

EC209 Assignment

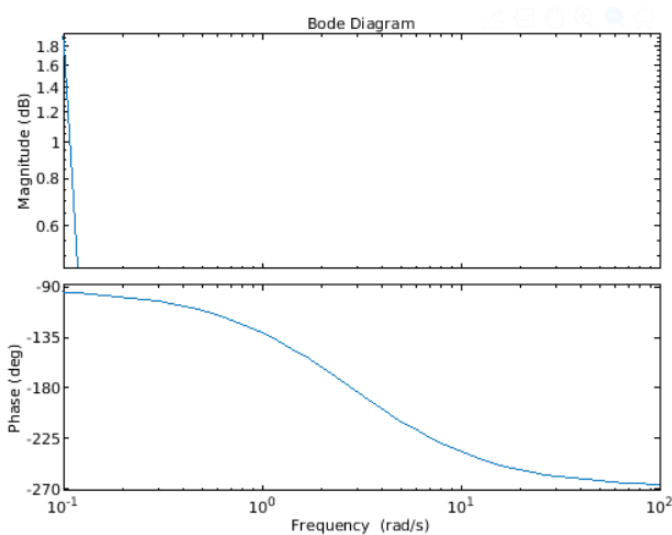
Zaid Khalifa – 191EC160

1. For each of the following functions, make a plot of the log-magnitude and the phase, using log-frequency in rad/s as the ordinate. Do not use asymptotic approximations.

a) $G(s) = \frac{1}{s(s+2)(s+4)}$

code:

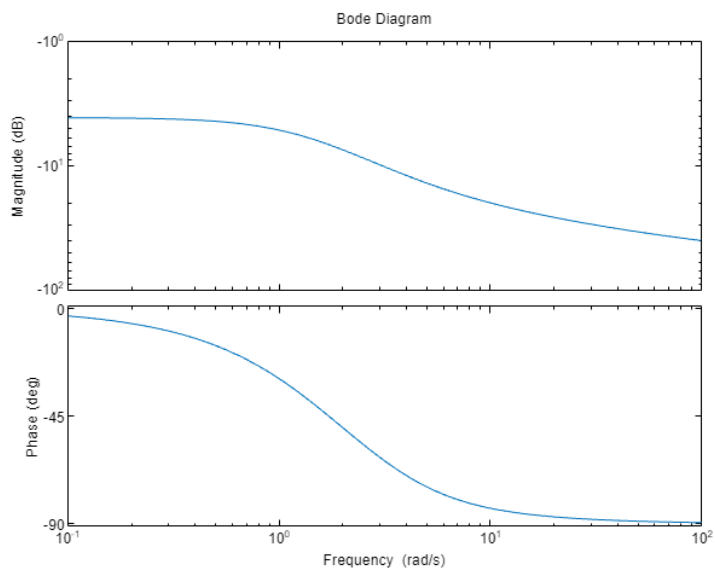
```
H = tf([1],[1 6 8 0]);  
opts = bodeoptions  
opts.MagScale = 'log'  
bode(H, opts)
```



b) $G(s) = \frac{(s+5)}{(s+2)(s+4)}$

code:

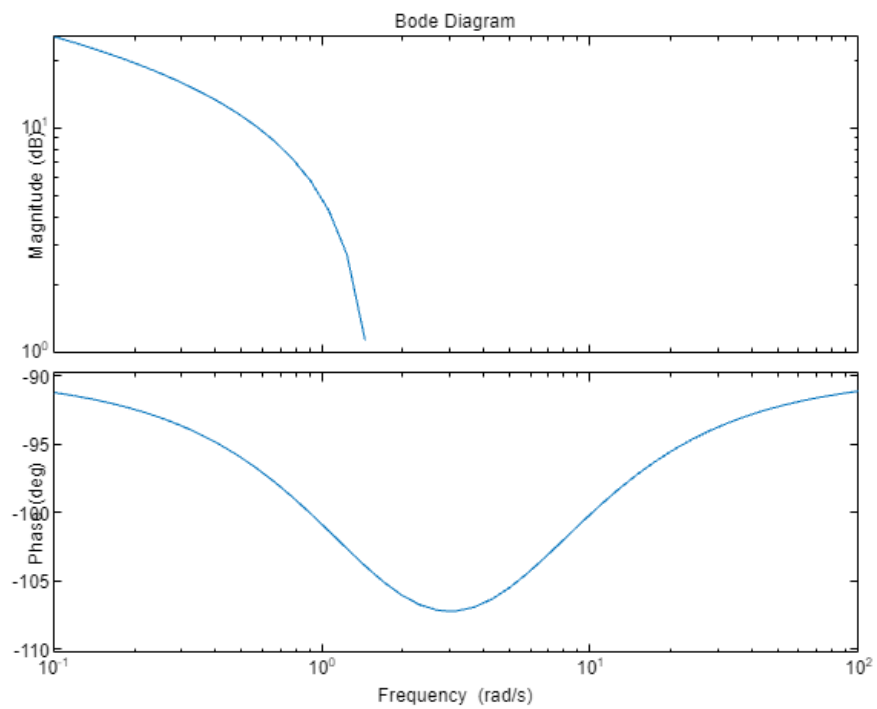
```
H = tf([1 5],[1 6 8]);  
opts = bodeoptions  
opts.MagScale = 'log'  
bode(H, opts)
```



c)
$$G(s) = \frac{(s+3)(s+5)}{s(s+2)(s+4)}$$

code:

```
H = tf([1 8 15],[1 6 8 0]);
opts = bodeoptions
opts.MagScale = 'log'
bode(H, opts)
```



2. For each function, make a polar plot of the frequency response.

a) $G(s) = \frac{1}{s(s+2)(s+4)}$

First replace $s = j\omega$

$$G(j\omega) = \frac{1}{j\omega(2 + j\omega)(4 + j\omega)}$$

Then obtain expression for magnitude and phase

$$|G(j\omega)| = \frac{1}{\omega(\sqrt{4 + \omega^2})(\sqrt{16 + \omega^2})}$$

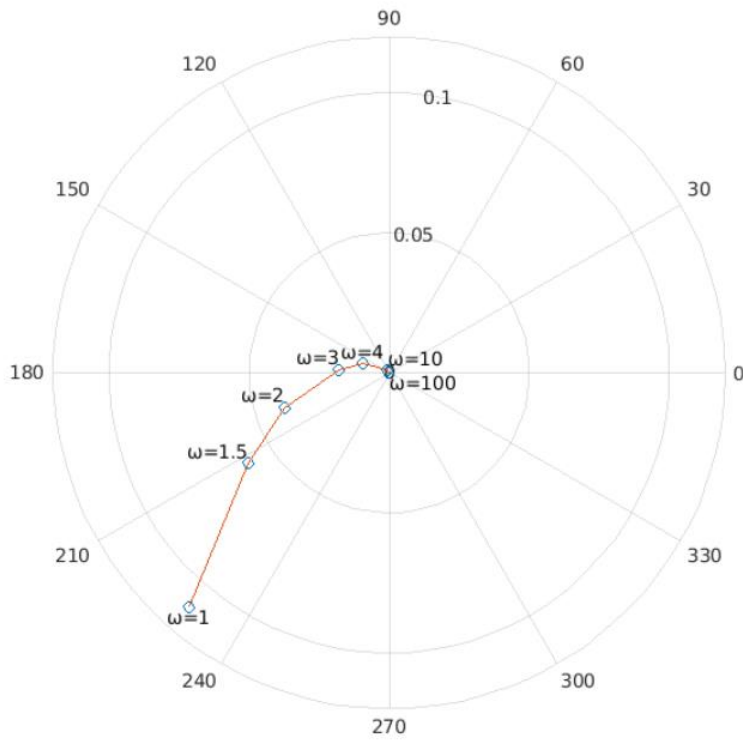
$$\varphi = \angle G(j\omega) = -\frac{\pi}{2} - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right)$$

Then tabulate the values of $|G(j\omega)|$ and φ for frequencies

ω	$ G(j\omega) $	φ
1	0.11	-2.28
1.5	0.06	-2.57
2	0.0395	-2.82
3	0.018	-3.20
4	0.01	-3.46
10	0.001	-4.13
100	0.000001	-4.55

Code:

```
x = [-4.55 -4.13 -3.46 -3.20 -2.82 -2.57 -2.28];
y = [0.000001 0.001 0.010 0.018 0.0395 0.06 0.11];
polarplot(x,y,'o',x,y)
labels = {'w=100',' ',' ',' ',' ',' ',' '};
labels2 = {' ','w=10',' ',' ',' ',' ',' '};
labels3 = {' ',' ','w=4',' ',' ',' ',' '};
labels4 = {' ',' ',' ','w=3','w=2','w=1.5',' '};
labels5 = {' ',' ',' ',' ',' ',' ','w=1'};
text(x,y,labels,'VerticalAlignment','top','HorizontalAlignment','left')
text(x,y,labels2,'VerticalAlignment','bottom','HorizontalAlignment','left')
text(x,y,labels3,'VerticalAlignment','bottom','HorizontalAlignment','center')
text(x,y,labels4,'VerticalAlignment','bottom','HorizontalAlignment','right')
text(x,y,labels5,'VerticalAlignment','top','HorizontalAlignment','center')
```



b) $G(s) = \frac{(s+5)}{(s+2)(s+4)}$

First replace $s = j\omega$

$$G(j\omega) = \frac{(5 + j\omega)}{(2 + j\omega)(4 + j\omega)}$$

Then obtain expression for magnitude and phase

$$|G(j\omega)| = \frac{(\sqrt{25 + \omega^2})}{(\sqrt{4 + \omega^2})(\sqrt{16 + \omega^2})}$$

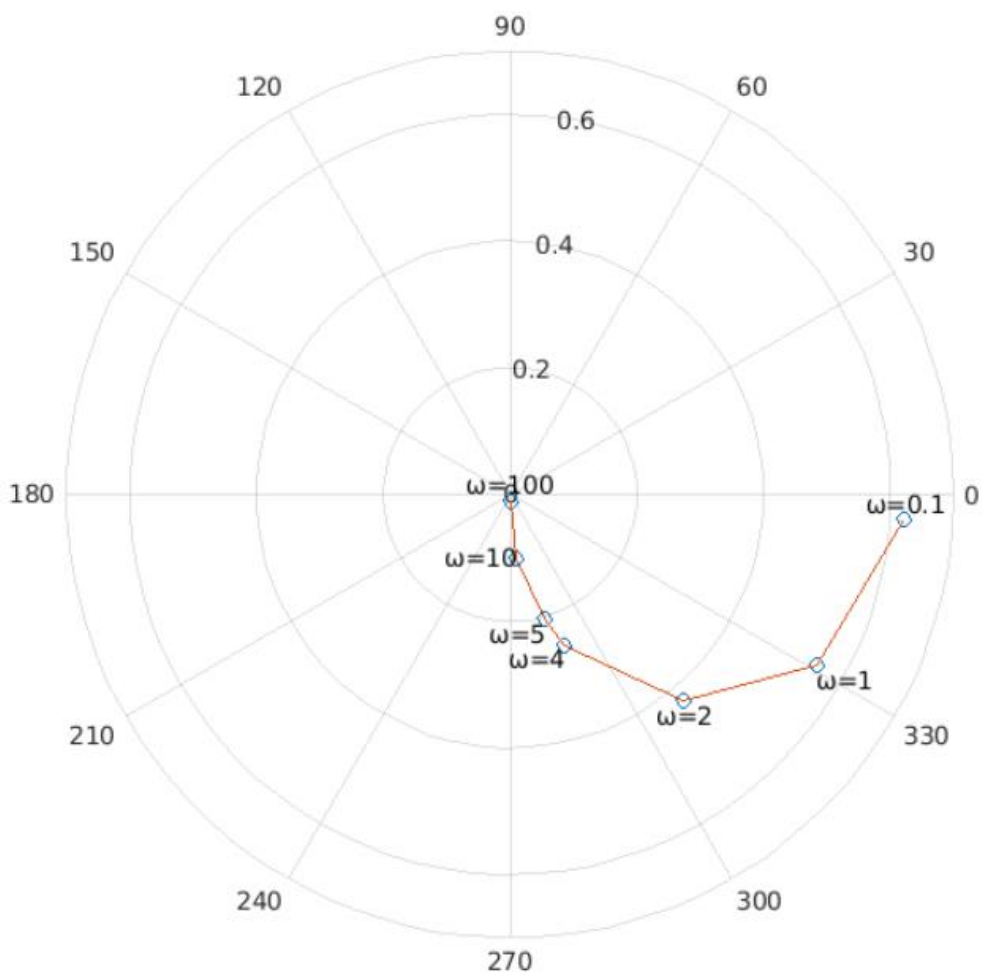
$$\varphi = \angle G(j\omega) = \tan^{-1}\left(\frac{\omega}{5}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right)$$

Then tabulate the values of $|G(j\omega)|$ and φ for frequencies

ω	$ G(j\omega) $	φ
0.1	0.624	-0.065
1	0.553	-0.51
2	0.426	-0.87
4	0.253	-1.22
5	0.205	-1.30
10	0.101	-1.46
100	0.01	-1.56

Code:

```
x = [-1.56 -1.46 -1.30 -1.22 -0.87 -0.51 -0.065];
y = [0.01 0.101 0.205 0.253 0.426 0.553 0.624];
polarplot(x,y,'o',x,y)
labels = {' $\omega=100$ ', '', '', '', '', '', ' $\omega=0.1$ '};
labels2 = {'', ' $\omega=10$ ', '', '', '', '', ''};
labels3 = {'', '', ' $\omega=5$ ', ' $\omega=4$ ', '', '', ''};
labels4 = {'', '', '', ' $\omega=2$ ', '', '', ''};
labels5 = {'', '', '', '', ' $\omega=1$ ', ''};
text(x,y,labels,'VerticalAlignment','bottom','HorizontalAlignment','center')
text(x,y,labels2,'VerticalAlignment','middle','HorizontalAlignment','right')
text(x,y,labels3,'VerticalAlignment','top','HorizontalAlignment','right')
text(x,y,labels4,'VerticalAlignment','top','HorizontalAlignment','center')
text(x,y,labels5,'VerticalAlignment','top','HorizontalAlignment','left')
```



$$c) \quad G(s) = \frac{(s+3)(s+5)}{s(s+2)(s+4)}$$

First replace $s = j\omega$

$$G(j\omega) = \frac{(3 + j\omega)(5 + j\omega)}{j\omega(2 + j\omega)(4 + j\omega)}$$

Then obtain expression for magnitude and phase

$$|G(j\omega)| = \frac{(\sqrt{9 + \omega^2})(\sqrt{25 + \omega^2})}{\omega(\sqrt{4 + \omega^2})(\sqrt{16 + \omega^2})}$$

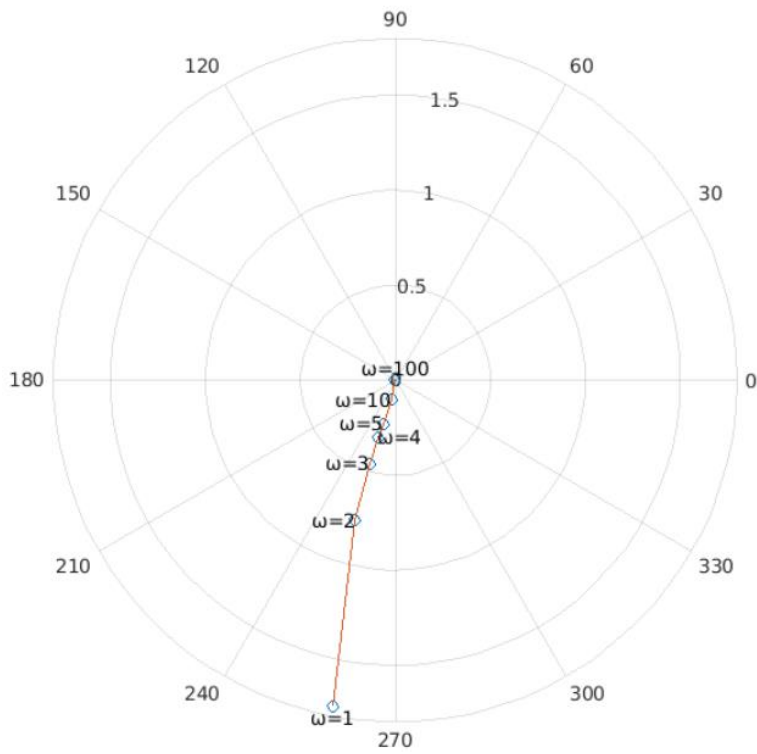
$$\varphi = \angle G(j\omega) = \tan^{-1}\left(\frac{\omega}{3}\right) + \tan^{-1}\left(\frac{\omega}{5}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right)$$

Then tabulate the values of $|G(j\omega)|$ and φ for frequencies

ω	$ G(j\omega) $	φ
1	1.749	-1.760
2	0.768	-1.851
3	0.458	-1.871
4	0.316	-1.861
5	0.239	-1.842
10	0.106	-1.748
100	0.001	-1.591

Code:

```
x = [-1.591 -1.748 -1.842 -1.861 -1.871 -1.851 -1.760];
y = [0.001 0.106 0.239 0.316 0.458 0.768 1.749];
polarplot(x,y,'o',x,y)
labels = {'w=100',' ',' ',' ',' ',' ',' '};
labels2 = {' ','w=10','w=5',' ','w=3','w=2',' '};
labels3 = {' ',' ',' ','w=4',' ',' ',' '};
labels4 = {' ',' ',' ',' ',' ',' ','w=1'};
text(x,y,labels,'VerticalAlignment','bottom','HorizontalAlignment','center')
text(x,y,labels2,'VerticalAlignment','middle','HorizontalAlignment','right')
text(x,y,labels3,'VerticalAlignment','middle','HorizontalAlignment','left')
text(x,y,labels4,'VerticalAlignment','top','HorizontalAlignment','center')
```

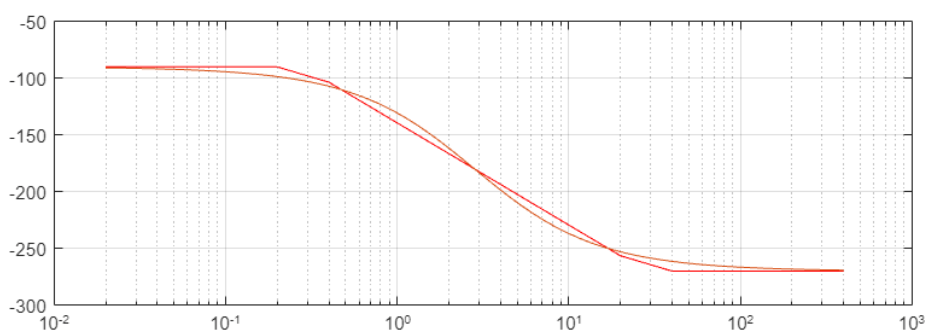
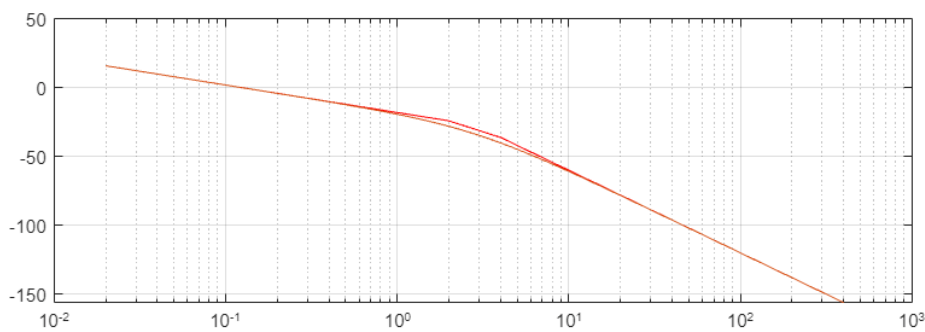


3. For each of the given functions, sketch the Bode asymptotic magnitude and asymptotic phase plots.

a) $G(s) = \frac{1}{s(s+2)(s+4)}$

Code:

```
addpath(['bode_asymptotic.m']);
num = [1];
den = [1 6 8 0];
bode_asymptotic(num, den)
```

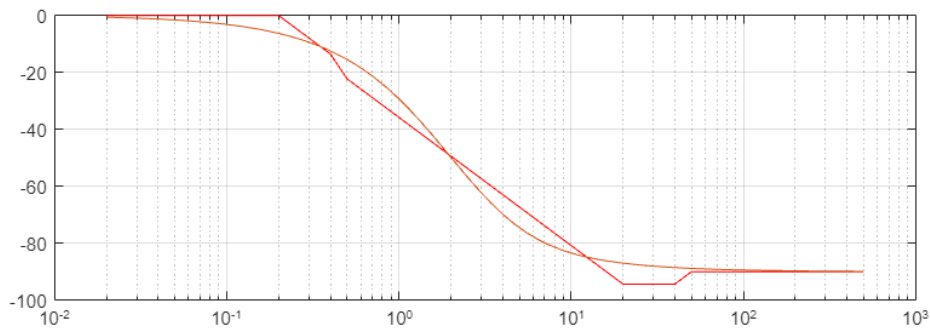
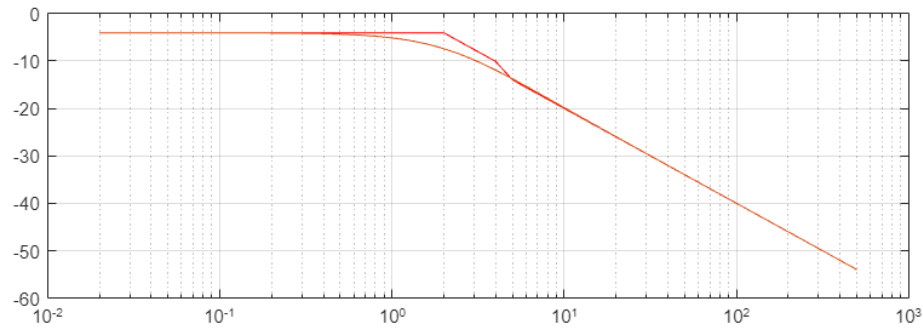


b) $G(s) = \frac{(s+5)}{(s+2)(s+4)}$

code:

```
addpath(['bode_asymptotic.m']);
```

```
num = [1 5];  
den = [1 6 8];  
bode_asymptotic(num, den)
```

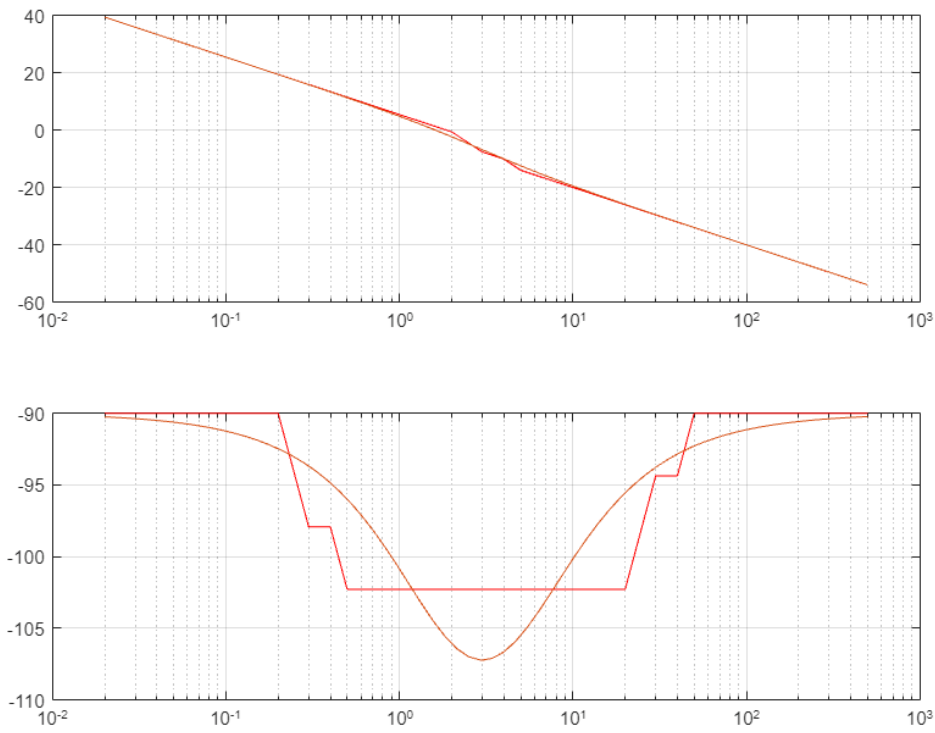


c) $G(s) = \frac{(s+3)(s+5)}{s(s+2)(s+4)}$

Code:

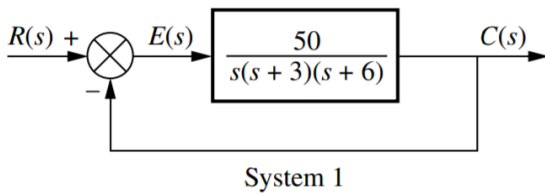
```
addpath(['bode_asymptotic.m']);
```

```
num = [1 8 15];  
den = [1 6 8 0];  
bode_asymptotic(num, den)
```



4) Sketch the Nyquist diagram for each of the systems given.

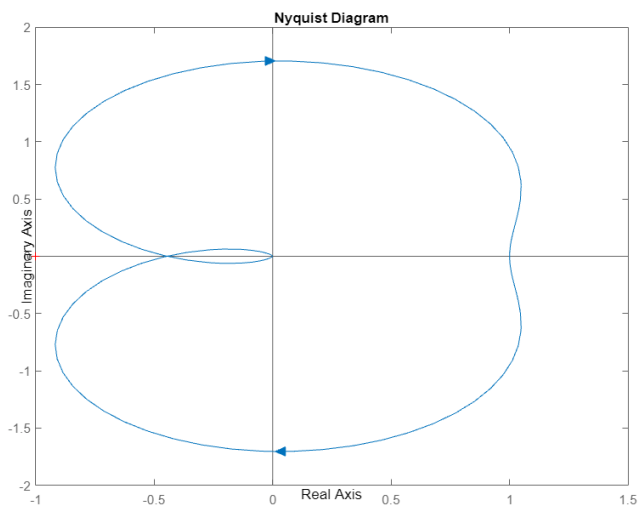
a)



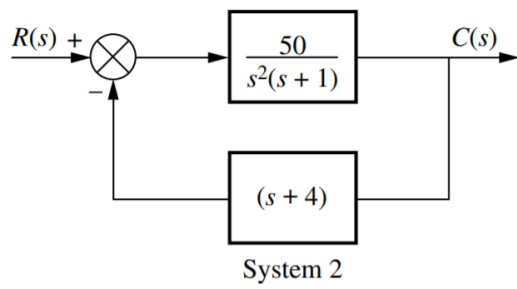
$$\text{Closed loop transfer function} = \frac{50}{s(s+3)(s+6)+50}$$

Code:

```
H = tf([50],[1 9 18 50]);
nyquist(H)
```



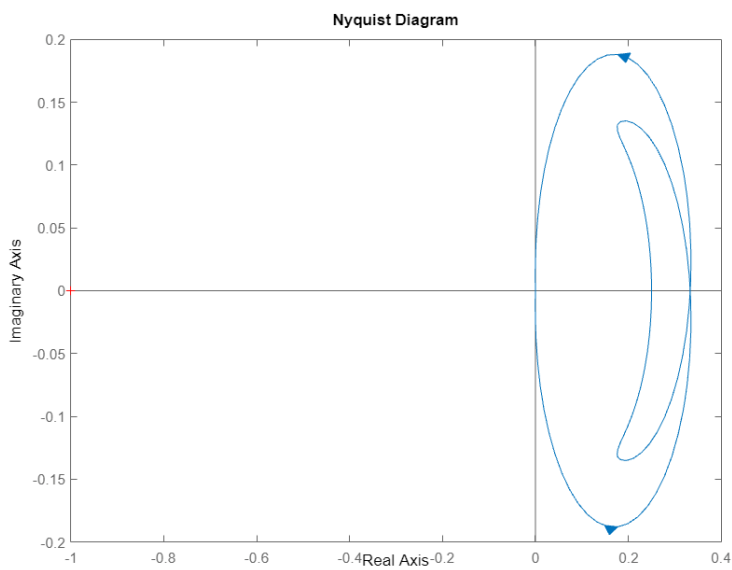
b)



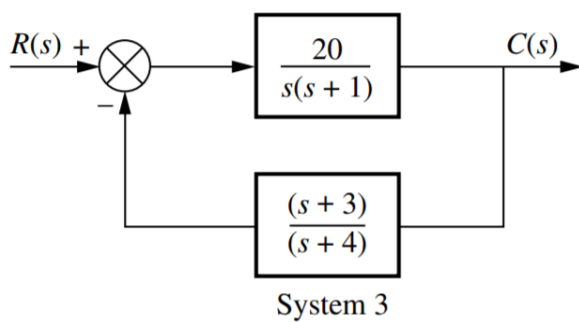
$$\text{Closed loop transfer function} = \frac{50}{s^2(s+1)+50(s+4)}$$

Code:

```
H = tf([50],[1 1 50 200]);
nyquist(H)
```



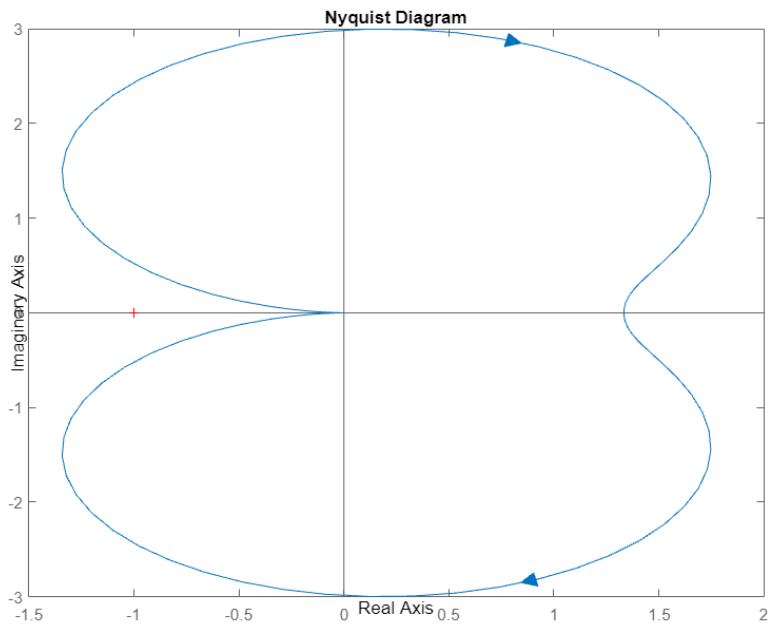
c)



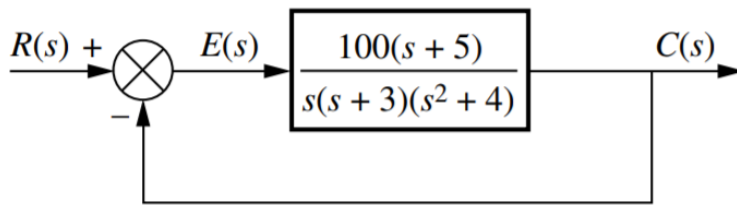
$$\text{Closed loop transfer function} = \frac{20(s+4)}{s(s+1)(s+4)+20(s+3)}$$

Code:

```
H = tf([20 80],[1 5 24 60]);
nyquist(H)
```



d)

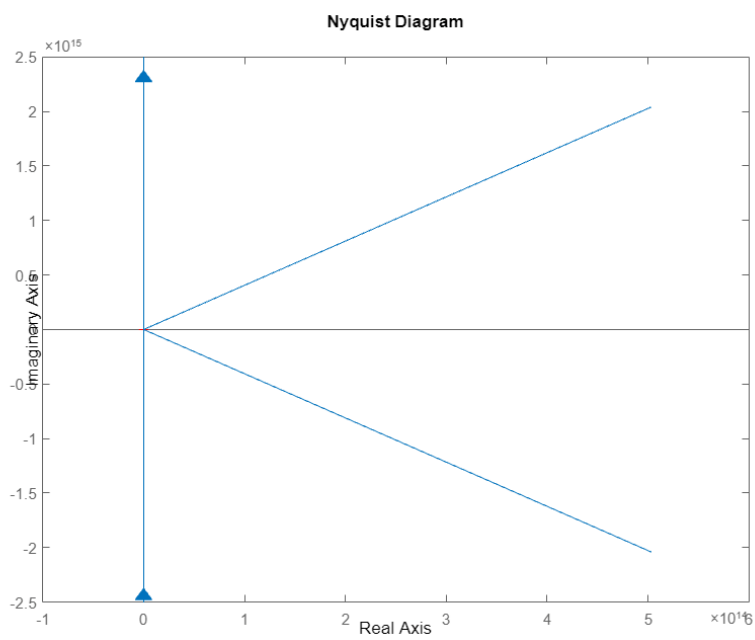


System 4

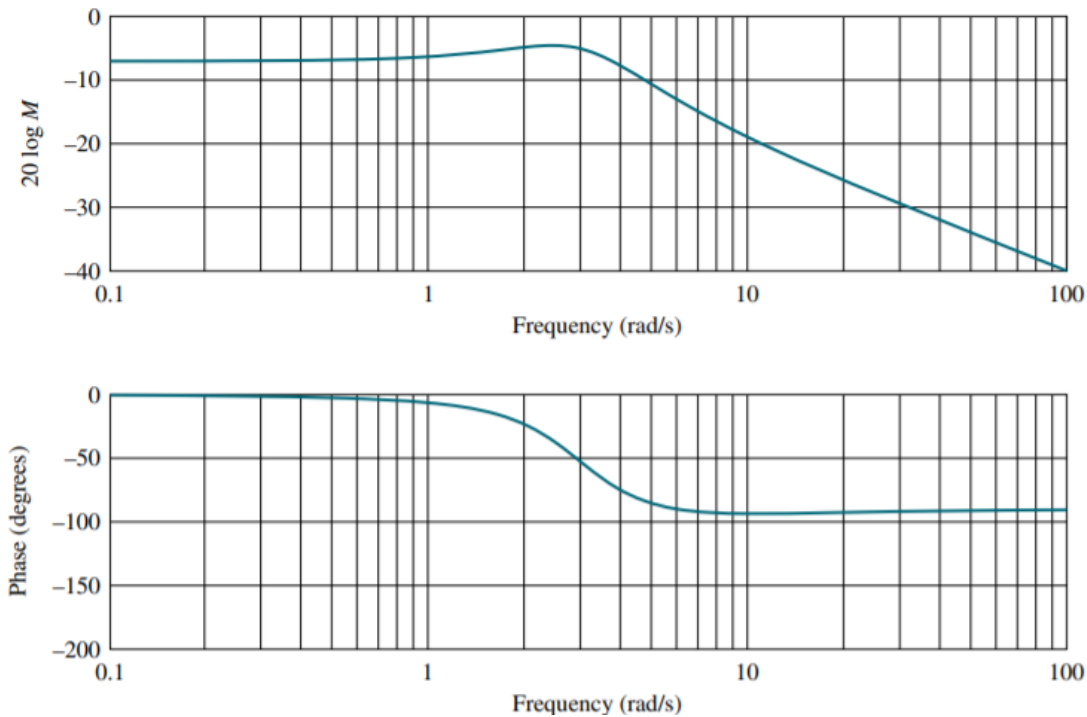
$$\text{Closed loop transfer function} = \frac{100(s+5)}{s(s+3)(s^2+4)}$$

Code:

```
H = tf([100 500],[1 3 4 12 0]);
nyquist(H)
```



5) Draw the polar plot from the separate magnitude and phase curves shown in this figure



From the plots given it can be estimated that the system has a pole at $s = -3$.

Further, system gain $20\text{Log}(K) = -7$

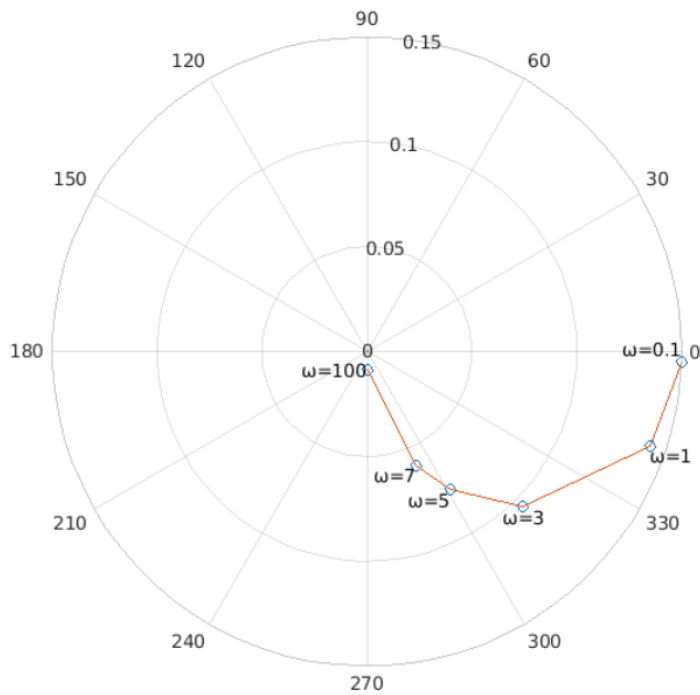
This implies $K = 0.45$

$$\therefore G(s) = \frac{0.45}{s + 3}$$

ω	$ G(j\omega) $	φ
0.1	0.15	-0.033
1	0.142	-0.322
3	0.105	-0.785
5	0.077	-1.030
7	0.059	-1.166
100	0.009	-1.541

Code:

```
x = [-1.541 -1.166 -1.030 -0.785 -0.322 -0.033];
y = [0.009 0.059 0.077 0.105 0.142 0.15];
polarplot(x,y,'o',x,y)
labels = {'w=100',' ',' ',' ',' ',' '};
labels2 = {' ','w=7','w=5',' ',' ',' '};
labels3 = {' ',' ','w=3',' ',' ',' '};
labels4 = {' ',' ',' ','w=1',' ',' '};
labels5 = {' ',' ',' ',' ','w=0.1',' '};
text(x,y,labels,'VerticalAlignment','middle','HorizontalAlignment','right')
text(x,y,labels2,'VerticalAlignment','top','HorizontalAlignment','right')
text(x,y,labels3,'VerticalAlignment','top','HorizontalAlignment','center')
text(x,y,labels4,'VerticalAlignment','top','HorizontalAlignment','left')
text(x,y,labels5,'VerticalAlignment','bottom','HorizontalAlignment','right')
```



6) For each closed-loop system with the following performance characteristics, find the closed-loop bandwidth.

a. $\zeta = 0.2$, $T_s = 3$ seconds

$$\omega_{BW} = \frac{4}{T_s \zeta} \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

Using the above equation and solving , $\omega_{BW} = 10.06$ rad/s.

b. $\zeta = 0.2$, $T_p = 3$ seconds

$$\omega_{BW} = \frac{\pi}{T_p \sqrt{1 - \zeta^2}} \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

Using The above equation and solving , $\omega_{BW} = 1.613$ rad/s

c. $T_s = 4$ seconds, $T_p = 2$ seconds

First find ζ . Since $T_s = 4 / \zeta \omega_n$ and $T_p = \pi / \omega_n \sqrt{1 - \zeta^2}$.

$$T_p / T_s = \zeta \pi / 4 \sqrt{1 - \zeta^2} .$$

Solving for ζ with $T_p / T_s = 0.5$ yields $\zeta = 0.537$.

Then using the below equation,

$$\omega_{BW} = \frac{\pi}{T_p \sqrt{1 - \zeta^2}} \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

$\omega_{BW} = 2.29$ rad/s.

d. $\zeta = 0.3$, $T_r = 4$ seconds

Using $\zeta = 0.3$, $\omega_n T_r = 1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1 = 1.3217$.

Hence, $\omega_n = 1.3217/T_r = 1.3217/4 = 0.3304$ rad/s.

Hence using the equation below,

$$\omega_{BW} = \omega_n \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

$\omega_{BW} = 0.4803$ rad/s.

7) The Bode plots for a plant, $G(s)$, used in a unity feedback system are shown in the figure

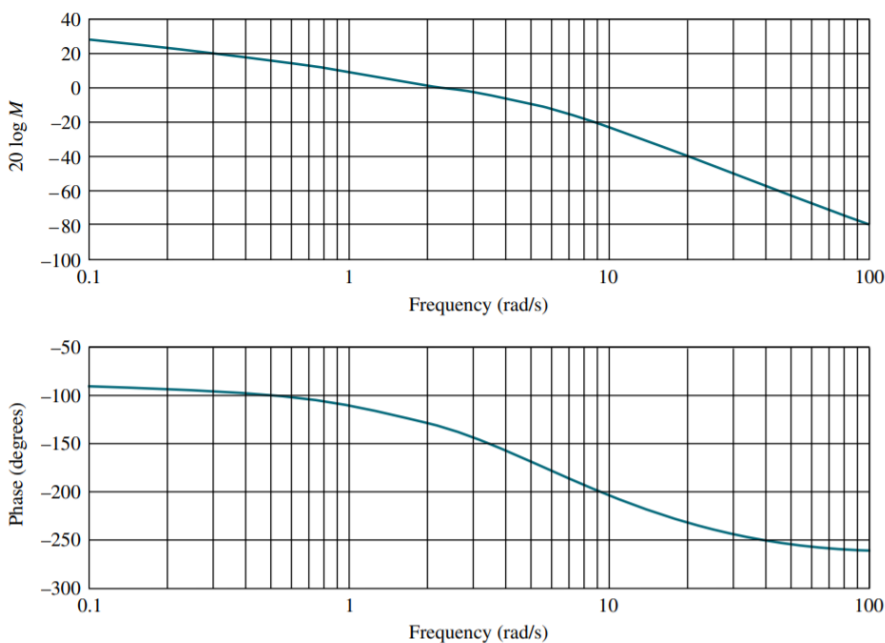


FIGURE P10.7

a) Find the gain margin, phase margin, zero dB frequency, 180° frequency, and the closed-loop bandwidth.

By inspection from the figure, we find that the **zero dB frequency** is **2.152 rad/s**.

And the **180° frequency** is **6.325 rad/s**.

Gain margin is the negative of the gain at 180° frequency (phase crossover frequency)

Therefore, **Gain margin = +14.96 dB**.

Phase margin is defined as:

180° + phase at crossover frequency

Therefore, **phase margin = 180° - 130.42° = 49.57°**

Closed loop bandwidth at -7dB point = **3.8 rad/s**

b) Use your results in Part a to estimate the damping ratio, percent overshoot, settling time, and peak time.

Using the formula:

$$\phi_M = \tan^{-1} \frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}$$

Damping ratio = **0.48**

%Offset = **17.93%**.

Setting time T_s = **2.84s**.

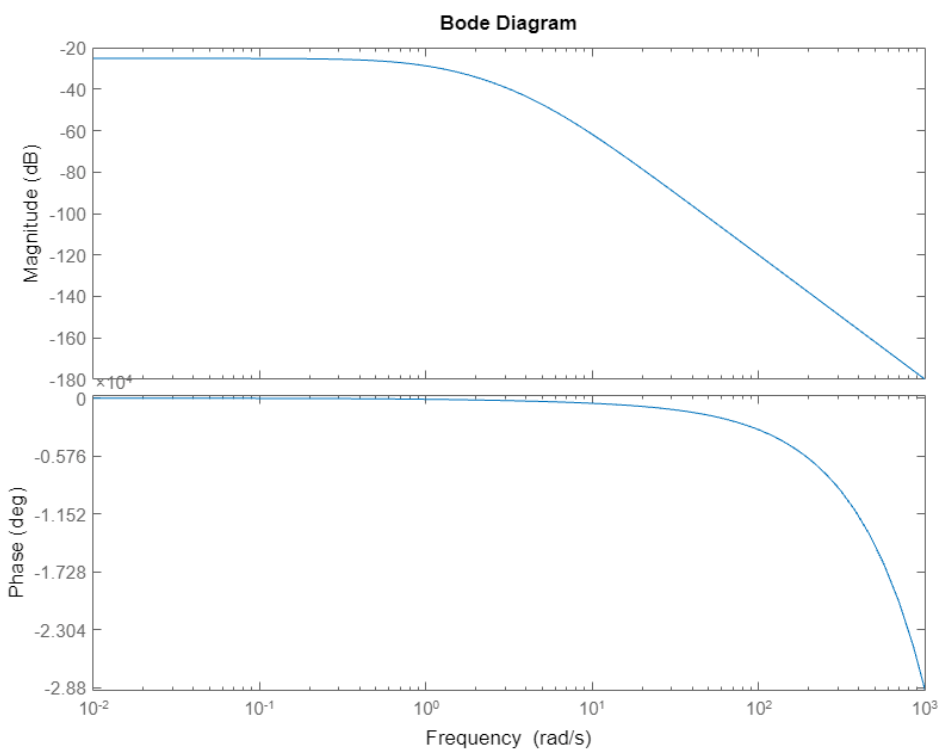
Peak time T_p = **1.22s**.

8) Given a unity feedback system with the forward-path transfer function $G(s) = \frac{K}{(s+1)(s+3)(s+6)}$ and a delay of 0.5 second, find the range of gain, K, to yield stability. Use Bode plots and frequency response techniques.

Adding phase delay, $G(s) = \frac{K e^{-0.5s}}{(s+1)(s+3)(s+6)}$

Code:

```
s=tf('s');  
H = tf([1],[1 10 27 18]);  
H2=H*exp(-0.5*s)  
bode(H2)
```



From the graph, it can be observed that the phase crossover frequency is approximately -34 dB. Then the gain can be increased by +34 dB (54.71).

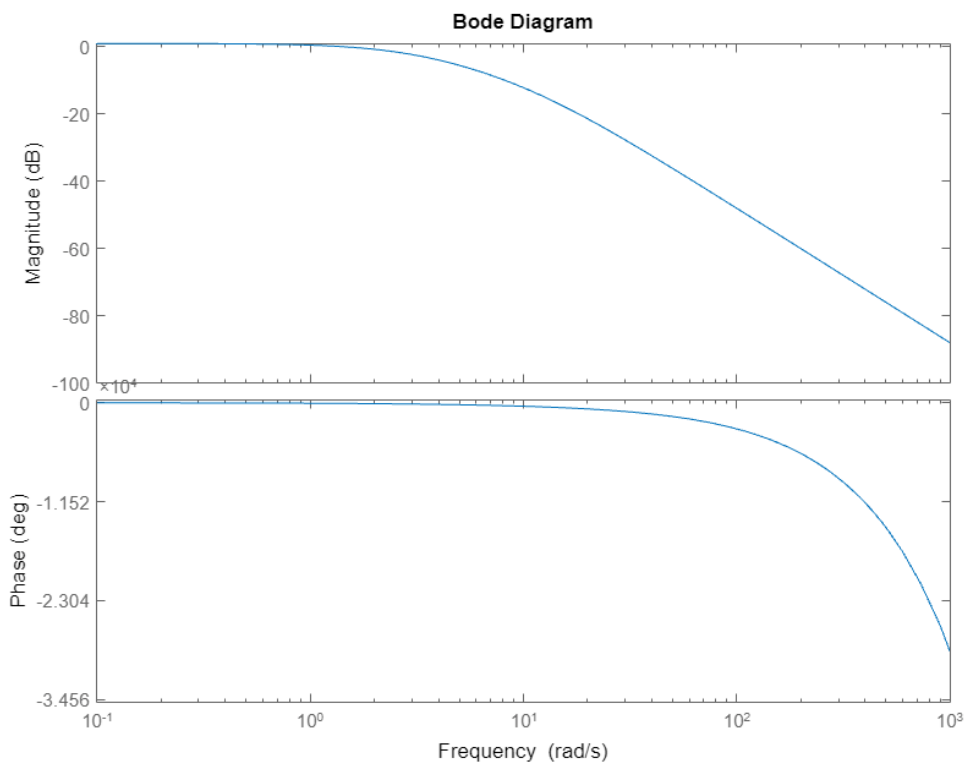
For stability the condition is $0 < K < 54.71$

9) Given a unity feedback system with the forward path transfer function $G(s) = \frac{K}{(s+3)(s+12)}$ and a delay of 0.5 second, make a second-order approximation and estimate the percent overshoot if $K \approx 40$. Use Bode plots and frequency response techniques.

Adding phase delay, $G(s) = \frac{40e^{-0.5s}}{(s+3)(s+12)}$

Code:

```
s=tf('s');
H = tf([40],[1 15 36]);
H2=H*exp(-0.5*s);
bode(H2)
```



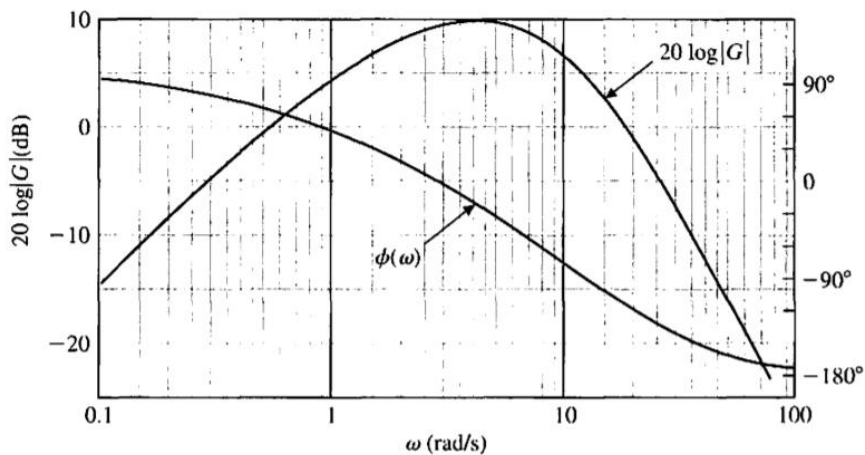
From the graph, we obtain the phase margin to be $\pi - 2.518 = 0.624$

$$\gamma = 0.624 = \tan^{-1}\left(\frac{2\delta}{\sqrt{-2\delta^2 + \sqrt{1 + 4\delta^4}}}\right)$$

$$\delta = 0.33$$

Thus % offset = 33.3%

10) The frequency response for a process of the form $G(s) = \frac{Ks}{(s+a)(s^2+20s+100)}$ is shown in the given figure. Determine K and a by examining the frequency response curves.



We can observe that $\varphi = 0^\circ$ at $\omega = 3$,

and that $\varphi = +90^\circ - \tan^{-1}\left(\frac{\omega}{a}\right) - 2 \tan^{-1}\left(\frac{\omega}{10}\right)$.

Substituting $\omega = 3$ and solving yields, $a = 2$.

Similarly, from the magnitude relationship we determine that $K = 353.67$

11) A feedback system has a loop transfer function $L(s) = G_c(s)G(s) = \frac{100(s-1)}{(s^2+25s+100)}$

a) Determine the corner frequencies (break frequencies) for the Bode plot,

The function can be written as $L(s) = \frac{100(s-1)}{(s+20)(s+5)}$

Therefore, break frequencies occur at:

$\omega = 1 \text{ rad/s}$; $\omega = 20 \text{ rad/s}$; $\omega = 5 \text{ rad/s}$

b) Determine the slope of the asymptotic plot at very low frequencies and at high frequencies,

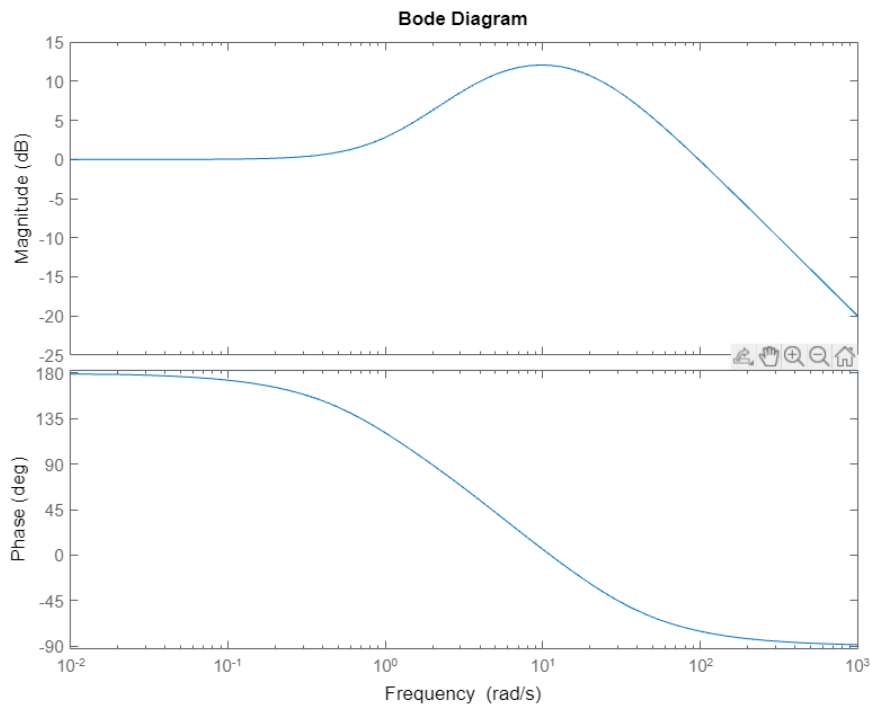
The slope of the asymptotic plot at low frequencies is 0 dB/decade.

And at high frequencies the slope of the asymptotic plot is -20 dB/decade.

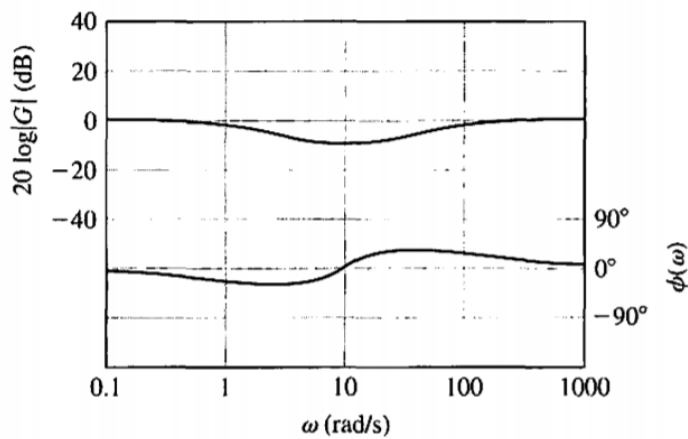
c) Sketch the Bode magnitude plot.

Code:

```
H = tf([100 -100],[1 25 100]);
bode(H)
```



12) The Bode diagram of a system is shown in the given figure. Determine the transfer function $G(s)$.



From the bode diagram given above, we can observe that

Two poles at 1rad/s and 80rad/s

Two zeros at 5 rad/s and 20 rad/s

Therefore

$$G(s) = \frac{(1 + \frac{s}{5})(1 + \frac{s}{20})}{(1 + s)(1 + \frac{s}{80})}$$