Differentiation

Imagine a bug that moves with constant speed on a circular path of radius r around the origin. The angle of the bug's position vector with the +x axis can be written as

$$\theta = \omega t + a.$$

Assume a = 0, so that the bug is on the +x axis at time 0. Then the position vector of the bug is

$$X(t) = (r\cos(\omega t), r\sin(\omega t)).$$

Now imagine the bug lives in \mathbb{R}^3 with

$$X(t) = (\cos(t), \sin(t), t).$$

This lifts the circular path into a helix.