HW 4

Exercise 1. Find the equation of the tangent plane and normal line to each of the following surfaces at the specific point.

- $xy + yz + zx - 1 = 0$ at $(1, 1, 0)$.
- $\sin(xy) + \sin(yz) + \sin(xz)$ at $(1, \pi/2, 0)$.
- $x = e^{2y-z}$ at $(1, 1, 2)$.

Exercise 2. (a) A differentiable curve $C(t)$ lies on the surface

$$x^2 + 4y^2 + 9z^2 = 14,$$

and is parametrized so that $C(0) = (1, 1, 1)$. Let

$$f(x, y, z) = x^2 + 4y^2 + 9z^2,$$

and let $h(t) = f(C(t))$. Find $h'(0)$.

(b) Let $g(x, y, z) = x^2 + y^2 + z^2$ and let $k(t) = g(C(t))$. Suppose also that $C'(0) = (4, -1, 0)$. Find $k'(0)$.

Exercise 3. Let $f(x, y, z) = z - e^x \sin(y)$ and $P = (\ln(3), 3\pi/2, -3)$. Find:

- the directional derivative of f at P in the direction of $(1, 2, 2,)$.
- the maximum and minimum values for the directional derivative of f at P (i.e. considering all possible directions).

Exercise 4. Find the critical points of the function

$$x^2 + 4xy - y^2 - 8x - 6y.$$

Exercise 5. Find the maxima and minima of the function $xy - (1 - x^2 - y^2)^{1/2}$ in the region $x^2 + y^2 \leq 1$. (Naturally, you'll first find the critical points in the interior of the disk. For the boundary (which is a circle), you can either parametrize the circle as $X(t)$ and then min/max $f(X(t))$, *or* one could also use Lagrange multipliers.)

Exercise 6. Find the points on the surface $z^2 - xy = 1$ closest to the origin. (Suggestion: you'll be optimizing a distance; it will probably be easier to optimize the *square* of the distance. Nothing is lost by doing this, as a point being a minimum for the distance squared is the same as being a minimum for the distance itself.)