Surface Reconstruction for Unstructured Meshes

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PDF Version

Introduction

Problem: take an arbitrary pre-existing mesh and insert a prescribed internal boundaries into its topology.

Goal: obtain internal boundaries with minimal mesh modification

Motivation: remeshing to match the surfaces of a newly deposited layers of material in the simulation of the semiconductor device manufacturing process.

Other Approaches to Similar Problems

Mesh Cleaving (Levine, Clemson, Whitaker, Utah)

- Builds a background unstructured mesh based on density of [biomedical] data, and conforms it by moving points using a template.
- "Provable" quality bounds, though in practice, not great, especially for non-manifold domains.
- Works well with level set data, but does not easily conform to prescribed boundary curves.
- Requires initially "well-sized" initial mesh.

Other Approaches to Similar Problems

Universal Meshes (Lew, Stanford)

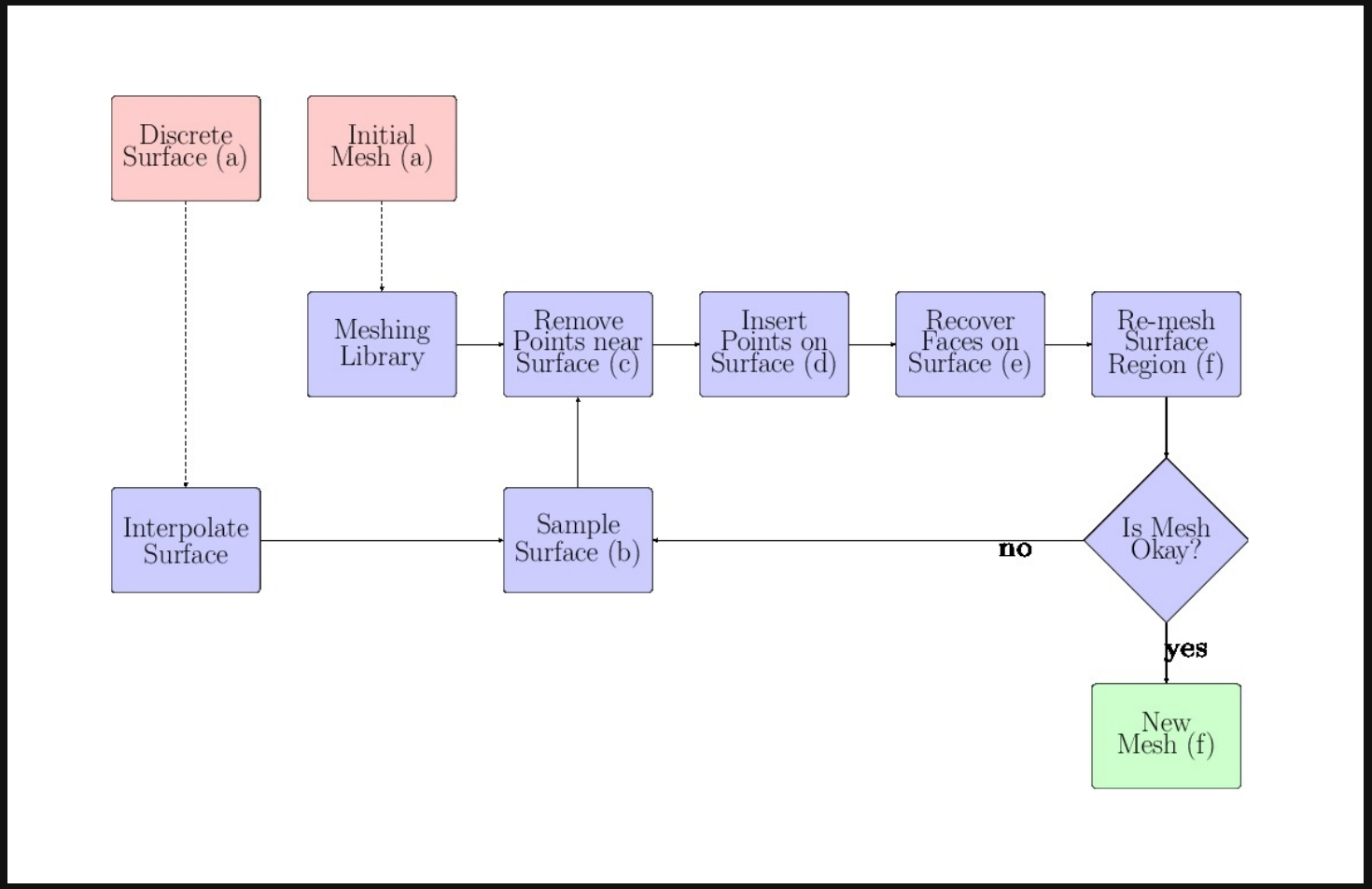
- Starting with a initially uniform unstructured mesh, conform boundary.
- Can prove the mesh is boundary conforming (for conditions on the geometry).
- Requires post-processing for smoothing, and no guarantees for quality.
- Not yet extended for arbitrary unstructured meshes, nor conforming internal boundaries.

Other Approaches to Similar Problems

Both other methods rooted in Isosurface Stuffing.
Both face the challenge of moving existing points onto the surface.
Our approach is closer to traditional mesh generation, using point insertion.

Our challenge is in surface recovery.

Review – Our Approach



Surface Sampling - Length Scale

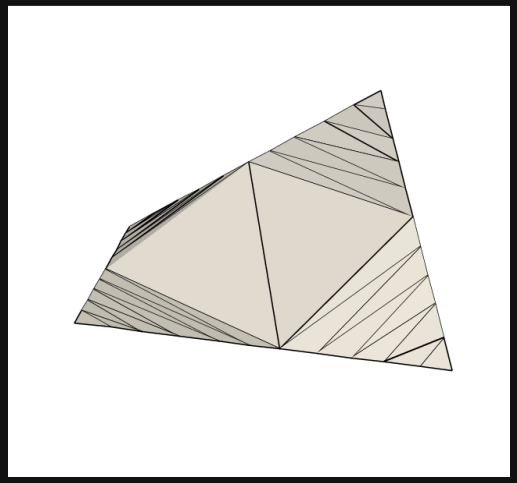
Define the length scale of a d-dimensional vertex by summing over adjacent simplices

$$LS(v) = \left(d \sum_{i=1}^{N} V_i \Big/ \sum_{i=1}^{N} \theta_i \right)^{\frac{1}{d}}$$

Where θ_i is a solid angle, and V_i is the volume (area) of the simplex.

The length scale at any point in the mesh can be determined using bary centric interpolation inside its containing simplex.

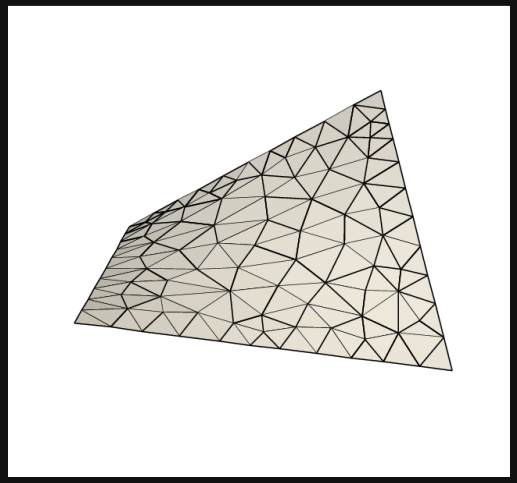
Surface Sampling - Boundary Curves



Define the computational domain by ξ running from 0 to N, and the length of the curve in this space as $\ell = \ell(\xi)$ and solve the following to equidistribute points

$$\ell_i^{n+1} = \frac{LS\left(\ell_{i-\frac{1}{2}}^{n+1}\right)\ell_{i+1}^{n} + LS\left(\ell_{i+\frac{1}{2}}^{n}\right)\ell_{i-1}^{n+1}}{LS\left(\ell_{i-\frac{1}{2}}^{n+1}\right) + LS\left(\ell_{i+\frac{1}{2}}^{n}\right)}$$

Surface Sampling - Surfaces



Once the boundary curve has been sampled, refine the surface mesh until the following is satisfied

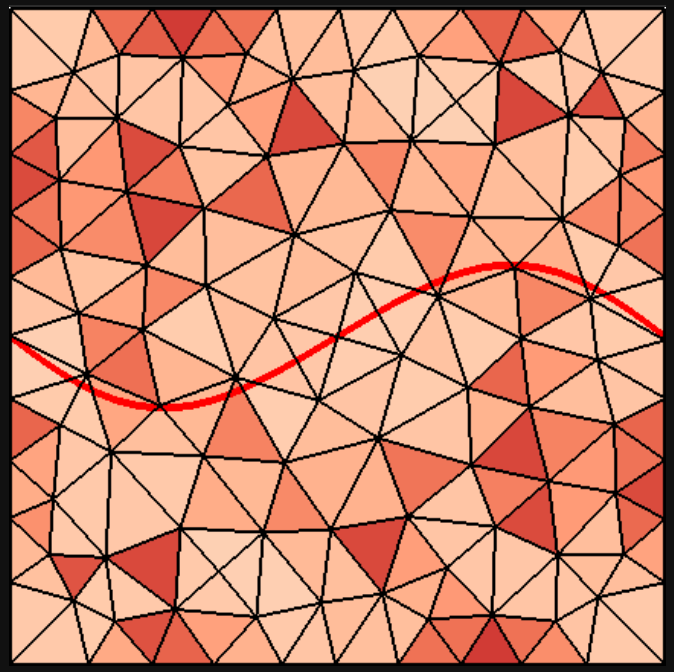
$$\sqrt{2}r_{circum} < \frac{1}{3}\sum_{i=1}^{3} LS(\mathbf{x}(v_i)$$

Vertex Removal

After sampling, vertices that are either within 1/2 length scale of a sampled vertex or would encroach on the face are removed

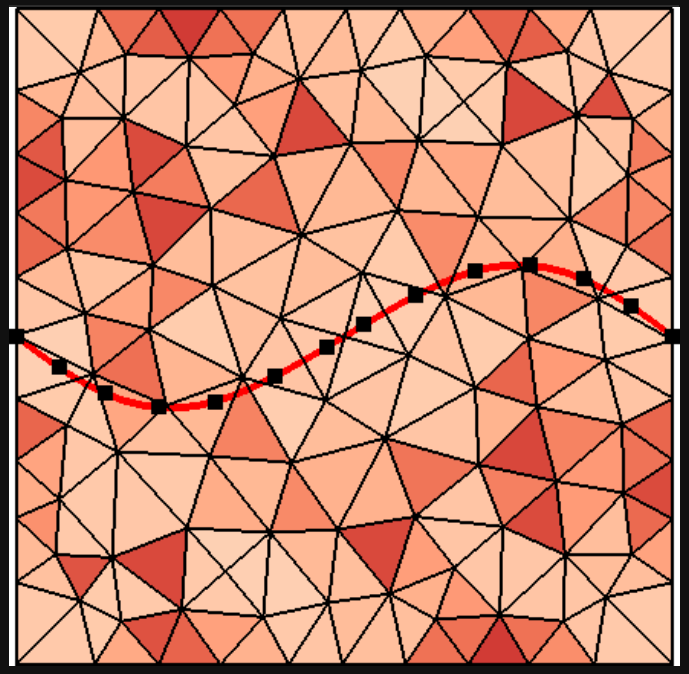
We rely on being locally Delaunay to guarantee recovery

A 2D example



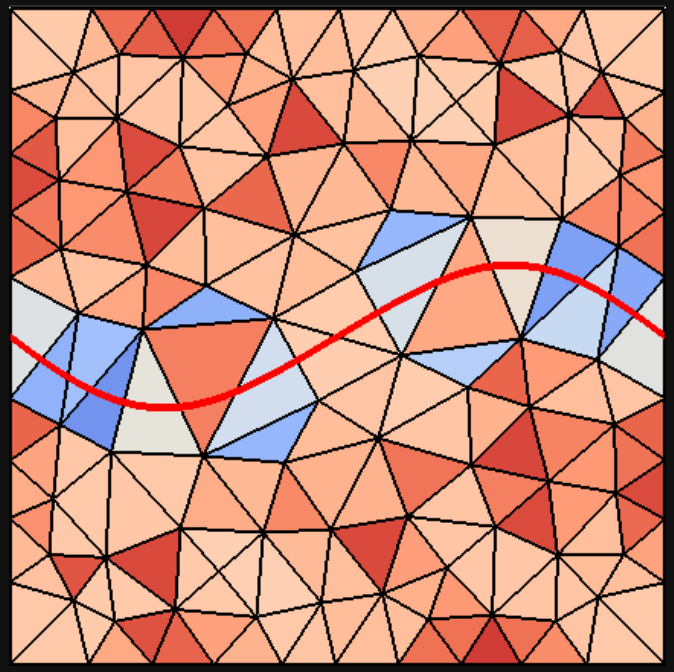
Initial Mesh, 121 Vertices.
Minimum angle: 44.02° .
Cubic Spline in red.

A 2D example



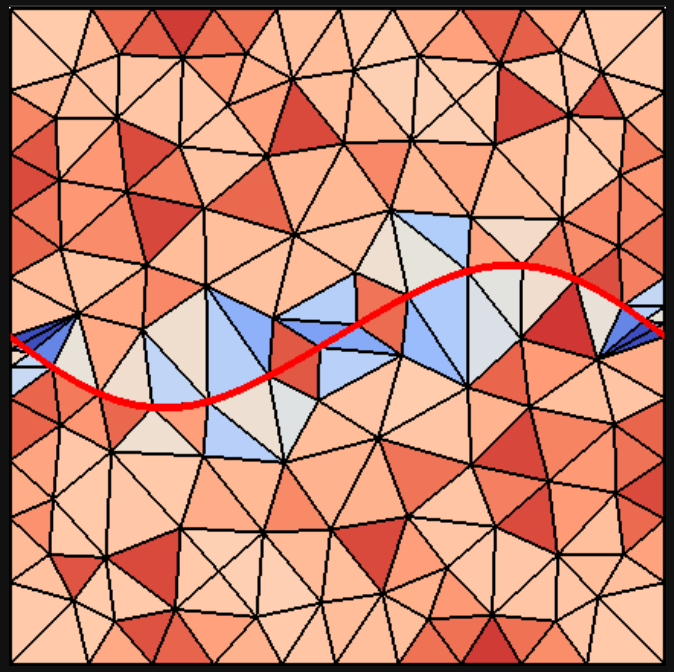
Sample for new points based length scale of underlying mesh.
Length scale is a measure of distance to neighboring vertices.
Vertex locations in black after optimization performed.

A 2D example



With the locations of the new vertices on the curve known, remove vertices too close to the curve, one at a time to maintain a connected mesh.

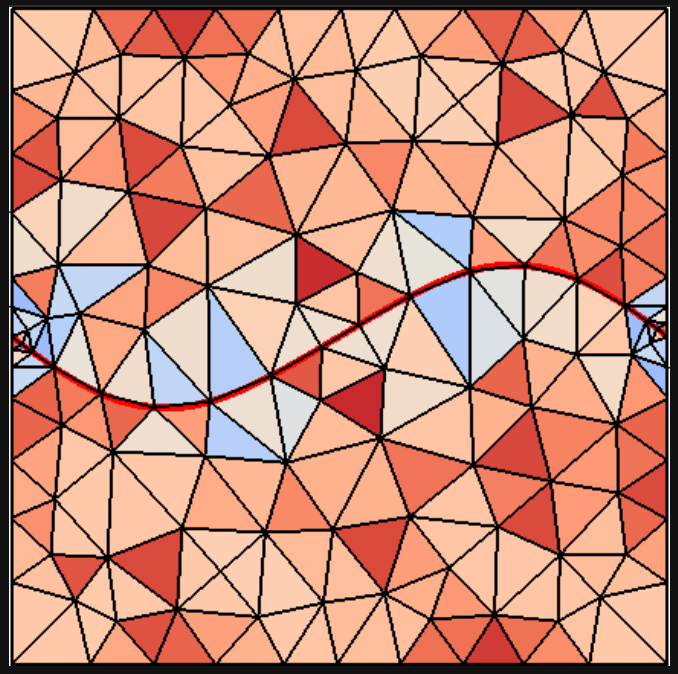
A 2D example



With vertices near the curve removed, insert the new vertices into the the mesh

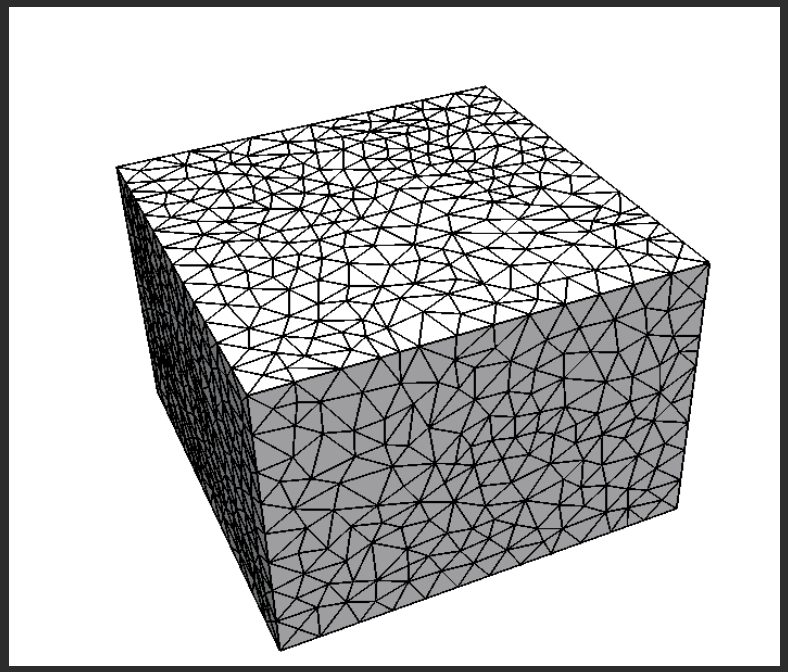
Small angles are protected at this stage, to ensure termination of the refinement algorithm.

A 2D example



Finally, by swapping edges, recover the curve and assess mesh quality. The final mesh has 140 Vertices, with 89% of the initial vertices unchanged and minimum angle: 29.57° .

A 3D example

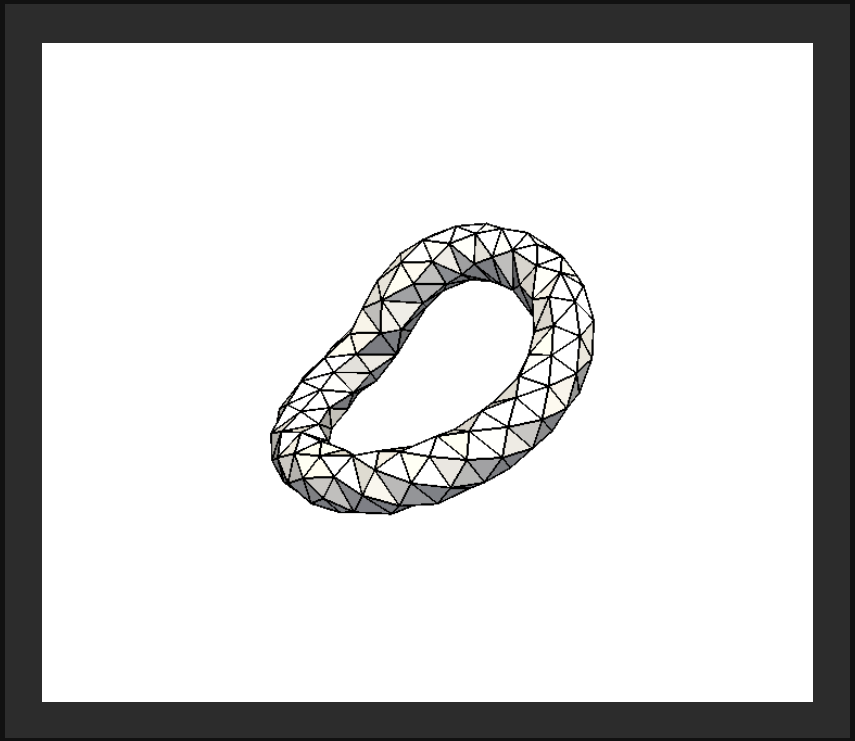


Initially, an isotropic mesh is created.

A 3D example

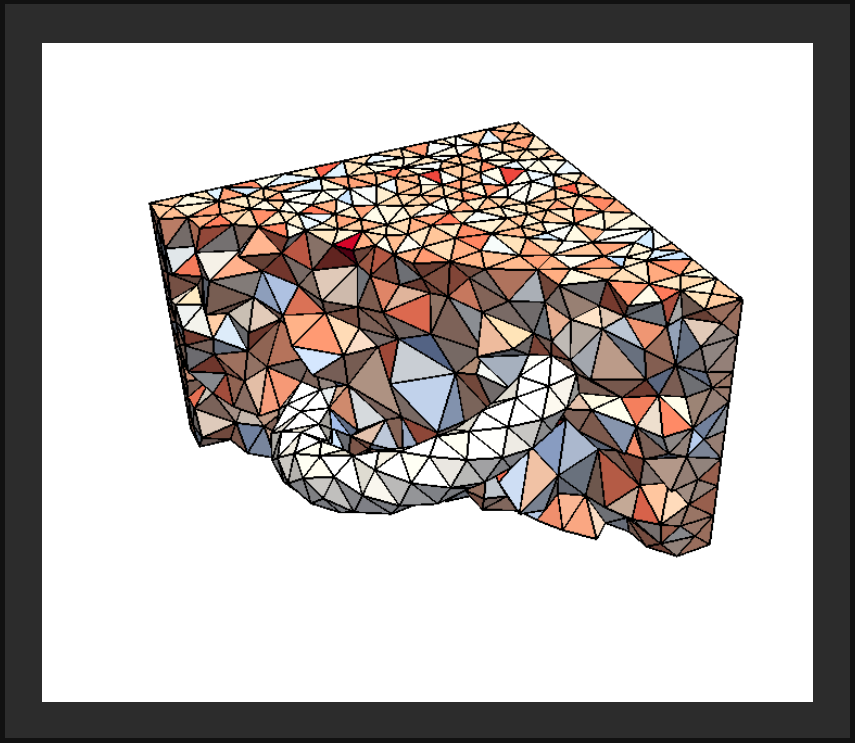
The surface to insert is defined as a cubic bezier surface, though any geometric representation is valid.

A 3D example



The surface is sampled based on the length scale of of initial mesh.

A 3D example



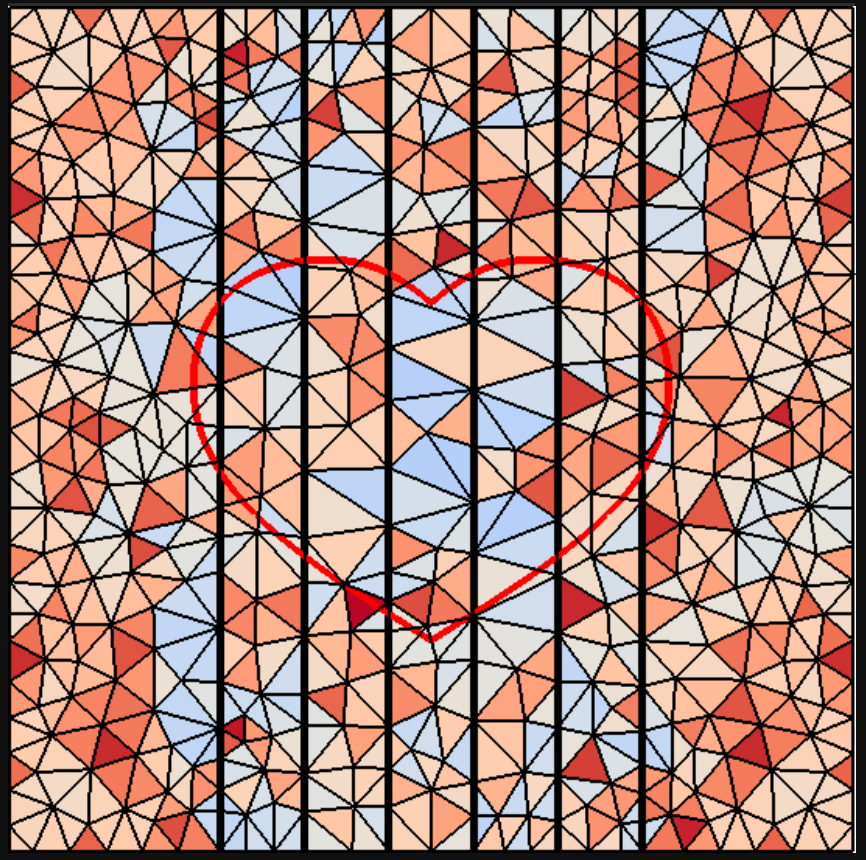
After vertices are removed, the surface is inserted and the final mesh is shown.

Extensions

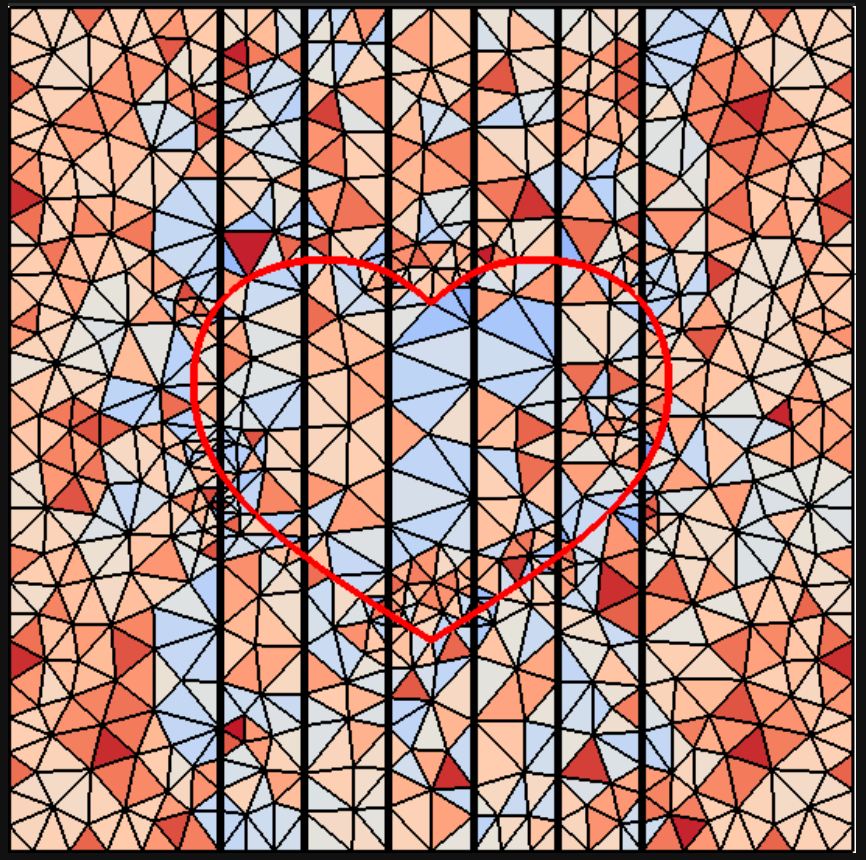
Inserting curves/surfaces into 2D/3D Isotropic Meshes that intersect with existing internal boundaries.

Sampling extended to sample along internal boundary intersections.

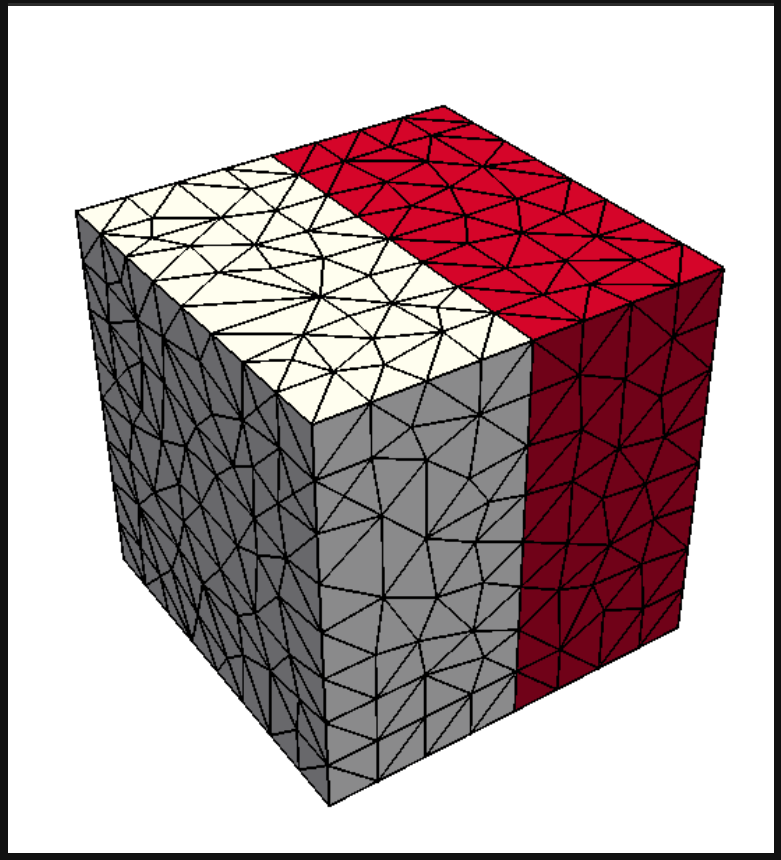
Extensions - 2D Initial



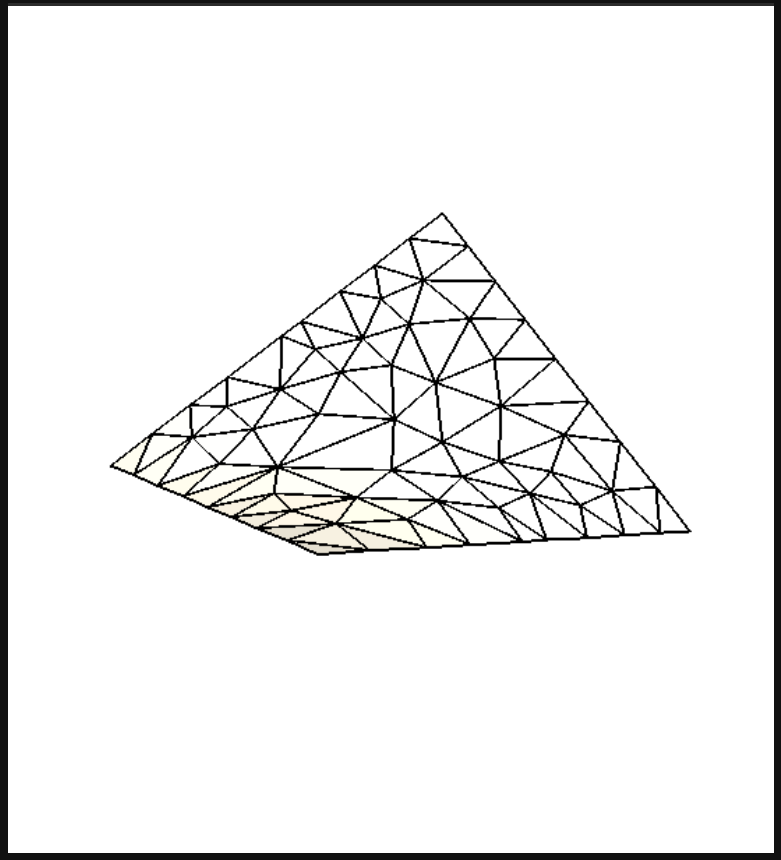
Extensions - 2D Final



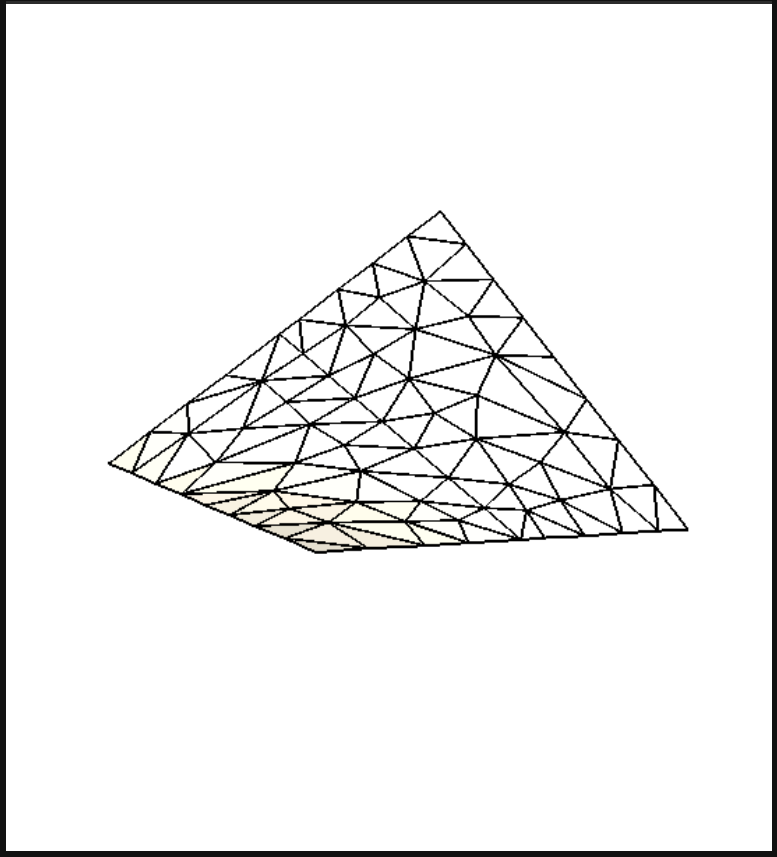
Extensions - 3D Initial



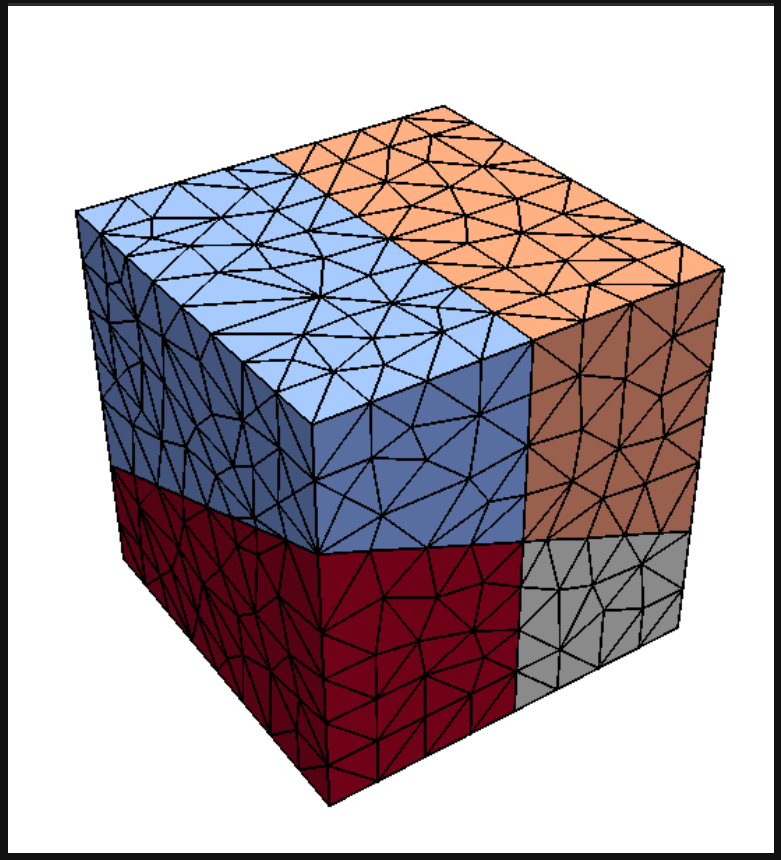
Extensions – 3D Sampled



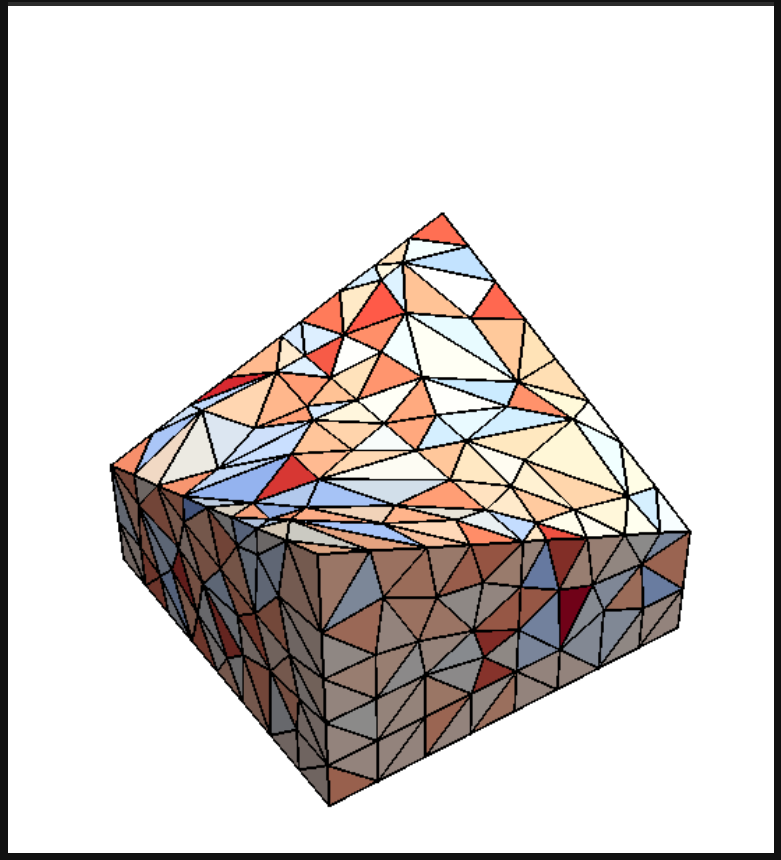
Extensions – 3D Added Intersection



Extensions - 3D Final



Extensions -3D Final



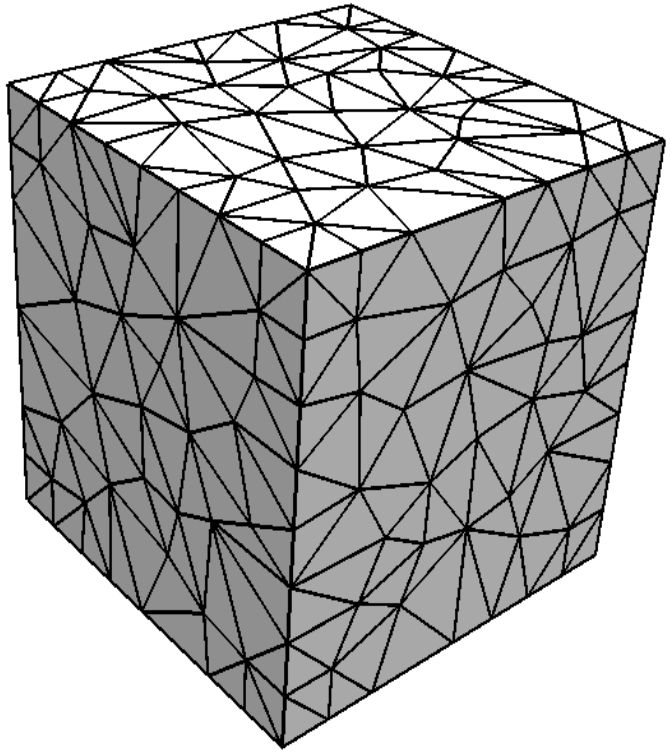
Extensions

Inserting layers into 2D/3D Isotropic Meshes

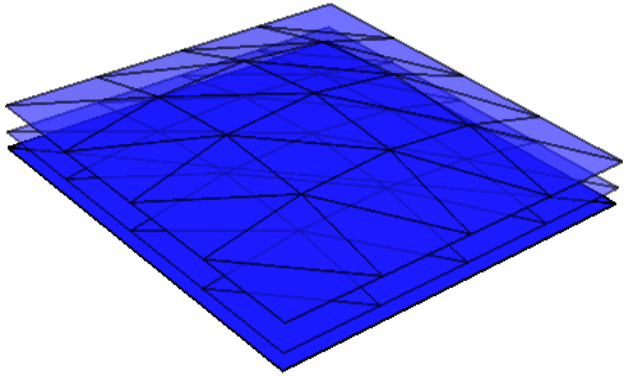
Insert one surface, project sampling onto second nearby surface and build thin layer between.

Can contain prismatic elements (quadrilaterals) in between.

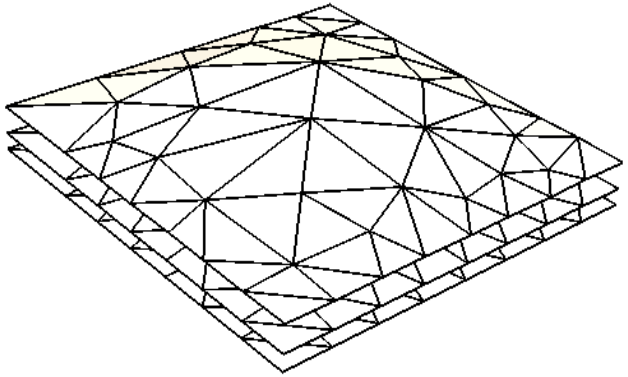
Extensions - 3D Initial



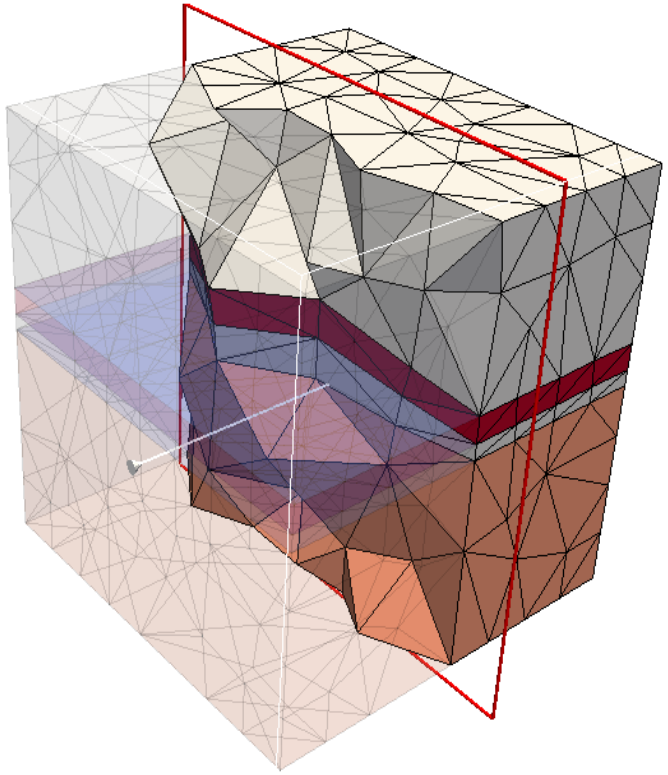
Extensions - 3D Surface



Extensions – 3D Sampled



Extensions - 3D Final

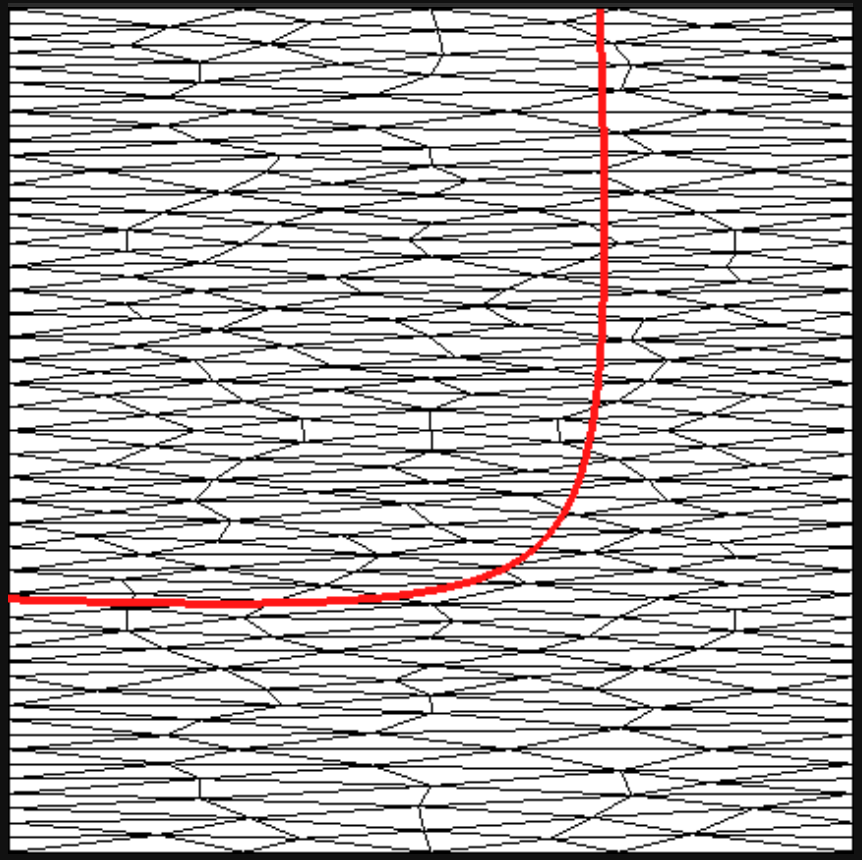


Extensions

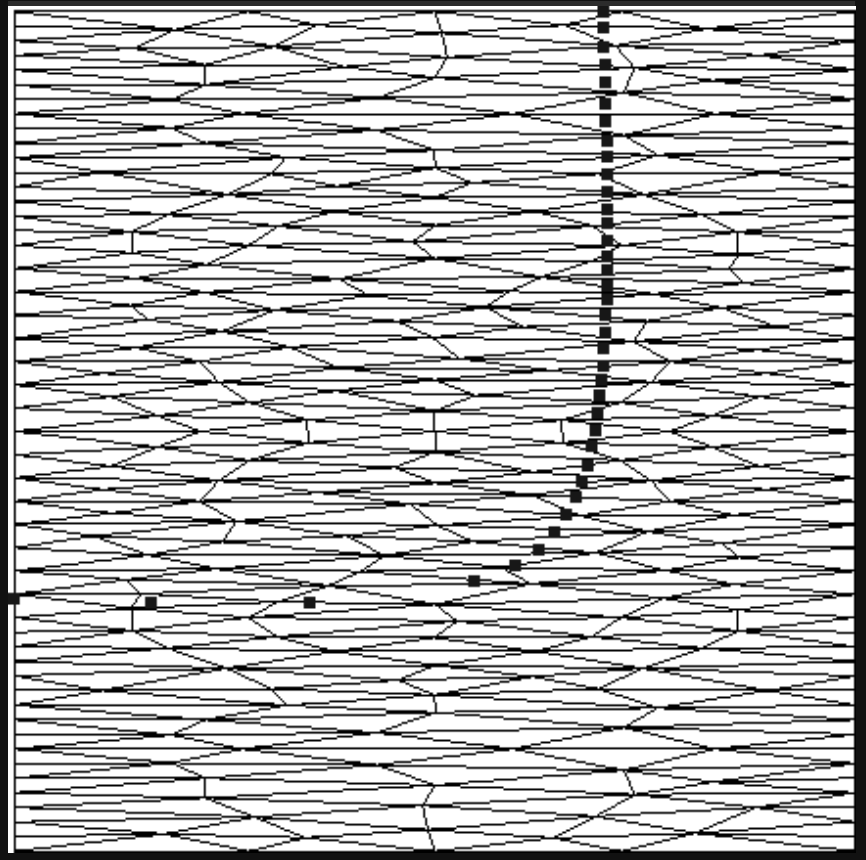
Inserting curves into 2D Anistropic Meshes

Replaces length scale function with metric based function using second moment of area on initial mesh (or original metric, if known)

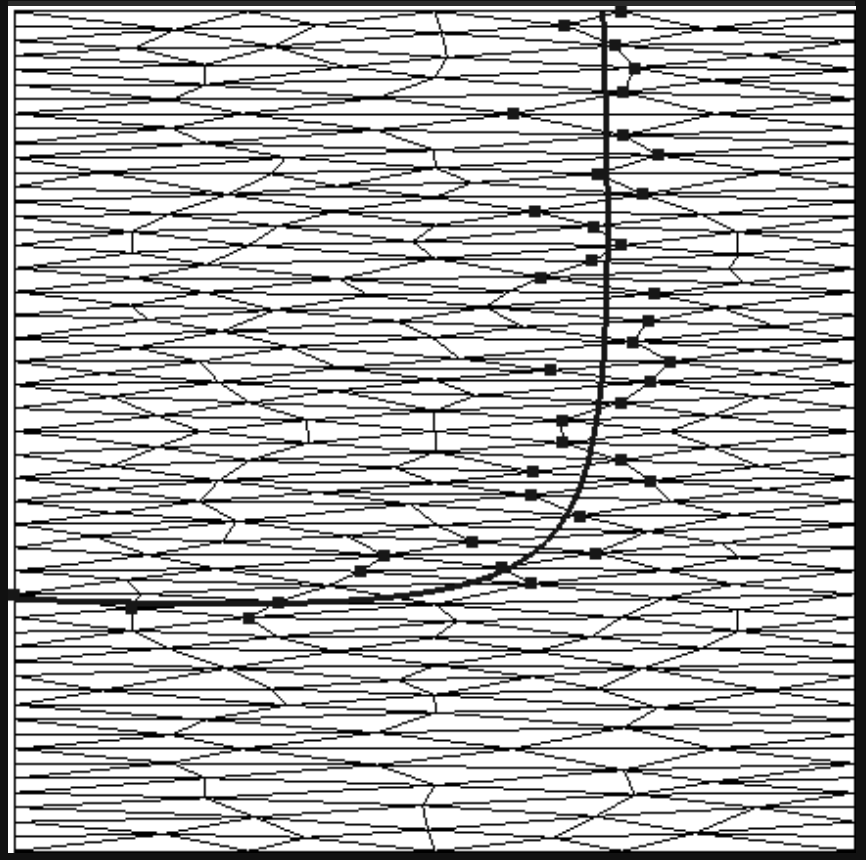
Extensions - 2D Aligned Initial



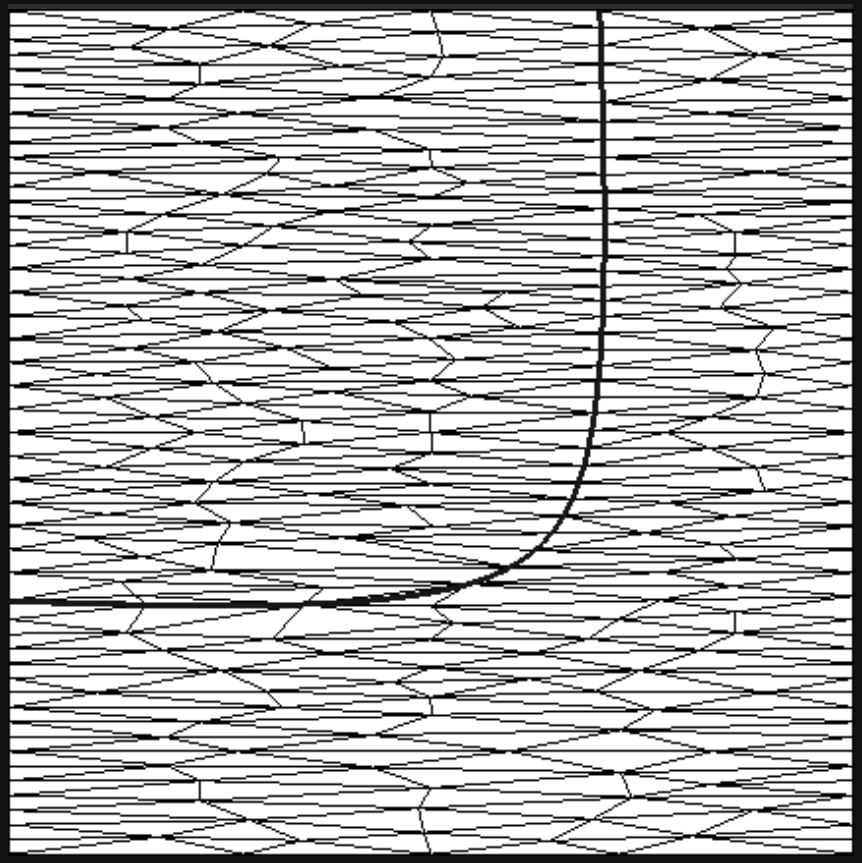
Extensions - 2D Aligned Sampled



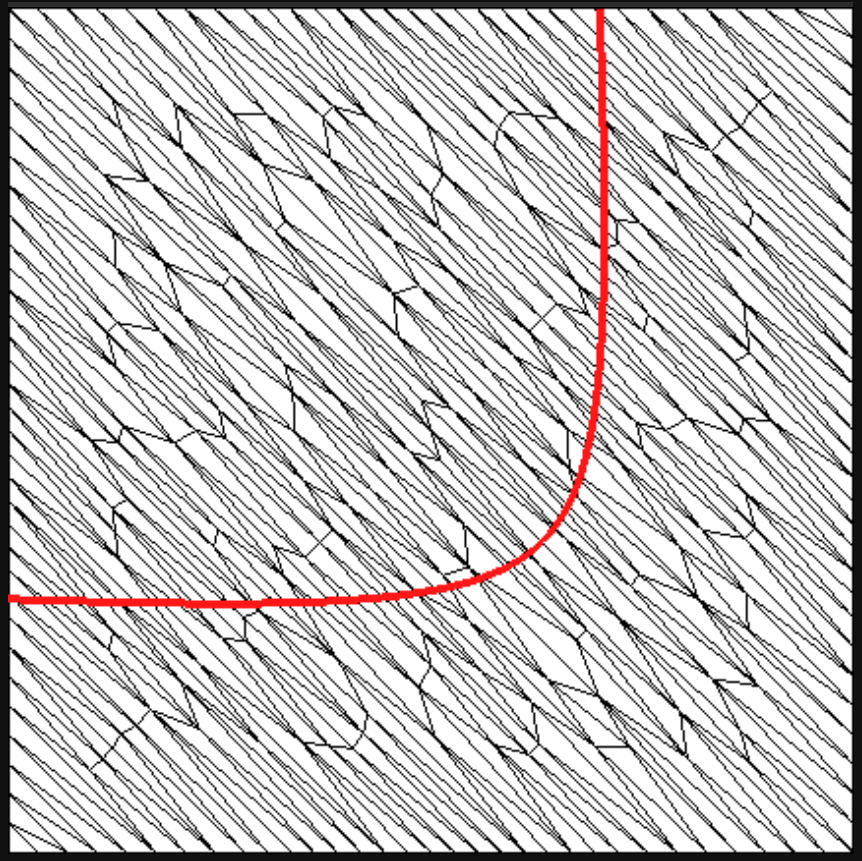
Extensions - 2D Aligned Sampled



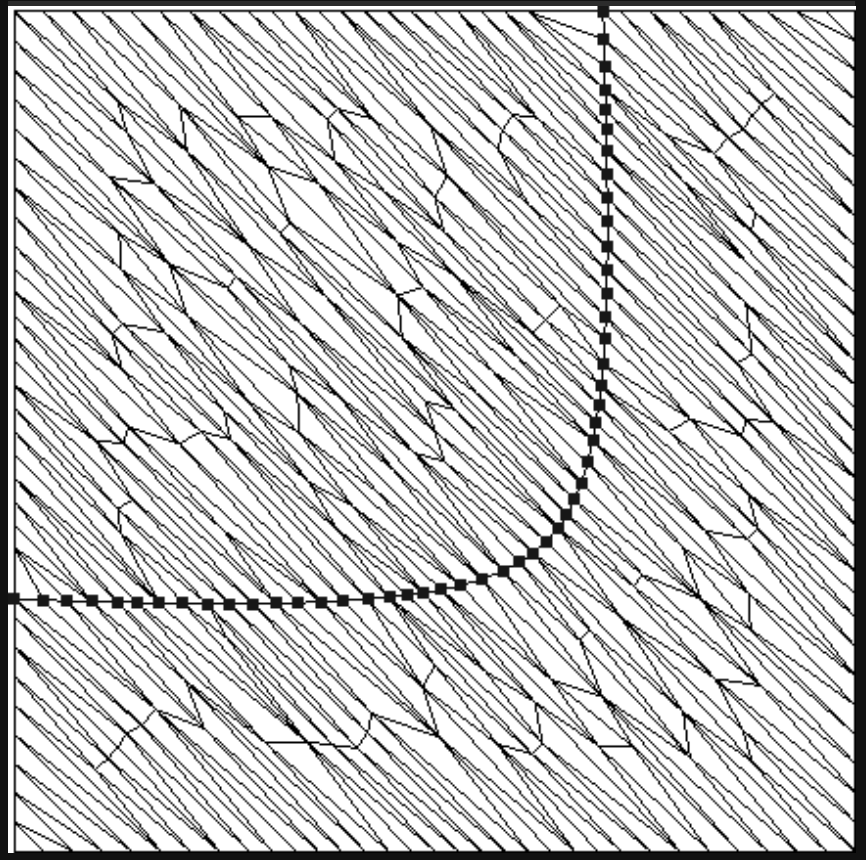
Extensions - 2D Aligned Final



Extensions - 2D Diagonal Initial



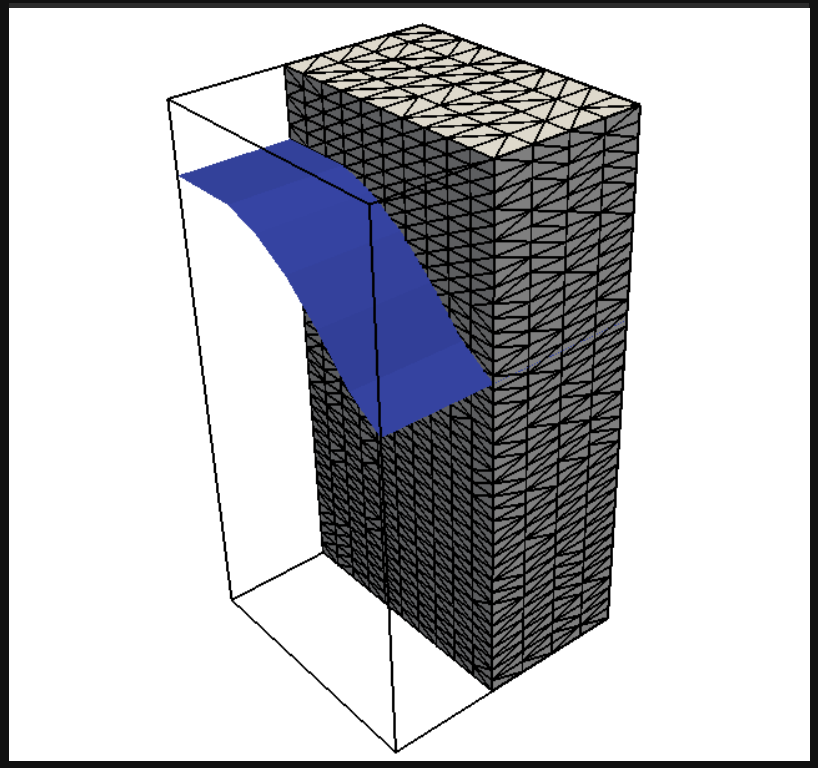
Extensions - 2D Diagonal Final



Current Work

Within the current algorithm, extensions come naturally.

Current focus on inserting into quasi-structured anisotropic meshes.



Questions?

WHAT A RIP-OFF! THEY SAY IF YOU CONNECT THESE DOTS YOU GET A PICTURE, BUT LOOK! I DID IT AND IT'S JUST A BIG MESS!
I THINK YOU'RE SUPPOSED TO CONNECT THEM IN THE ORDER THAT THEY'RE NUMBERED.
OH.
EVERYTHING'S GOTTA HAVE RULES, RULES, RULES!
10-4