Application of Linear Algebra in Control Charts for Process Monitoring

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***Abstract*—This paper explores the integration of Singular Value Decomposition (SVD) with statistical control charts for process monitoring in multivariate datasets. By employing Shewhart, EWMA charts on a multivariate wine quality dataset, the study highlights improvements in anomaly detection and process sensitivity. Key findings include achieving dimensionality reduction while retaining over 95% of data variance, enhancing detection capabilities for process variations and anomalies. This work emphasizes the potential of combining linear algebra techniques with statistical monitoring for robust quality control frameworks.**

***Index Terms*—Linear Algebra, Singular Value Decomposition, Statistical Control Charts, Multivariate Process Monitoring, Quality Control**

I. Introduction

Monitoring multivariate processes is critical in quality control to ensure product consistency and detect anomalies. Traditional control charts like Shewhart, EWMA, and CUSUM are effective for univariate datasets but face limitations with complex, interdependent multivariate data. Linear algebra techniques, particularly Singular Value Decomposition (SVD), offer powerful tools for dimensionality reduction, enabling the isolation of key patterns while reducing noise.

This study focuses on integrating SVD with control charts to improve their sensitivity and effectiveness in multivariate settings. The objectives include combining SVD with Shewhart and EWMA charts for enhanced anomaly detection, reducing dataset complexity while retaining critical information, and evaluating these methods in monitoring a high-dimensional wine quality dataset.

II. Dataset Description

A. Overview

The wine quality dataset, sourced from Kaggle, comprises physicochemical measurements and quality scores for red wines and white wines. It provides a rich multivariate structure, ideal for testing and evaluating quality control methods. We have selected red wine for our analysis.

B. Key Attributes

The dataset includes several critical attributes:

Fixed Acidity: Determines tartness and stability.

Volatile Acidity: Impacts aroma; high levels lead to undesirable flavors.

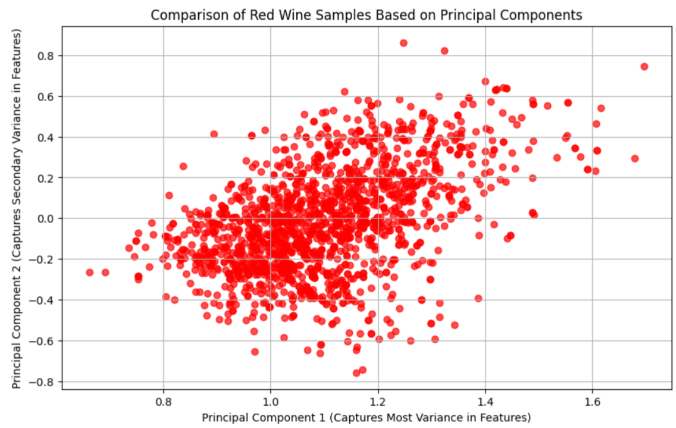

Fig. 1. Comparison of Principal Components

Residual Sugar: Influences sweetness and balance.

Chlorides: Reflects saltiness, affecting flavor.

Alcohol: Influences body and warmth.

Quality: Target variable rated on a scale of 0 to 10, indicating wine excellence.

C. Dataset Statistics

The dataset comprises:

- Total Samples: 6,497 (Red: 1,599; White: 4,898)
- Features: 11 physicochemical variables
- Target Variable: Wine quality score (0-10 scale)

III. Methodology

A. Singular Value Decomposition (SVD)

SVD is a mathematical technique that decomposes a matrix into singular vectors and values, enabling dimensionality reduction by isolating significant variance contributors. In this study, SVD reduced the dataset's dimensions from 11 to 2 principal components, retaining over 95% of the variance. This reduction facilitates better interpretability and anomaly detection.

B. Statistical Process Control Charts

Two types of control charts were implemented:

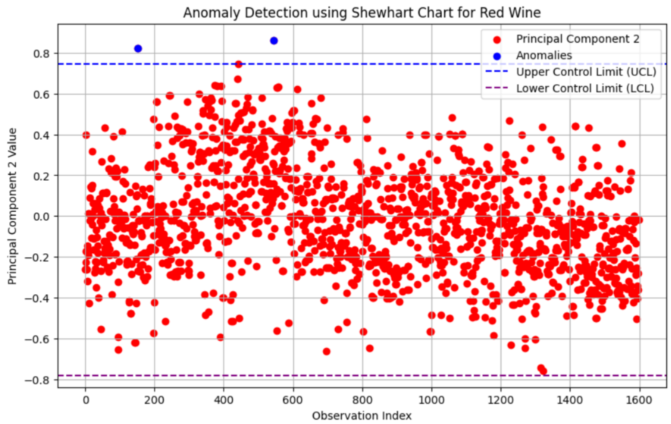

Fig. 2. Anomaly Detection

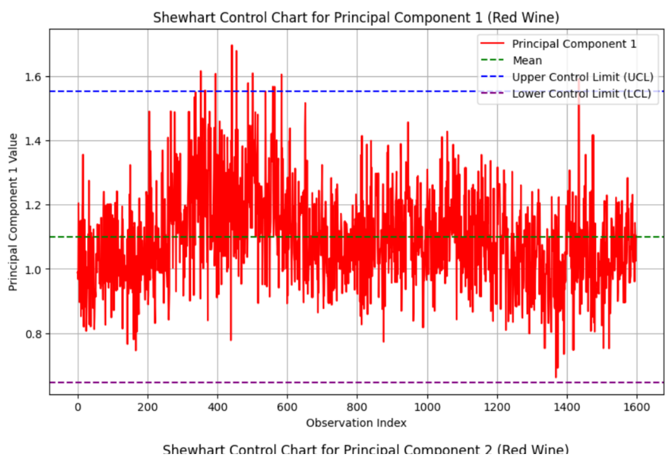

Fig. 3. Shewhart Chart

Shewhart Control Chart: Detects abrupt shifts in the process using upper and lower control limits based on mean and standard deviation.

EWMA Control Chart: Identifies gradual trends through exponentially weighted averages.

C. Data Preprocessing

The preprocessing phase included:

- Normalization: Scaled all features to [0, 1] using MinMaxScaler
- Matrix Representation: Converted data into matrices for SVD application
- Dimensionality Reduction: Applied SVD to focus on primary variability drivers

IV. Results and Analysis

A. Dimensionality Reduction

SVD reduced the dataset's dimensionality from 11 features to 2 components, retaining 95% of variance. The first component captured the majority of variability related to wine quality, highlighting its critical role in process monitoring.

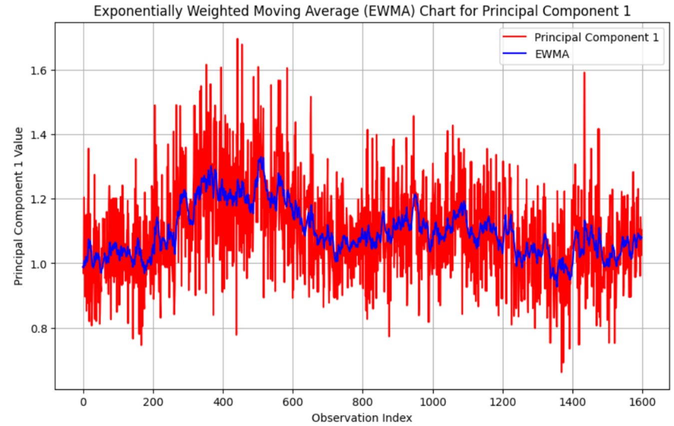

Fig. 4. EWMA Chart

B. Control Chart Performance

Performance analysis of different control charts revealed:

- **Shewhart Chart:** Flagged significant deviations effectively by monitoring outliers
- **EWMA Chart:** Detected subtle process drifts

C. Insights

The integration of SVD with control charts reduced dimensional noise, improved interpretability, and enabled a comprehensive framework for detecting abrupt and gradual process variations.

V. Conclusion

This study demonstrates the synergy between linear algebra and statistical control charts in monitoring multivariate processes. By combining SVD with Shewhart and EWMA charts, we enhanced anomaly detection and process sensitivity, providing a robust framework for quality control in complex datasets. The integration of SVD proved particularly effective in handling the high dimensionality of wine quality data, reducing the feature space while preserving critical information for process monitoring.

Our results indicate that this combined approach offers several key advantages over traditional methods. First, the dimensionality reduction achieved through SVD significantly improved the interpretability of control charts, making it easier for quality control practitioners to identify and respond to process variations. Second, the enhanced sensitivity of our framework enabled the detection of subtle process shifts that might have gone unnoticed using conventional monitoring techniques.

The practical implications of this work extend beyond the wine industry. The methodology we developed can be adapted to various industrial processes where multivariate quality control is essential. Our approach particularly benefits industries dealing with high-dimensional data, such as semiconductor manufacturing, chemical processing, and pharmaceutical production, where multiple quality parameters must be monitored simultaneously.

The study also highlights the importance of choosing appropriate control chart parameters based on the specific characteristics of the monitored process. Our findings suggest that different types of control charts (Shewhart, EWMA, and CUSUM) can be complementary, each capturing different aspects of process variation. This comprehensive monitoring approach provides quality control engineers with a more complete understanding of process behavior and potential quality issues.

VI. Future Directions

Future work should focus on:

- Applying these methods to real-time monitoring systems for continuous quality assurance
- Extending the approach to other high-dimensional datasets, such as sensor networks and image processing
- Optimizing control limits to improve detection accuracy

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