Problem Set 2

Zain Ali (zaa23)
Michael Saunders (mbs189)

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Problem 1

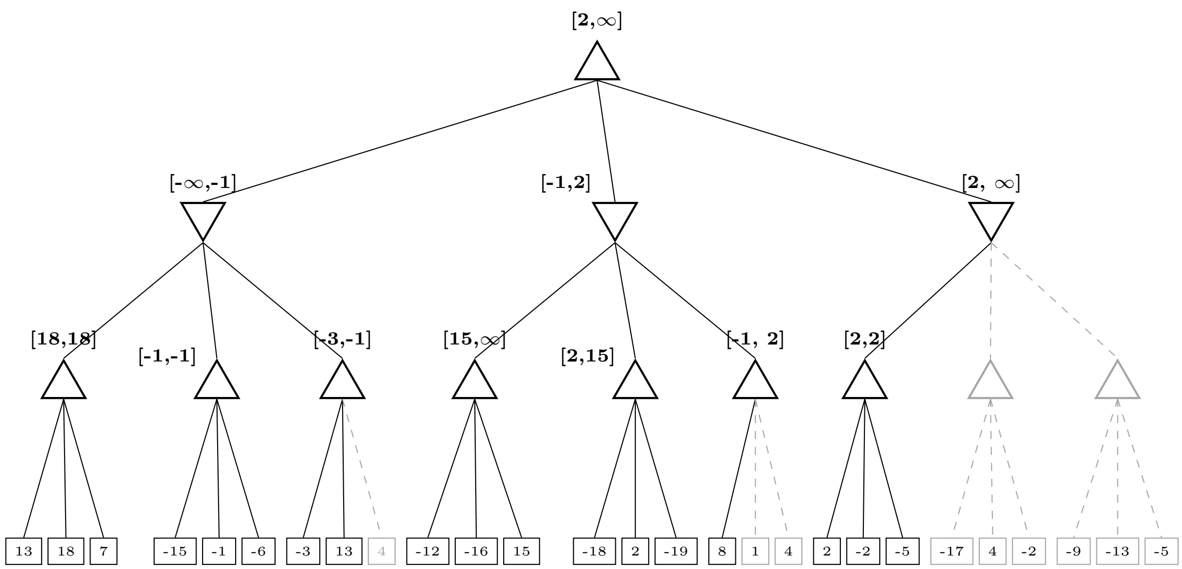

Figure 1: MIN-MAX Search Tree with Alpha-Beta Pruning.

Problem 2

To find the probability of a person having Lyme disease given a positive test ($P(A|B)$) we must first define the following:

A = has Lyme disease
B = positive test
$P(A)$ = probability of having Lyme disease = 0.008
$P(\neg A)$ = probability of not having Lyme disease = $1 - P(A) = 0.992$
$P(B|A)$ = probability of a positive test given the person has Lyme disease = 0.93
$P(B|\neg A)$ = probability of a positive test given the person does not have Lyme disease = 0.16

To solve this we will use Bayes's Rule

Theorem 1 (Bayes's Rule)

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$$

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B|A) * P(A) + P(B|\neg A) * P(\neg A)}$$

Using Bayes's Rule:

$$P(A|B) = \frac{0.93 * 0.008}{0.93 * 0.008 + 0.16 * 0.992}$$

$$P(A|B) = \frac{0.00744}{0.00744 + 0.15872}$$

$$P(A|B) = 0.04477611 = 4.477611\%$$

According to Bayes's Rule, the probability of a person taking a test and finding out they have Lyme disease due to a positive test ($P(A|B)$) is 4.477611%.

Problem 3

Suppose there are 5 balls with distinct colors (blue, yellow, read, green, and pink) in a box. Each time I randomly pick a ball, I record the color of the ball I picked then put it back. I repeat this behavior several times.

(a) What is the expected number of picking attempts it takes so as to see all colors of balls?

Assuming X is the number of picking attempted it takes to see all colors of balls we will find $E[X]$ using linearity of expectation and expected attempts.

$$\text{Golden Rule 1: } E[X] = E[X_1 + X_2] = E[X_1] + E[X_2]$$

$$\text{Golden Rule 2: } E[X_i] = \frac{1}{P(X_i)}$$

$$P(X_i) = \text{Probability to pick an i'th unique color}$$

The probability for the first unique color is $P(X_1) = 1$, the next color probability would be $P(X_2) = \frac{4}{5}$ followed by $P(X_3) = \frac{3}{5}$, $P(X_4) = \frac{2}{5}$, and $P(X_5) = \frac{1}{5}$. Using that, we can find out how many picks you would need for to get all 5 ball colors.

$$E[X] = \sum_{i=1}^{5} E[X_i] = \sum_{i=1}^{5} \frac{1}{P(X_i)}$$

$$\sum_{i=1}^{5} \frac{1}{P(X_i)} = \frac{1}{1} + \frac{1}{(\frac{4}{5})} + \frac{1}{(\frac{3}{4})} + \frac{1}{(\frac{2}{5})} + \frac{1}{(\frac{1}{5})} = 11\frac{5}{12} \approx 11.42 \text{ attempts}$$

It is expected to take about 11.42 attempts of picking the colored balls to see every color.

(b) If I repeat the picking behavior 6 times, what is the expected number of distinct colors I will see?

Let X = the number of distinct colors recorded after 6 repetitions of picking a ball, recording the color, and replacing the ball.

Define Random Variables x_i for $1 \leq i \leq 5$ such that:

$x_i = 1$ if color i was selected at least once, otherwise $x_i = 0$.

The number of distinct colors recorded is the sum of the Random Variables, x_i:

$$X = x_1 + x_2 + x_3 + x_5 + x_5$$

The expected number of distinct colors $E(X)$ is defined as such:

$$E[X] = E[x_1 + x_2 + x_3 + x_4 + x_5].$$

By *linearity of expectation* (golden rule 1),

$$E[X] = E[x_1] + E[x_2] + E[x_3] + E[x_4] + E[x_5]$$

Since the value of our random variables x_i can only be either 1 or 0, we can derive the following:

$$E[x_i] = 0 * P(x_i = 0) + 1 * P(x_i = 1)$$

$$E[x_i] = P(x_i = 1)$$

$$\implies E[X] = \sum_{i=1}^{5} P(x_i = 1)$$

To find $E[X]$ we now just need to find the probability of each random variable x_i having the value of 1.

$$P(x_i = 1) = P(\text{color } i \text{ is recorded at least once})$$

$$P(x_i = 1) = 1 - P(\text{color } i \text{ is never recorded})$$

Since we are replacing the balls we choose each time after their color is recorded, then selecting from the same set of 5 colored balls each time, *each color has the same probability of never being chosen.* So, we can simplify the equation further by factoring out a 5.

$$E[X] = 5 * P(x_i = 1)$$

$$E[X] = 5 * (1 - (\frac{4}{5})^6) \approx 3.69 \text{ colors}$$

The expected number of distinct colors recorded after 6 repetitions of the behavior is approximately 3.69.