Turbulent Radiative Shock Modeling with Low-Order Angular Moment Resolution

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ABSTRACT

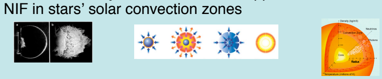
Preliminary stages of this investigation couple a hydrodynamic conservation system to radiative momentum and energy equations to develop a radiatively driven shock wave model. In particular, we are interested in low-order angular moment radiative angular closures. The second phase of this project is dedicated to the inclusion of turbulent phenomena. We augment the radiation gas-dynamics equations by the radiative turbulent kinetic energy and dissipation rate equations. Theoretical development and initial numerical investigations are considered. Applications of these studies include, but are not limited to, astrophysical events, advanced energy generation via inertial confinement fusion (ICF), and high energy density phenomena.

Motivation for Investigation

Shock wave applications occur in a broad variety of applications. Some representative examples where shock waves can occur are traffic flow, supersonic travel, and atmospheric events.

We are particularly interested in the physics of shock wave behavior driven and influenced by radiative effects in natural and controlled environments, Examples include astrophysical applications (supernovae and astrophysical jets) and inertial confinement fusion (ICF).

Trifecta of such phenomena - radiation, gas dynamics, turbulence - may be encountered in applications such as NIF in stars' solar convection zones



Investigative Goals and Relevance

- The main objective is to elucidate the physics for turbulent effects on radiative and non-radiative shocks via $K-\varepsilon$ model
- New endeavors include
  - Extend work on O(v/c) accurate transport equation by coupling the turbulent gas dynamics equations
  - Rather than constant or tabulated cross sections, we employ a medium-interactive cross section of the form $\Sigma_A \propto \frac{\rho}{T^3}$
  - Employ numerical methods as a tool to model and simulate such phenomena as experimental methods can be limited/expensive

Radiative Description: O(v/c) Accurate Radiative Energy and Momentum Equations

The transport equation contains terms dependent on velocity, of which the largest are O(v/c) and are necessary to conserve energy correctly and account for Doppler Shifts, aberration, and advection.

Assuming $a_R T_\infty^4/\rho_\infty c_S = O(1)$ so as to have moderate radiation in the system and neglecting scattering events, line emission, and resonance absorption, the O(v/c) accurate radiative transfer equation is

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+\Omega_i\frac{\partial}{\partial x_i}\right)I(\underline{\Omega},\upsilon)=-\gamma_L\left(1-\Omega_i\frac{v_i}{c}\right)\Sigma_A(\upsilon)I(\underline{\Omega},\upsilon)+\gamma_L^2\Sigma_A(\upsilon)B(\upsilon,T)$$
$$+\gamma_L^2\Omega_i\frac{v_i}{c}\left[2-\upsilon\frac{\partial}{\partial\upsilon}\right]\Sigma_A(\upsilon)B(\upsilon,T)\ ,$$

with radiative energy and momentum equations

$$\frac{\partial}{\partial t}\Theta+\frac{\partial}{\partial x_j}F_j=-\gamma_L\Sigma_A\left(c\Theta-3\frac{v_j}{c}F_j\right)+\gamma_L^2\Sigma_A a_R cT^4,$$

$$\frac{1}{c^2}\frac{\partial}{\partial t}F_i+\frac{\partial}{\partial x_j}P_{j,i}=-\frac{\gamma_L}{c}\Sigma_A\left(F_i-3v_jP_{j,i}\right)+\frac{\gamma_L^2}{c}\Sigma_A a_R v_i T^4.$$

Radiatively Driven Shock Waves: Theoretical and Preliminary Numerical Developments

We link radiation and hydrodynamics via the source moments and develop and open set of equations for *non-turbulent* radiative shocks

1. $\frac{\partial}{\partial t}\rho+\frac{\partial}{\partial x_i}\rho v_i=0\ ,$

2. $\frac{\partial}{\partial t}\rho v_i+\frac{\partial}{\partial x_j}\rho v_jv_i+\frac{\partial}{\partial x_i}(\gamma-1)\rho\left(E-\frac{1}{2}v_jv_j\right)=-\frac{1}{c^2}\frac{\partial}{\partial t}F_i-\frac{\partial}{\partial x_j}P_{j,i}\ ,$

3. $\frac{\partial}{\partial t}\rho E+\frac{\partial}{\partial x_j}\rho\left[\gamma E-(\gamma-1)\frac{1}{2}v_iv_i\right]v_j=-\frac{\partial}{\partial t}\Theta-\frac{\partial}{\partial x_j}F_j\ ,$

4. $\frac{\partial}{\partial t}\Theta+\frac{\partial}{\partial x_j}F_j=-\gamma_L\Sigma_A\left(c\Theta-3\frac{v_j}{c}F_j\right)+\gamma_L^2\Sigma_A a_R c\left[\frac{1}{c_V}\left(E-\frac{1}{2}v_iv_i\right)\right]^4\ ,$

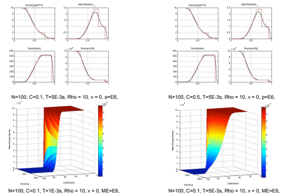
5. $\frac{1}{c^2}\frac{\partial}{\partial t}F_i+\frac{\partial}{\partial x_j}P_{j,i}=-\frac{\gamma_L}{c}\Sigma_A\left(F_i-3v_jP_{j,i}\right)+\frac{\gamma_L^2}{c}\Sigma_A a_R v_i\left[\frac{1}{c_V}\left(E-\frac{1}{2}v_iv_i\right)\right]^4\ ,\qquad \Sigma_A\cong\frac{\rho}{T^3}\equiv\frac{\rho}{c_V{}^3}\left(E-\frac{1}{2}v_iv_i\right)^{-3},$

where we define

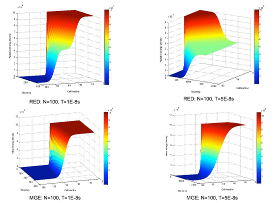
$\rho\to$ mass density
$\rho v\to$ momentum density
$\rho e=\frac{p}{(\gamma-1)}=\rho c_V T$
$\to$ Equation of State
$\gamma=\frac{c_P}{c_V}\to$ polytropic index
$\rho E\to$ mean energy density
$\Theta\to$ radiative energy density
$F\to$ radaitive flux
$P\to$ radiation pressure

We employ Riemann solvers for non-radiative and radiative shock modeling. We provide examples for both, respectively:

Non-Radiative Shock



Radiative Shock



Non-Radiative Turbulent Shocks

In modeling turbulence, we must employ the Navier-Stokes equations to account for viscosity and energy dissipation. Through Reynolds-Favre averaging and $K-\varepsilon$ gradient diffusion closures, we first arrive at a *non-radiative* turbulent gas-dynamics system

1. $\frac{\partial}{\partial t}\rho+\frac{\partial}{\partial x_i}\rho v_i=0,$

2. $\frac{\partial}{\partial t}\rho v_i+\frac{\partial}{\partial x_j}\rho v_jv_i+\frac{\partial}{\partial x_i}(\gamma-1)\rho\left(E-\frac{1}{2}v_jv_j\right)=\frac{\partial}{\partial x_j}\underbrace{\rho\tau_{j,i}}_{\text{Stress Tensor}}+\frac{\partial}{\partial x_i}(\gamma-1)\underbrace{\rho K}_{\text{T.K.E.}}\ ,$

3. $\frac{\partial}{\partial t}\rho E+\frac{\partial}{\partial x_j}\rho\left[\gamma E-(\gamma-1)\frac{1}{2}v_iv_i\right]v_j=\frac{\partial}{\partial x_j}\left[\frac{\mu_T}{\sigma_E}\frac{\partial}{\partial x_j}E+(\gamma-1)\rho\left(v_i\tau_{i,j}-Kv_j\right)\right]+(\gamma-1)\underbrace{\frac{\partial}{\partial x_j}\frac{\mu_T}{\sigma_K}\frac{\partial}{\partial x_j}K}_{\text{Turbulence Transport}}+\underbrace{\rho\varepsilon}_{\text{Dissipation}}\ ,$

4. $\frac{\partial}{\partial t}\rho K+\frac{\partial}{\partial x_j}\rho Kv_j=\underbrace{\rho\tau_{i,j}\frac{\partial}{\partial x_j}v_i}_{\text{Production}}-\underbrace{\rho\varepsilon}_{\text{Dissipation}}+\underbrace{\frac{\partial}{\partial x_j}\frac{\mu_T}{\sigma_K}\frac{\partial}{\partial x_j}K}_{\text{Turbulence Transport}}-\underbrace{\frac{\upsilon_T}{\sigma_\rho}\frac{1}{\rho}\frac{\partial}{\partial x_i}\rho\frac{\partial}{\partial x_j}(\gamma-1)\rho\left(E-\frac{1}{2}v_jv_j\right)}_{\text{Pressure Work}}+\underbrace{\left(\alpha_2\rho\tau_{i,j}\frac{\partial}{\partial x_j}v_i+\alpha_3M_T\rho\varepsilon+\frac{16}{3}\alpha_3M_TK\frac{\partial}{\partial x_j}v_j\right)M_T}_{\text{Pressure Dilatation}}\ ,$

5. $\frac{\partial}{\partial t}\rho\varepsilon+\frac{\partial}{\partial x_j}\rho\varepsilon v_j=\frac{\varepsilon}{K}\left[C_{\varepsilon,1}\rho\tau_{i,j}\frac{\partial}{\partial x_j}v_i-C_{\varepsilon,2}\rho\varepsilon-C_{\varepsilon,3}\frac{\upsilon_T}{\sigma_\rho}\frac{1}{\bar{\rho}}\frac{\partial}{\partial x_i}\rho\frac{\partial}{\partial x_j}(\gamma-1)\rho\left(E-\frac{1}{2}v_jv_j\right)\right]+\frac{\varepsilon}{K}C_{\varepsilon,4}\left(\alpha_2\rho\tau_{i,j}\frac{\partial}{\partial x_j}v_i+\alpha_3M_T\rho\varepsilon+\frac{16}{3}\alpha_3M_TK\frac{\partial}{\partial x_j}v_j\right)M_T+\frac{\partial}{\partial x_j}\frac{\mu_T}{\sigma_\varepsilon}\frac{\partial}{\partial x_j}\varepsilon\ .$

We gain two additional equations to account for the turbulent kinetic energy and dissipation rate. Combining them with the turbulent density, velocity, and energy equations leads to a closed model for *non-radiative* turbulent shock phenomena.

Radiative Turbulent Shocks

Just as we Reynolds-Favre averaged the gas-dynamics equations, we construct a mean radiative hydrodynamic model. It is essential this be done in order to couple radiation and turbulence correctly.

We develop an initial unclosed *radiative* turbulent shock wave model

1. $\frac{\partial}{\partial t}\rho+\frac{\partial}{\partial x_i}\rho v_i=0,$

2. $\frac{\partial}{\partial t}\left(\rho v_i+\frac{1}{c^2}F_i\right)+\frac{\partial}{\partial x_j}\left(\rho v_jv_i+P_{j,i}\right)+\frac{\partial}{\partial x_i}(\gamma-1)\rho\left(E-\frac{1}{2}v_jv_j\right)=\frac{\partial}{\partial x_j}\rho\tau_{j,i}+\frac{\partial}{\partial x_i}(\gamma-1)\rho K-\frac{\partial}{\partial t}\frac{1}{c^2}\overline{F_i''}-\frac{\partial}{\partial x_j}\overline{P_{j,i}''},$

3. $\frac{\partial}{\partial t}(\rho E+\Theta)+\frac{\partial}{\partial x_j}\left\{\rho\left[\gamma E-(\gamma-1)\frac{1}{2}v_iv_i\right]v_j+F_j\right\}=\frac{\partial}{\partial x_j}\left[\frac{\mu_T}{\sigma_E}\frac{\partial}{\partial x_j}E+(\gamma-1)\rho\left(v_i\tau_{i,j}-Kv_j\right)\right]+(\gamma-1)\frac{\partial}{\partial x_j}\frac{\mu_T}{\sigma_K}\frac{\partial}{\partial x_j}K+\rho\varepsilon-\frac{\partial}{\partial t}\overline{\Theta''}-\frac{\partial}{\partial x_j}\overline{F_j''},$

4. $\frac{\partial}{\partial t}\Theta+\frac{\partial}{\partial x_j}F_j=-\gamma_L\frac{\xi}{\hat{T}}\left(c\rho\Theta-\frac{3}{c}\rho v_jF_j\right)+\gamma_L^2\xi a_R\frac{c}{c_V}\left[\rho\left(E-\frac{1}{2}v_iv_i-K\right)\right]-\frac{\partial}{\partial t}\overline{\Theta''}-\frac{\partial}{\partial x_j}\overline{F_j''}+\gamma_L\frac{\xi}{\hat{T}}\frac{3}{c}\overline{\rho v_j''F_j''}\ ,$

5. $\frac{1}{c^2}\frac{\partial}{\partial t}F_i+\frac{\partial}{\partial x_j}P_{j,i}=-\frac{\gamma_L}{c}\frac{\xi}{\hat{T}}\left(\rho F_i-3\rho v_jP_{j,i}\right)+\frac{\gamma_L^2}{c}\xi a_R\frac{1}{c_V}\left[\rho v_i\left(E-\frac{1}{2}v_jv_j\right)-\left(\frac{\mu_T}{\sigma_E}\frac{\partial}{\partial x_i}E+\rho\left(v_j\tau_{j,i}-v_iK\right)+\frac{\mu_T}{\sigma_K}\frac{\partial}{\partial x_i}K\right)\right]$
$\quad-\frac{1}{c^2}\frac{\partial}{\partial t}\overline{F_i''}-\frac{\partial}{\partial x_j}\overline{P_{j,i}''}+\frac{\gamma_L}{c}\frac{\xi}{\hat{T}}3\overline{\rho v_j''P_{j,i}''}\ ,$

6. $\frac{\partial}{\partial t}\rho K+\frac{\partial}{\partial x_j}\rho Kv_j=\rho\tau_{i,j}\frac{\partial}{\partial x_j}v_i-\rho\varepsilon+\frac{\partial}{\partial x_j}\left(\overline{t_{j,i}v_i''}-\overline{\rho v_j\frac{1}{2}v_i''v_i''}-\overline{pv_i''}+\overline{P_{j,i}''v_i''}\right)+\overline{\frac{\partial}{\partial t}v_i''\frac{F_i''}{c^2}}$
$\quad-\frac{\upsilon_T}{\sigma_\rho}\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\rho\right)\left[\frac{\partial}{\partial x_i}(\gamma-1)\rho\left(E-\frac{1}{2}v_iv_i\right)-\frac{\partial}{\partial t}\frac{F_i}{c^2}-\frac{\partial}{\partial x_j}P_{j,i}\right]+\overline{\left(p'\frac{\partial}{\partial x_i}-P_{j,i}''\frac{\partial}{\partial x_j}-\frac{F_i''}{c^2}\frac{\partial}{\partial t}\right)v_i''}$

7. $\frac{\partial}{\partial t}\rho\varepsilon+\frac{\partial}{\partial x_j}\rho\varepsilon v_j=\frac{\varepsilon}{K}\left[-C_{\varepsilon,0}\frac{\upsilon_T}{\sigma_\rho}\frac{1}{\rho}\left(\frac{\partial}{\partial x_i}\rho\right)\left[\frac{\partial}{\partial x_i}(\gamma-1)\rho\left(E-\frac{1}{2}v_iv_i\right)-\frac{\partial}{\partial t}\frac{F_i}{c^2}-\frac{\partial}{\partial x_j}P_{j,i}\right]-C_{\varepsilon,1}\rho\tau_{i,j}\frac{\partial}{\partial x_j}v_i\right]$
$\quad+\frac{\varepsilon}{K}\left[-C_{\varepsilon,2}\rho\varepsilon+C_{\varepsilon,3}\overline{\left(p'\frac{\partial}{\partial x_i}-P_{j,i}''\frac{\partial}{\partial x_j}-\frac{F_i''}{c^2}\frac{\partial}{\partial t}\right)v_i''}\right]-\frac{\partial}{\partial x_j}\left(\overline{t_{j,i}v_i''}-\overline{\rho v_j\frac{1}{2}v_i''v_i''}-\overline{pv_i''}+\overline{P_{j,i}''v_i''}\right)+\overline{\frac{\partial}{\partial t}v_i''\frac{F_i''}{c^2}}$

where we employ the Reynolds-Favre averaged temperature as an initial approximation in our cross section

$$\hat{T}=\frac{1}{c_V}\left(E-\frac{1}{2}v_iv_i-K\right).$$

neutral $\to$ hydrodynamics
blue $\to$ radiation
yellow $\to$ turbulence
green $\to$ radiative turbulence

Ongoing and Future Investigations

- Improve numerics and switch from explicit to implicit solvers
- Develop blast wave Riemann solvers
- Generate an initial algorithm for non-radiative turbulent shocks
- Seek closure approximations for *radiative* turbulent contributions
- Figure how to best Reynolds-Favre average the cross section
- Couple radiative hydrodynamic and turbulent hydrodynamic models as one full system