1. Suppose that x and y are real numbers for which $2\log_{10}(x-2y) = \log_{10} x + \log_{10} y$. Determine all possible values of $\dfrac{x}{y}$.

$$2\log_{10}(x-2y) = \log_{10}(x) + \log_{10}(y) \quad -(1)$$

$$\Rightarrow \log_{10}(x-2y)^2 = \log_{10}(xy) \quad -(2)$$

$\because$ all terms have $\log_{10}$, $\log_{10}$ cancels out $\Rightarrow (x-2y)^2 = xy$

$$x^2 - 4xy + 4y^2 = xy$$

$$\Rightarrow x^2 - 5xy + 4y^2 = 0$$

$$\Rightarrow x = \frac{-(-5y) \pm \sqrt{(5y)^2 - 4(1)(4y^2)}}{2(1)} \quad \text{(using quadratic formula)}$$

$$\Rightarrow x = \frac{5y \pm \sqrt{25y^2 - 16y^2}}{2}$$

$$\Rightarrow x = \frac{5y \pm \sqrt{9y^2}}{2} \Rightarrow x = \frac{5y \pm 3y}{2}$$

$$\Rightarrow x = \frac{5y+3y}{2} \text{ or } x = \frac{5y-3y}{2}$$

$$\Rightarrow x = \frac{8y}{2} \text{ or } x = \frac{2y}{2}$$

$$\Rightarrow x = 4y \text{ or } x = y$$

$$\Rightarrow \frac{x}{y} = 4 \text{ or } \frac{x}{y} = 1$$

$\therefore$ Answer) $\dfrac{x}{y} = 4$ or $\dfrac{x}{y} = 1$