Shock Capturing Anomalies and the Jump Conditions in One Dimension

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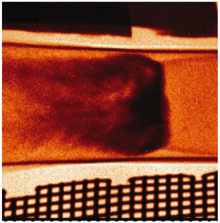
Introduction

- Understanding shockwaves is critical in the prediction and study of many phenomena, where the abrupt changes in material properties due to shockwaves can greatly affect regions of interest such as surfaces and activate other physical mechanisms, such as combustion or ionization.



Drake, 2011

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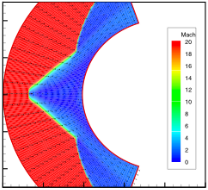
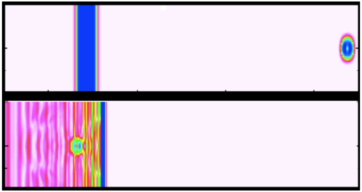
Introduction

- Previous work with shock capturing schemes has classified some of these errors as errors in shock position, spurious waves, or unstable shock behavior.
- We will refer to these errors as **Shockwave Anomalies**.
- These are numerical artifacts formed due to the presence of shockwaves within the flow solution.



(a) Ismail, 2006


(b) Johnsen, 2011

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Outline

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The Euler Equations

$$\mathbf{u}_t + \mathbf{f}(\mathbf{u})_x = 0.$$

$$\frac{\partial}{\partial t}\begin{bmatrix} \rho \\ \rho u \\ E \end{bmatrix} + \frac{\partial}{\partial x}\begin{bmatrix} \rho u \\ \rho u^2 + p \\ u(E+p) \end{bmatrix} = \mathbf{0}$$

- The equation of state is

$$p = (\gamma - 1)\left(E - \frac{1}{2}\rho u^2\right)$$

- The speed of sound is then $a = \sqrt{\frac{\gamma p}{\rho}}$.

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The Euler Equations

Rankine-Hugoniot Jump Conditions

- For a shockwave moving in 1D with speed S, the Rankine-Hugoniot jump conditions are given by $[\mathbf{f}] = S[\mathbf{u}]$.
- Given the left preshock state $\mathbf{u}_L$ and the postshock density, ρ_R.

$$
\begin{aligned}
\frac{p_R}{p_L} &= \frac{(\gamma+1)\rho_R - (\gamma-1)\rho_L}{(\gamma+1)\rho_L - (\gamma-1)\rho_R} \\
u_R - u_L &= (p_L - p_R)\sqrt{\frac{2}{\rho_L\left((\gamma-1)p_L + (\gamma+1)p_R\right)}}
\end{aligned}
$$

- Taking $\rho_R > \rho_L$ results in the physical Hugoniot, and $\rho_R < \rho_L$ results in the nonphysical Hugoniot.

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Godunov-Type Finite Volume Methods

- In this work, the explicit Forward Euler in space and time scheme is used,

$$\mathbf{u}_i^{n+1} = \mathbf{u}_i^n - \frac{\Delta t}{\Delta x}(\mathbf{f}_{i+\frac{1}{2}}^n(\mathbf{u}_i^n, \mathbf{u}_{i+1}^n) - \mathbf{f}_{i-\frac{1}{2}}^n(\mathbf{u}_{i-1}^n, \mathbf{u}_i^n))$$

  with the flux $\mathbf{f}_{i+\frac{1}{2}}^n(\mathbf{u}_i^n, \mathbf{u}_{i+1}^n)$ coming from a Riemann solver.
- In this work, both an exact and approximate (Roe) Riemann solver are used.
- 2^{nd}-order (or "high-resolution") methods do not alleviate shock anomalies; they instead preserve errors for much longer than 1^{st}-order methods.

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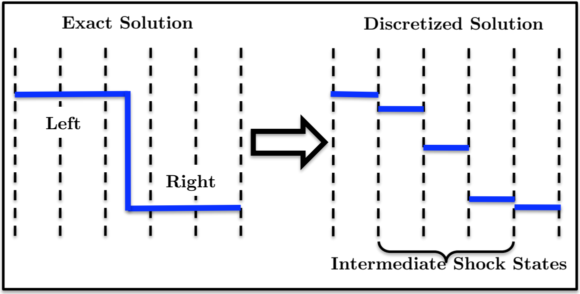
Intermediate Shock States



- For a single captured shock to be located anywhere on a 1D grid, at least one internal value is needed.
- While the number of required intermediate states varies from scheme to scheme, all conservative schemes produce them.

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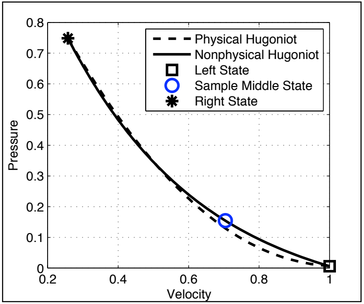
Stationary Shocks

- The stationary shockwave is the simplest shockwave and can be achieved by setting $S = 0$.
- To allow the shock to sit sub-cell, there must be at least one intermediate shock state.
- For the exact or Roe Riemann solvers, this intermediate state lies on the non-physical Hugoniot curve.



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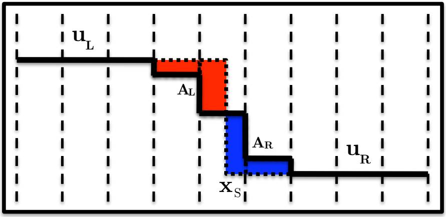
Stationary Shocks

Shock Position

- So what's anomalous about the stationary shock? Lets compute shock position.



- To compute shock position, we can use the equal area rule.
- For the conserved variable, u, the shock position, x_S, divides the discrete solution such that A_L and A_R are equal.

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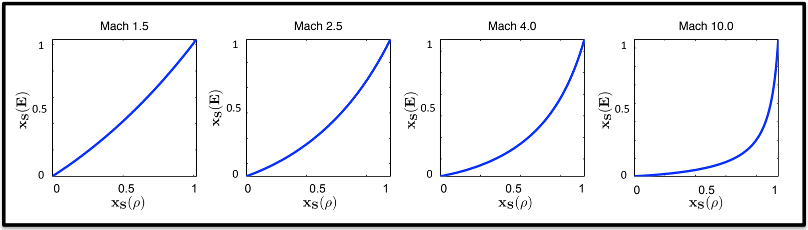
Stationary Shocks

Shock Position - An Ambiguity

- For a system of conservation laws, we would hope that shock positions calculated from each conserved variable would agree.
- Unfortunately this is not the case; plots of $x_S(E)$ versus $x_S(\rho)$ are shown below for M_0 =1.5, 2.5, 4.0 and 10.0.



- Shock positions from conservation of mass and energy are

$$x_S(\rho) = \frac{\rho_M - \rho_R}{\rho_L - \rho_R}, \quad x_S(E) = \frac{E_M - E_R}{E_L - E_R}$$

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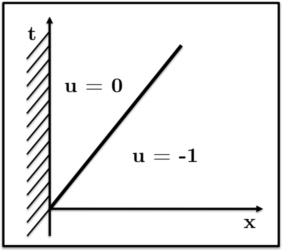
Wall Heating

The Noh Problem

- Notorious difficulty encapsulated by William Noh (1986)
- Describes a uniform gas driven into a wall and the resulting strong shock.
- Exists in three dimensions with radial, cylindrical, and spherical symmetry.
- We look at the 1D problem.



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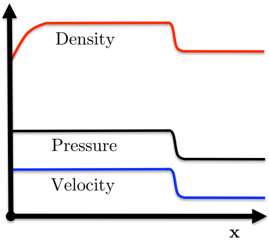
Wall Heating

Representative Solutions

- Virtually all shock-capturing methods provide quite good solutions for pressure and velocity, but predict too small a density in a small region at the origin.
- In consequence the temperature there is too high, so that this and related phenomena have been called *wall heating*.



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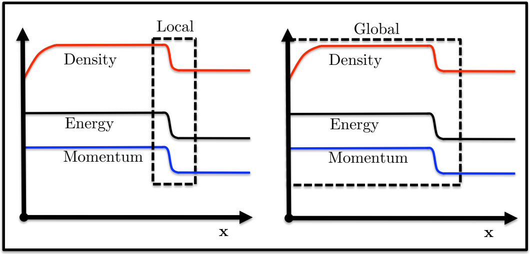
Wall Heating

Shock Position

- To show how shock position plays a role in wall heating, we can calculate it using two different control volumes:
  - **Local** - contains only the region immediately around the shock.
  - **Global** - contains the whole domain, including the density defect in the calculation.



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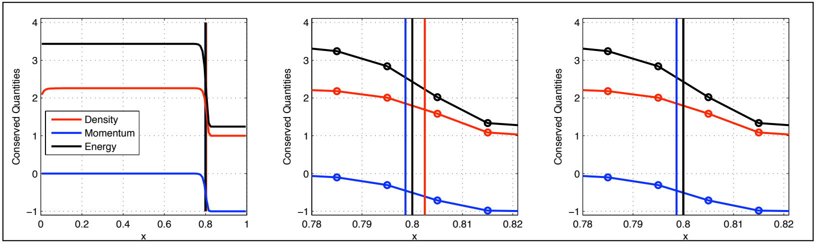
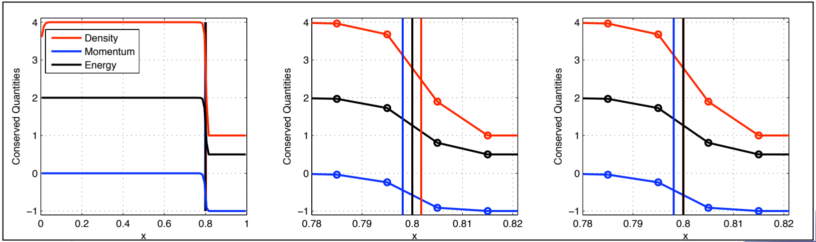
Wall Heating

Conserved quantities for a Mach 1.1 shock (Top) and Mach 10^6 shock (Bottom).





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Slowly Moving Shockwave Phenomenon

- The slowly moving shockwave phenomenon results in spurious waves developing at the fronts of slowly moving shocks.
- These waves are purely numerical, but upon creation by the method, they appear as physical waves and will be propagated as such.
- The cause of this phenomenon is that all intermediate states within the shock do not satisfy the Rankine-Hugoniot jump conditions of the initial states.
- The solution of future Riemann problems results in a family of waves being created.

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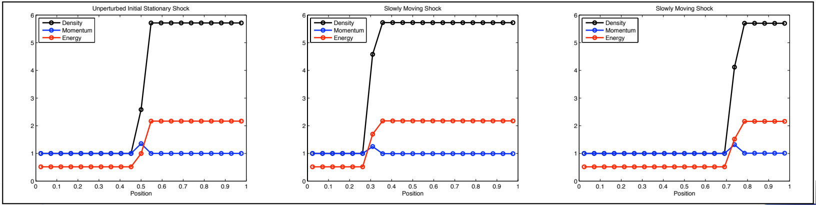
Slowly Moving Shockwave Phenomenon

- While it is known that these intermediate states do not lie directly on the Hugoniot, it is not obvious that the nonlinearity is directly responsible for the spurious waves.
- Here, it can be directly related by examining the limit of the stationary shock.
- Starting with a stationary shock, by slightly perturbing this right state, slowly moving shocks in either direction can be created.



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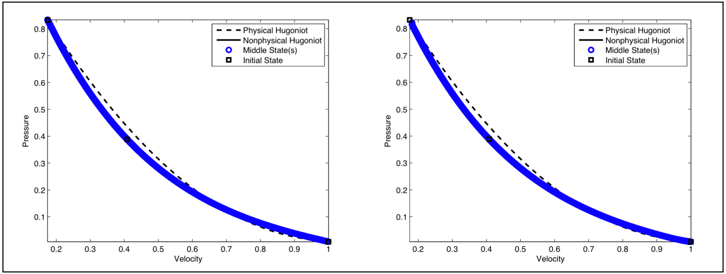
Slowly Moving Shockwave Phenomenon

Intermediate States from the Slowly Moving Shock



- (Left) Intermediate states of left moving shock.
  (Right) Intermediate states of the right moving shock.
- While the intermediate states do not lie exactly on the nonphysical Hugoniot, they do lie slightly off of it, and will coincide within the limit.

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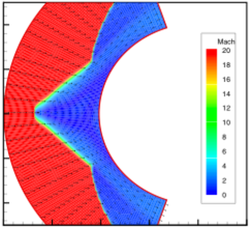
The Carbuncle

- The **carbuncle** is the standard name given to the anomaly produced when computing the flow past a blunt body in the hypersonic flow regime.
- Instead of the expected smooth shock profile, a pair of oblique shocks are observed ahead of the surface.
- This solution is an entropy satisfying, and experimentally producible solution to the Euler equations, thus it cannot be excluded by any simple physical property.



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The Carbuncle

Initial Setup

- In one dimension, the carbuncle manifests itself as an initially stationary shock that does not remain stationary.
- This problem can be set up exactly as a stationary shock problem, with the intermediate state varied as

$$\rho_M = x_S \rho_L + (1 - x_S)\rho_R$$

  and the remaining variables computed to lie on the nonphysical Hugoniot.
- To prevent waves from leaving the domain, a fixed mass outflow boundary condition is used.

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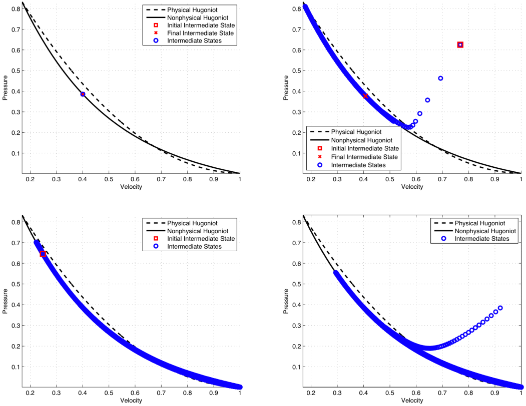
The Carbuncle

Numerical Results

- (Top) Stable Equilibrium. (Bottom) Unstable Equilibrium.



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What if the Hugoniot were linear?

- Here we present a set of nonlinear conservation laws complete with a linear shock curve.

$$\begin{bmatrix} u \\ a \\ b \end{bmatrix}_t + \begin{bmatrix} \frac{1}{2}\left(u^2 + a^2 + b^2\right) \\ ua \\ ub \end{bmatrix}_x = 0$$

- This system has a propagation speed, $c = \sqrt{a^2 + b^2}$, genuinely nonlinear "acoustic waves" that move with speeds $u \pm c =$ constant and a contact discontinuity with speed u.

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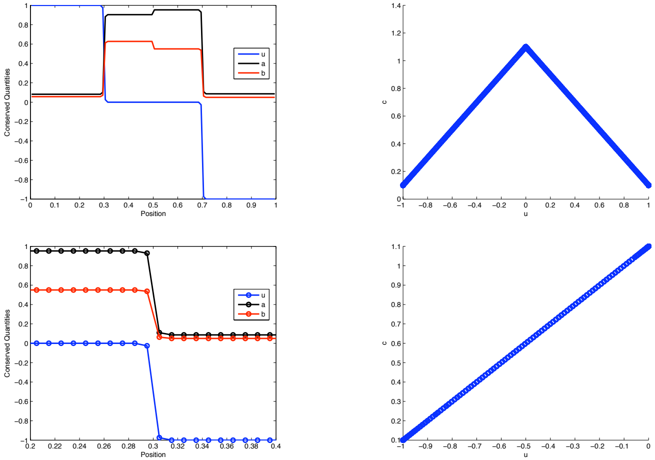
What if the Hugoniot were linear?

(top) Noh Problem. (bottom) Slowly moving shock.



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Concluding Thoughts

- In this work, we have established connections between four common defects of shock-capturing methods, by linking each of them to the nonlinearity of the Hugoniot curve.
- Since that nonlinearity is a physical fact, it would seem that an over-reliance on physical derivations may have some drawbacks, especially if they rely on assumptions of thermodynamic equilibrium.
- Of note, is that wherever these intermediate states lie, they are always assumed to satisfy local thermal equilibrium (LTE) in a cell-averaged sense, and therefore to obey the equilibrium EOS.

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Concluding Thoughts

- From the exact solution to the NS equations, we know that LTE does not hold inside the shock itself.
- This suggests a form of 'interpretation' error, an error made in the philosophical approach we take in assigning meaning to the shock-captured solution.
- At present, the benefits of schemes that crisply resolve discontinuities seem to outweigh the drawbacks, but at the cost of various fixes and workarounds.

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Future Work

- Future work will focus on strengthening the connections made, further examining the relationship of stationary shocks to their transient relatives.
- By instituting a form of shock structure control, we hope to position intermediate shock states to minimize the creation of spurious waves.
- Since all of problems have multi-dimensional counterparts, $1\frac{1}{2}$D and 2D versions will be analysed.

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