Differentiation

Imagine a bug that moves with constant speed on a circular path of radius r around the origin. The angle of the bug's position vector with the $+x$ axis can be written as

$$\theta = \omega t + a.$$

Assume $a = 0$, so that the bug is on the $+x$ axis at time 0. Then the position vector of the bug is

$$X(t) = (r\cos(\omega t), r\sin(\omega t)).$$

Now imagine the bug lives in $\mathbb{R}^3$ with

$$X(t) = (\cos(t), \sin(t), t).$$

This lifts the circular path into a helix.

In general, a **parametrized curve** $X : I \to \mathbb{R}^n$ is a vector-valued function that maps points from an interval I into n-space. In the examples above, I is the entire real line $\mathbb{R}$ (which we consider to be an interval). We can write $X(t)$ as its individual coordinate functions

$$X(t) = (x_1(t), \ldots, x_n(t)).$$

Just as with ordinary real-valued function, we can take derivatives by looking at the limit

$$\lim_{h\to 0} \frac{X(t+h) - X(t)}{h}.$$

Writing out components, this is simply

$$\lim_{h\to 0} \frac{(x_1(t+h) - x_1(t), \ldots, x_n(t+h) - x_n(t))}{h}.$$

If the individual components are all differentiable, we obtain a new vector-valued function

$$X'(t) = ()$$