Principles of Modern CDMA/MIMO/OFDM Wireless Communications

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Lecture – 08
Bit Error Rate of Rayleigh Fading Wireless Channel

Hello welcome, to this MOOC on a Principles of CDMA MIMO OFDM Wireless Communications. So, today we are going to start looking at the bit error rate performance of wireless communication system, previously we looked at the bit error rate performance of a wire line communication system, which could be modeled as simple AWGN, that is an Additive White Gaussian Noise Channel. Now, let us start looking at the bit error rate performance of a wireless communication system, so, that we can compare the performance of a wire line and wireless communication system.

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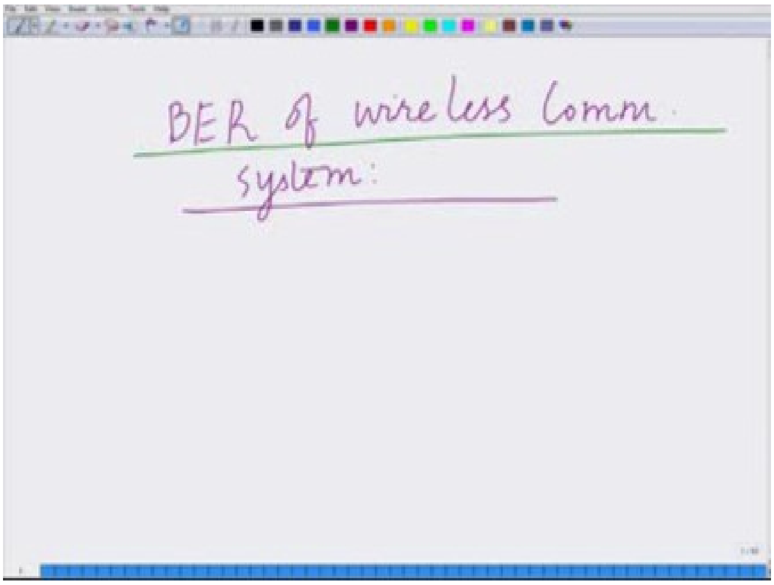

So, we are going to short looking at bit error rate BER of wireless communication system, and as we said previously in a wireless communication system there is fading due to multi path nature of the propagation of the signal in the wireless communication environment.

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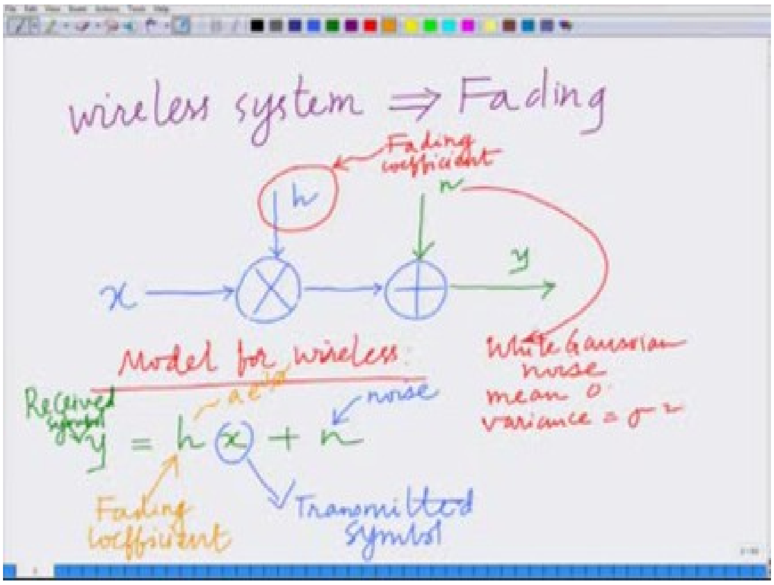

So, wireless communication system there is basically, fading the signal is fading in nature because of the multi path propagation nature of the wireless channel and therefore, the channel can be modeled as follows; let say I have a transmitted signal x and this signal is now transmitted over the wireless channel, and as we know that this signal undergoes fading. Therefore, it is multiplied by the fading channel coefficient h and now at the receiver as before there is the additive noise n and finally, we have the received signal y.

So, the new aspect of this wireless communication system is this fading, fading nature of the channel which is modeled by the fading coefficient this h is the fading coefficient, as usual we are assuming this n to be Gaussian noise or white Gaussian noise with 0 mean that is mean 0 and variance or basically power variance equal to σ^2 . And therefore, the channel or the model for this wireless communication system the model for this wireless system can be written as we have y that is the receive symbol equals

$$y = h\,x + n$$

So, where y this is the received symbol, x is my transmitted, n is the noise, and most importantly my h is the fading coefficient. And also we said that h can be modeled as

$$h = a\;e^{j\phi}$$

where a is the magnitude the amplitude of the fading coefficient and ϕ is the phase of the fading coefficient and we also said that a follows the rely density or the rely distribution right this is the fading channel coefficient.

So, I have $y = h x + n$, where h is the fading channel coefficient therefore, now if I look at this signal model.

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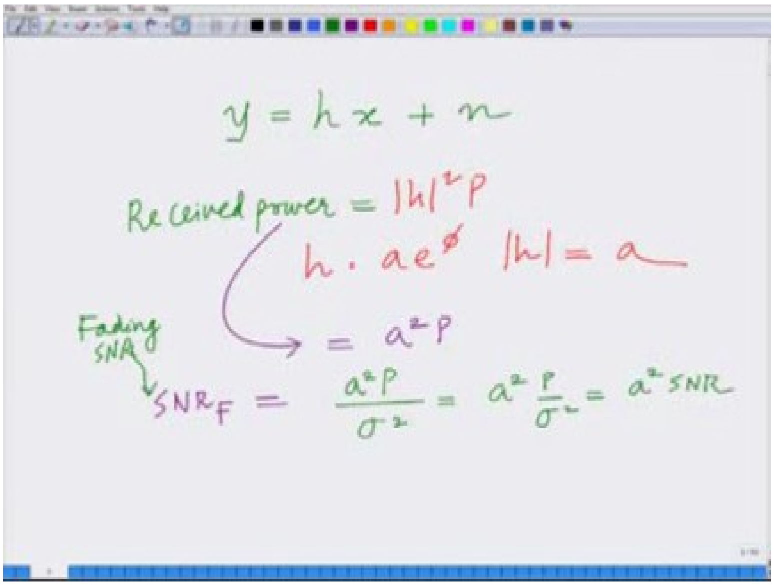

I have, what do I have? I have $y = h x + n$. So, this signal x is multiplied h and therefore, I have the received signal power = $|h|^2$ P

but h = a $e^{j\phi}$ which means |h| = a and therefore, we have

Received power = a^2 P

and therefore, the received SNR or the fading SNR, which I am going to denote by SNR_F , this I am going to call as the fading SNR, SNR_F is basically the received power

$$SNR_F = \frac{a^2 P}{\sigma^2} = a2\frac{P}{\sigma^2} = a^2 \, SNR$$

So, because of the fading nature of the wireless communication channel the fading SNR, SNR_F is a^2 SNR which is $\frac{P}{\sigma^2}$. Where a is the magnitude of the fading channel coefficient h, and now what we have is as we can as you remember from the bit error rate expression derived for BPSK modulation.

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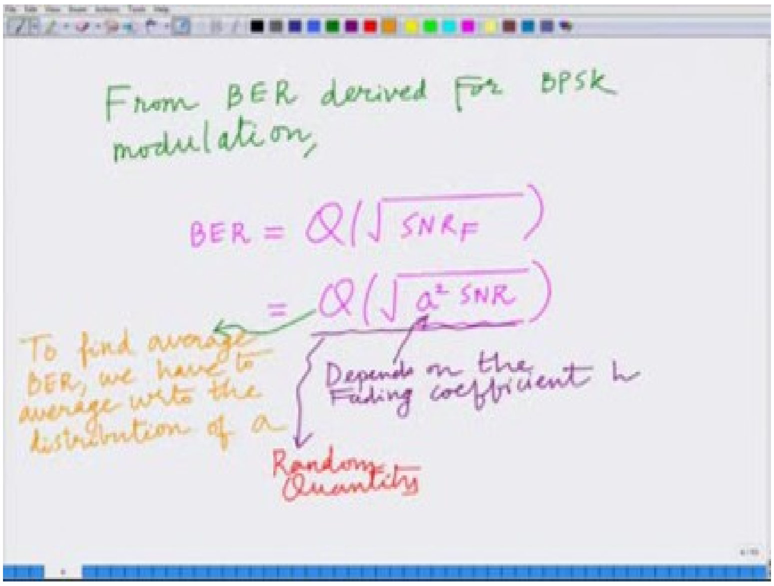

From the bit error rate derived for BPSK modulation, what do we have, we have bit error rate Q ($\sqrt{SNR}$), but here the SNR is the fading SNR, because the transmitted signal is multiplied by the fading coefficient h therefore, the bit error rate is Q ($\sqrt{SNR_F}$), SNR_F which is also equal to Q ($\sqrt{a^2 SNR}$), where SNR equals $\frac{P}{\sigma^2}$.

Therefore, what you can see is the this quantity under the square root this depends on the magnitude of the fading coefficient, this depends on the fading coefficient h, and since the fading coefficient h is random in nature therefore, this fading SNR that is $a^2 SNR$ is also random in nature, hence the resulted bit error rate Q ($\sqrt{a^2 SNR}$) is also random quantity, because a which is random in nature hence therefore, this entire quantity this is basically this is a random quantity, because of the random nature of the fading channel coefficient this is the random quantity which arises, because of the random nature of the fading channel coefficient.

And therefore, to find the average bit error rate in order to find the average bit error rate one has to average with respect to the distribution of the fading channel coefficient, or the distribution of a. To find average bit error rate therefore, to find average BER, we have to average with respect to the distribution of a, we have to average with respect to find the average bit error rate, we have to take this function because this function is a random quantity, because it depends on the random channel amplitude that is a. So, this is random in nature therefore, to find average bit error rate in this fading channel condition one has to average with respect to, one has to average this expression with respect to the distribution of the fading channel coefficient amplitude a.

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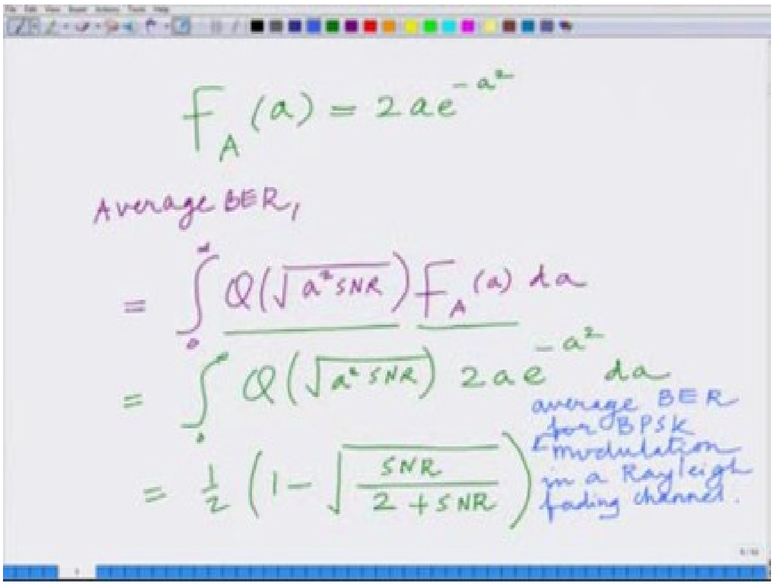

And therefore, and we already know that the distribution of a that is a fading channel amplitude it follows relay distribution, you can look at the previous modules this is given as

$$F_A(a) = 2a\ e^{-a^2}$$

therefore the average bit error rate is

$$\text{Average BER} = \int_0^\infty Q(\sqrt{a^2 SNR})\, F_A(a)\, d\, a$$

So, what I am doing, I am taking this random bit error rate, I am multiplying this with the distribution of the fading channel amplitude a, and I am averaging it by integrating it from 0 to infinity that will give me the average bit error rate average with respect to the distribution of the fading channel amplitude that is a.

Or in other words, this is equal to

$$= \int_0^{\infty} Q\left(\sqrt{a^2 SNR}\right) 2a \; e^{-a^2} \; d\,a$$

and now I am not going to evaluate this expression explicitly in this module, because the evaluating this integral is slightly complicated. So, I am not going to evaluate it right now in this module, I am going to skip this derivation at this point and I am going to give you the result directly, but for the more advanced viewer who is interested in knowing the complete derivation of this expression if you look at subsequent module (Refer Time : 11:26) for a sought of a beginner, who is probably not interested in all the details all the minor details involved in this derivation you can simply follow the result, and the result is given as

$$= \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right)$$

So, the average bit error rate the final result which have not derive here thoroughly for which, I have not given the complete derivation here, I will do this in the next module.

So, the expression that is an expression for the average bit error rate, when an average this bit error rate over the distribution of the fading channel coefficient a is given as

$\frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right)$. So, let me remind you this is the expression for average bit error rate of BPSK modulation in a fading wireless in a Rayleigh fading wireless channel. So, this is the average bit error rate for BPSK modulation in a Rayleigh fading channel.

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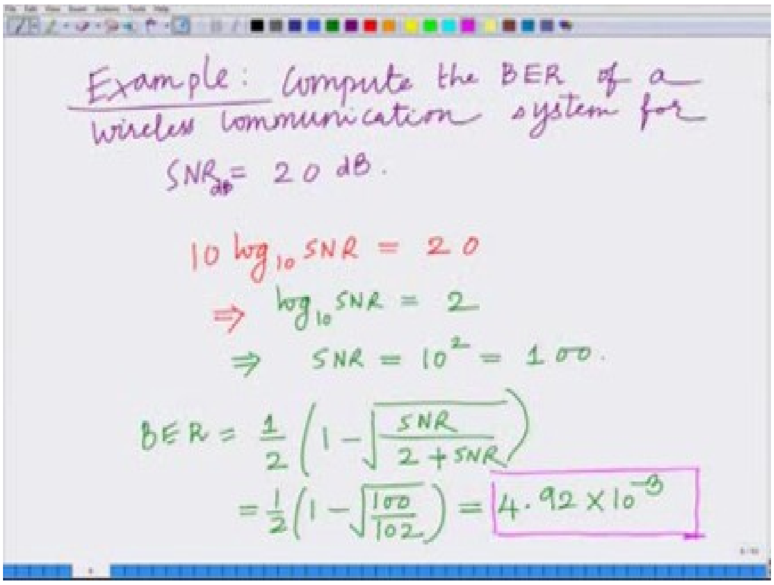

And let us do a few examples, to understand this better for instance, let us again do the examples similar to what we have you done in the case of a simple wire line, or a simple AWGN base communication channel. So, compute the bit error rate of a wireless communication systems, so, we will we want to compute the bit error rate of a wireless communication system for SNR equals 20 dB. So, we want to compute the bit error rate that is the SNR dB SNR equals 20 dB, which means

$$10 \log_{10} SNR = 20 \text{ dB}$$

$$\log_{10} SNR = 2$$

$$SNR = 100$$

So, if the dB SNR is 20; that means, the actual SNR is 100 and now my bit error rate I have to substitute in my formula for a wireless communication channel, that is

$$BER = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right)$$

$$= \frac{1}{2}\left(1 - \sqrt{\frac{100}{2+100}}\right)$$

$$= \frac{1}{2}\left(1 - \sqrt{\frac{100}{102}}\right)$$

$$= 4.92 \times 10^{-3}$$

So, the bit error rate in a wireless channel at 20 dB SNR is 4.92×10^{-3}.

Right, so, the bit, so, what we have done is we have computed the bit error rate in a wireless communication system with fading for BPSK transmission at SNR of 20 dB and the answer is 4.92×10^{-3} now.

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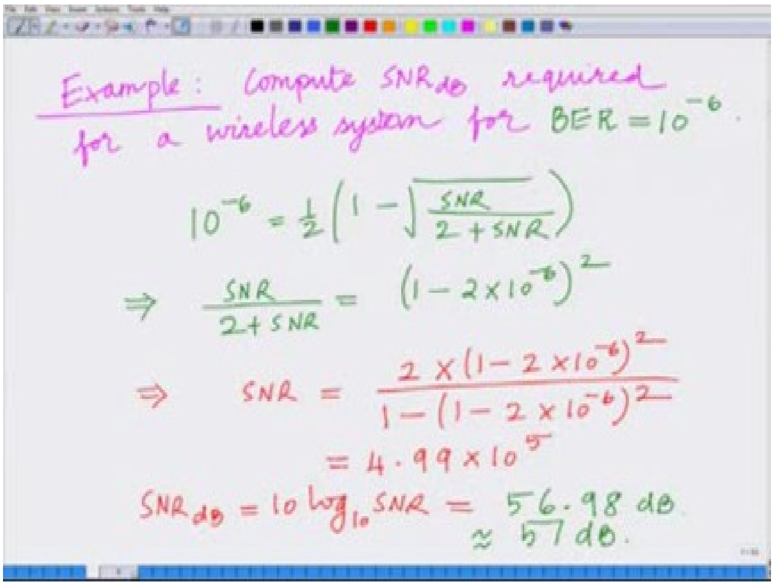

Let us calculate do another example to understand this better, and we would like to compute SNR, the dB SNR required for a wireless communication system, for a bit error rate equals 10^{-6}, that is what is the SNR in dB in a wireless communication system required to achieve a bit error rate of 10^{-6}. So, this is the reverse problem we required to previously we are given the SNR and ask to calculate the bit error rate, now we are given the bit error rate and ask to calculate the required SNR in dB.

Therefore, we have

$$10^{-6} = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right)$$

$$\frac{SNR}{2+SNR} = (1 - 2 \times 10^{-6})^2$$

$$SNR = 4.99 \times 10^{-5}$$

$$SNR_{dB} = 10 \log_{10} SNR$$

$$= 56.98 \text{ dB}$$

$$\approx 57 \text{ dB}$$

and if you remember from our previous discussion on the AWGN that is a wire line communication system, remember from our previous discussion that for 10^{-6} bit error rate in an AWGN channel we require a bit error we require a SNR approximately 14.19 dB for an AWGN.

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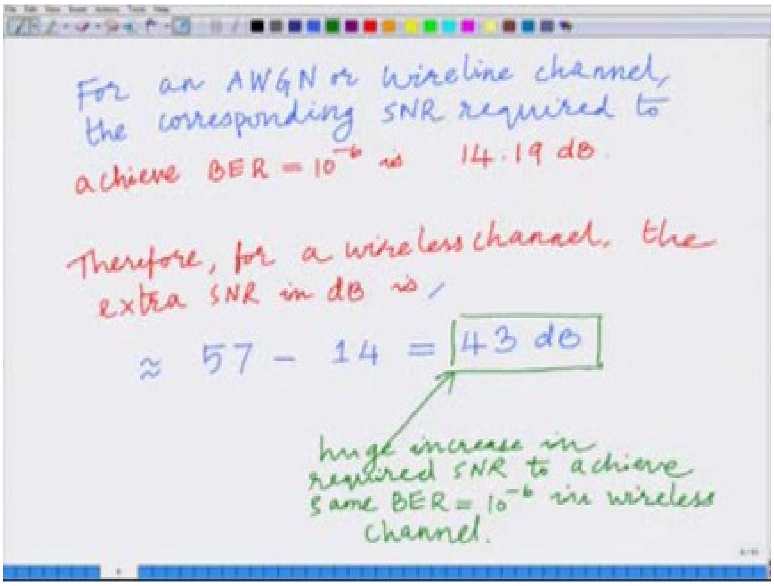

Or a wireless channel, the corresponding SNR required to achieve BER equals 10 power minus 6 is 14.19 dB.

For a fading wireless channel as we have just seen previously it is just 57 dB therefore, approximately for a wireless channel how much more SNR dB need to achieve 10^{-6} bit error rate therefore, for a wireless channel the extra SNR in dB is my 56.98, which is approximately

$\approx$ 57 - 14.19 $\approx$ 43 dB

So, you can see this 43 dB is the extra SNR that is required in a wireless channel approximately 14 dB is required in AWGN or wireline channel.

So, on top of that additionally require an SNR of around 43 dB in a wireless channel to achieve the same bit error rate, so, you require. So, if the noise power is the same at the receiver you require 43 dB more transmit power in the wireless channel to achieve the same bit error rate of 10^{-6} . So, there is a huge increase, in required SNR to achieve same bit error rate equal to 10^{-6} . So, what we are saying is in a wireless channel to achieve a bit error rate of 10^{-6} we need 43 dB more SNR approximately, and if the noise for remember SNR is ratio of senior power to noise power. Therefore, if the noise power is same in the both the systems remain transmit power has to increase by 43 dB, 43 dB if you converted into a normal scale 43 dB is log scale, in normal scale it means $10^{4.3}$ times more power is required in a wireless communication. So, that is a huge amount of extra power that is required in a wireless communication system, that is how is it requires the huge increase in the SNR to achieve the same bit error rate.

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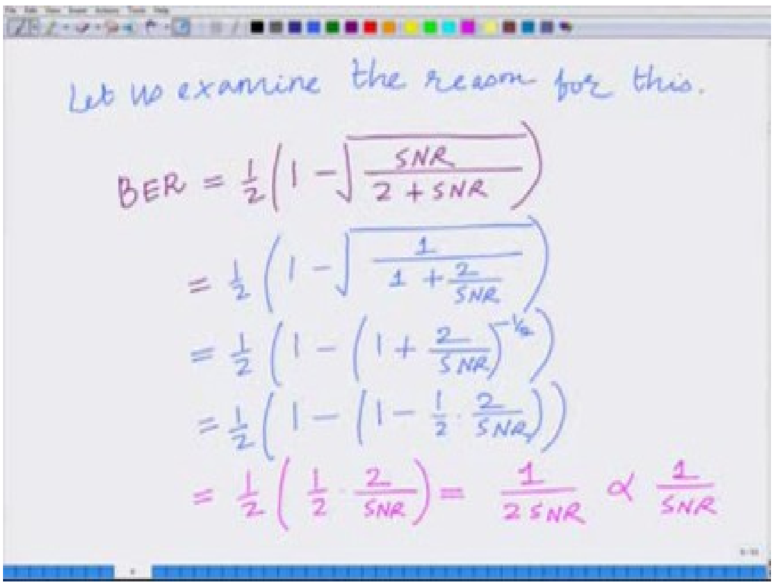

And why is this the case let us examine, the reason for this and if you look at the bit error rate expression you have bit error rate equals,

$$BER = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right)$$

$$= \frac{1}{2}\left(1 - \sqrt{\frac{1}{1+\frac{2}{SNR}}}\right)$$

$$= \frac{1}{2}\left(1 - (1 + \frac{2}{SNR}¿^{-0.5}\right)$$

$$= \frac{1}{2}\left(1 - (1 - \frac{1}{2}.\ \frac{2}{SNR}\right)$$

$$= \frac{1}{2}.\frac{1}{2}.\ \frac{2}{SNR}$$

$$= \frac{1}{2\,SNR}$$

So, the bit error rate is decreased is proportional to $\frac{1}{SNR}$. So, in the wireless communication system the bit error rate is decreasing at $\frac{1}{SNR}$.

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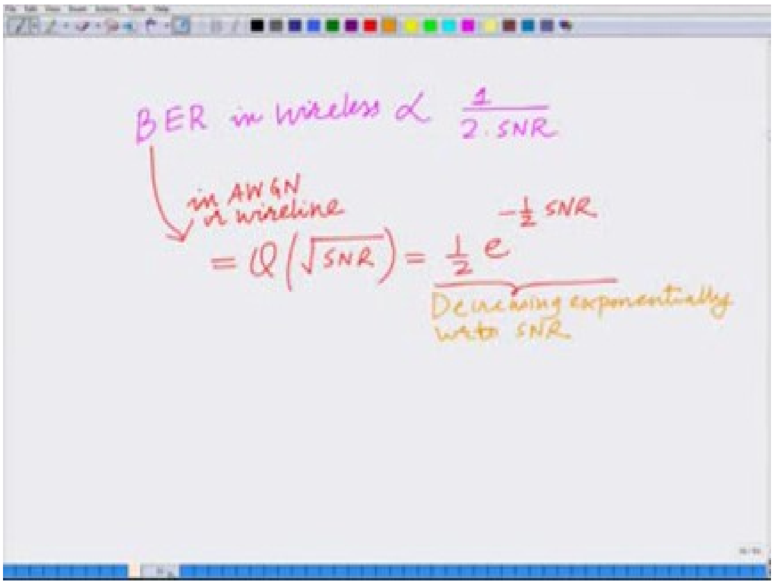

So, if you look at the net result BER in wireless is proportional to $\frac{1}{2\,SNR}$; however, if you remember the bit error rate in AWGN or the wired, wire line communication system. This is equal to approximately Q ($\sqrt{SNR}$, which you remember this is equal to $\frac{1}{2}e^{\frac{-1}{2}SNR}$ therefore, in a wire line communication system it is decreasing as exponentially with respect to SNR. So, if you look at the bit error rate in a wire line communication system bit error rate in the wire line communication system is decreasing exponentially, with respect to SNR while in a wireless system it is only decreasing as $\frac{1}{SNR}$.

So, there is a huge gap in the bit error rate of a wire line communication system versus a wireless communication system. This is because, in a wire line system while it is decreasing exponentially in a wireless communication system it is only decreasing as $\frac{1}{SNR}$ and that is the very important difference between the performance of wire line and wireless communication system.

So, wireless communication systems have a very high bit error rate, because of the sluggish decrease in the bit error rate verses SNR which is only decreasing as $\frac{1}{SNR}$, and that is the result you see in order to achieve the same bit error rate in a wireless communication system you need a huge amount of extra transmit power that is 10^{-6} bit error rate for BPSK you need a approximately 43 dB more transmitter power all right.

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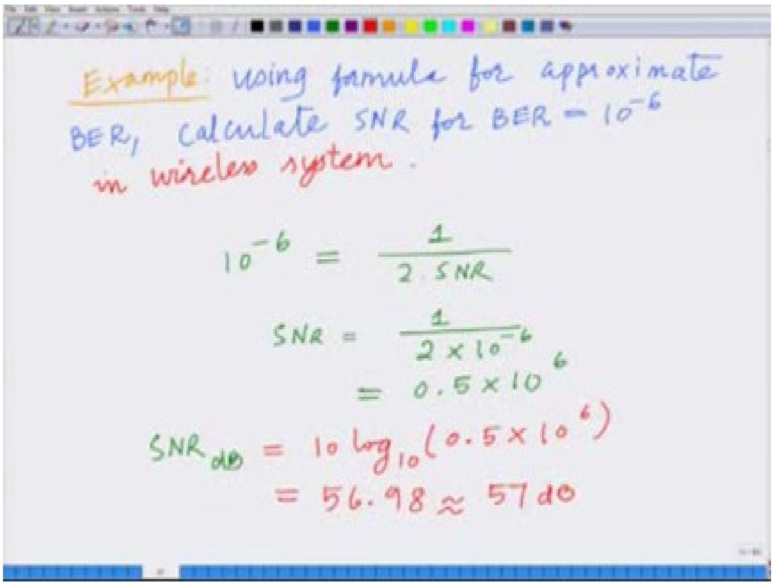

So, let us do another simple example to understand this better. So, another example is now using our approximate formula, using our approximation that we have recently developed using the formula for approximate bit error rate in the wireless system, again calculate SNR for bit error rate equals 10^{-6} in or wireless system and for this purpose what do we have we have 10^{-6}, remember now we have to use approximate formula, which is approximate formula is if you look at is $\frac{1}{2\,SNR}$.

$$10^{-6} = \frac{1}{2\,SNR}$$

$$SNR = \frac{1}{2} \cdot 10^{6} = 0.5 \times 10^{6}$$

$$SNR_{dB} = 10 \log_{10} SNR = 10 \log_{10} (0.5 \times 10^{6}) = 56.98 \approx 57 \text{ dB}$$

So, this is the same answer that we get for the dB SNR that is a required to achieve a bit error rate of 10^{-6} in a wireless communication system. So, therefore, we have 2 expressions for the bit error rate wireless communication system.

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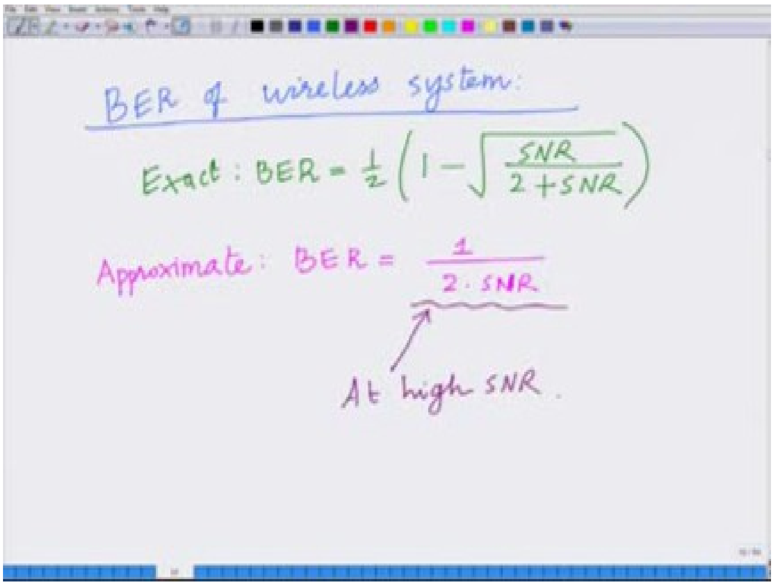

Bit error rate, so, in this module let us summarize the bit error rate of wireless system the first formula is the exact formula, the exact expression for the bit error rate equals

$$\text{BER} = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right)$$

and then we also have an approximate expression in the approximate expression the

$$\text{Approximate BER} = \frac{1}{2\,SNR},$$

So, we have 2 expressions. So, 1 is the exact expression and the approximate expression is bit error rate equals $\frac{1}{2\,SNR}$ and this is especially approximate expression is valid at high SNR remember we employed the assumption that $\frac{1}{SNR}$ is the small quantity close to approximately equal to 0 close to 0 to derive this approximation. So, this approximation

$\frac{1}{2\,SNR}$ that is a bit error rate for wireless system is equal to $\frac{1}{2\,SNR}$ is valid at high SNR.

So, these are the 2 expressions that we have derived for the bit error rate performance of a wireless communication system. So, we will end this module here and take up other topics in subsequent modules.

Thank you very much.