HW 1

Exercise 1. Let $A = (1, 2)$, $B = (3, 1)$. Draw the points $A+B$, $A+2B$, $A+3B$, $A-B$, $A-2B$, and $A-3B$ on a sheet of graph paper (or a reasonably drawn set of axes).

Exercise 2. Which of the following pairs of vectors are perpendicular?

- $(1, -1, 1), (2, 1, 5)$
- $(1, -1, 1), (2, 3, 1)$
- $(-5, 2, 7), (3, -1, 2)$
- $(\pi, 2, 1), (2, -\pi, 0)$

Exercise 3. Suppose $A = (a_1, a_2, a_3)$ is perpendicular to every vector X. Show that A is the zero vector. (Hint: if this holds for every X, it holds in particular for E_1, E_2, and E_3)

Exercise 4. Determine the interior angles of the triangle whose vertices are $(2, -1, 1)$, $(1, -3, -5)$, and $(3, -4, -4)$. (Hint: label the points as P, Q, and R. Then, for instance, one of the angles can be found by computing the angle between the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. Then you can do this for the other angles.)

Exercise 5. Let $A_1, \ldots, A_r$ be *nonzero* vectors which are mutually perpendicular (i.e. $A_i \cdot A_j = 0$ whenever $i \neq j$). Suppose $c_1, \ldots, c_r$ are numbers such that

$$c_1 A_1 + \cdots + c_r A_r = 0.$$

Show that we must have $c_i = 0$ for each $i = 1, \ldots, r$.

Exercise 6. Let $P = (1, 3, -1)$ and $Q = (-4, 5, 2)$. Determine the coordinates of the following points

- The midpoint of the line segment between P and Q
- The point on this line segment that is two thirds of the way from P to Q.

Exercise 7. Find the equation of the plane passing through the points $(2, 1, 1)$, $(3, -1, 1)$, and $(4, 1, -1)$. (Hint: to obtain a normal vector to this plane, label the points P, Q, and R and form the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. What is true of $\overrightarrow{PQ} \times \overrightarrow{PR}$?)

Exercise 8. Find a parametric representation for the line of intersection of the planes

$$\begin{aligned} 2x + y + 5z &= 2 \\ 3x - 2y + z &= 3. \end{aligned}$$

(Hint: notice that when two planes intersect, the line of intersection is perpendicular to the normals of both planes.)

Exercise 9. Compute the area of the parallelogram spanned by the vectors $(3, -2, 4)$ and $(5, 1, 1)$.