Investigating the Numerical Wall Heating Phenomenon

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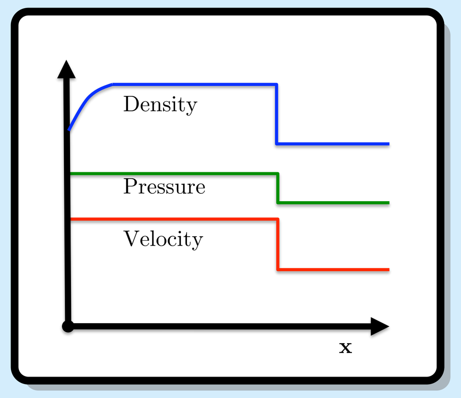
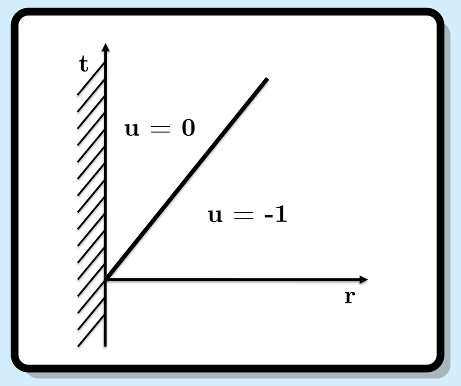
Introduction

- There is a common difficulty in the numerical solution of gasdynamic problems with strong shockwaves.
- At boundaries and near intersections of shockwaves, regions of anomalously high temperature appear.
- This is universal among shock-capturing codes and casts doubt on the use of these methods if an assessment of temperature variation is important.
- This is the case in CRASH, where incorrect wall temperatures can cause unnecessary wall ablation.

The Noh Problem

- A problem that specifically highlights this difficulty was devised by Noh (1986).
- It describes the implosion of a strong shockwave by imposing an inward velocity.





- Two things are typically observed:
  - The pressure near the wall quickly reaches a constant level (which is close to correct).
  - The density is too small and the resulting temperature is too high, hence the 'wall heating phenomenon'.

Methodology

- The Euler equations can be written in one dimension as $\mathbf{u}_t + \mathbf{f}(\mathbf{u})_x = 0$ or expanded

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ \rho u \\ E\end{bmatrix} + \frac{\partial}{\partial x}\begin{bmatrix}\rho u \\ \rho u^2 + p \\ u(E+p)\end{bmatrix} = \begin{bmatrix}0 \\ 0 \\ 0\end{bmatrix}.$$

- To isolate the problem, use a first-order in space and time finite-volume method of the form

$$\mathbf{u}_i^{n+1} = \mathbf{u}_i^n - \frac{\Delta t}{\Delta x}(\mathbf{f}^n_{i+\frac{1}{2}}(\mathbf{u}_i^n, \mathbf{u}_{i+1}^n) - \mathbf{f}^n_{i-\frac{1}{2}}(\mathbf{u}_{i-1}^n, \mathbf{u}_i^n)).$$

Pressure Convexity

- Start the simple case of the Noh problem under planar symmetry, with initial conditions $\mathbf{u}_0 = [\rho_0, \rho_0 u_0, E_0]$.
- Define the uniform state produced by the shock as $\mathbf{u}_1$.
- Assume an ideal gas with convex equation of state

$$p = (\gamma - 1)\left[E - \frac{\rho u^2}{2}\right].$$

and define a shock Courant number $\nu = \frac{S\Delta t}{\Delta r}$ for some shock speed S.

- After one timestep, the new state in the cell next to the wall will be

$$\mathbf{u}(\nu) = \nu\mathbf{u}_1 + (1-\nu)\mathbf{u}_0.$$

- All conserved variables change by an amount directly proportional to the timestep, however the average pressure does not change linearly.
- Instead, the average pressure is

$$p(\nu) = \nu p_1 + (1-\nu)p_0 + \frac{\nu(1-\nu)\rho_0\rho_1 u_0^2}{2(\nu\rho_1 + (1-\nu)\rho_0)},$$

which is greater than a linearly interpolated pressure.

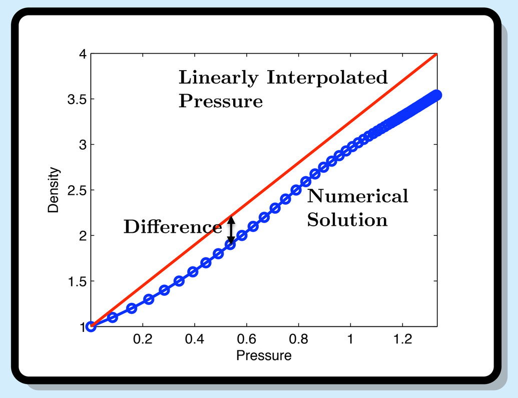
- This higher pressure is then used in the second timestep and results in an excessive pressure gradient that expels gas away from the center, resulting in the removal of too much mass.
- Although the density is correct at the first timestep, it will be too low in the second step and in further steps this process continues, resulting in wall heating.

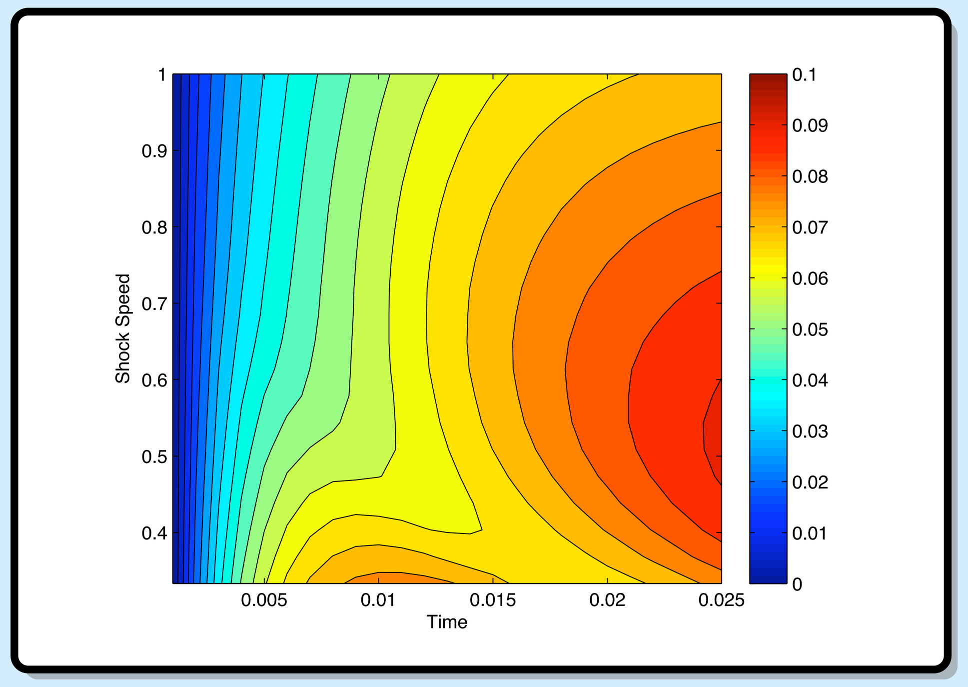
A Linear Equation of State (EOS)

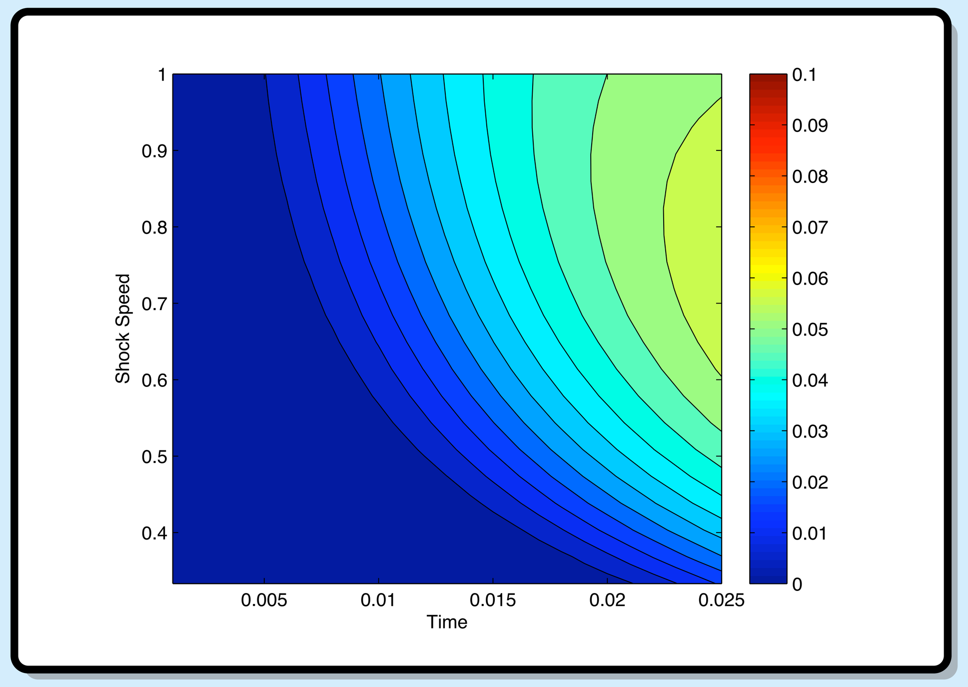
- To see if this is the driving mechanism, introduce a linear equation of state of the form

$$p = (\gamma - 1)E.$$

- For this EOS, a Noh problem emerges and the results can be compared to the ideal EOS.
- To make a comparison, look at $p - \rho$ space for the cell closest to the wall and measure the difference, as shown to the right.



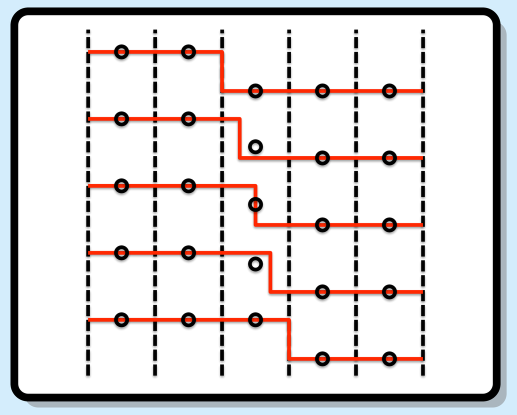




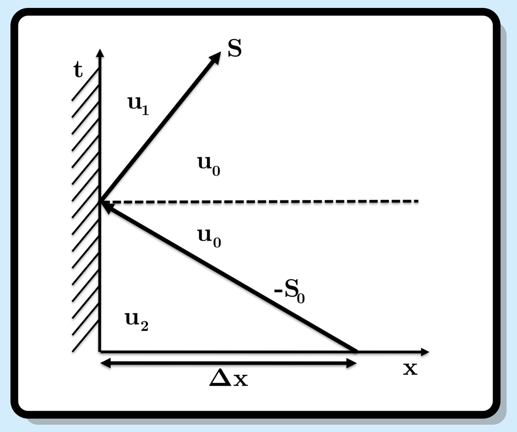
- Above, contour plots of the difference between a linearly interpolated pressure and the numerical solution for a range of shock speeds are shown, for ideal EOS (top) and linear EOS (bottom).
- Taking a closer look, using a linear EOS seems to eliminate one mechanism of error.
- However, some error at the wall still remains.

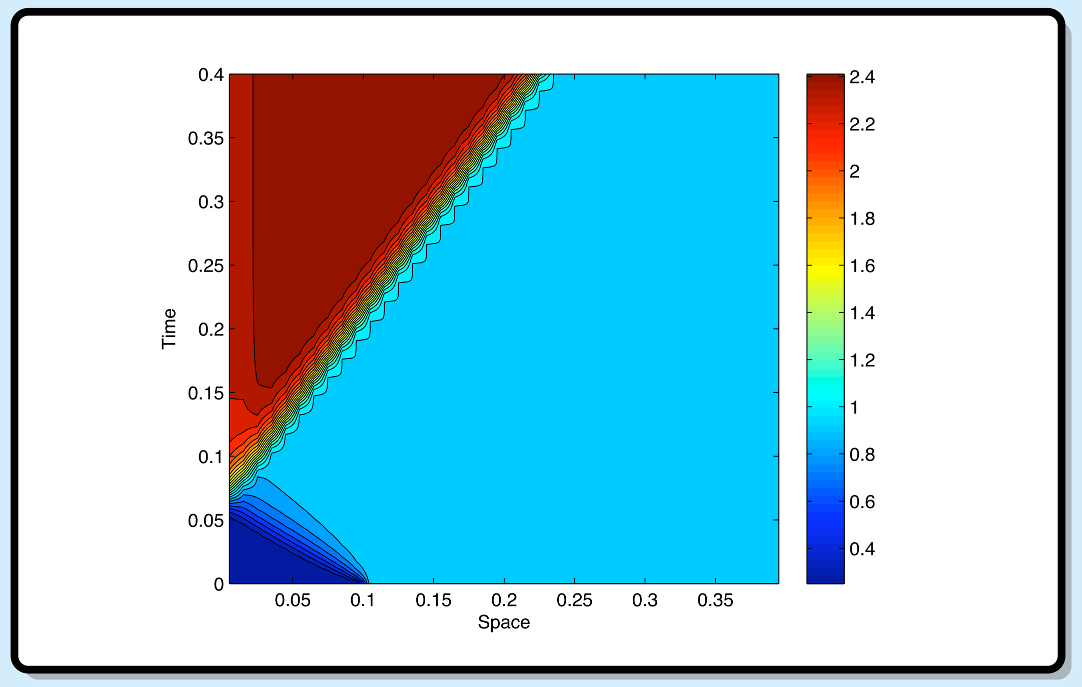
Shock Transition with an Ideal EOS

- There is at least one more mechanism at work.
- Since the EOS is only one mechanism, we can switch back to the ideal EOS for the following examination.
- Previous work (Arora and Roe, 1996, Karni and Čanić, 1997, Xu and Hu, 1998) suggests that this could be due to the slowly moving shock or similar phenomena.
- For a captured shock to be located anywhere on a 1D grid, at least one internal value is needed.
- If we regard the figure as snapshots of the moving shock, it appears that this shock structure will be periodic.

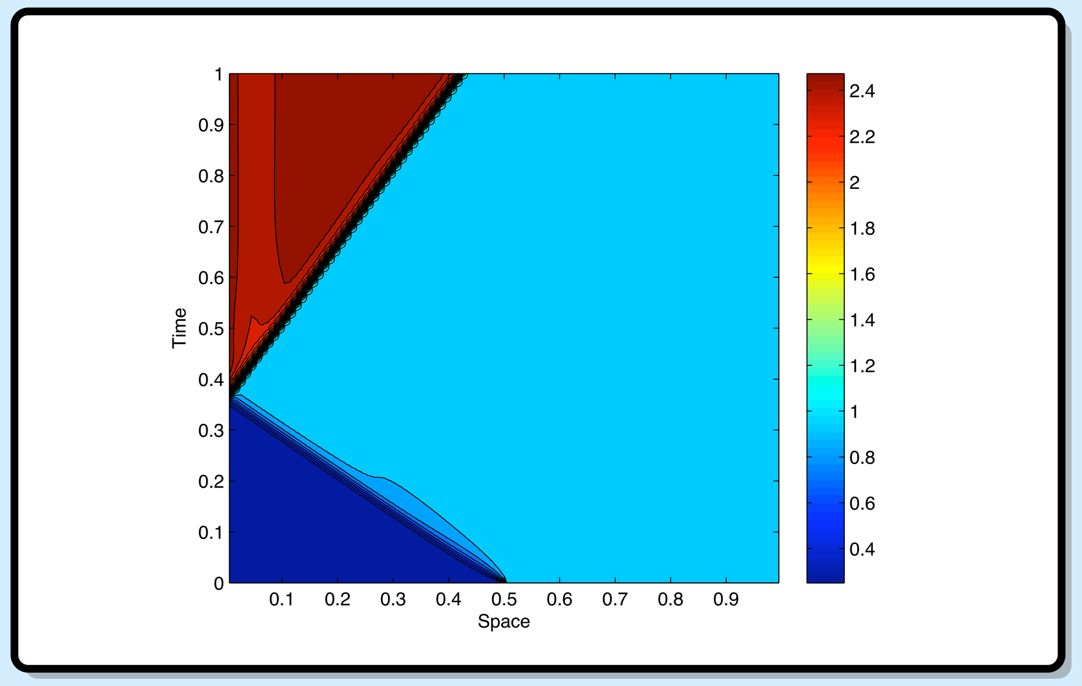


- In order to get into a periodic structure, a shock has to 'transition' into this state from its initial stages.
- To investigate shock transition as a mechanism, the reflecting Noh problem has been developed.
- This is similar to the original Noh problem, but allows for the shock to transition before contacting the wall.





- In both figures, density contours are shown.
- The shock is started at $\Delta x = 0.1$ and does not have time to go through transition before reflecting off the wall, and wall heating is observed.



- In this case, the shock is started at $\Delta x = 0.5$ and does fully develop, and no wall heating is observed, however an alternate density defect does occur.
- **Future work** revolves around the questions:
  - "Can we start a shock in a stable cycle, without it going through transition?"
  - "Now that we think we understand wall heating, how do we eliminate it?"