Delta\mathbf{u} = \mathbf{u}$ results in proving that $\mathbf{v}^T\mathbf{u} \geq 0$. Positivity is shown above. **Note** I strongly believe that scalability fails.

2 Entropy Distance Definition

In 1D, the entropy distance I use is defined across a cell interface as

$$D_{i-1/2} = (\mathbf{v}_i - \mathbf{v}_{i-1})^T(\mathbf{u}_i - \mathbf{u}_{i-1}) \tag{2}$$

where we can associate that distance with the interface located at $x_{i-1/2}$. A different question is what is the entropy distance associated in a particular cell. In my work with moving meshes, I complete this by averaging as

$$D_i = \frac{1}{2}(D_{i-1/2} + D_{i+1/2}) \tag{3}$$

This further reduces to, dropping the transpose for notational simplicity, that $\mathbf{ab} = \mathbf{a}^T\mathbf{b} = \mathbf{a}\cdot\mathbf{b}$

$$D_i \;=\; \frac{1}{2}\left(\mathbf{v}_i\mathbf{u}_i - \mathbf{v}_{i-1}\mathbf{u}_i - \mathbf{v}_i\mathbf{u}_{i-1} + \mathbf{v}_{i-1}\mathbf{u}_{i-1} + \mathbf{v}_{i+1}\mathbf{u}_{i+1} - \mathbf{v}_i\mathbf{u}_{i+1} - \mathbf{v}_{i+1}\mathbf{u}_i + \mathbf{v}_i\mathbf{u}_i\right) \tag{4}$$

which may or may not simplify to something more physically intuitive. An alternate view may to be to ask what happens if we take the differences between fluxes at the interface and evaluate (with some linearizations?)

$$D_{12} = (\mathbf{F}_{i+1/2} - \mathbf{F}_{i-1/2})^T(\mathbf{f}_{i+1/2} - \mathbf{f}_{i-1/2}) \tag{5}$$

where $\mathbf{f}_{i+1/2}$ is the solution to the conserved variable riemann problem and $\mathbf{F}_{i+1/2}$ is the solution to the entropy variable riemann problem.