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12.11

1. Multiple Linear Regression Model.

$$\hat{y} = B_0 + B_1x_1 + B_2x_2 + \beta_3x_3 + \beta_4x_4 + \beta_5x_5$$

as $\hat{\beta} = (X^TX)^{-1}X^Ty$.

$\beta_0 = -6.4443$ $\beta_3 = 2.4111$
$\beta_1 = 1.7719$ $\beta_4 = 5.3967$
$\beta_2 = -3.6573$ $\beta_5 = 14.6205$

Putting values in equation.

$$\hat{y} = -6.4443 + 1.7719x_1 - 3.6573x_2 + 2.4111x_3 + 5.3967x_4 + 14.6205x_5.$$

2. Variable Screening with $\alpha = 0.05$ using t-test

for β_1:

$H_0: \beta_1 = 0$
$H_1: \beta_1 \neq 0$

Test Statistics $t_0 = \frac{\hat{\beta}_1}{\sqrt{s^2 C_{11}}} \Rightarrow \frac{1.7719}{2.487} \Rightarrow \boxed{0.712}$

Critical Region $v = N - k - 1 \Rightarrow 20$

critical value $t_{\alpha/2} \Rightarrow 2.086$.

Therefore we accept H_0.

We can say that the x_1 regressor is insignificant.

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for β_2:

$H_0: \beta_2 = 0$
$H_1: \beta_2 \neq 0$

$t_0 \Rightarrow \frac{-3.6573}{2.784} \Rightarrow \boxed{-1.3136}$

Critical value: $v = N-k-1 \Rightarrow 20$

$t_{\alpha/2} \Rightarrow 2.086.$

Ho Accepted

Therefore the x_2 regressor is insignificant.

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for $\beta 3$:

$H_0 : \beta_3 = 0$

$H_1 : \beta_3 \neq 0$

$t_0 \Rightarrow \frac{2.411}{6.361} \Rightarrow \boxed{0.3790}$

Critical value: $v = N-k-1 \Rightarrow 20$

$t_{\alpha/2} \Rightarrow 2.086$

Ho Accepted

Therefore the x_3 regressor is insignificant.

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for $\beta 4$:

$H_0 : \beta 4 = 0$

$H_1 : \beta 4 \neq 0$

$t_0 \Rightarrow \frac{5.3967}{3.588} = \boxed{1.504}$

critical value: $v = N-k-1 \Rightarrow 20.$

$t_{\alpha/2} \Rightarrow 2.086$

Ho Accepted

Therefore the X4 regressor is also insignificant.

for β_5:

$H_0 : \beta_5 = 0$

$H_1 : \beta_5 \neq 0$

$$t_0 \Rightarrow \frac{14.6205}{4.818} \Rightarrow \boxed{3.0345}$$

Critical value $\Rightarrow$ $V = N - K - 1 \Rightarrow 20$

$t_{\alpha/2} \Rightarrow 2.086$

H_0 Rejected

Therefore the X_5 regression is SIGNIFICANT

12.12

(I) Multiple Linear Regression

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5.$$

$$\hat{\beta} = (X^T X)^{-1} (X^T Y)$$

$\beta_0 = 1710.7679$

$\beta_1 = -9.6249$

$\beta_2 = 0.0563$

$\beta_3 = 1.3772$

$\beta_4 = -3.9881$

$\beta_5 = -358.0028$

$$\hat{y} = 1710.7679 - 9.6249 x_1 + 0.0563 x_2 + 1.3772 x_3 - 3.9881 x_4 - 358.0028 x_5$$

(2) Variable Screening
with $\alpha = 0.05$

for β_1:

$H_0 : \beta_1 = 0$

$H_1 : \beta_1 \neq 0$

$$t_0 \Rightarrow \frac{-9.6249}{96.210} \Rightarrow \boxed{-0.100}$$

critical values $\Rightarrow$ $V = N-K-1 \Rightarrow 15$

$t_{\alpha/2} \Rightarrow \boxed{2.131}$

Ho Accepted

Therefore the x_1 regressor is insignificant.

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for β_2:

$H_0 : \beta_2 = 0$

$H_1 : \beta_2 \neq 0$

$$t_0 \Rightarrow \frac{0.0563}{0.021} \Rightarrow \boxed{2.685}$$

critical values $\Rightarrow$ $V = N-K-1 \Rightarrow 15$

$t_{\alpha/2} \Rightarrow \boxed{2.131}$

Rejected Ho

Therefore we can say that the x_2 regressor is <u>SIGNIFICANT</u>

for β_3 : $H_0 : \beta_3 = 0$

$H_1 : \beta_3 \neq 0$

$t_0 \Rightarrow \frac{1.3772}{3.047} \Rightarrow \boxed{0.452}$

Critical values $\Rightarrow$ $V = N - K - 1 \Rightarrow 15$

$t_{\alpha/2} \Rightarrow \boxed{2.131}$

Ho Accepted → X3 regressor insignificant.

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for β_4 : $H_0 : \beta_4 = 0$

$H_1 : \beta_4 \neq 0$

$t_0 \Rightarrow \frac{-3.9881}{7.061} \Rightarrow \boxed{-0.565}$

critical values $\Rightarrow \boxed{2.131}$

Ho Accepted → X4 regressor insignificant.

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for β_5: $H_0 : \beta_5 = 0$

$H_1 : \beta_5 \neq 0$

$t_0 \Rightarrow \frac{-358.0028}{207.106} \Rightarrow \boxed{-1.729}$

Critical values $\Rightarrow t_{\alpha/2} \Rightarrow \boxed{2.131}$

Ho Accepted → X5 is also insignificant.

Therefore we can conclude that the X_2 regressor is the most significant.

12.14

① Multiple Linear Regression Line/Model.

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4.$$

$$\hat{\beta} = (X^T X)^{-1}(X^T Y)$$

$\beta_0 = -884.6670$
$\beta_1 = -0.8381$
$\beta_2 = 4.9066$
$\beta_3 = 1.3311$
$\beta_4 = 11.9313$

Putting values in equation.

$$\hat{y} = -884.6670 - 0.8381x_1 + 4.9066x_2 + 1.3311x_3 + 11.9313x_4$$

② Variable Screening.

for β_1 :

$H_0 : \beta_1 = 0$
$H_1 : \beta_1 \neq 0$

$$t_0 \Rightarrow \frac{-0.8381}{1.414} \Rightarrow \boxed{-0.593}$$

Critical values $\Rightarrow$ $v = N - k - 1 \Rightarrow 9$

$$t_{\alpha/2} \Rightarrow \boxed{2.262}$$

H_0 Accepted

X_1 regressor Insignificant.

for β_2 :

$H_0 : \beta_2 = 0$

$H_1 : \beta_2 \neq 0$

$t_0 \Rightarrow \frac{4.9066}{2.937} \Rightarrow \boxed{1.671}$

critical value $\Rightarrow \boxed{2.262}$

H_0 Accepted $\rightarrow$ X_2 regressor insignificant.

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for β_3 :

$H_0 : \beta_3 = 0$

$H_1 : \beta_3 \neq 0$

$t_0 \Rightarrow \frac{1.3311}{3.230} \Rightarrow \boxed{0.412}$

Critical value $\Rightarrow \boxed{2.262}$

H_0 Accepted $\rightarrow$ X_3 regressor insignificant.

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for β_4 :

$H_0 : \beta_4 = 0$

$H_1 : \beta_4 \neq 0$

$t_0 \Rightarrow \frac{11.9313}{5.601} \Rightarrow \boxed{2.130}$

critical value $\Rightarrow \boxed{2.262}$

H_0 Accepted $\rightarrow$ X_4 regressor insignificant.

Since all came as insignificant. We will count X_4 regressor as the most significant because it has the highest t_0 (2.130).

12.56

① Multiple Linear Regression Model

$\hat{y} = \beta_0 + \beta_1 x_2 + \beta_2 x_2 + \beta_3 x_3.$

$\beta_0 = 344.4549$

$\beta_1 = 0.0141$

$\beta_2 = -0.1270$

$\beta_3 = 0.0014$

Putting the values in equation.

$\hat{y} = 344.4549 + 0.0141 x_1 - 0.1270 x_2 + 0.0014 x_3$

② Variable Screening

for β_1 :

$H_0 : \beta_1 = 0$

$H_1 : \beta_2 \neq 0$

$t_0 \Rightarrow \dfrac{0.0141}{0.014} \Rightarrow \boxed{0.997}$

Critical value $\Rightarrow$ $V = N-k-1 \Rightarrow 31$

$t_{\alpha/2} \Rightarrow \boxed{2.042}$

Ho Accepted $\rightarrow$ X1 Regressor insignificant.

for B_2 :

$H_0 : B_2 = 0$

$H_1 : B_2 \neq 0$

$t_0 \Rightarrow \dfrac{-0.1270}{0.131} \Rightarrow \boxed{-0.970}$

Ho Accepted

X3 regressor insignificant

critical value $\Rightarrow \boxed{2.042}$

for β_3: $H_0: \beta_3 = 0$
$H_1: \beta_3 \neq 0$

$t_0 \Rightarrow \frac{0.0014}{0.001} \Rightarrow \boxed{1.387}$ Critical value $\Rightarrow \boxed{2.042}$

H_0 Accepted → X_3 regressor insignificant.

All regressors are Insignificant
Therefore we select X_3 as the most significant
because of the highest t_0 score $\boxed{1.387}$.

12.57

(1) Multiple Linear Regression Model.

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6$$

$\beta_0 = 3.1368$
$\beta_1 = 0.6444$
$\beta_2 = -0.0104$
$\beta_3 = 0.5046$
$\beta_4 = -0.1197$
$\beta_5 = -2.4618$
$\beta_6 = 1.5044$ Now putting values in equation

$$\hat{y} = 3.1368 + 0.6444x_1 - 0.0104x_2 + 0.5046x_3 - 0.1197x_4 - 2.4618x_5 + 1.5044x_6$$

(2) Variable Screening

for β_1:

$H_0: \beta_1 = 0$

$H_1: \beta_1 \neq 0$

$t_0 \Rightarrow \frac{0.6444}{0.589} \Rightarrow \boxed{1.094}$

critical values $\Rightarrow V = N-k-1 = 17$

$t_{\alpha/2} \Rightarrow \boxed{2.110}$

H_0 Accepted $\longrightarrow$ X_1 Regressor insignificant.

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for β_2:

$H_0: \beta_2 = 0$

$H_1: \beta_2 \neq 0$

$t_0 \Rightarrow \frac{-0.0104}{0.268} = \boxed{-0.039}$

Critical values $\Rightarrow \boxed{2.110}$

H_0 Accepted $\rightarrow$ X_2 regressor insignificant.

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for β_3:

$H_0: \beta_3 = 0$

$H_1: \beta \neq 0$

$t_0 \Rightarrow \frac{0.5046}{0.142} \Rightarrow \boxed{3.545}$

Critical values $\Rightarrow \boxed{2.110}$

H_0 Rejected $\rightarrow$ X_3 Regressor SIGNIFICANT

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for β_4:

$H_0: \beta_4 = 0$

$H_1: \beta_4 \neq 0$

$t_0 \Rightarrow \frac{-0.1197}{0.056} \Rightarrow \boxed{-2.128}$

critical values $\Rightarrow \boxed{2.110}$

H_0 Rejected $\rightarrow$ X_4 Regressor SIGNIFICANT

<u>for $\beta 5$</u> :

$H_0 : \beta_5 = 0$

$H_1 : \beta_5 \neq 0$

$t_0 \Rightarrow \frac{-2.4618}{2.598} \Rightarrow \boxed{-0.948}$

critical value $\Rightarrow \boxed{2.110}$

Ho Accepted → X_5 Regressor insignificant

<u>for $\beta 6$</u> :

$H_0 : \beta_6 = 0$

$H_1 : \beta_6 \neq 0$

$t_0 \Rightarrow \frac{1.5044}{1.519} = \boxed{0.990}$

critical value $\Rightarrow \boxed{2.110}$

Ho Accepted → X_6 regressor insignificant

Therefore we can conclude that X_3 is the most significant because of the highest t_0 (3.545)