

Inequalities, absolute value, distance, etc.

Let $x = (x_1, \dots, x_n)$ and $y = (y_1, \dots, y_n)$ be points. The distance between them is

$$d(x, y) = \|x - y\| = \sqrt{(x_1 - y_1)^2 + \dots + (x_n - y_n)^2}.$$

In the case where $n = 1$ (i.e. when x and y are just numbers), we see the distance is the absolute of the difference $x - y$.

Recall that $\sqrt{x^2} = |x|$. If a and b are *nonnegative* and $a \leq b$, then $\sqrt{a} \leq \sqrt{b}$. Conversely, if $\sqrt{a} \leq \sqrt{b}$, then $a \leq b$. In particular, we have $|x| = \sqrt{x^2} \leq \sqrt{x^2 + y^2}$. In the same way, $|y| \leq \sqrt{x^2 + y^2}$, and of course, this works for more than just two variables.

Limits

Let $f : X \rightarrow Y$ be a function taking points in X to points in Y . In Calc I, X and Y were usually both \mathbb{R} . Now we allow X to be \mathbb{R}^2 or \mathbb{R}^3 .

Recall the idea of a function having a limit at a point $x_0 \in X$. Intuitively, this means there is some value L such that when x gets closer to x_0 , $f(x)$ gets closer to this value L . But what does this really mean? Consider the following game.

Definition 0.1. We say that the **limit** of $f(x, y)$ as (x, y) approaches (x_0, y_0) equals L if, for every positive number ϵ , one can find a corresponding positive number δ_ϵ such that $d((x_0, y_0), (x, y)) < \delta_\epsilon$ guarantees $d(f(x, y), L) < \epsilon$.

In other words, for a limit to exist, the game to play is this: you have function $f(x)$, where