

Bubble Entropy: An Entropy Almost Free of Parameters

George Manis , Md Aktaruzzaman, and Roberto Sassi

Biomedical Signal Processing Final 25/26

Zaina Abushaban

TABLE OF CONTENTS

A decorative graphic on the left side of the page consisting of three overlapping circles. The largest circle is light blue, and the two smaller circles are a gradient of blue and orange. They are arranged in a cluster, with one circle partially overlapping the other two.

01

OBJECTIVES

What is entropy and why is it important?

02

METHODOLOGY

What is the state of the art? And why is this new method valuable?

A decorative graphic on the left side of the page consisting of three overlapping circles. The largest circle is light blue, and the two smaller circles are a gradient of blue and orange. They are arranged in a cluster, with one circle partially overlapping the other two.

03

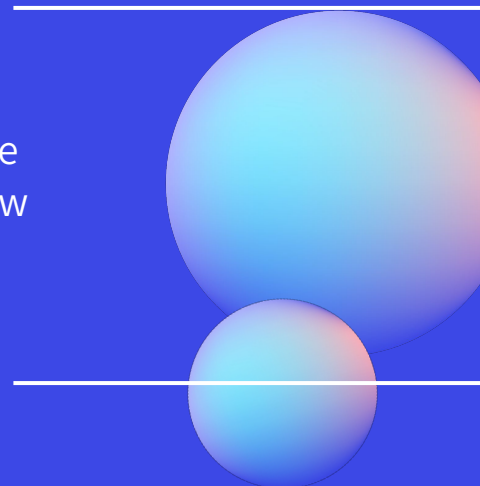
RESULTS ANALYSIS

Results of our experiments

04

CONCLUSIONS

What can we conclude?
What are some interesting problems to consider in the future?

A decorative graphic on the right side of the page consisting of three overlapping circles. The largest circle is light blue, and the two smaller circles are a gradient of blue and orange. They are arranged in a cluster, with one circle partially overlapping the other two.



INTRODUCTION

What is entropy?

Entropy

Thermodynamics



The unavailability of a system's thermal energy for conversion into mechanical work



Information Theory

a measure of the unpredictability



Time Series Analysis

a measure of complexity

Problem with current entropy measures



Strong Dependence
on parameters

Small parameter changes \rightarrow large entropy changes

Results become subjective and dataset-dependent

Sample Entropy

$$\text{SampEn}(m, r) = -\ln \left(\frac{A}{B} \right)$$

where:

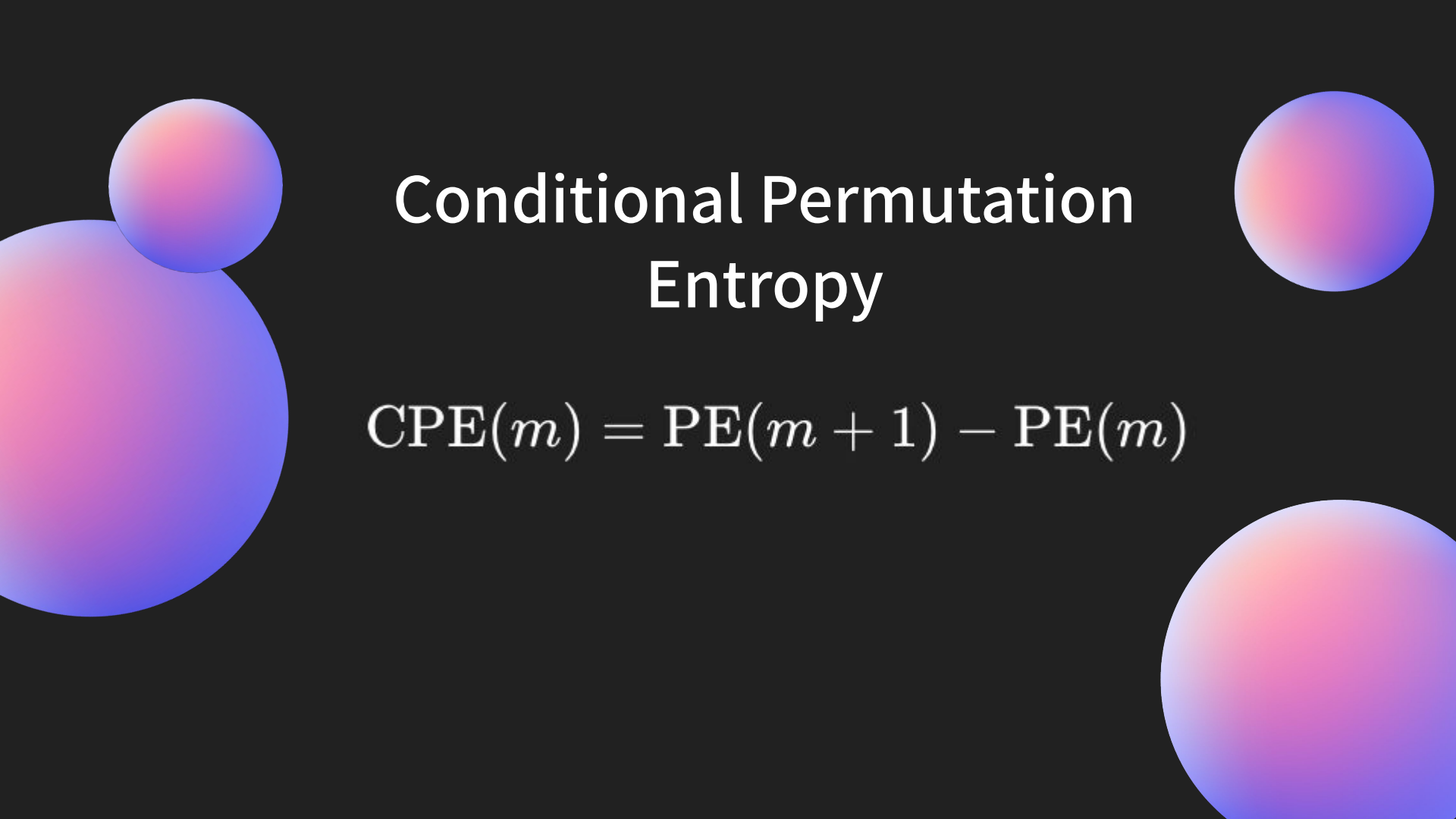
- B : number of pairs of vectors X_i^m, X_j^m with distance $\leq r$
- A : number of pairs of vectors X_i^{m+1}, X_j^{m+1} with distance $\leq r$
- Self-matches are **excluded**

Permutation Entropy

$$\text{PE}(m) = - \sum_{\pi} p(\pi) \log p(\pi)$$

where:

- $p(\pi)$ is the relative frequency of permutation pattern π

The background features four large, overlapping circles with a pink-to-blue gradient. One circle is on the left, one on the right, and two are positioned higher up, one on the left and one on the right.

Conditional Permutation Entropy

$$\text{CPE}(m) = \text{PE}(m + 1) - \text{PE}(m)$$

Renyi Permutation Entropy

$$\text{RpEn}_{\alpha}(m) = \frac{\log \left(\sum_{j=1}^{m!} p_j^{\alpha} \right)}{(1 - \alpha) \log(m)}$$

Bubble Entropy

$$x = \{x_1, x_2, \dots, x_N\}$$

Bubble Entropy

$$\mathbf{X}_i^{(m)} = (x_i, x_{i+1}, \dots, x_{i+m-1}), \quad i = 1, \dots, N - m + 1$$

Bubble Entropy

$$n_i^{(m)} \in \left[0, \frac{m(m-1)}{2} \right]$$

Bubble Entropy

$$p_k^{(m)} = \frac{\#\{i : n_i^{(m)} = k\}}{N - m + 1}$$

Bubble Entropy

$$H_{\text{swaps}}^{(m)} = -\log \left(\sum_k \left(p_k^{(m)} \right)^2 \right)$$

Bubble Entropy

$$\text{bEn}(m) = \frac{H_{\text{swaps}}^{(m+1)} - H_{\text{swaps}}^{(m)}}{\log(m+1)}$$


$$x = [4, 1, 3, 2, 0, 5, 6] \quad m = 3$$

1. $(4, 1, 3)$ swaps = 2
2. $(1, 3, 2)$ swaps = 1
3. $(3, 2, 0)$ swaps = 3
4. $(2, 0, 5)$ swaps = 1
5. $(0, 5, 6)$ swaps = 0

$$n^{(3)} = [2, 1, 3, 1, 0]$$

1. $(4, 1, 3, 2)$ swaps = 4
2. $(1, 3, 2, 0)$ swaps = 4
3. $(3, 2, 0, 5)$ swaps = 3
4. $(2, 0, 5, 6)$ swaps = 1

$$n^{(4)} = [4, 4, 3, 1]$$



$$k = 0: 1/5$$

$$k = 1: 2/5$$

$$k = 2: 1/5$$


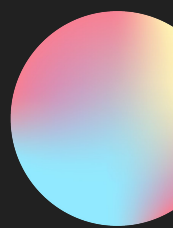
$$k = 3: 1/5$$

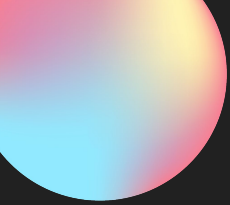
$$\sum_k (p_k^{(3)})^2 = (1/5)^2 + (2/5)^2 + (1/5)^2 + (1/5)^2 = \frac{7}{25}$$

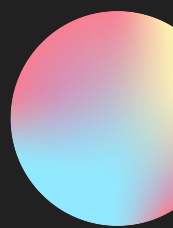
$$k = 1: 1/4$$


$$k = 3: 1/4$$

$$k = 4: 2/4$$

$$\sum_k (p_k^{(4)})^2 = (1/4)^2 + (1/4)^2 + (2/4)^2 = \frac{6}{16} = \frac{3}{8}$$



$$H_{\text{swaps}}^{(m)} = -\ln \left(\sum_k (p_k^{(m)})^2 \right)$$

$$H_{\text{swaps}}^{(3)} = -\ln(7/25) = \ln(25/7) \approx 1.2730$$


$$H_{\text{swaps}}^{(4)} = -\ln(3/8) = \ln(8/3) \approx 0.9808$$




numerator

$$0.9808 - 1.2730 = -0.2922$$

denominator

$$1 + \frac{m(m-1)}{2} = 1 + \frac{3 \cdot 2}{2} = 4$$

$$1 + \frac{(m+1)m}{2} = 1 + \frac{4 \cdot 3}{2} = 7$$

$$\ln(7/4) \approx 0.5596$$


Four decorative circles with a pink-to-blue gradient are positioned in the corners of the slide: top-left, top-right, bottom-left, and bottom-right.
$$bEn(3) \approx \frac{-0.2922}{0.5596} = -0.522$$

Why bubble sort?

Deterministic and interpretable



Measure

Number of swaps needed to sort, not the actual permutations



**Parameter Fine
Tuning**

Only one parameter, with not a lot of importance



**Unlike other entropy measures: Bubble
Entropy does NOT measure Entropy rate!**

Datasets used

1.

Synthetic Signals

Autoregressive (AR) models

White noise input \rightarrow maximum entropy

Used for stability and convergence analysis

2.

Real Signals (PhysioNet)

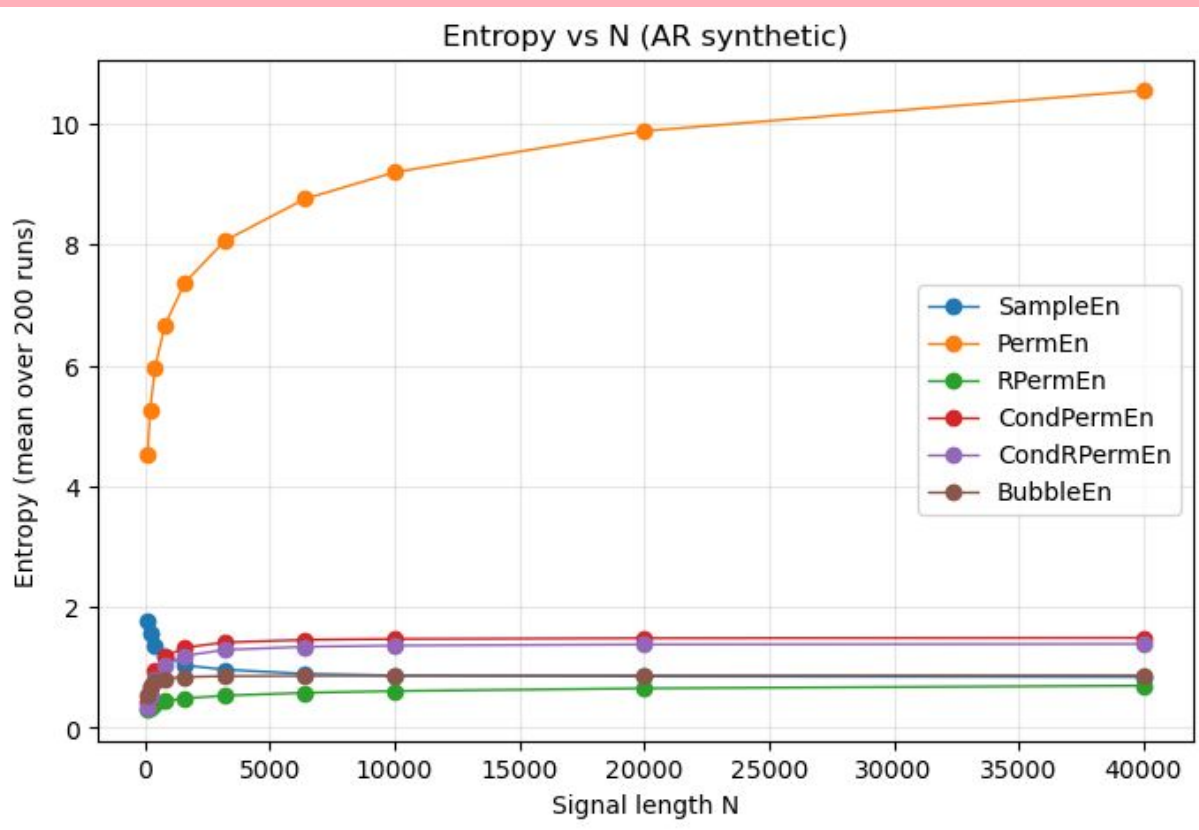
NSR: Normal Sinus Rhythm

CHF: Congestive Heart Failure patients

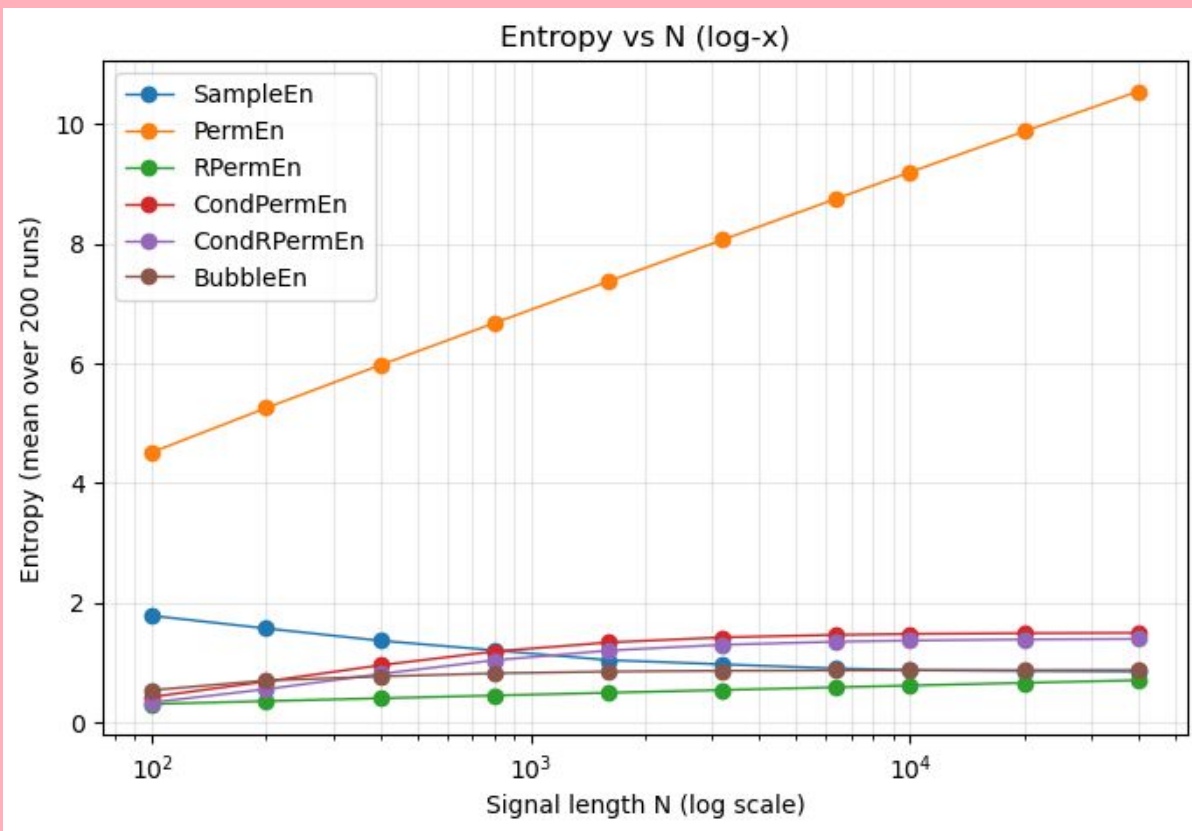
Sampling rate of 128Hz



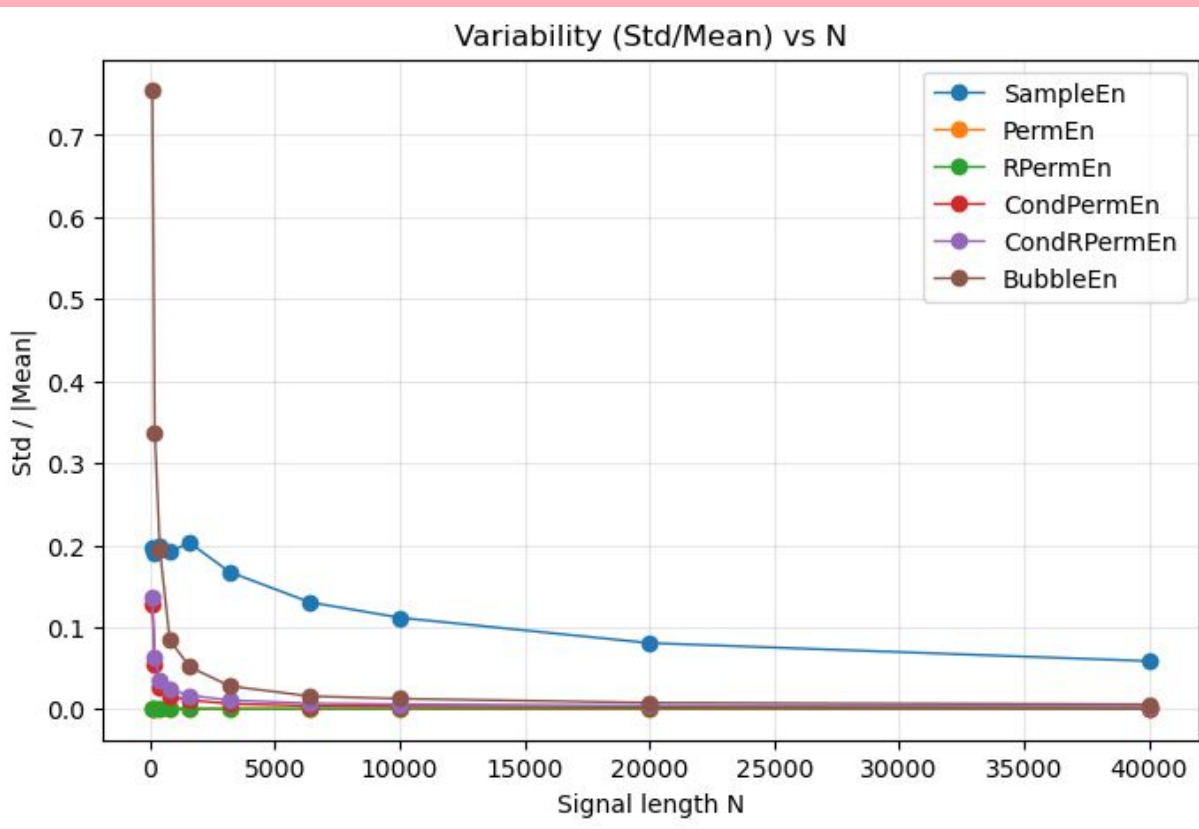
Stability Results



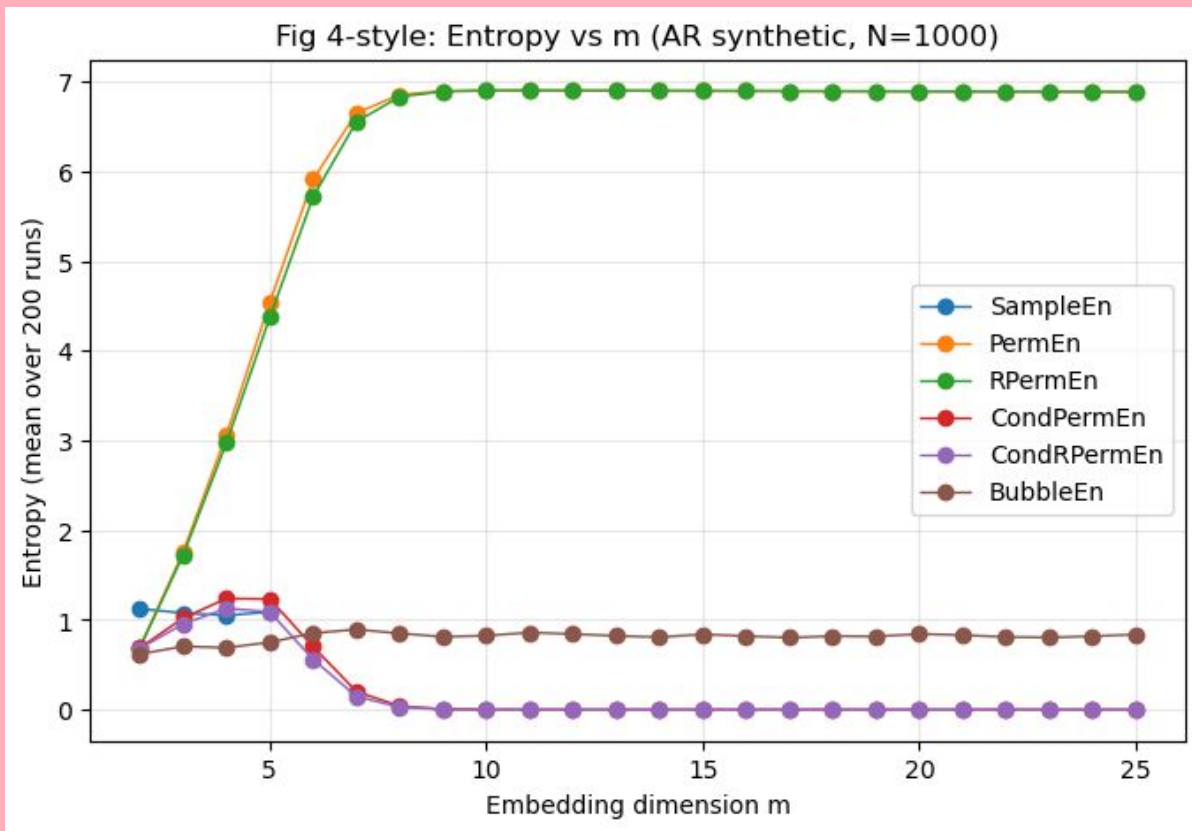
Stability Results



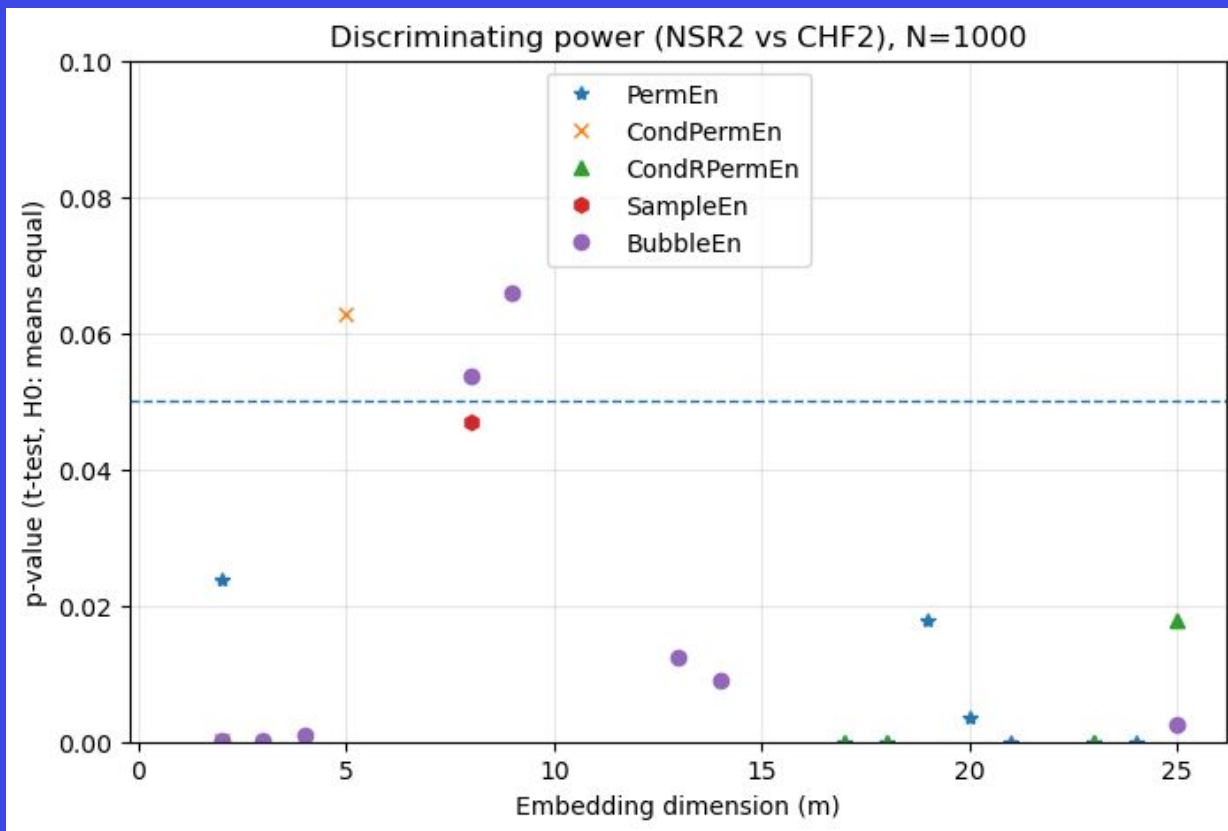
Stability Results



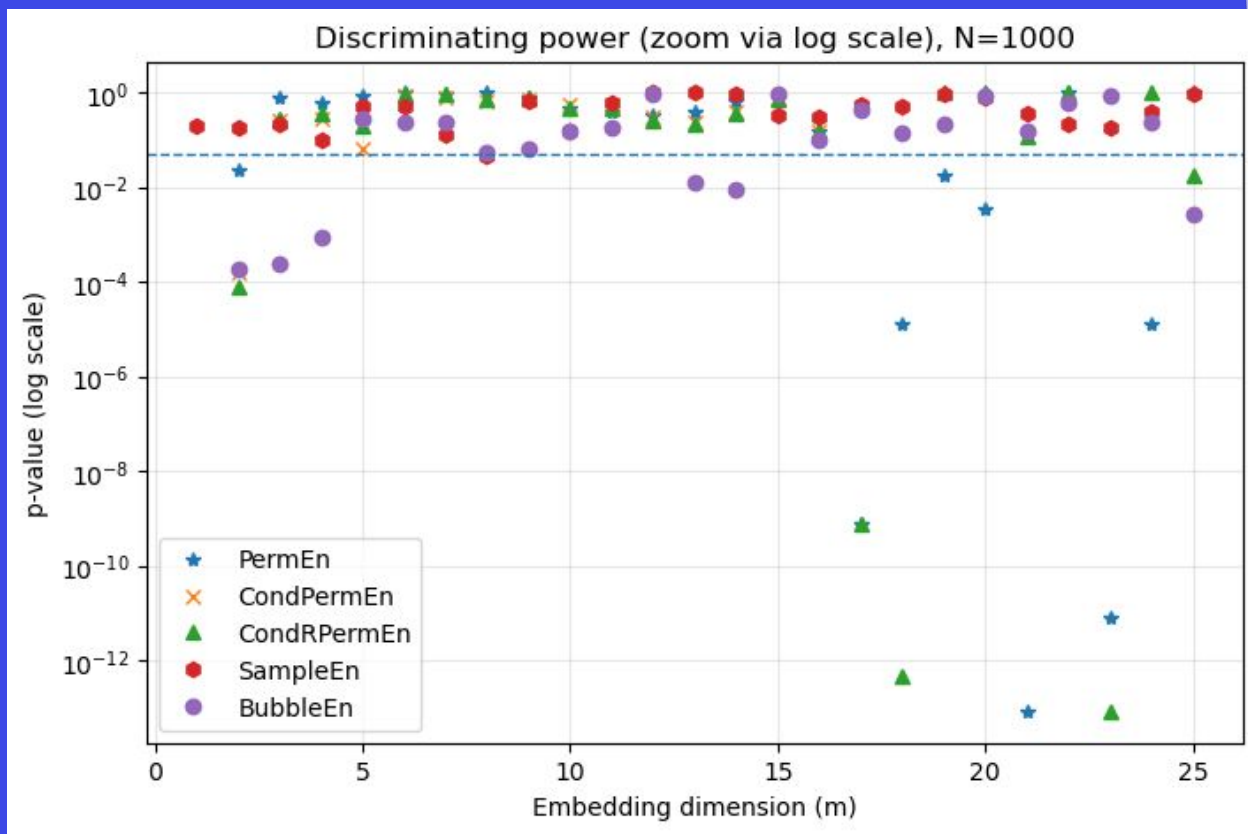
Stability Results



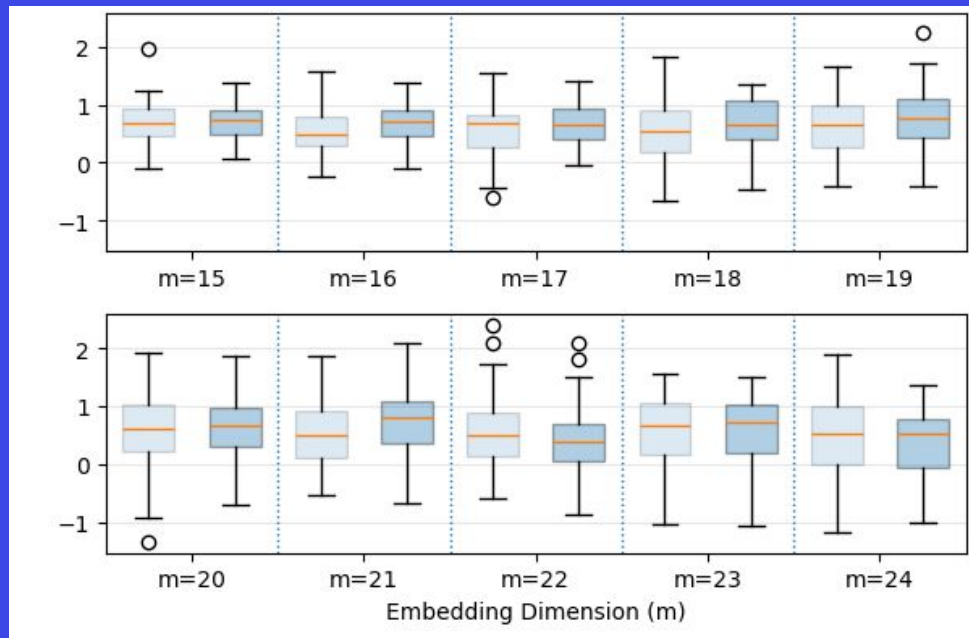
Discriminating Power (NSR vs CHF)



Discriminating Power (NSR vs CHF) Box plot



Discriminating Power (NSR vs CHF) Box plot



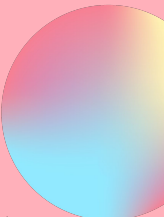


Naive Approach

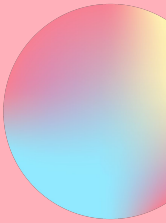
Bubble sort:
 $O(m^2)$ per vector

Optimized Complexity

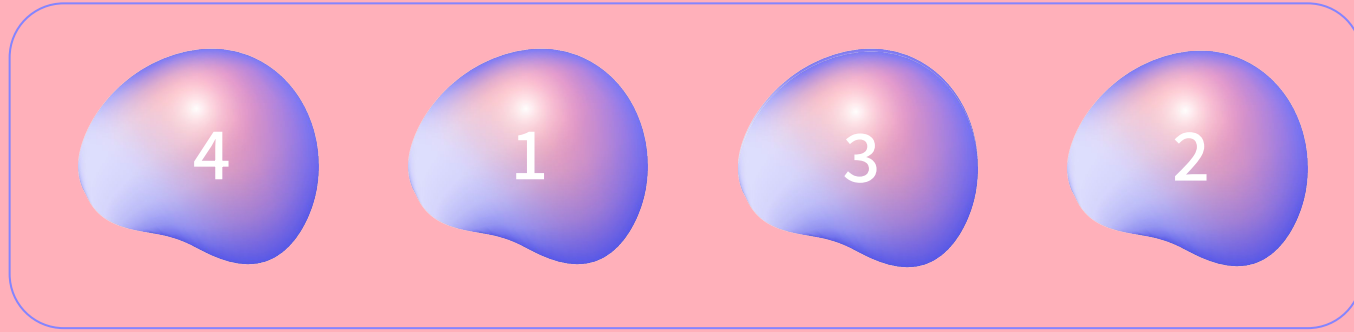
Exploit overlap between
consecutive vectors
Incremental insertion $\rightarrow O(m)$
 $O(mN)$



Bubble Sort: optimized

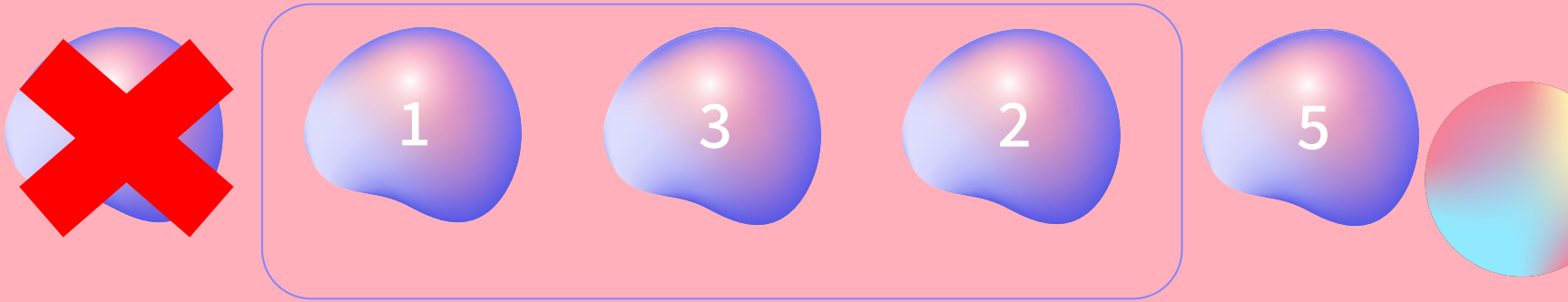


Bubble Sort: optimized



4 inversions

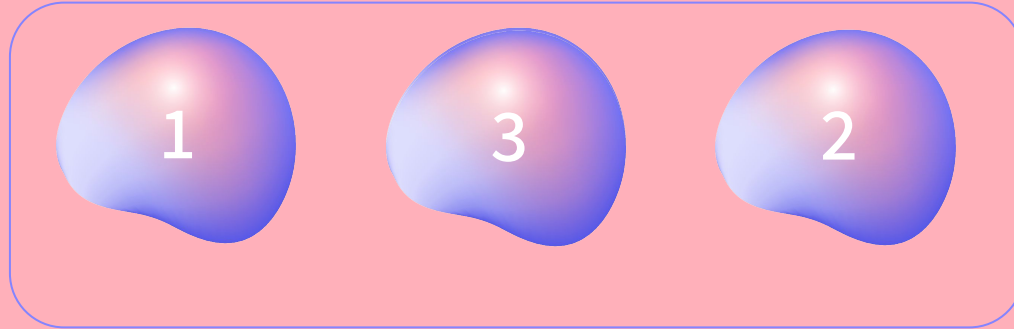
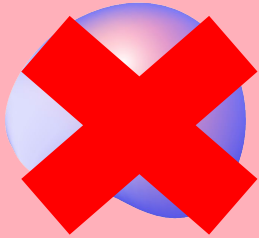
Bubble Sort: optimized



**Step 1: Remove 4, and decrease number of inversions by
number of inversions involving the 4**

$O(m)$

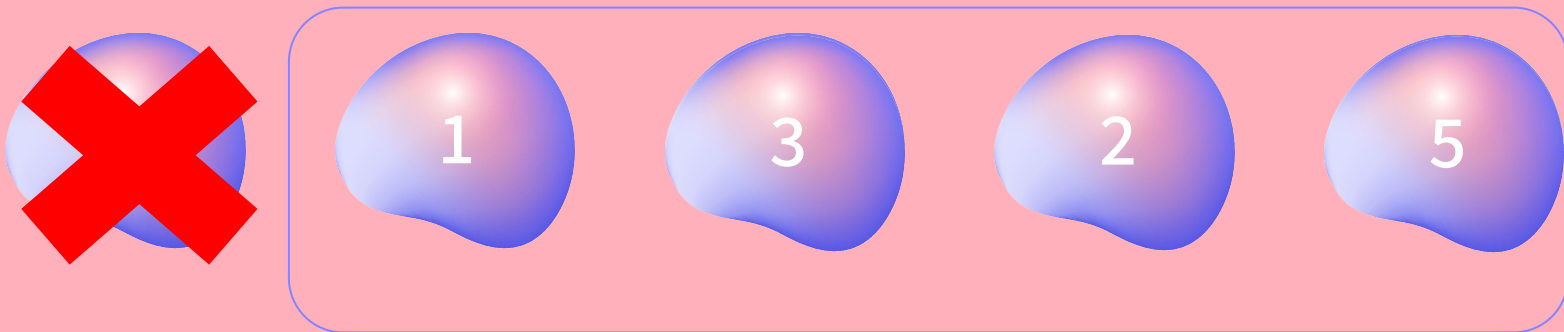
Bubble Sort: optimized



$$4 - 3 = 1$$

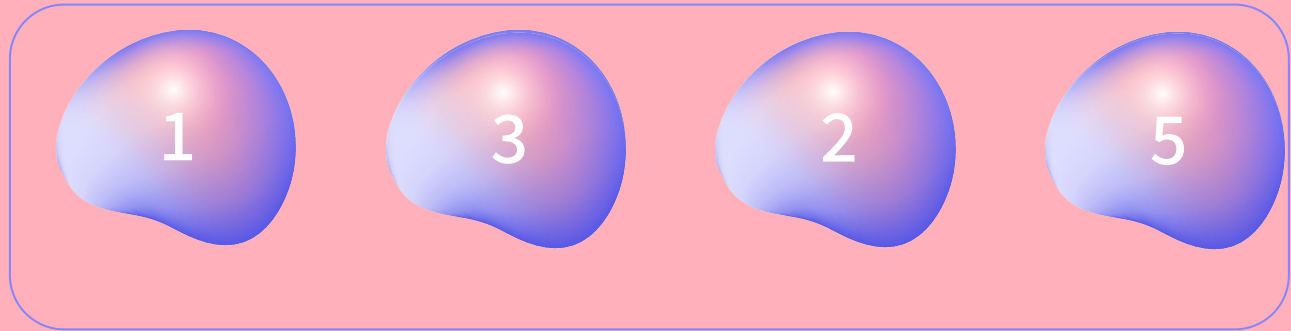
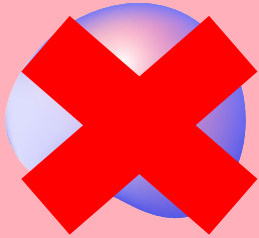
1 inversion left in our list of m-1

Bubble Sort: optimized



Step 2: Add inversions introduced by the new value
 $O(m)$

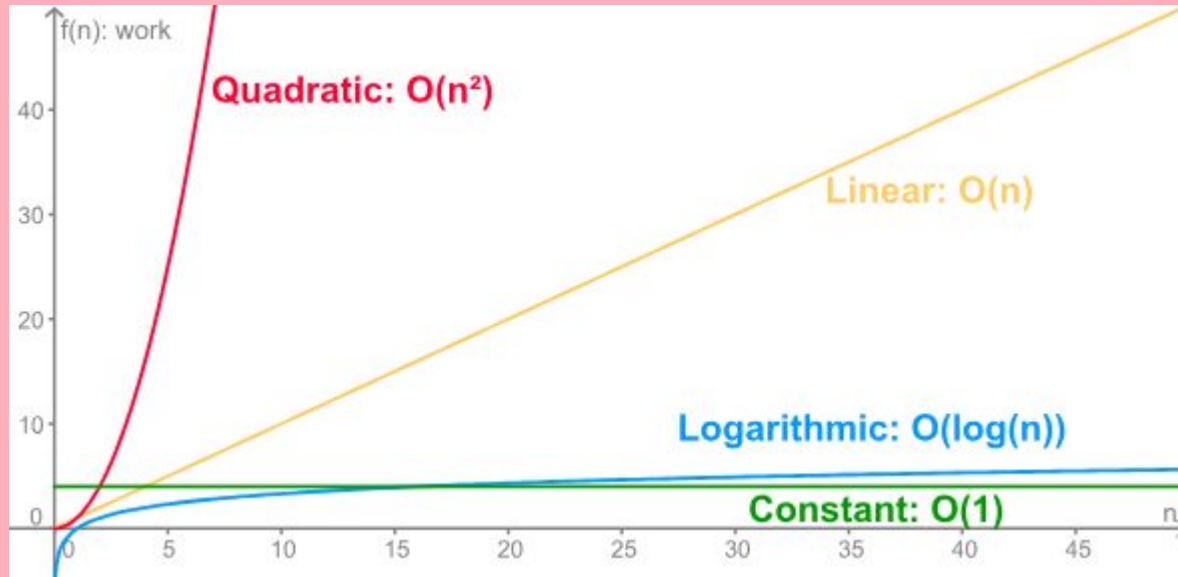
Bubble Sort: optimized



$$1 + 0 \Rightarrow 1$$

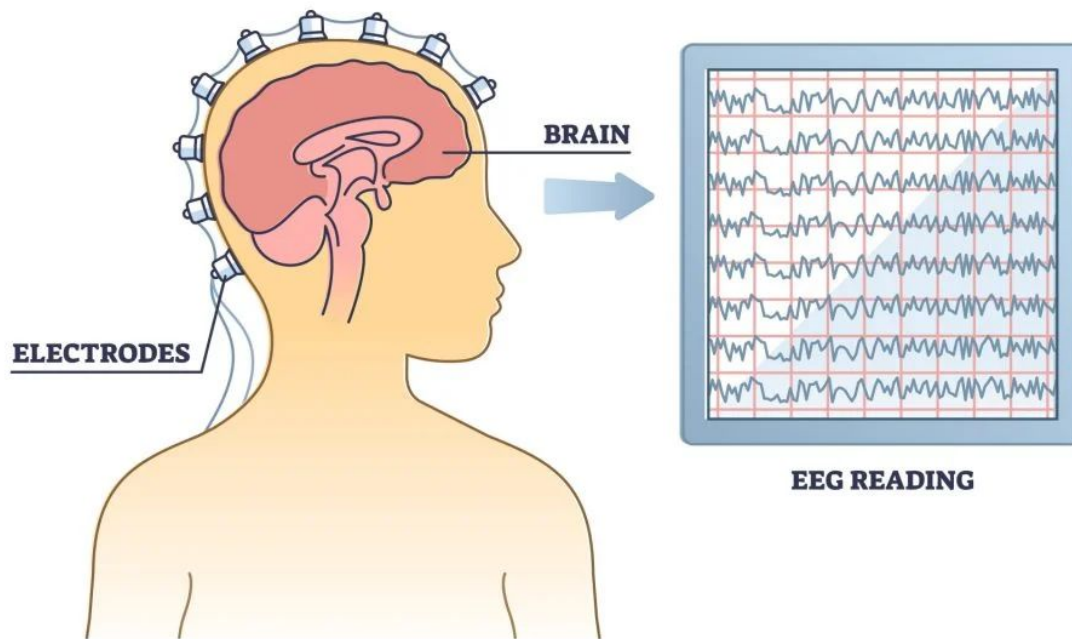
Bubble Sort: optimized

Becomes $O(Nm)$ instead of $O(Nm^2)$

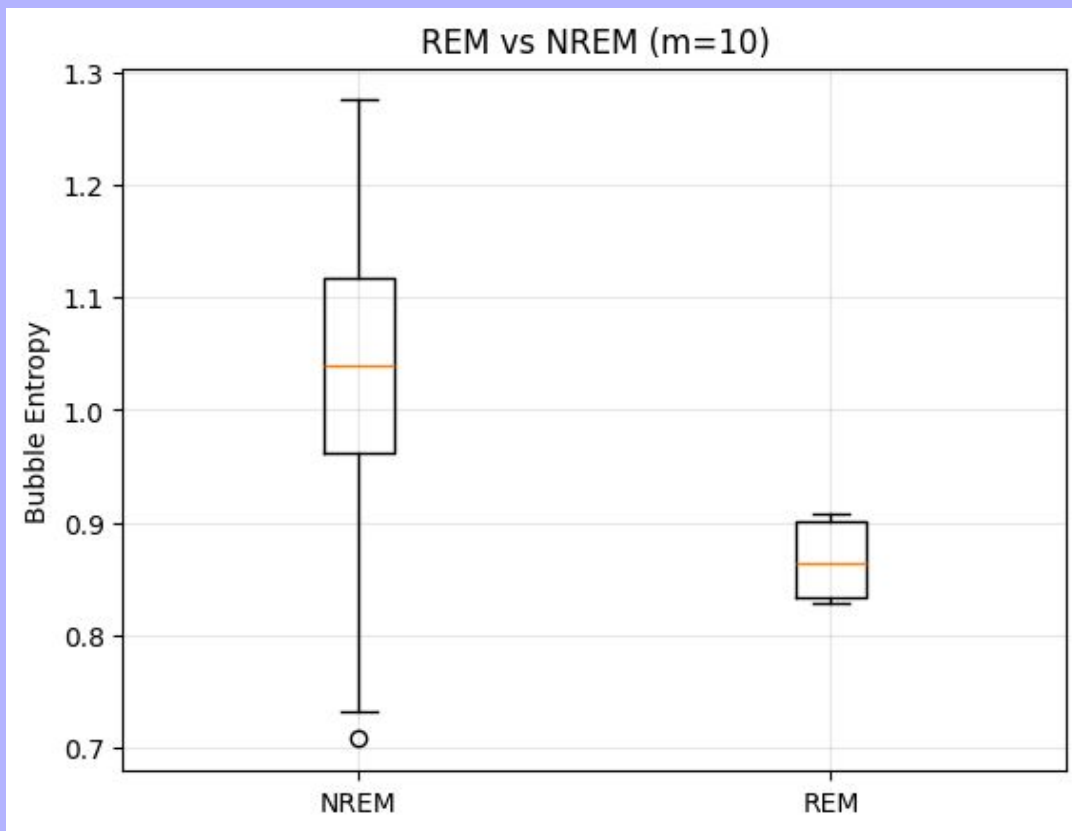


Trying discrimination on another dataset

ELECTROENCEPHALOGRAPHY



Trying discrimination on another dataset



Trying discrimination on another dataset

Best threshold : 1.275

Best accuracy (single subject): 0.95

Trying discrimination on another dataset

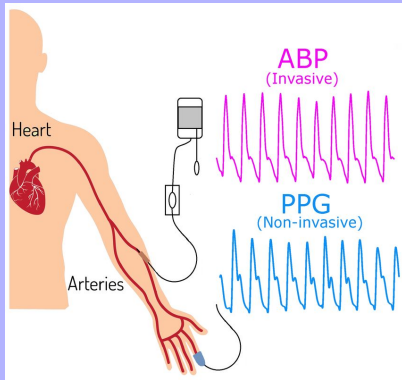
ECG and EEG are very different signal modalities

Bubble Entropy:

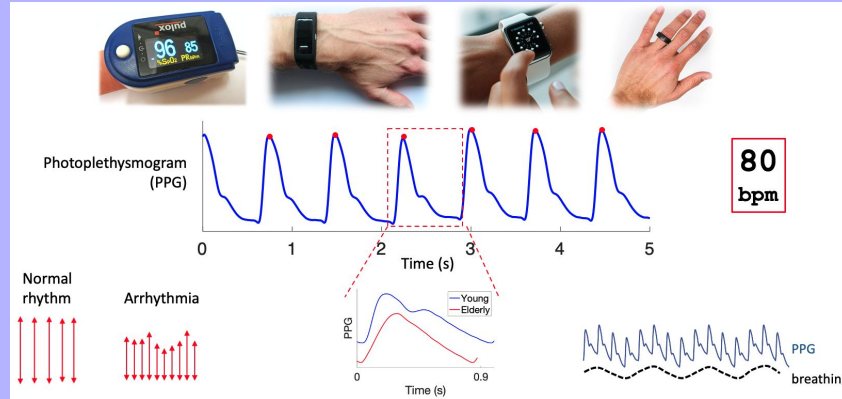
- does not rely on amplitude
- does not rely on scale parameter r
- captures ordinal structure

The method transfers without modification

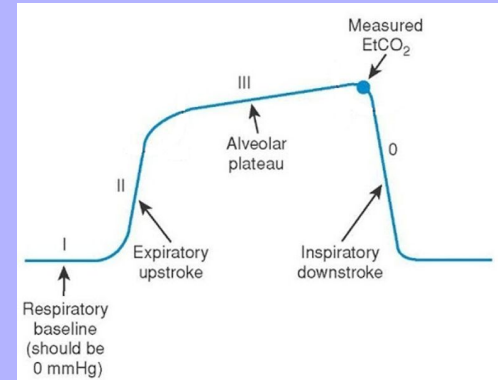
Other possible applications for Entropy based measures



Blood Pressure (ABP / PPG-derived)



Photoplethysmography (PPG)



Capnography (CO₂ waveform)

Key Advantages of Bubble Entropy

- No threshold parameter r
- Reduced dependence on m
- High stability
- Strong discriminating power
- Interpretable physical meaning (sorting effort)

Limitations & Open Issues

Time delay τ fixed to 1

Focused mainly on HRV

Needs validation on:

- EEG
- Other biomedical signals
- Multiscale settings



Conclusion

Bubble Entropy is
a robust, stable,
and nearly
parameter-free
entropy measure

