PROBLEM SOLVING & SEARCH STRATEGY Part 1

Dr. Emad Natsheh

Problem solving

- We want:
  - To automatically solve a problem
- We need:
  - A representation of the problem
  - Algorithms that use some strategy to solve the problem defined in that representation

Problem Description

- Components
  - ✓State space
    - ✓Initial state
    - ✓Goal state
  - ✓Actions (operators)
  - ✓Path cost

States

- A problem is defined **by its elements and their relations**
- A state is a representation of those elements in a given moment.
- Two special states are defined:
  - **Initial state** (starting point)
  - **Goal state**

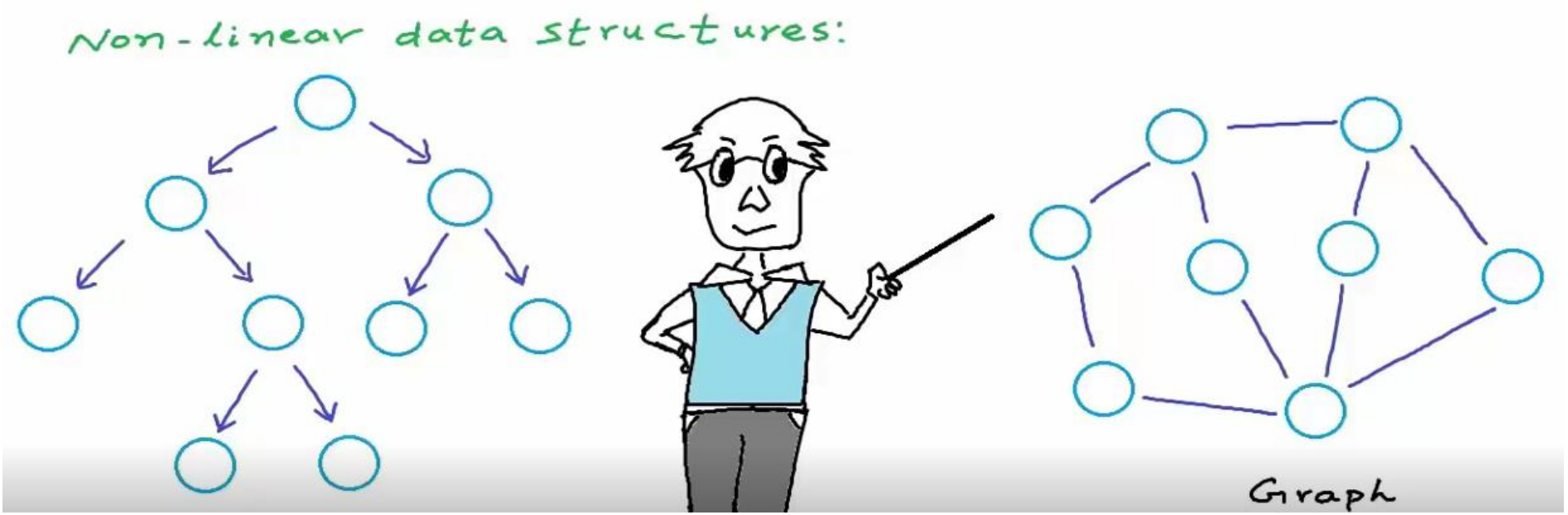
State Modification: Successor Function

- A successor function is needed to move between different states.
- A successor function is a description of possible actions a set of operators. It is a transformation function on a state representation, which move it into another state.
- The successor function defines a relation of accessibility among states.

State space

- The state space is the set of all states reachable from the initial state
- Its form a graph (or tree) in which the nodes are states and the arcs between nodes are actions.
- A path in the state space is a sequence of states connected by a sequence of actions.
- The solution of the problem is part of the map formed by the state space.

Tree vs Graph



if N nodes
then (N-1) edges
One edge for each
parent-child relationship

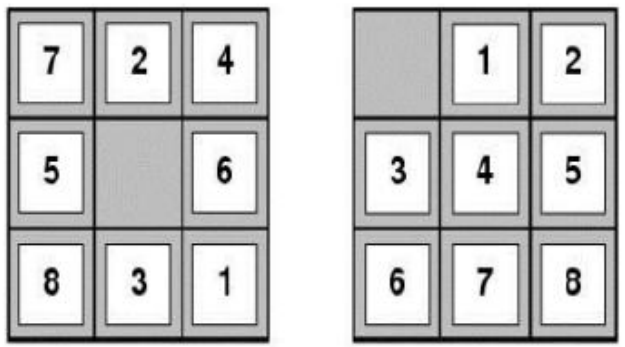
Problem Solution

- A solution in the state space is a path from the initial state to a goal state
- Path/solution cost: function that assigns a numeric cost to each path, the cost of applying the operators to the states
- Solution quality is measured by the path cost function, and an optimal solution has the lowest path cost among all solutions.

There is difference between Test Path and Solution Path

Example 8-puzzle

- **State space**: configuration of the eight tiles on the board
- **Initial state *as shown***
- **Goal state**: as shown
- **Operators or actions**: "blank moves"
  - Condition: the move is within the board
  - Transformation: blank moves *Left*, *Right*, *Up*, or *Down*
  - Performance measure: minimize total moves
- **Find solution**: Sequence of pieces moved: 3,1,6,3,1,…
  - optimal sequence of operators

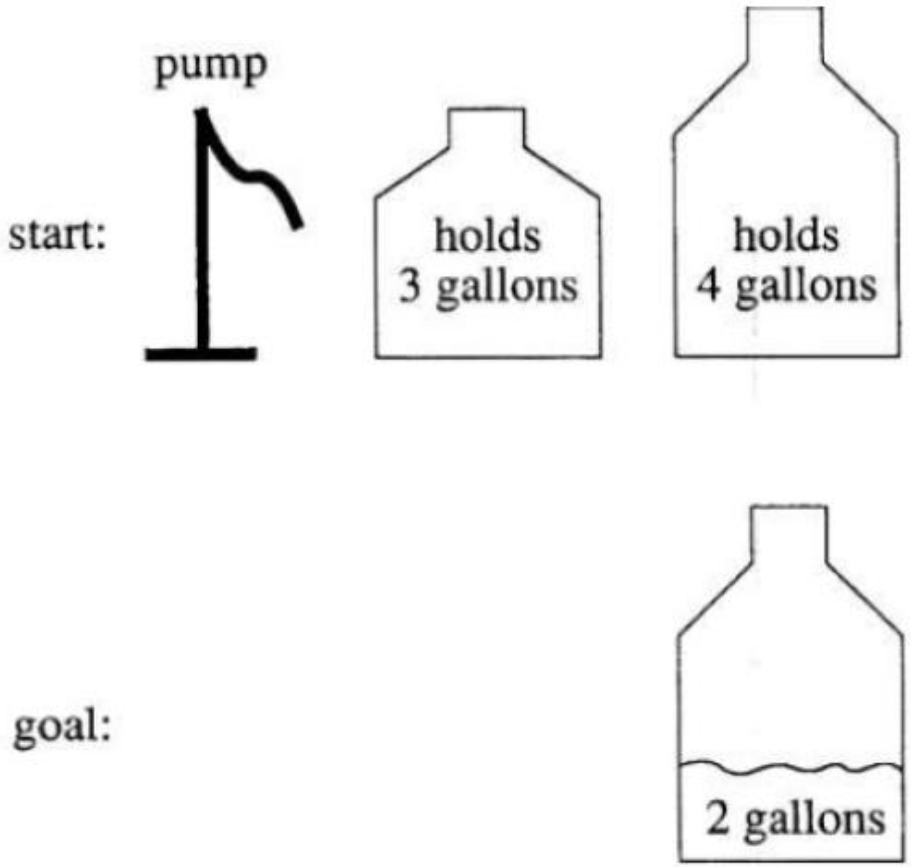


Example: Travelling

Problem Representation

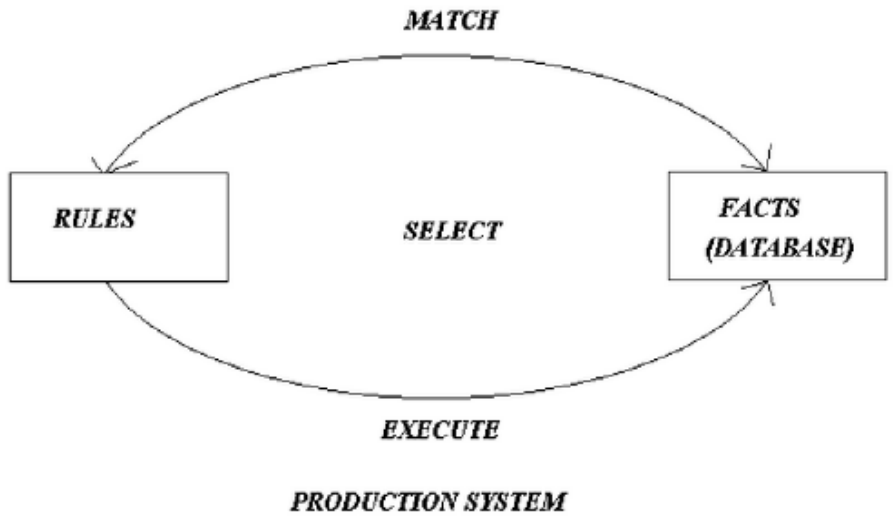
Example **Water Jug Problem**

- You have a 4-gallon and a 3-gallon water jug
- You have a faucet with an unlimited amount of water
- You need to get exactly 2 gallons in 4-gallon jug



Problem Description

- State representation**: (x, y)**
  - x: Contents of four gallon
  - y: Contents of three gallon
- Start state**: (0, 0)**
- Goal state **(2, n)**



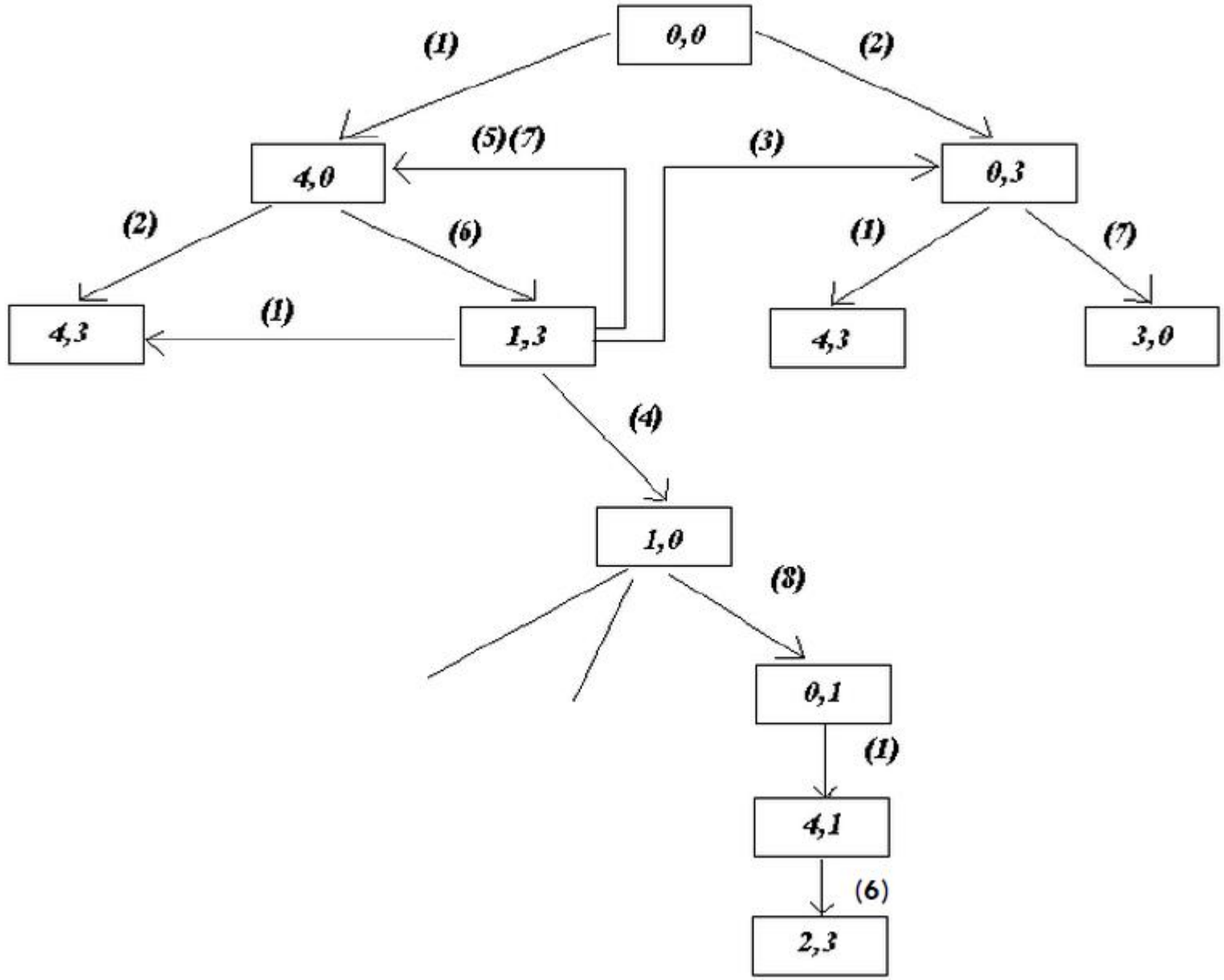
- Operators
  - Fill 3-gallon from faucet, fill 4-gallon from faucet
  - Fill 3-gallon from 4-gallon , fill 4-gallon from 3-gallon
  - Empty 3-gallon into 4-gallon, empty 4-gallon into 3-gallon
  - Dump 3-gallon down drain, dump 4-gallon down drain

Operations (Actions)

#	Actions
1	Fill X from Pump
2	Fill Y from Pump
3	Empty X into Ground
4	Empty Y into Ground
5	Get water from Y into X until X is full
6	Get water from X into Y until Y is full
7	Get all water from Y into X
8	Get all water from X into Y

Rules

#	Rules
1	
2	
3	
4	
5	
6	
7	
8	


0,0
(1)
(2)
4,0
(5)(7)
(3)
0,3
(2)
(6)
(1)
(7)
4,3
(1)
1,3
4,3
3,0
(4)
1,0
(8)
0,1
(1)
4,1
(6)
2,3

Another Solution to the Water Jug Problem

Gallons in the 4-Gallon Jug	Gallons in the 3-Gallon Jug	Rule Applied
0	0	2
0	3	9
3	0	2
3	3	7
4	2	5
0	2	9
2	0	

Algorithm for Problem Solving

1. **Initialize the search tree using the initial state of the problem**
2. **Choose a terminal node for expansion according to certain search strategy**
   - **If no terminal node is available for expansion return failure**
   - **If the chosen node contains a goal return the node**
3. **Expand the chosen node (according to the rules) and add the resulting node to the search tree**
4. **Go to step 2**

Missionaries & Cannibals Problem

Problem Description

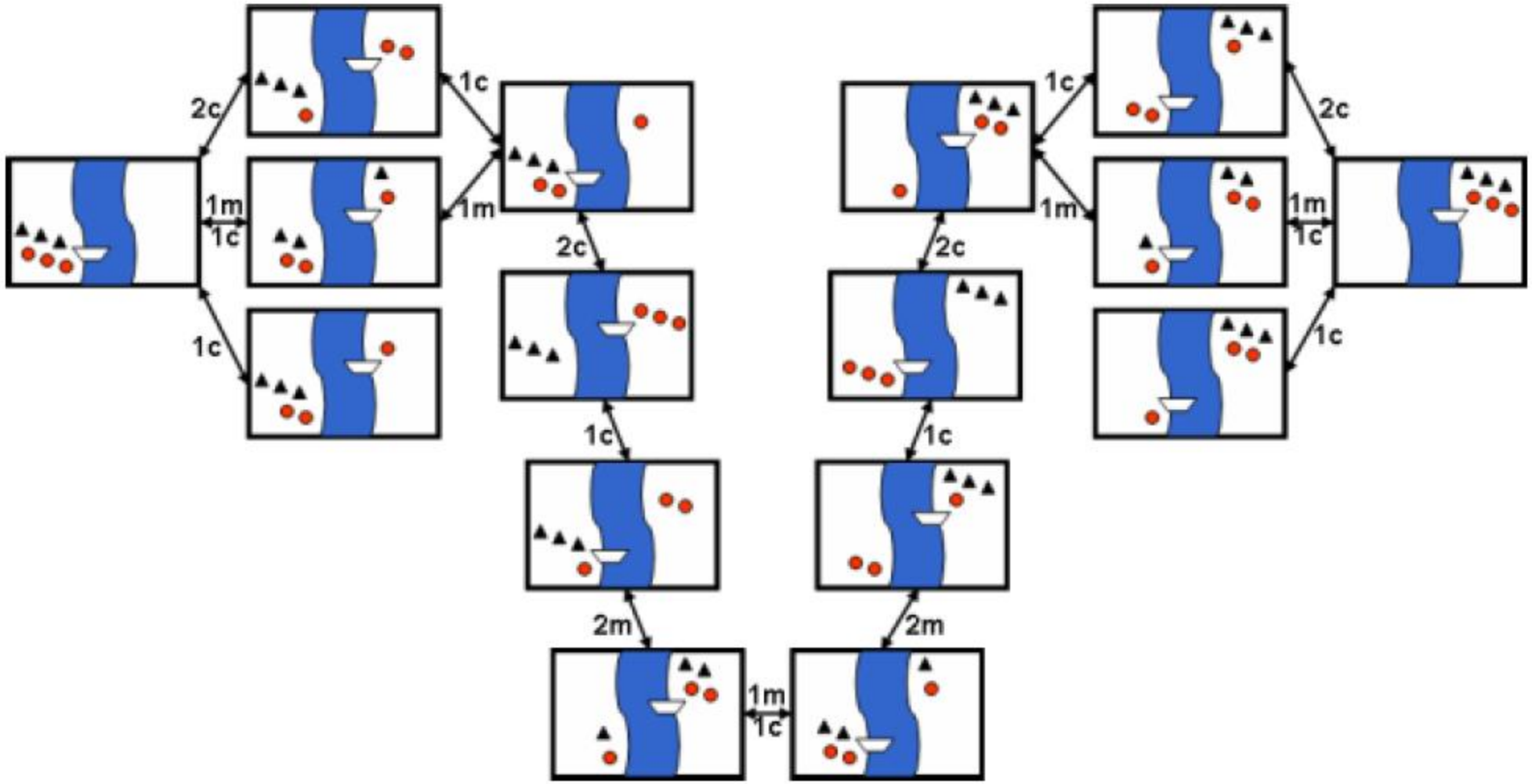
- **State(# of missionaries Left, # of cannibals Left, # of missionaries Right, # of cannibals Right, side_of_the_boat)**
- **Initial State => State (3, 3, 0, 0, 0)**
- **Final State => State (0, 0, 3, 3, 1).**

M1 C1 M2 C2 BS

- **Actions**
  - Carry (2, 0).
  - Carry (1, 0).
  - Carry (1, 1).
  - Carry (0, 1).
  - Carry (0, 2).
    Where Carry (M, C) means the boat will carry M missionaries and C cannibals on one trip.

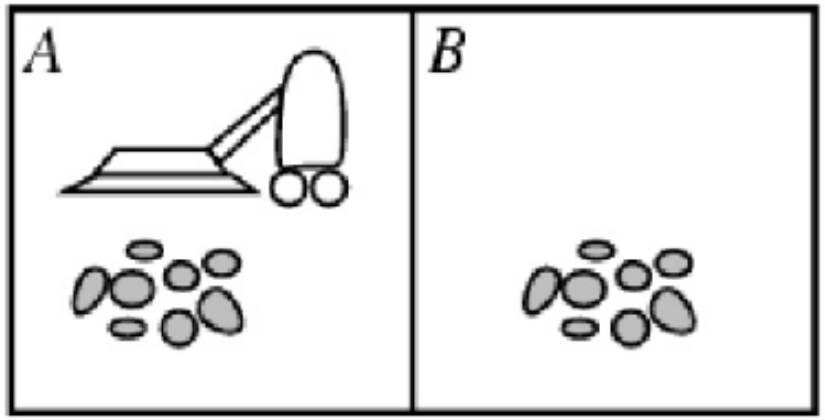
Rules

#	Rules
1	**One missionaries can move only when _____ in one side And _____ in the other**
2	**Two missionaries can move only when _____in one side And _____ in the other**
3	**One cannibals can move only when _____ in one side And _____ in the other**
4	**Two cannibals can move only when _____in one side And _____ in the other**
5	**One missionary and one cannibal can move only when _____ in one side And _____ in the other**


2c
1c
1m
1c
1c
1m
2c
1c
2m
1m
1c
2m
1c
1c
1m
2c
2c
1c
1m
1c

Vacuum Cleaner

- **World state space**
- **State**
- **Actions**
- **Goal**
- **Path costs:**



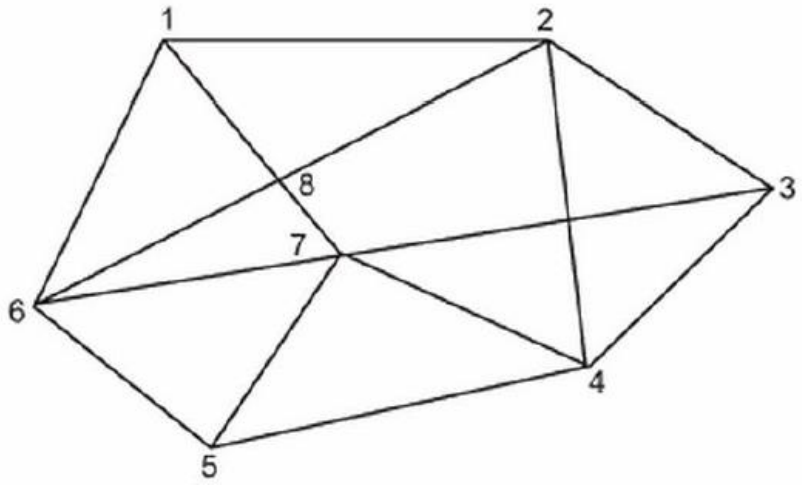
PROBLEM SOLVING & SEARCH STRATEGY
Part 2

Dr. Emad Natsheh

Traveling Salesman Problem

This problem falls in the category of path finding problems. The problem is defined as follows:

> *"Given 'n' cities connected by roads, and distances between each pair of cities. A sales person is required to travel each of the cities exactly once. We are required to find the route of salesperson so that by covering minimum distance, he can travel all the cities and come back to the city from where the journey was started".*



Traveling Salesman Problem

- The basic traveling salesman problem comprises of computing the shortest route through given a set of cities

Number of cities	Possible Routes
1	1
2	1 -2-1
3	1 -2 -3 1
	1 -3 -2 1
4	1- 2- 3- 4-1
	1- 2- 4- 3- 1
	1- 3- 2- 4- 1
	1- 3- 4- 2- 1
	1- 4- 2- 3-1
	1- 4- 3- 2- 1

(n-1)!

Traveling Salesman Problem

- The number of routes between cities is proportional to the factorial of the (number of cities -1)
  - E.g. For three cities the number of routes will be 2*1 and for 4 well be 4! and so on.
- The number of routes increase so rapidly !!
- If it take 1 hour from a CPU to solve 30 cities it will take 30 hour for 31 cities and 330 hour for 32 cities !!!
- The proper solution of this problem is done using Neural Network
  - The neural network can solve the 10 city case just as fast as the 30 city case.

Search Technique

Search

- The problem can be solved by using the rules, in combination with an appropriate control strategy, to move through the **problem space** until a path from an initial state to a goal state is found. This process is known as **search**.

- A very large number of AI problems are formulated as search problems.

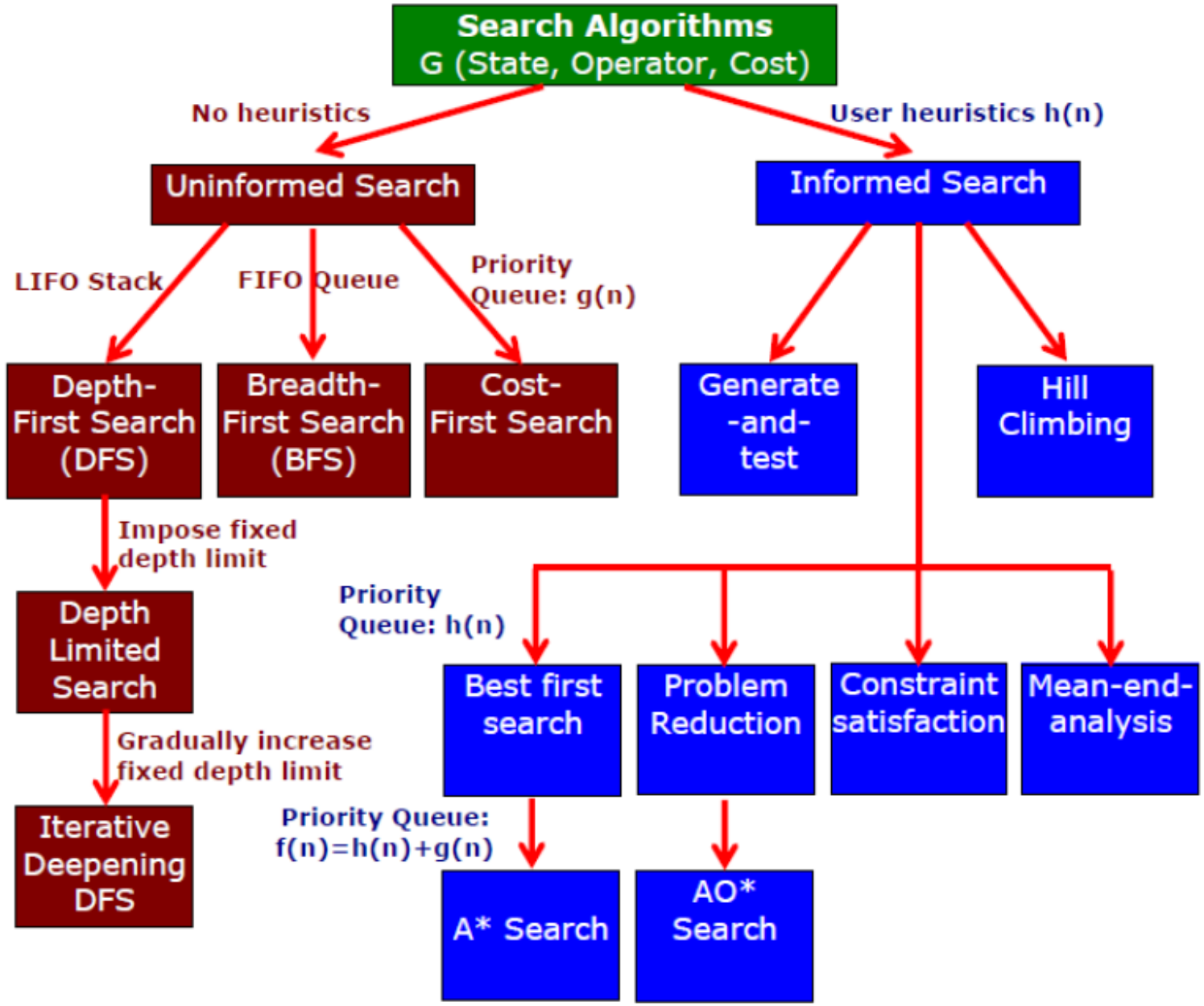
Search Technique

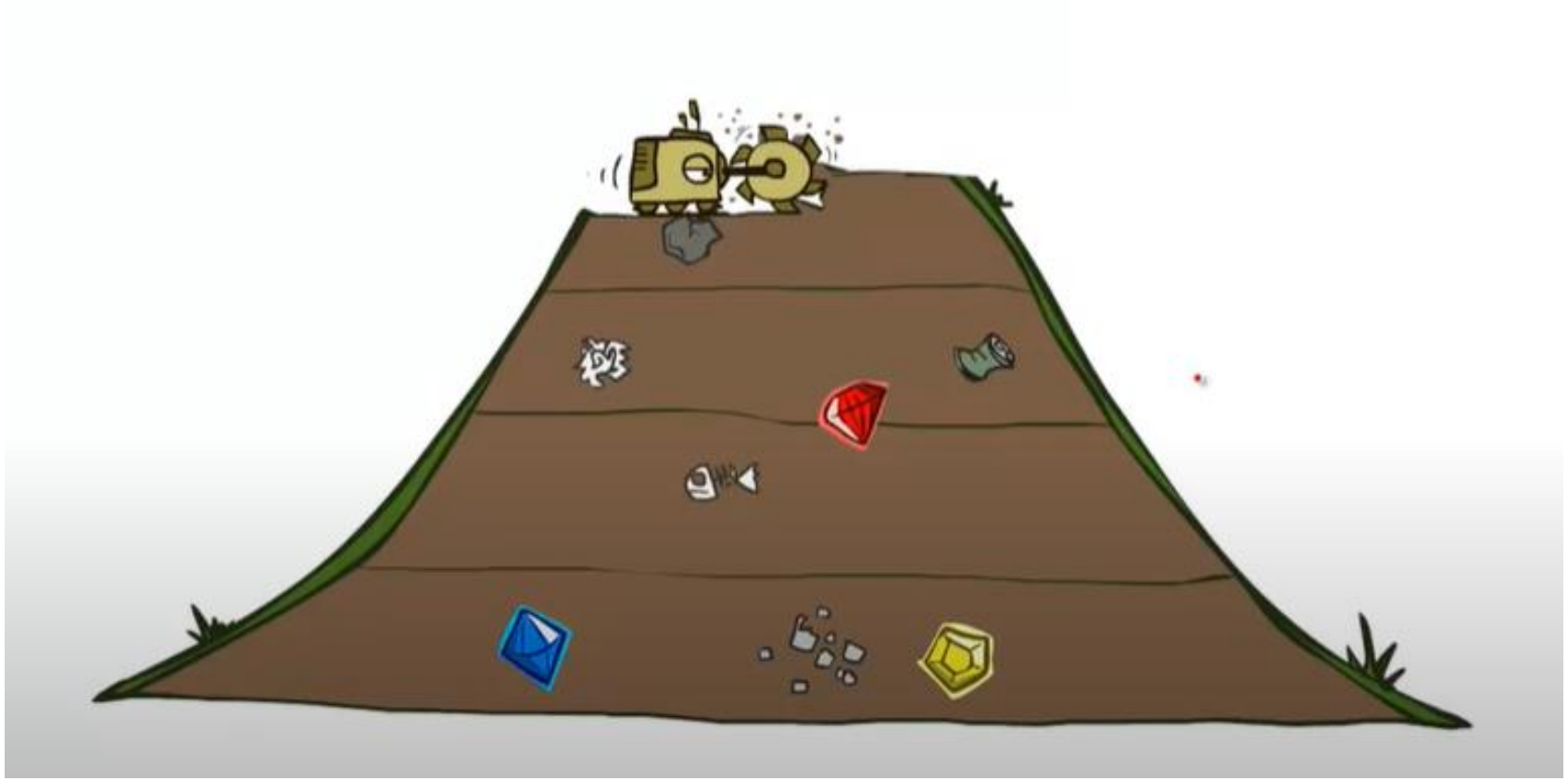
Not AI algorithms

- Blind (uninformed) search: is the search methodology having no additional information about states beyond that provided in the problem definitions
  - In this search total search space is looked for solution

AI algorithms

- Heuristic (informed) search: these are the search techniques where additional information about the problem is provided in order to guide the search in a specific direction


Search Algorithms
G (State, Operator, Cost)
No heuristics
User heuristics h(n)
Uninformed Search
Informed Search
LIFO Stack
FIFO Queue
Priority Queue: g(n)
Depth-First Search (DFS)
Breadth-First Search (BFS)
Cost-First Search
Generate-and-test
Hill Climbing
Impose fixed depth limit
Depth Limited Search
Gradually increase fixed depth limit
Iterative Deepening DFS
Priority Queue: h(n)
Best first search
Problem Reduction
Constraint satisfaction
Mean-end-analysis
Priority Queue: f(n)=h(n)+g(n)
A* Search
AO* Search

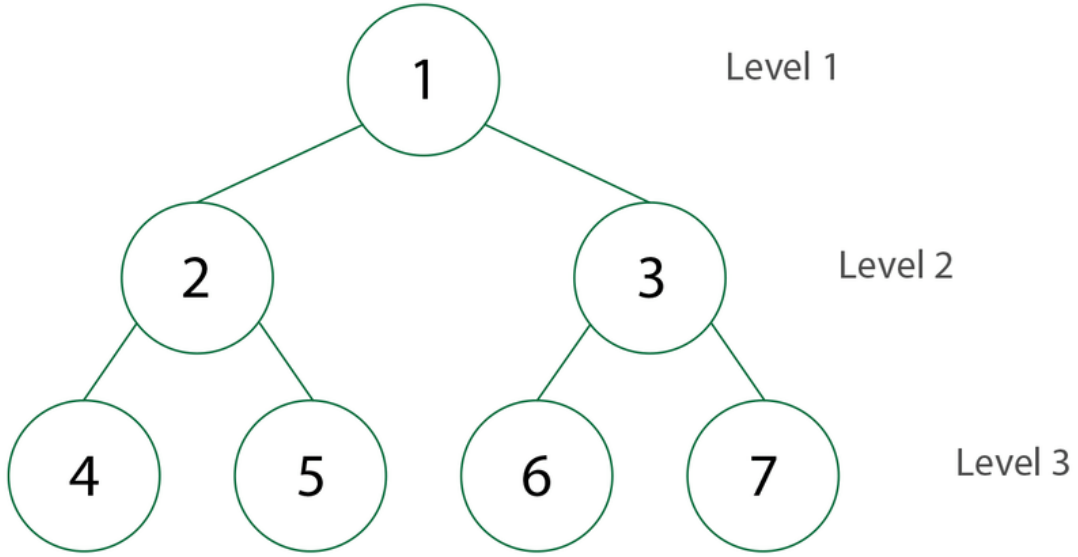


Breadth First Search

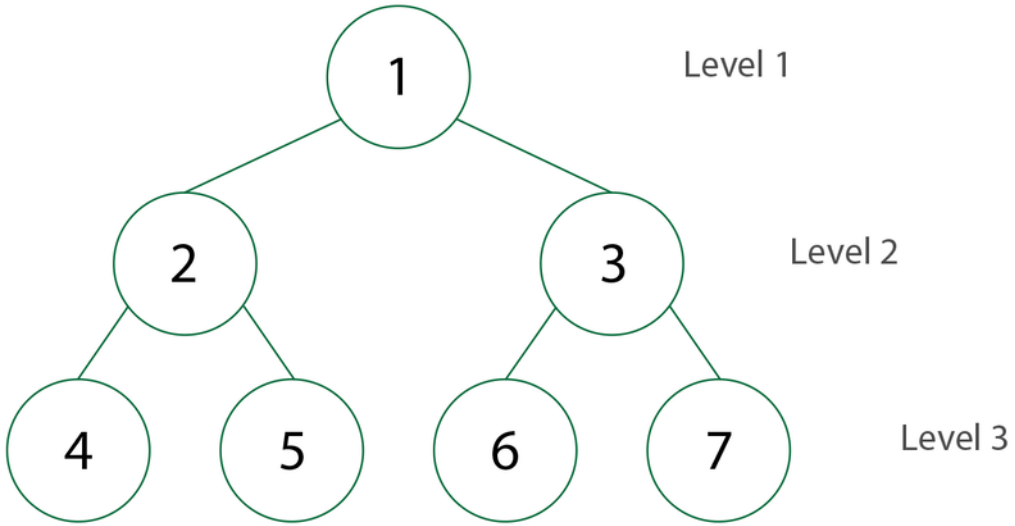
Breadth First Search

- Implementation: FIFO queue
- Strategy: expand a shallowest node first

The node closest to the root



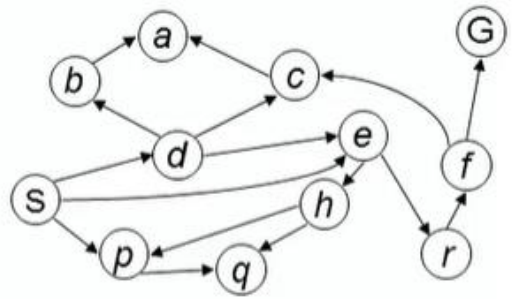
Breadth First Search (Example)

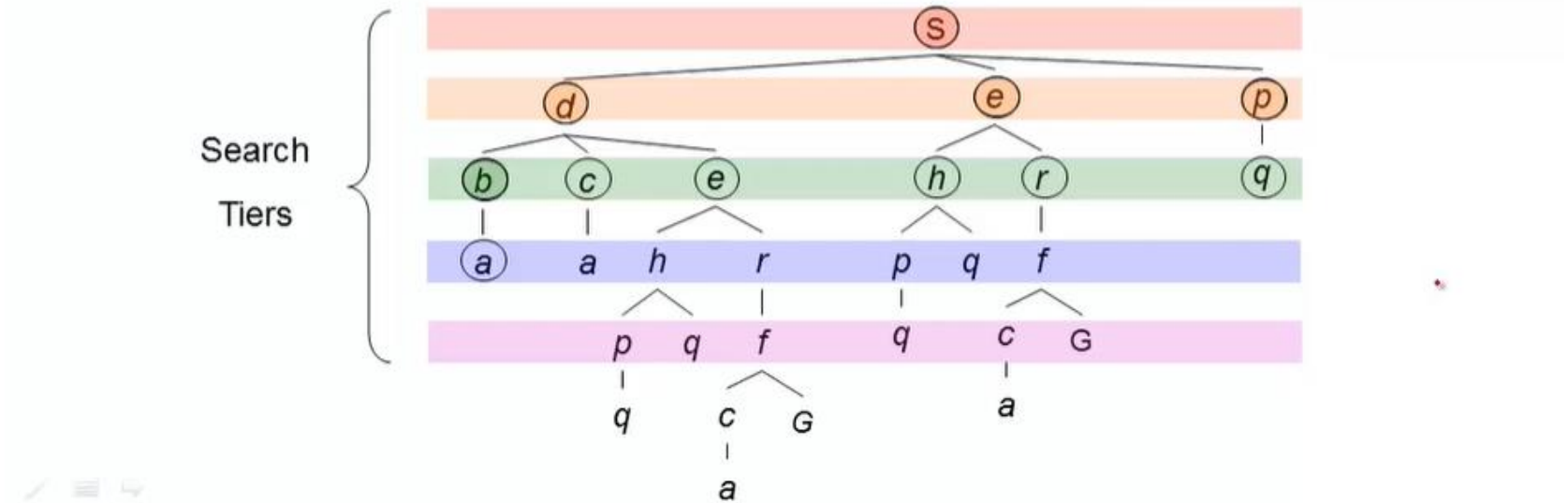


Breadth First Search (Example)

Strategy: expand a shallowest node first

Implementation: Fringe is a FIFO queue





BFS Analysis

- Complete?
- Optimal?

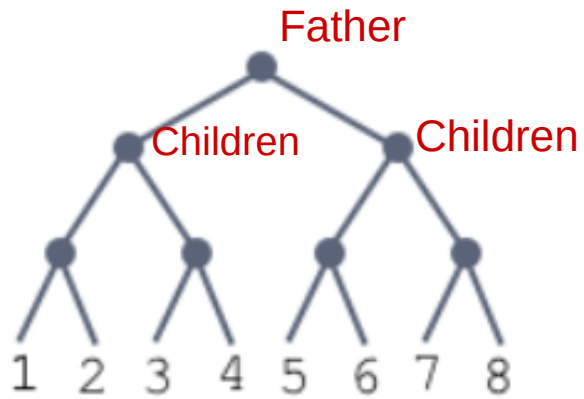
Time vs Space Complexity

- Time Complexity:
  - It is the amount of time need to generate the nodes
- Space Complexity:
  - It is the amount of space or memory required for getting a solution
- **Big O notation**
  - Is a mathematical notation that describes the limiting behavior of a function
  - Used in Computer Science to describe the performance or complexity of an algorithm

Branching Factor

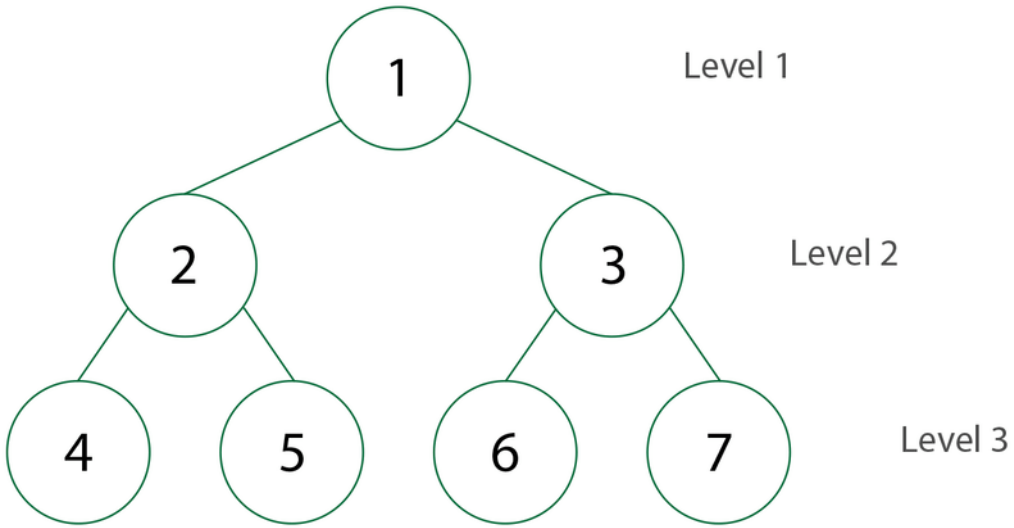
- The branching factor of a node in a tree is the number of children it has.

b = 2

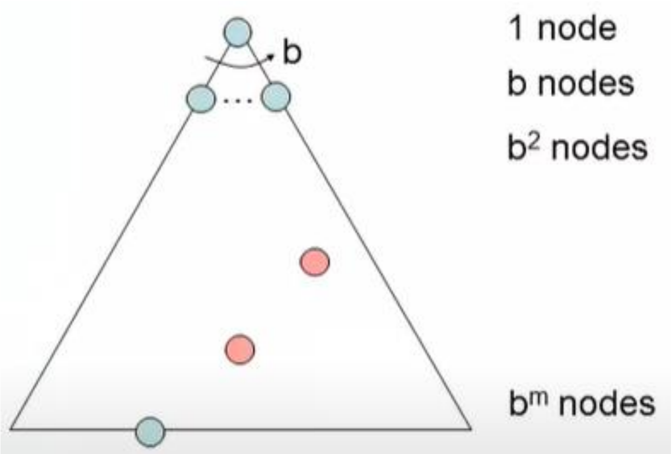


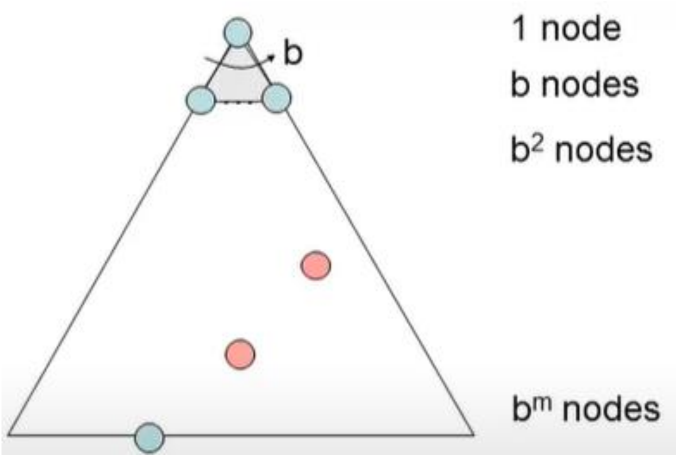
b = 8

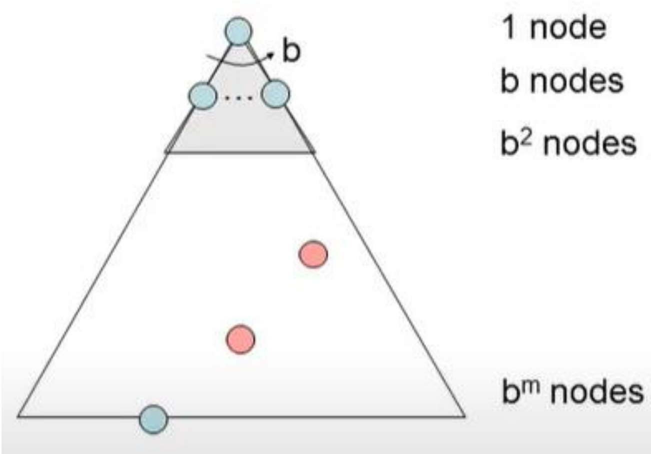
BFS Time and Space Complexity

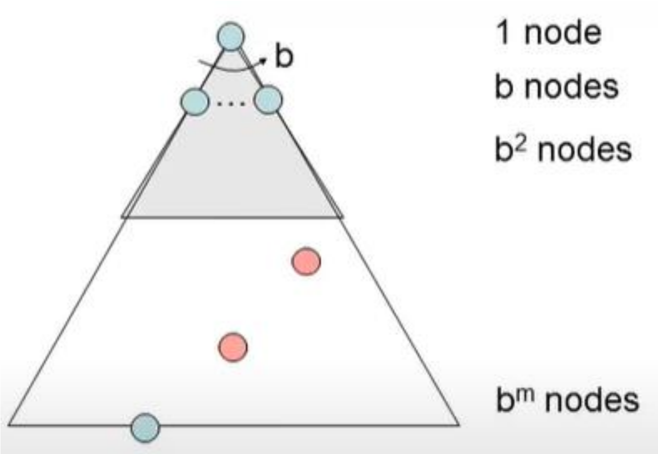


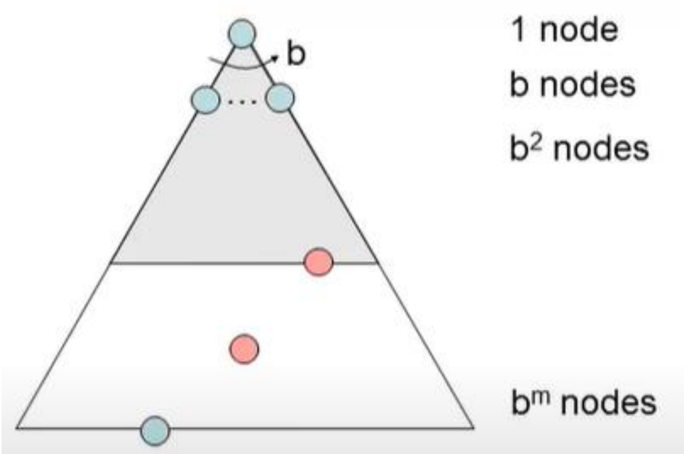
BFS Time and Space Complexity











$$1 + b + b^2 + b^3 + \cdots + b^d = O(b^d)$$

BFS Analysis

- Time Complexity → $1 + b + b^2 + b^3 + \cdots + b^d = O(b^d)$
- Space Complexity → O(b^d)
- Where b is branching factor and d is depth of solution

BFS Analysis

- Assume (branching factor) b =10
- CPU can expand 1 nodes/ms
- Each node size = 100 byte

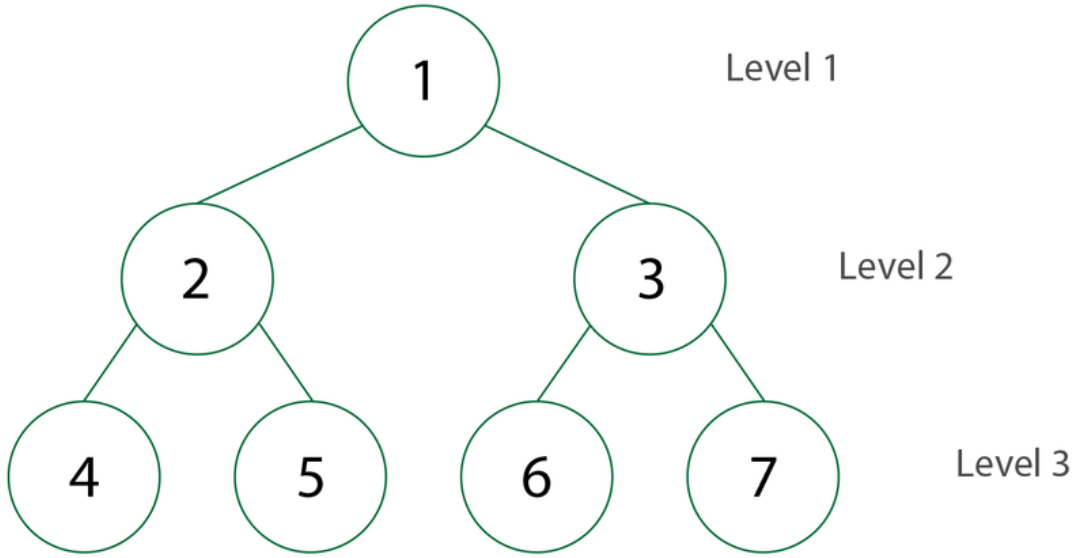
d	Node expand	Time	Memory
0	1	1 ms	100 bytes
2	111	0.1 s	10 KB
4	11111	11 sec	1 MB
8	≅ 10^8	31 hour	11 GB
12	≅10^12	35 year	111 TB
14	≅10^14	3500 year	11111 TB

Depth First Search

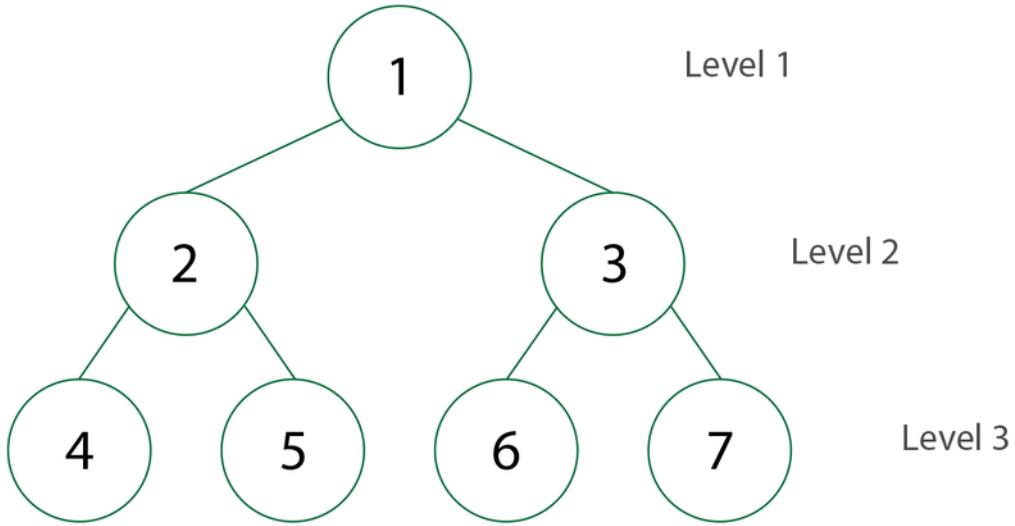
Depth First Search

There is a difference in space

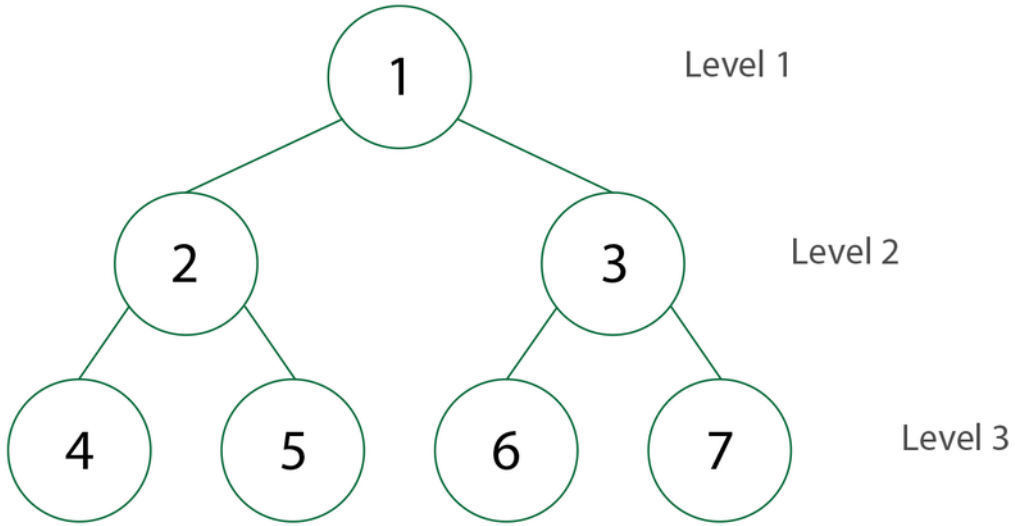
- Implementation: LIFO stack , queue
- Strategy: find the deepest solution



Depth First Search (Example- Stack)



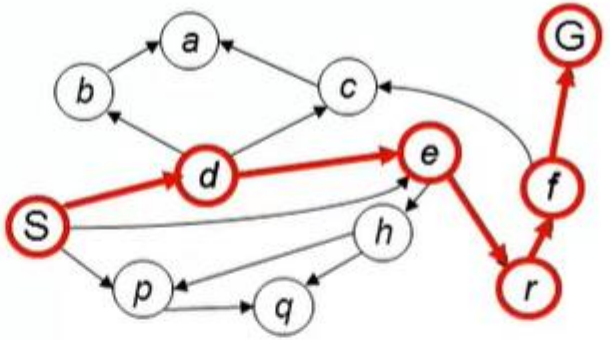
Depth First Search (Example- Queue)

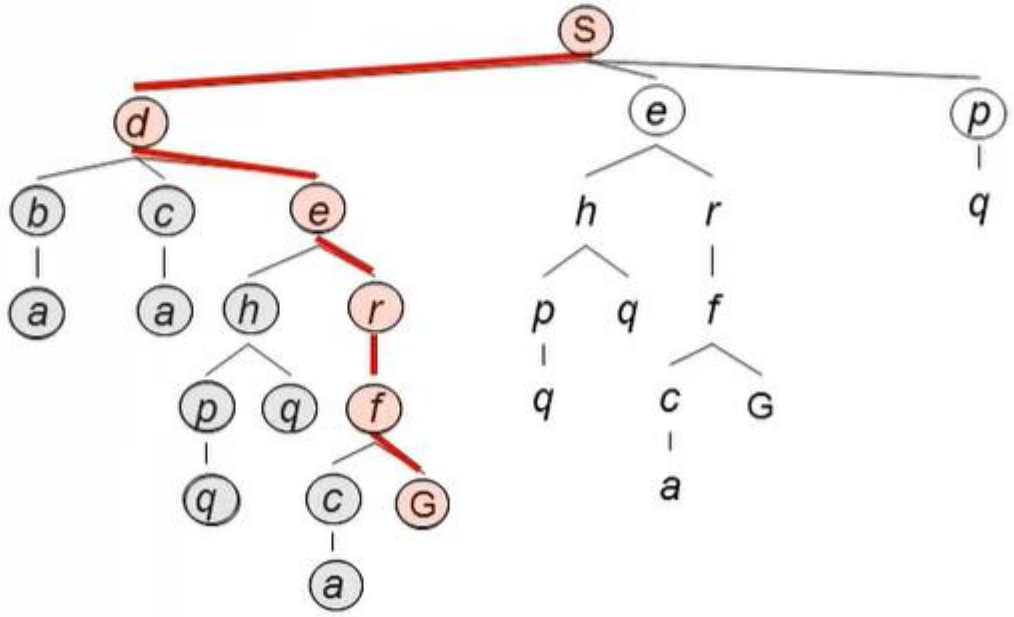


Depth First Search (Example)

Strategy: expand a deepest node first

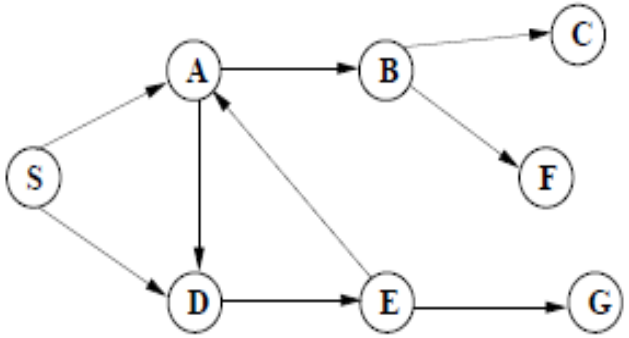
Implementation: Fringe is a LIFO stack

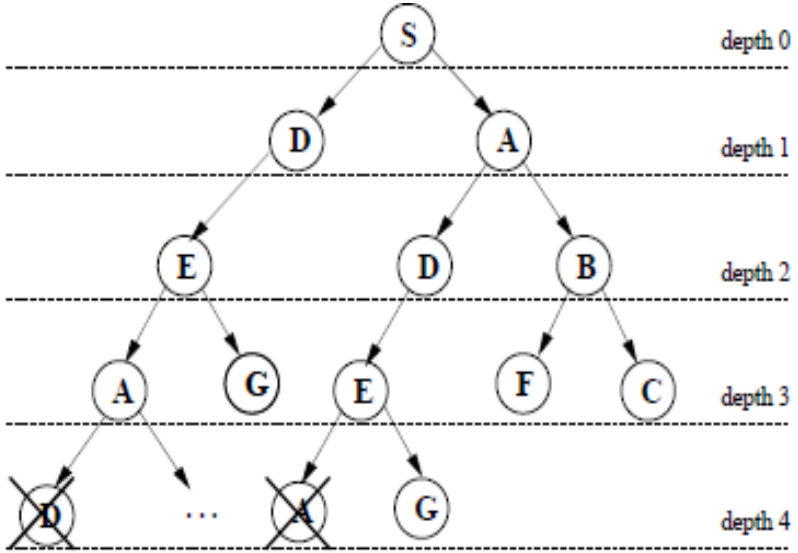




DFS Analysis

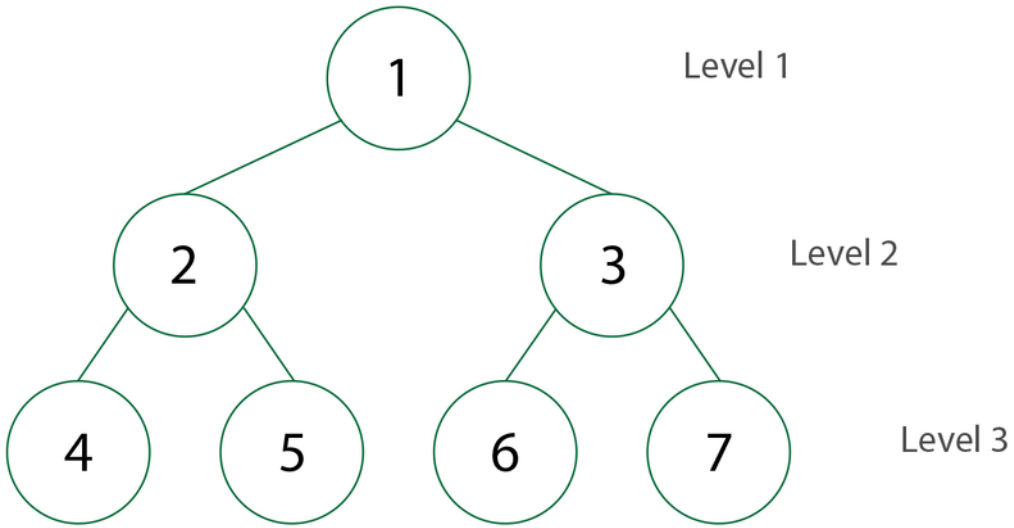
- Complete?



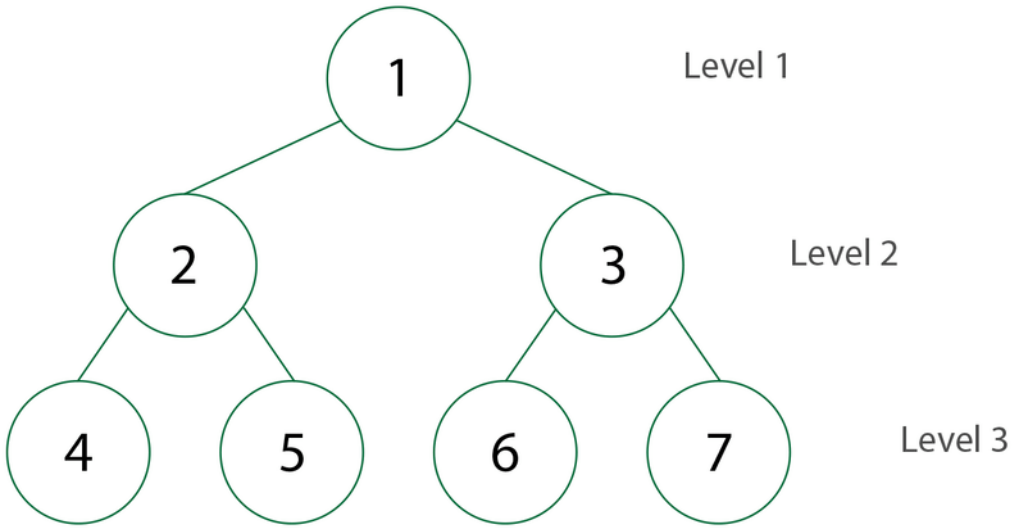


- Optimal?

DFS Time and Space Complexity (Stack)



DFS Time and Space Complexity (Queue)



DFS Analysis

- Time Complexity → O(b^m)
- Stack
  - Space Complexity → O(m)
- Queue
  - Space Complexity → O(bm)

- Where b is branching factor and m is maximum depth

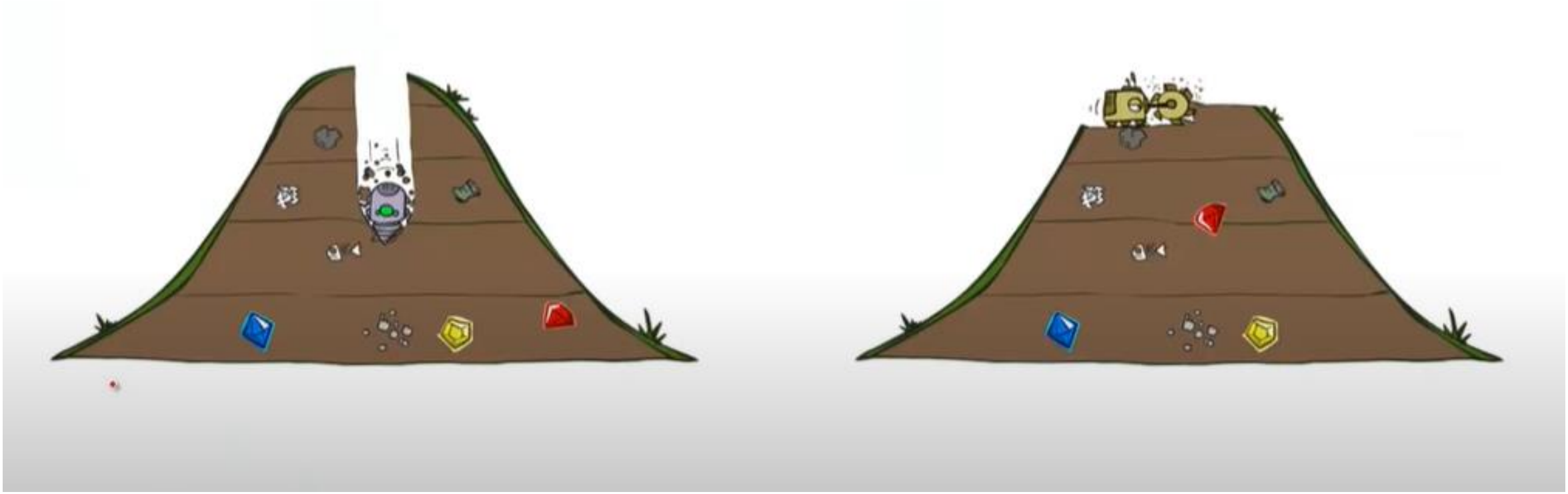
Quiz: DFS vs BFS

- When will BFS outperform DFS?

  1- Shallowest (target is closer to Root)
  2- Complete
  3- Sparse Graph (the number of branches is low)

- When will DFS outperform BFS?

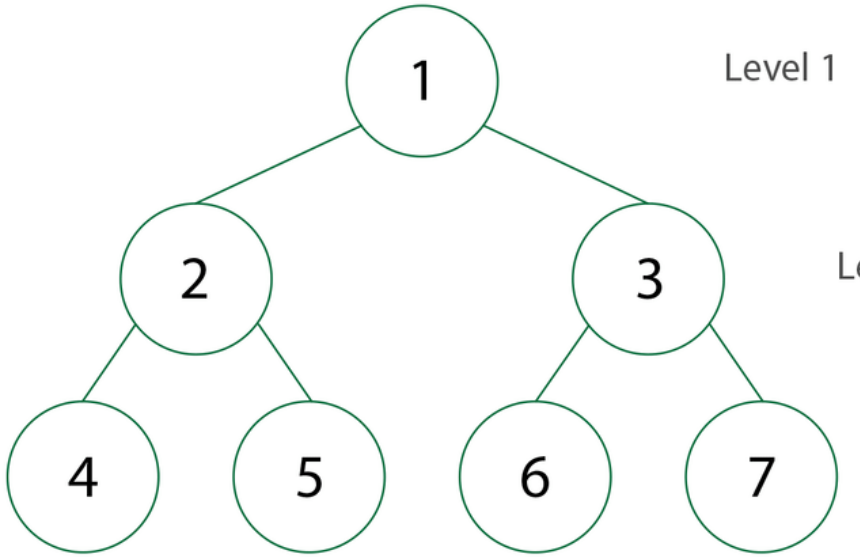
  1- High branch factor
  2- Deepest solution
  3- Memory



Iterative Deepening Depth First Search

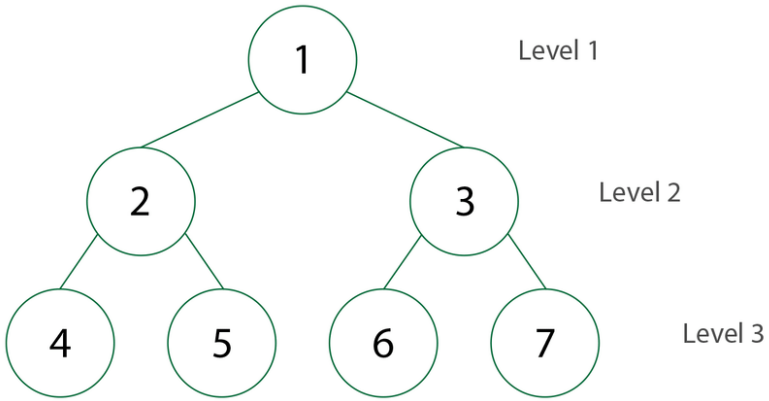
Iterative Deepening Depth First Search

- Its depth-limited version of depth-first search is run repeatedly with increasing depth limits until the goal is found.
- Find the shallowest solution

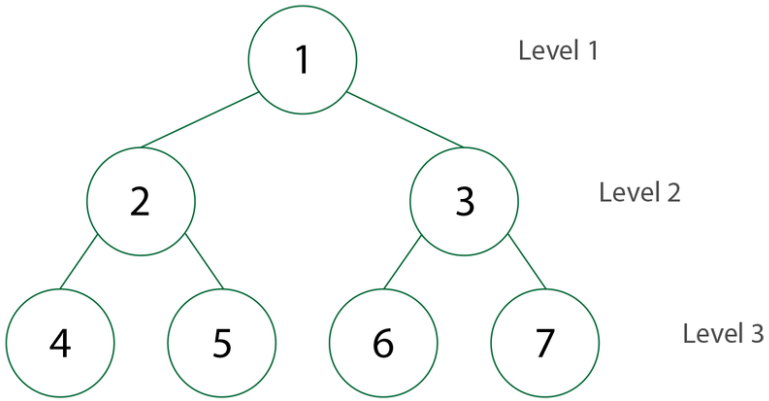


- Run a DFS with depth limit 1. If no solution...
- Run a DFS with depth limit 2. If no solution...
- Run a DFS with depth limit 3. .....

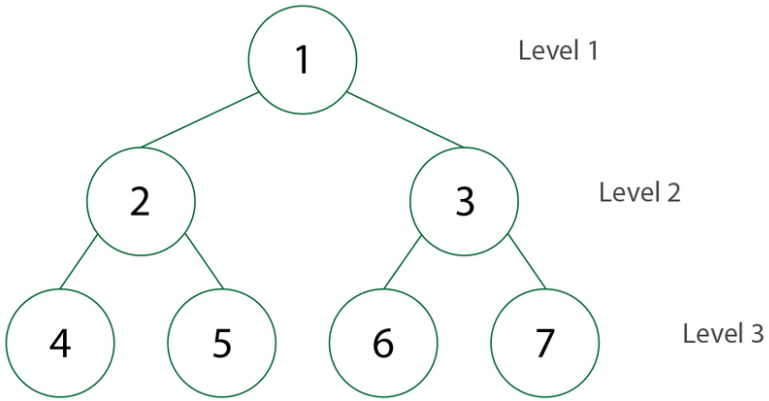
IDDFS(Example depth=0)



IDDFS(Example depth=1)



IDDFS(Example depth=2)



IDDFS Pseudocode

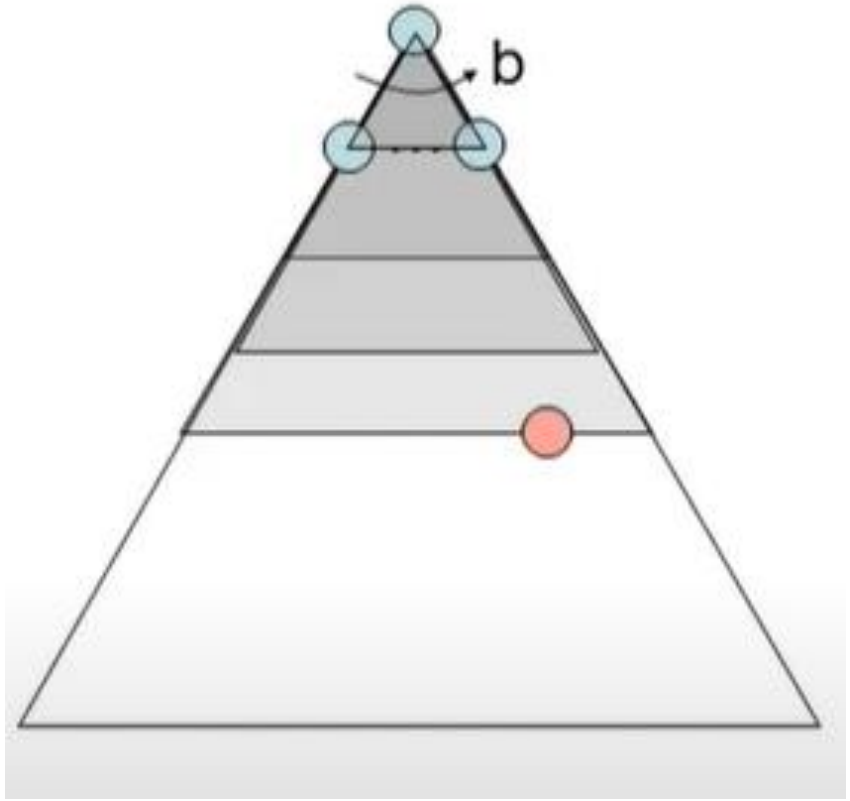
```
function IDDFS(root) is
    for depth from 0 to ∞ do
        found, remaining ← DLS(root, depth)
        if found ≠ null then
            return found
        else if not remaining then
            return null

function DLS(node, depth) is
    if depth = 0 then
        if node is a goal then
            return (node, true)
        else
            return (null, true)    (Not found, but may have children)

    else if depth > 0 then
        any_remaining ← false
        foreach child of node do
            found, remaining ← DLS(child, depth−1)
            if found ≠ null then
                return (found, true)
            if remaining then
                any_remaining ← true    (At least one node found at depth, let IDDFS deepen)
        return (null, any_remaining)
```

IDDFS Analysis

- Complete?

- Optimal?

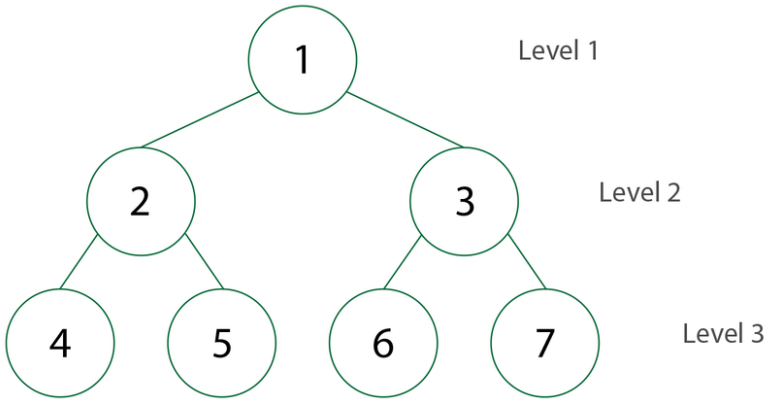
IDDFS Time Complexity



$$(d+1) + d(b) + (d-1)b^2 + \cdots + 3b^{d-2} + 2b^{d-1} + b^d$$

$$= \sum_{i=0}^{d} (d+1-i)\, b^i$$

IDDFS Space Complexity



IDDFS Analysis

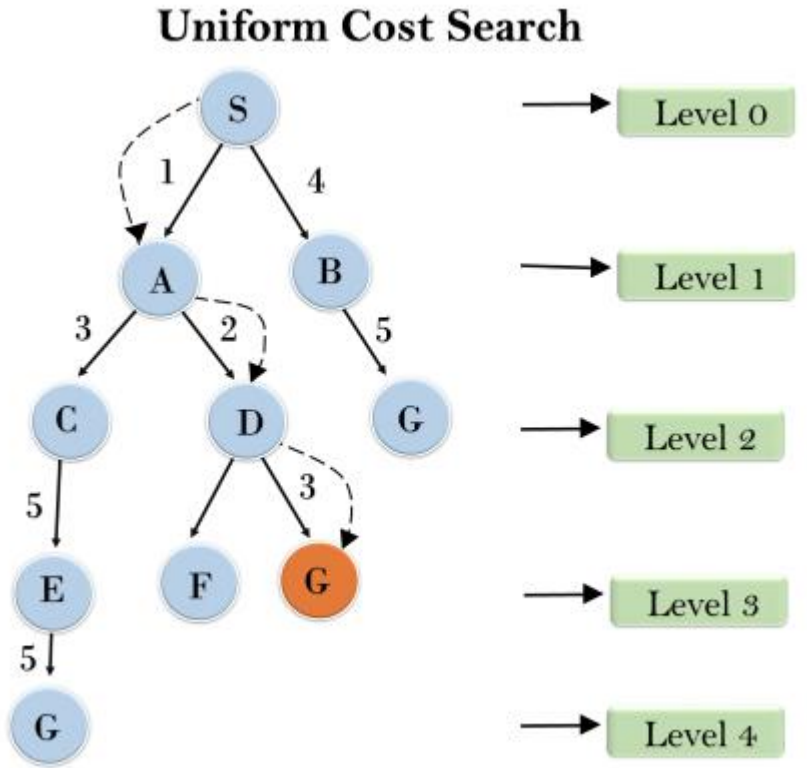
- Time Complexity → O(b^d)
- Stack
  - Space Complexity → O(d)
- Queue
  - Space Complexity → O(bd)

- Where b is branching factor and d is depth of solution

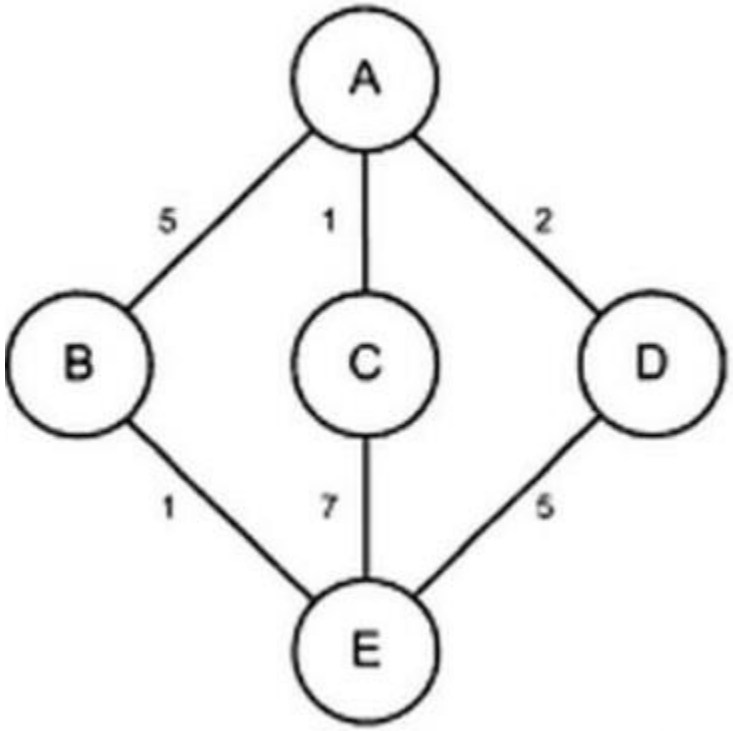
Uniform Cost Search

UCS (Branch and Bound Search)

- Implementation: priority queue (sort by cost function **g(n)**)
- **g(n)** is cost from root node to current node n
- Uniform cost search vs Dijkstra !!!!

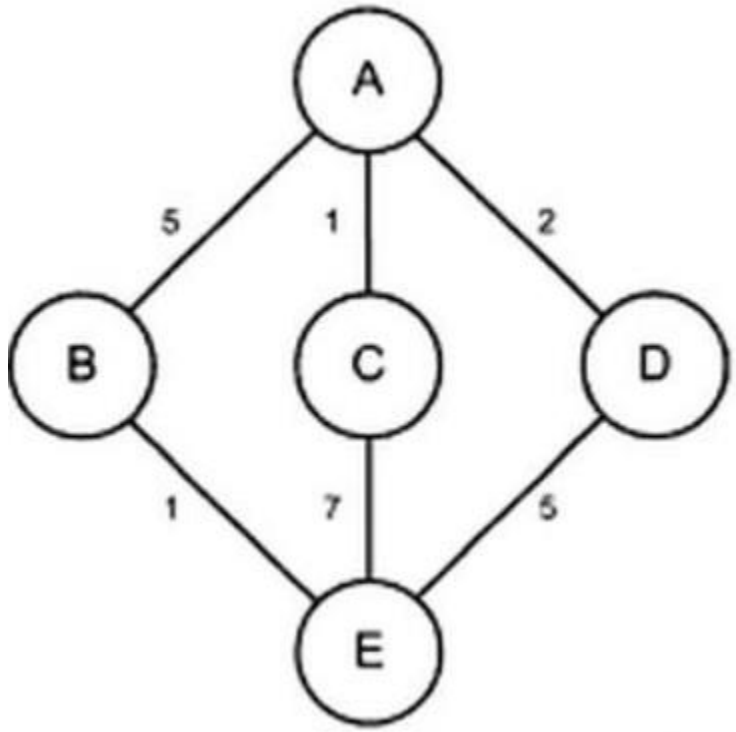


UCS (Example Table)

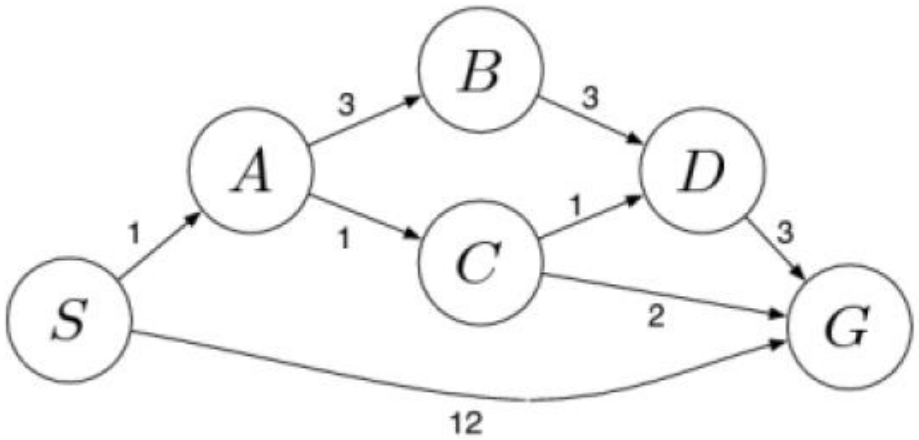


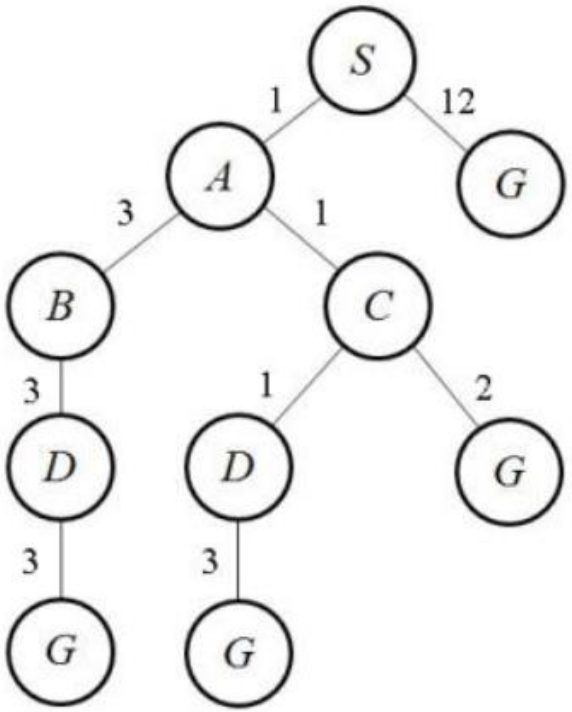
Visited Nodes (Close)	Priority Queue (Open)
-	A

UCS (Example Tree)

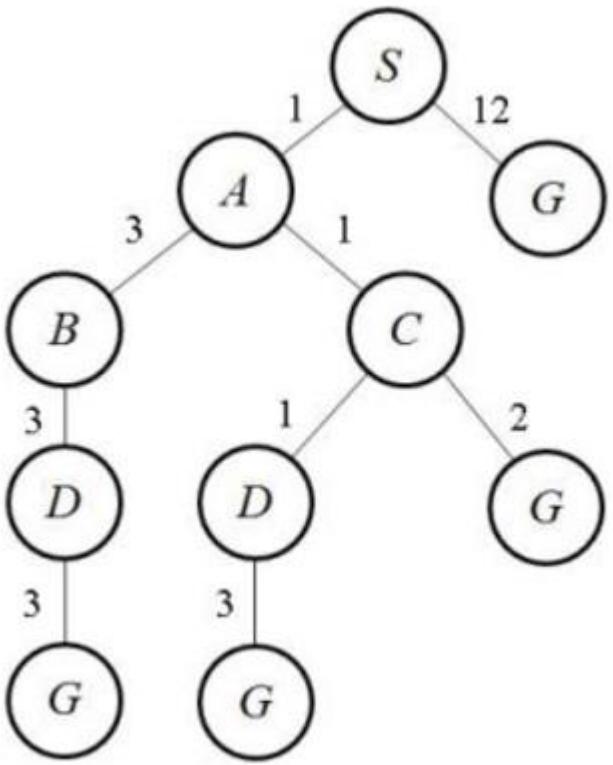


Class Exercise



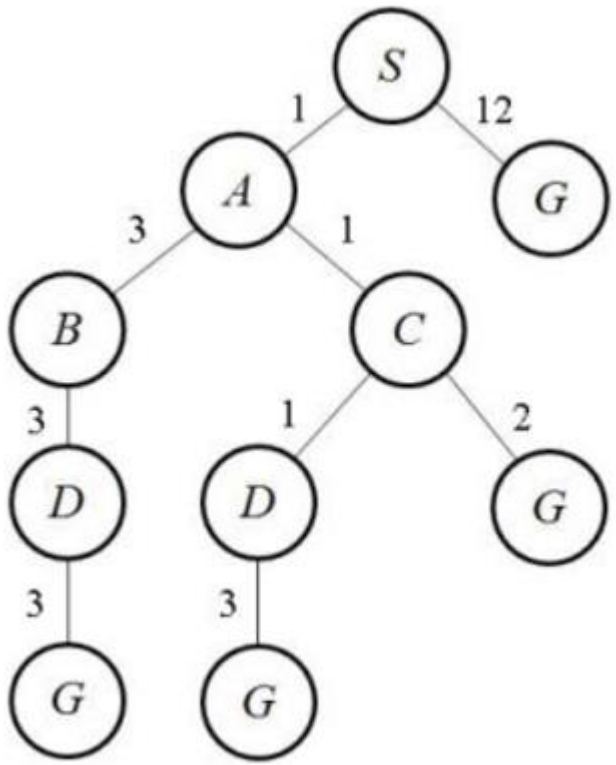


Class Exercise (Solution Table)



Visited Nodes (Close)	Priority Queue (Open)
-	S

Class Exercise (Solution Tree)



UCS Pseudocode

```
function UNIFORM-COST-SEARCH(problem) returns a solution, or failure

    node ← a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
    frontier ← a priority queue ordered by PATH-COST, with node as the only element
    explored ← an empty set
    loop do
        if EMPTY?(frontier) then return failure
        node ← POP(frontier)   /* chooses the lowest-cost node in frontier */
        if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
        add node.STATE to explored
        for each action in problem.ACTIONS(node.STATE) do
            child ← CHILD-NODE(problem, node, action)
            frontier ← INSERT(child, frontier)
```

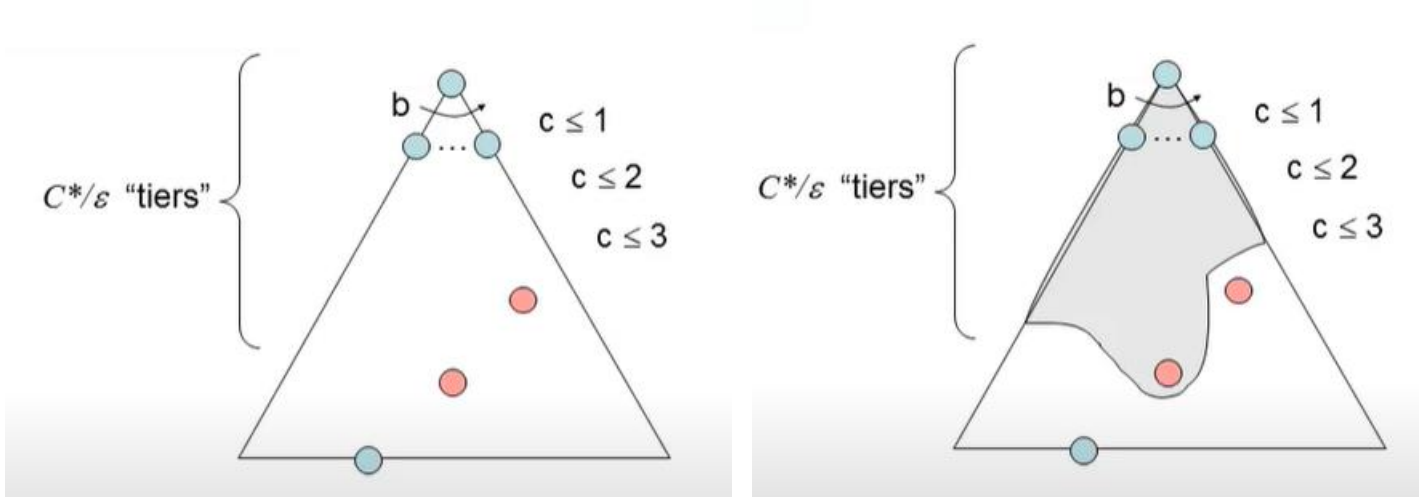
UCS Analysis

- Complete?

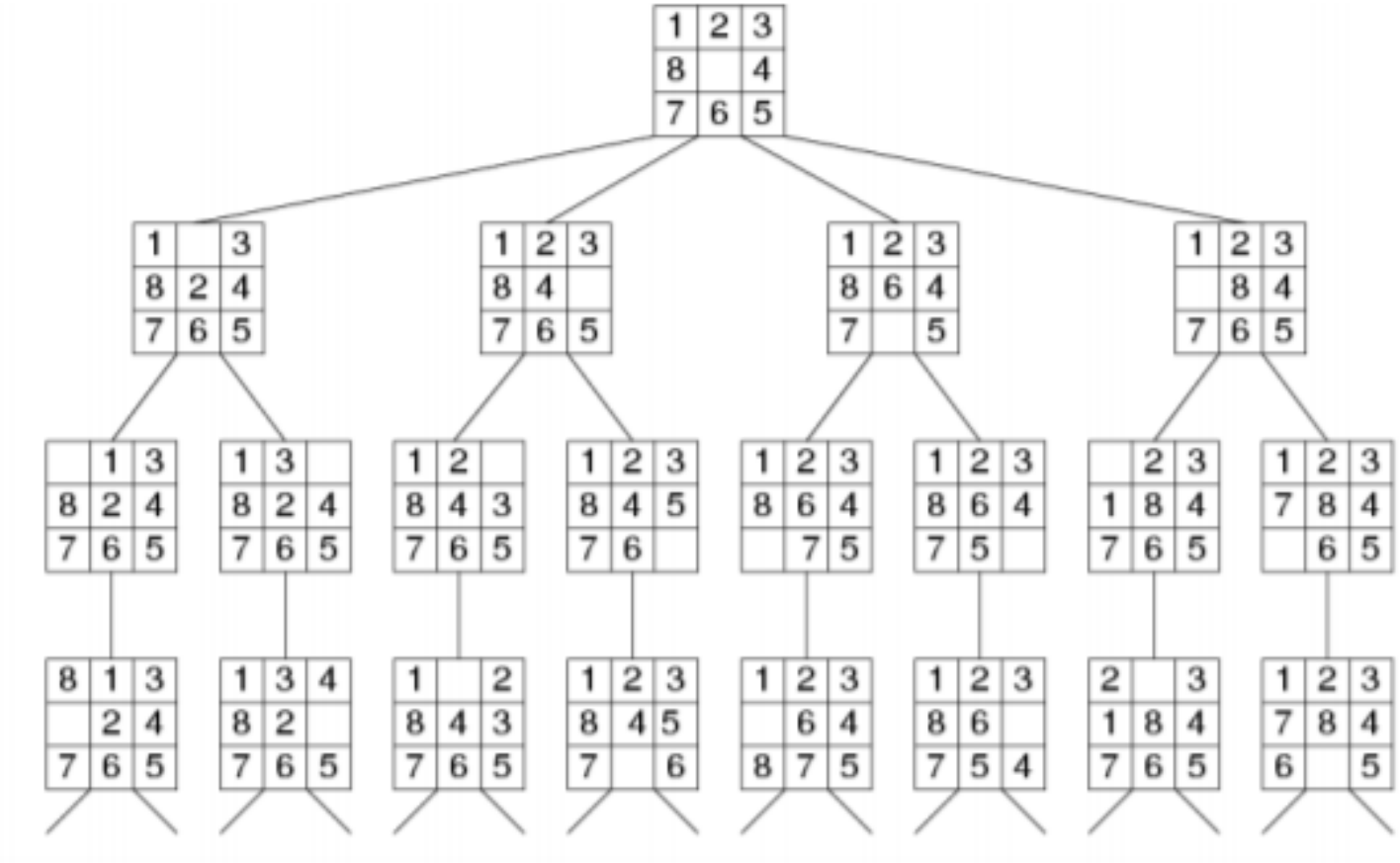
- Optimal?

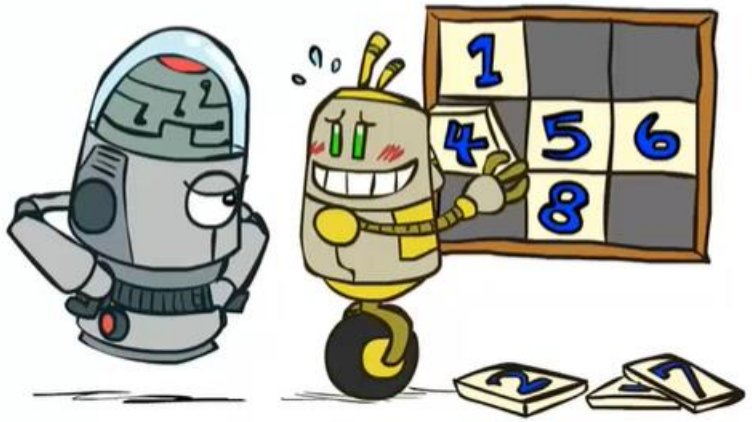
UCS Analysis

- Time Complexity
  - $O(b^d)$
  - $O(b^{C*/\varepsilon})$
- Space Complexity
  - $O(b^d)$
  - $O(b^{C*/\varepsilon})$



UCS and 8 puzzle





PROBLEM SOLVING & SEARCH STRATEGY
Part 3

Dr. Emad Natsheh

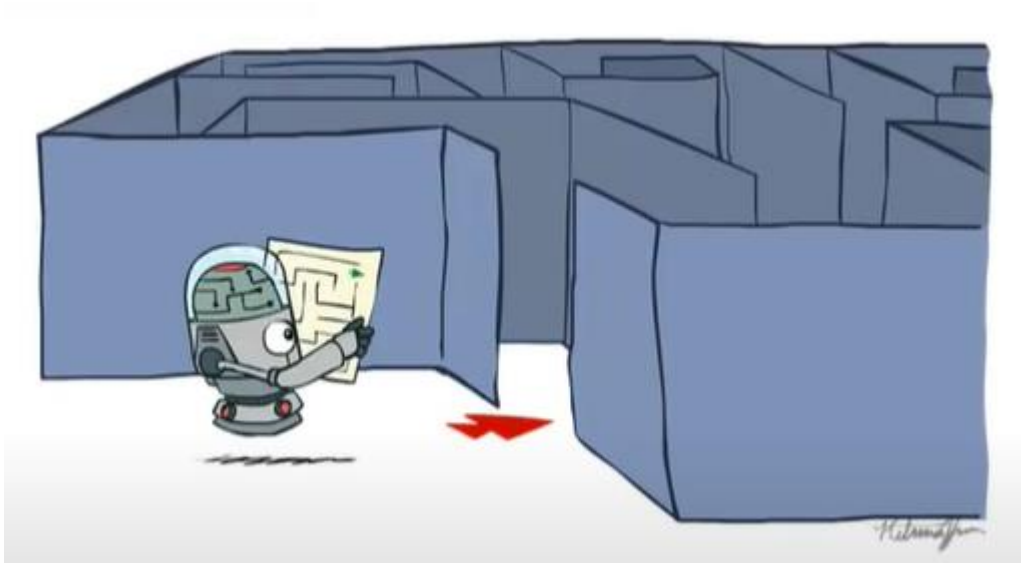
Informed (Heuristic) Search Technique

Informed search

- Heuristic
- Best First Search (Greedy search)
- A*

Recap: Search

- Search problem
  - State
  - Action or cost
  - Successor function
  - Start goal and end goal
- Represent problem as tree/graph
- Search algorithm
  - Choose node for expanding
    - Blind search
    - Inform search



Informed Search

- The assumption behind **blind search** is that we have no way of telling whether a particular search direction is likely to lead us to the goal or not

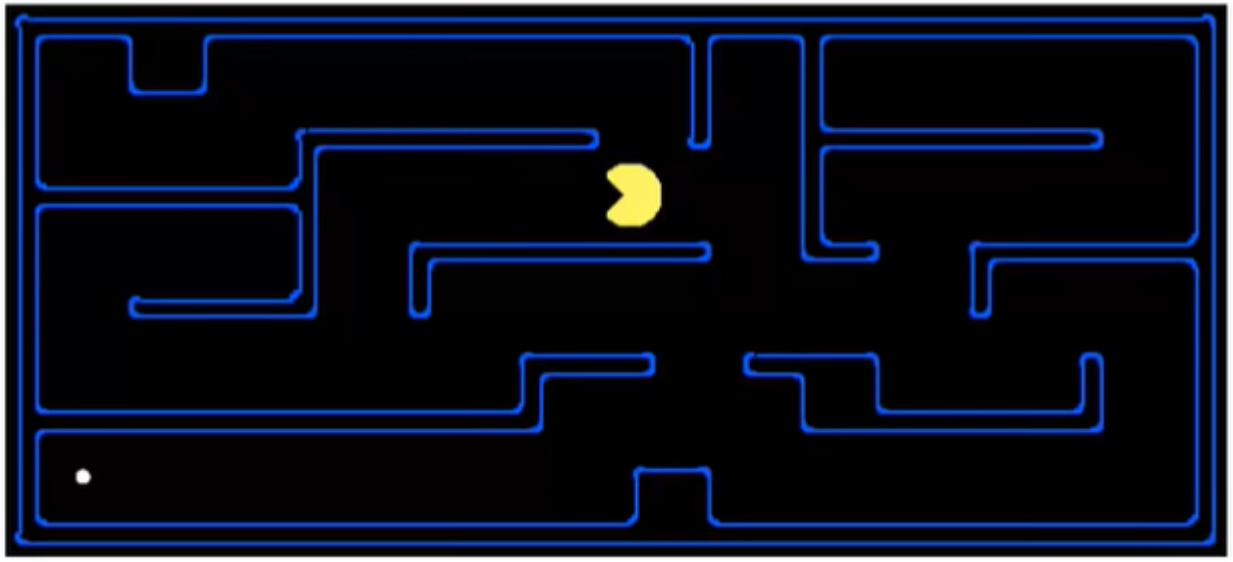
- The key idea behind **informed** or **heuristic** search algorithms is to exploit a task specific measure of goodness to try to either reach the goal more quickly or find a more desirable goal state.

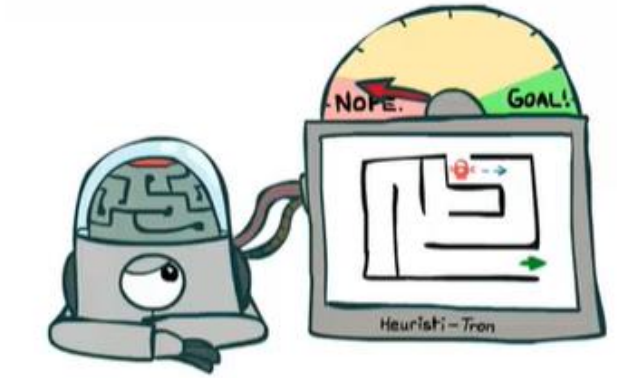
- Heuristic: From the Greek for “find”, “discover”.
  - Heuristics are criteria, methods, or principles for deciding which among several alternative courses of action promises to be the most effective in order to achieve some goal”.

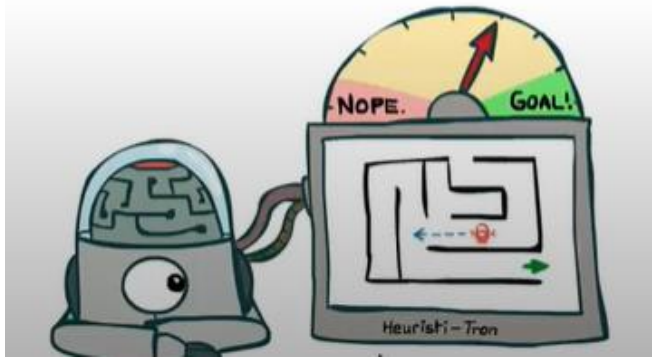
    Judea Pearl “ Heuristics” 1984

Heuristic Function h(n)

- **Heuristic h(n) is** a function that estimates how close a state is to a goal
- Designed for a particular search problem
- We assume that h(n) is non-negative,
  and that h(n)=0 if n is a goal node.

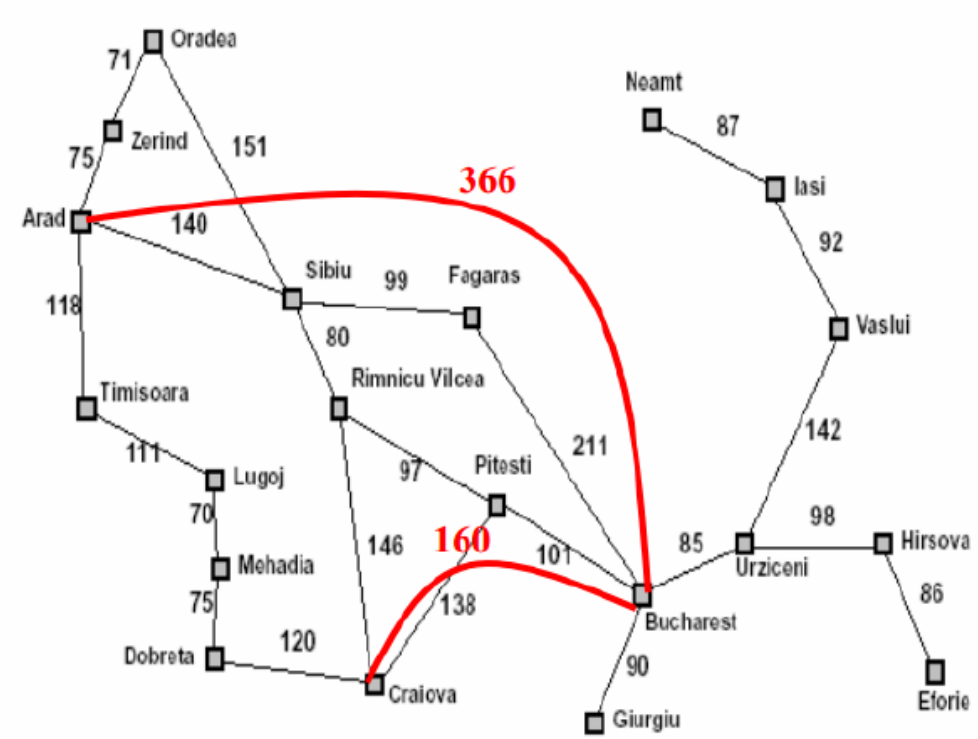






Heuristic Function (Example - Travel Planning)

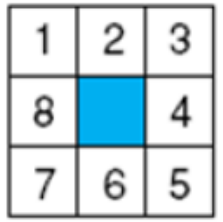
- h'(n) = straight-line distance between n and goal node



Straight-line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

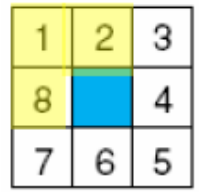
Heuristic Function (Example- 8Puzzle)

- Number of tiles out of places
  - Compared with the goal state
  - How many tiles are not in the right place?
- Sum of taxicab distances
  - For each tile, how far is it from the goal position? How many squares away? What is its distance?
- The objective is to minimize this heuristic value as you search. As it approaches 0, you are closer to the goal.

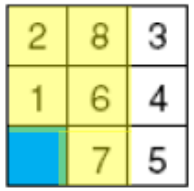
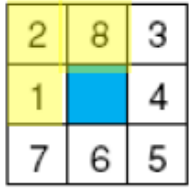
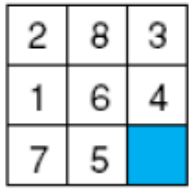


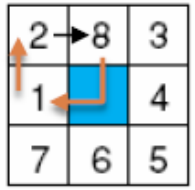
Goal

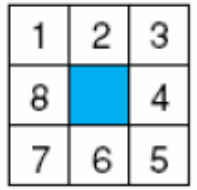
Heuristic Function (Example- 8Puzzle)



Goal

n	$h_1(n)$	$h_2(n)$
	5	6
	3	4
	5	6
	Tiles out of place	Sum of distances out of place



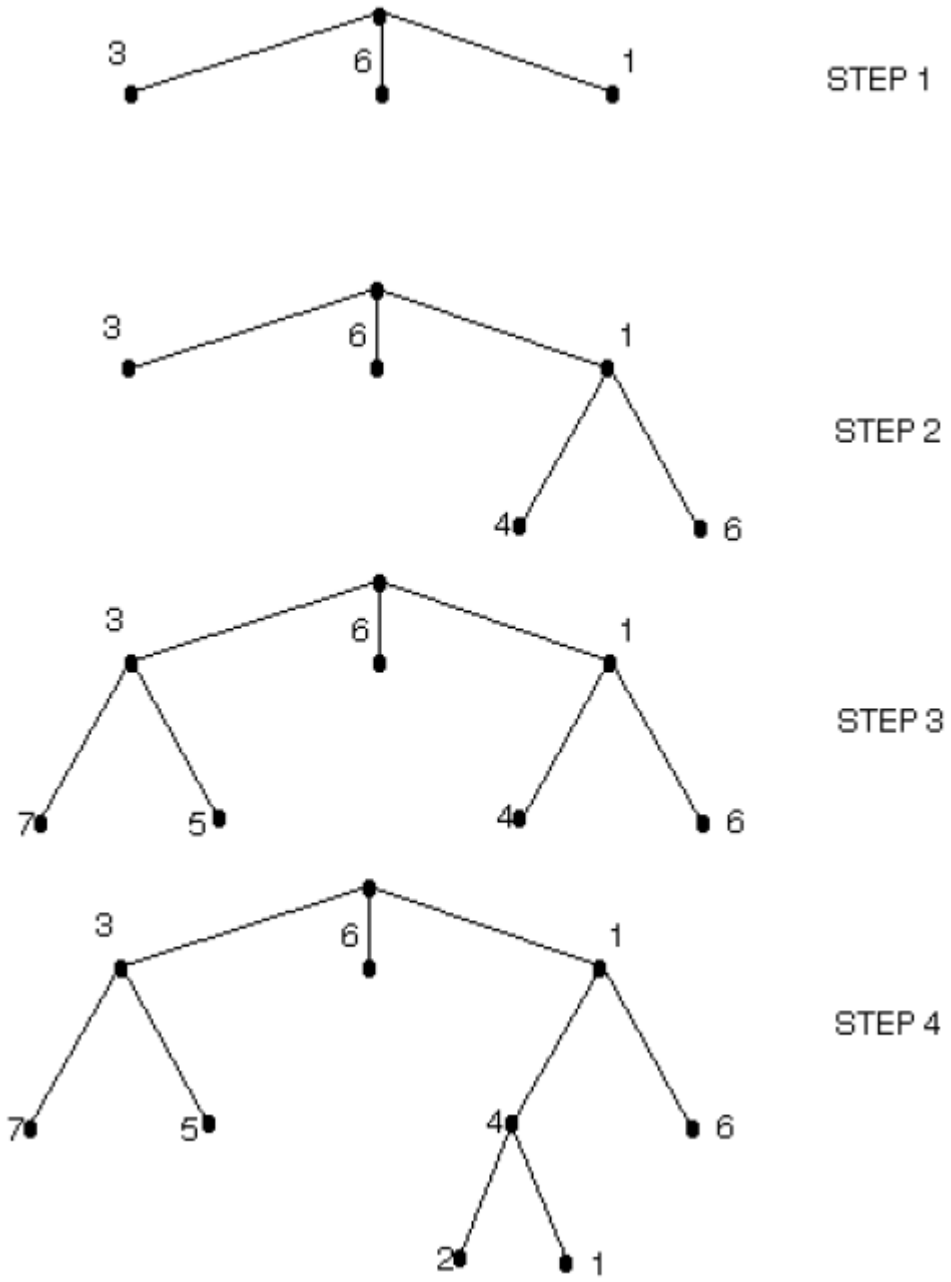


Goal

Best First Search (Greedy search)

Best First Search

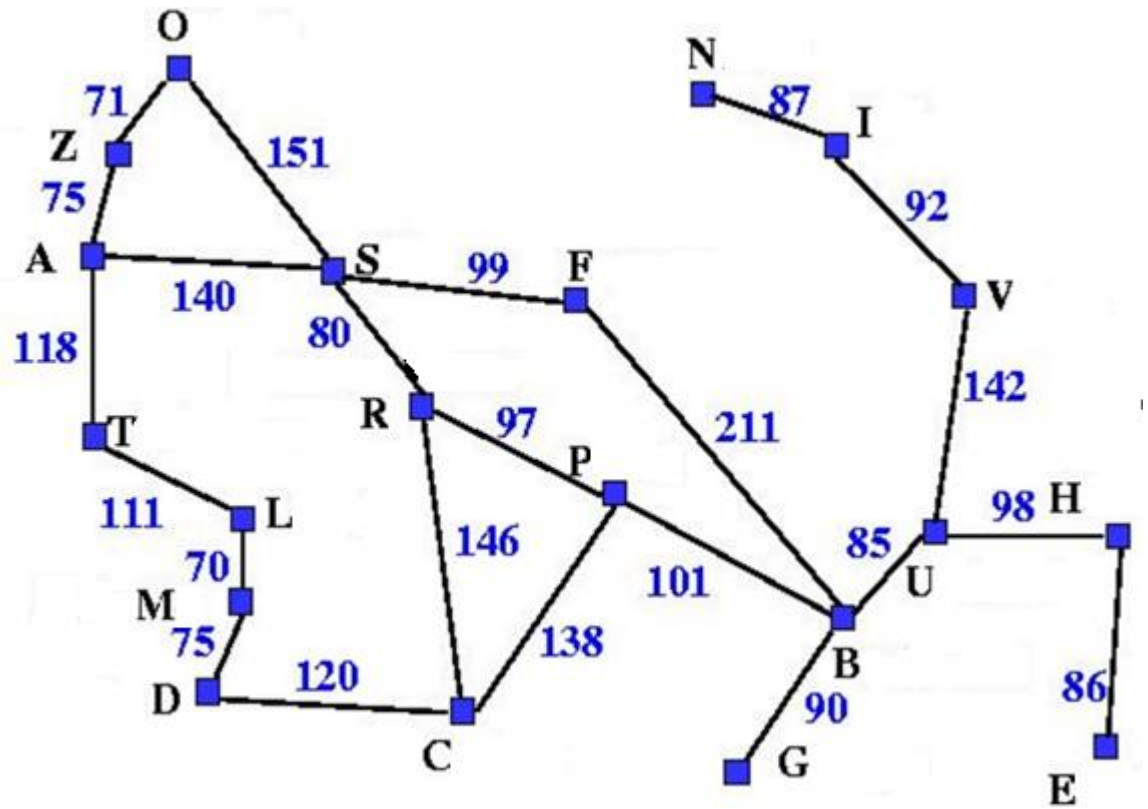
- Implementation: priority queue
- Very similar to UCS but sort the queue regarding to the h(n)
- Strategy: expand the node that you think is closest to the goal



```
Best-first search {

closed list = [ ]
open list = [start node]

    do {
        if open list is empty then{
            return no solution
        }
        n = heuristic best node
        if n == final node then {
            return path from start to goal node
        }
        foreach direct available node do{

            add current node to open list and calculate heuristic
            set n as his parent node

        }
        delete n from open list
        add n to closed list
    } while (open list is not empty)
}
```

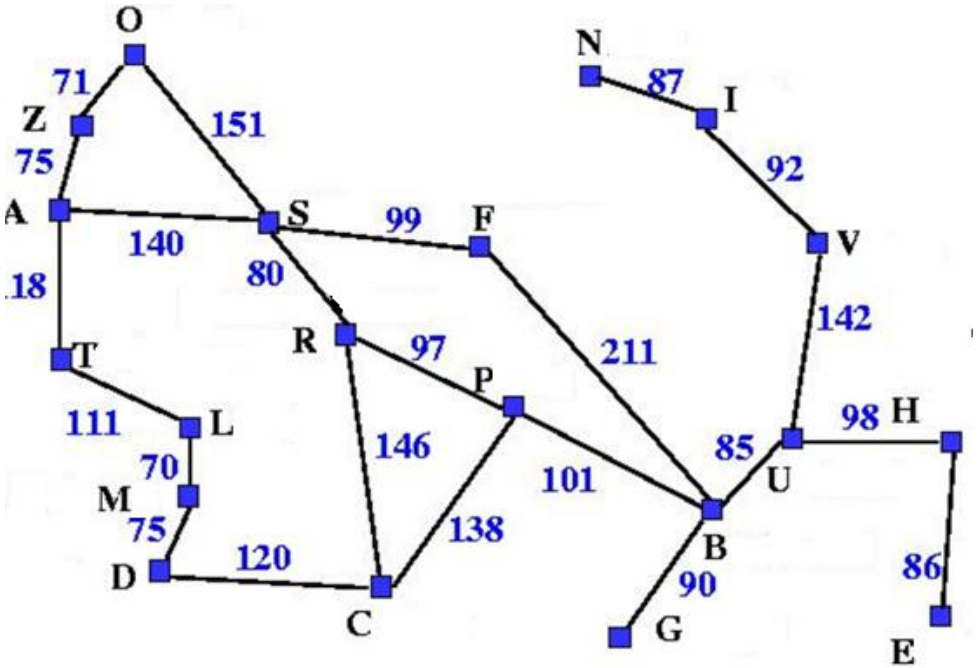
BFS Pseudocode

Best First Search (Example)

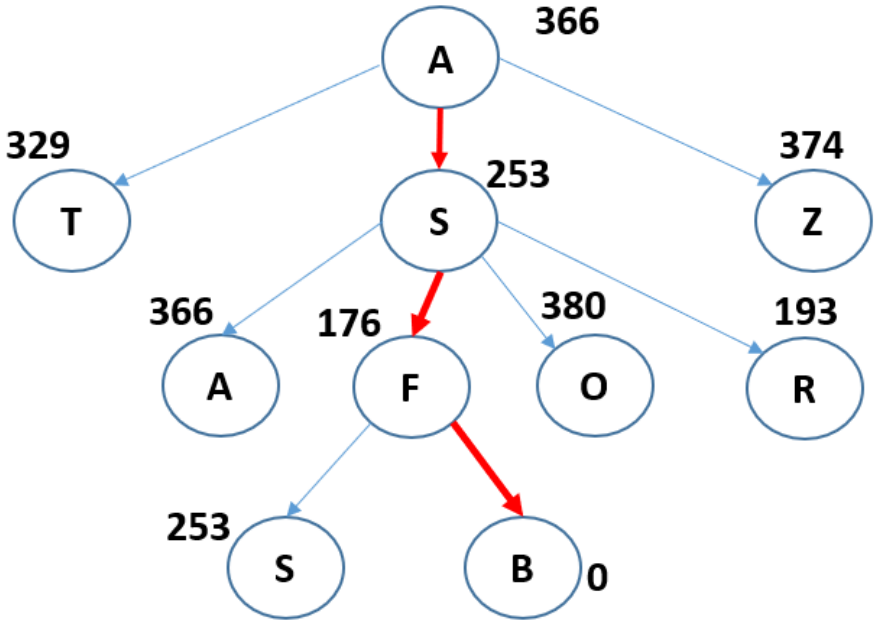


Town	h(n)
A	366
B	0
C	160
D	242
E	161
F	176
G	77
H	151
I	226
L	244
M	241
N	234
O	380
P	100
R	193
S	253
T	329
U	80
V	199
Z	374

Town	h(n)
A	366
B	0
C	160
D	242
E	161
F	176
G	77
H	151
I	226
L	244
M	241
N	234
O	380
P	100
R	193
S	253
T	329
U	80
V	199
Z	374



Best First Search (Example) cont.

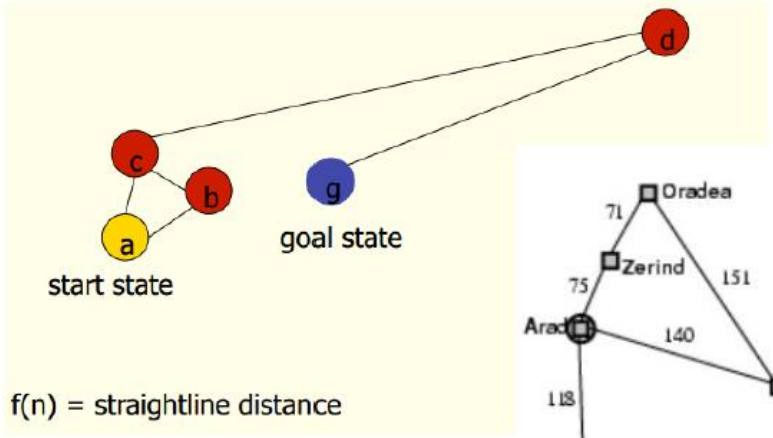


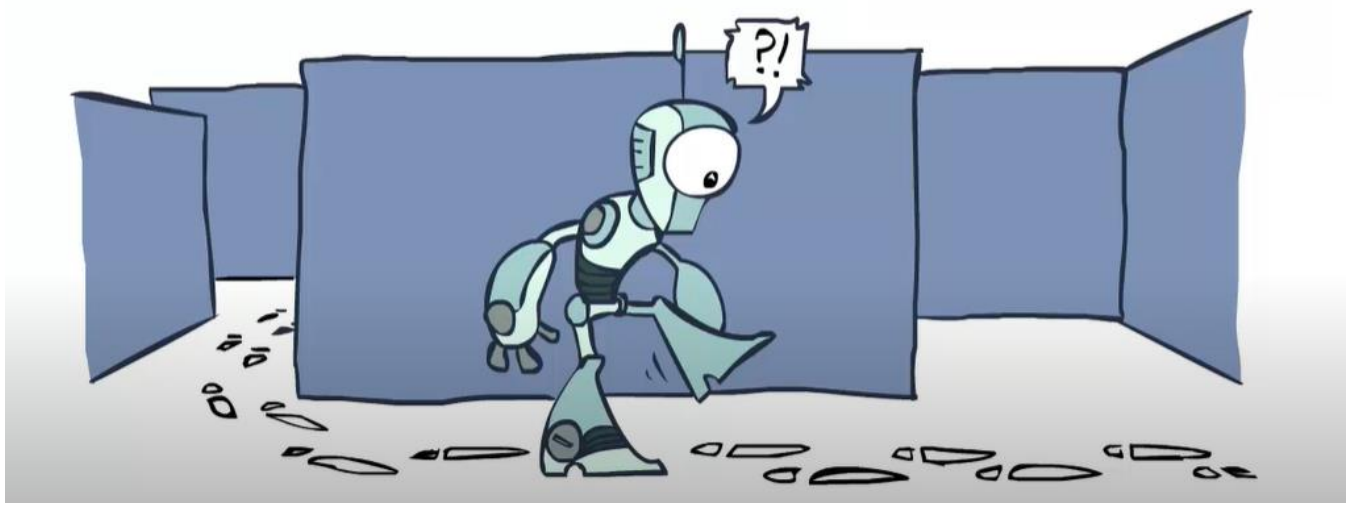
Path find cost =450

Optimal path cost = 418

Best First Search Analysis

- Complete?



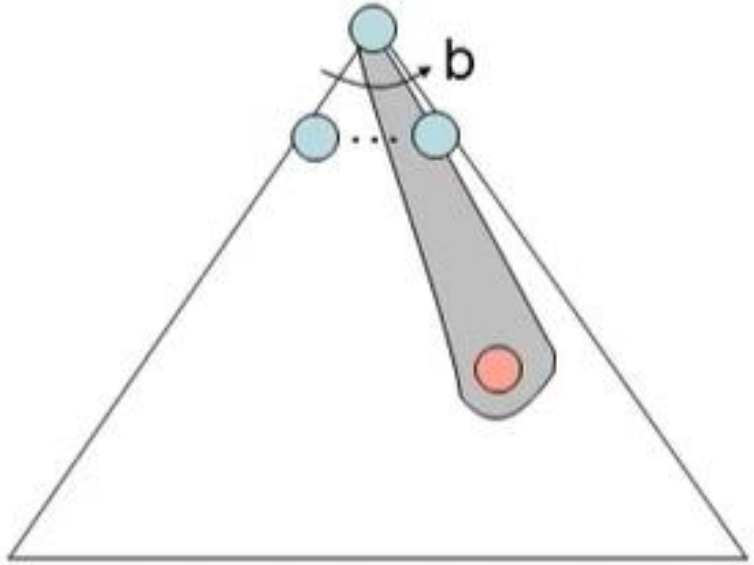


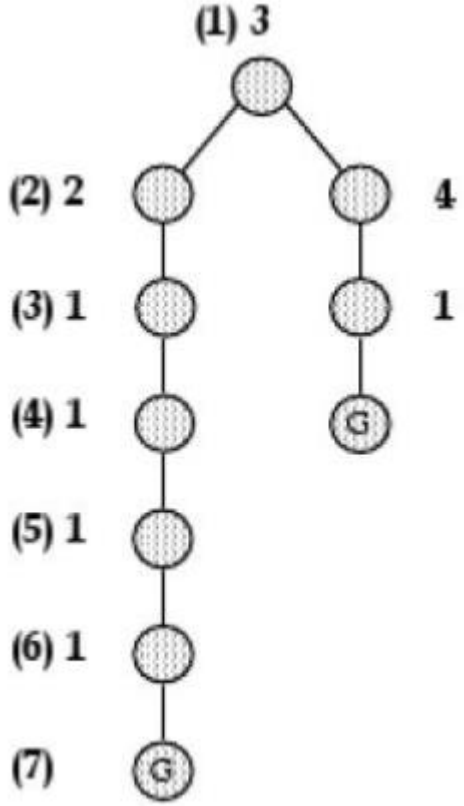
BFS Pseudocode 2

```
Best-first search {

closed list = [ ]
open list = [start node]

    do {
        if open list is empty then{
            return no solution
        }
        n = heuristic best node
        if n == final node then {
            return path from start to goal node
        }
        foreach direct available node do{
            if node not in open and not in closed list do {
                add node to open list
                set n as his parent node
        }
        delete n from open list
        add n to closed list
    } while (open list is not empty)
}
```

Best First Search Analysis

- Optimal?
- A Common case
  - Best first takes you straight to the (wrong) goal

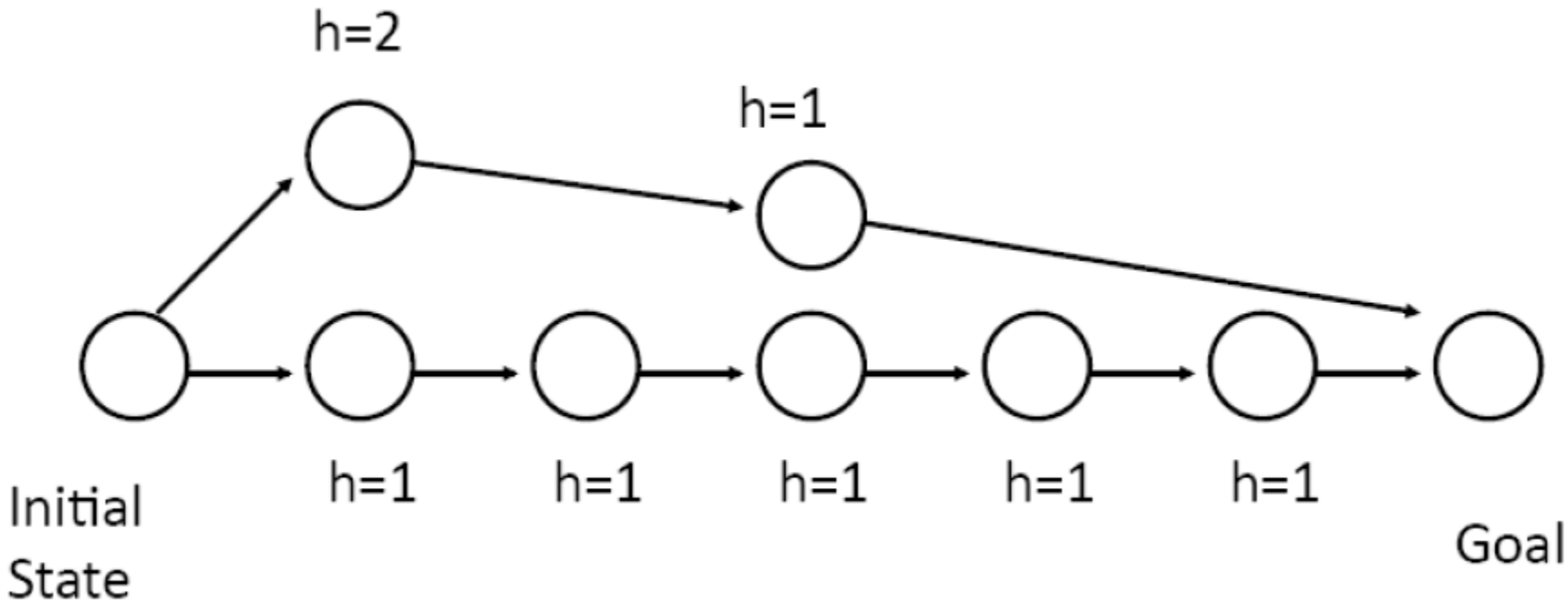




Best First Search Analysis

- Time Complexity → O(b^d)
- Space Complexity → O(b^d)
  - Where b is branching factor and d is depth of solution
  - But a good heuristic can give dramatic improvement

How can we fix the greedy problem?



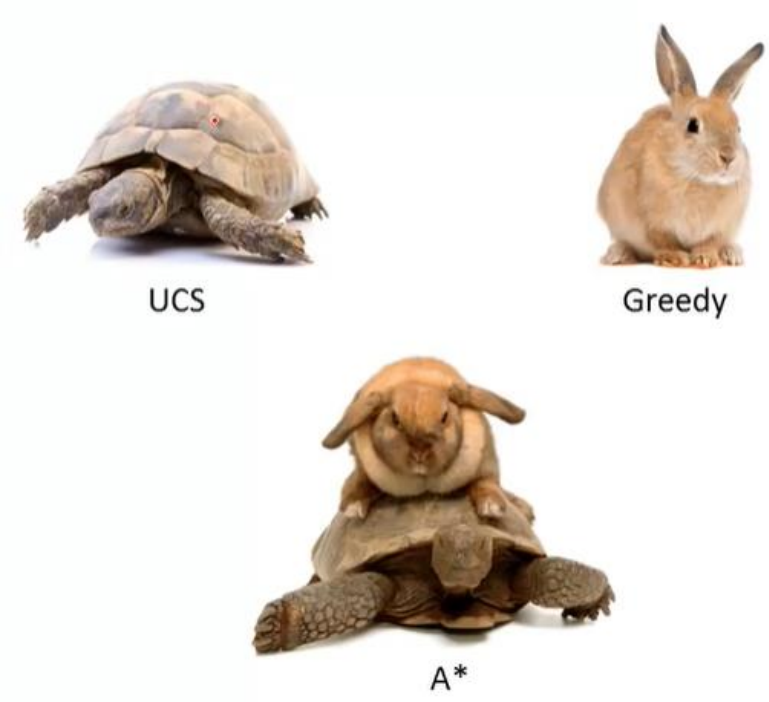
A* Search

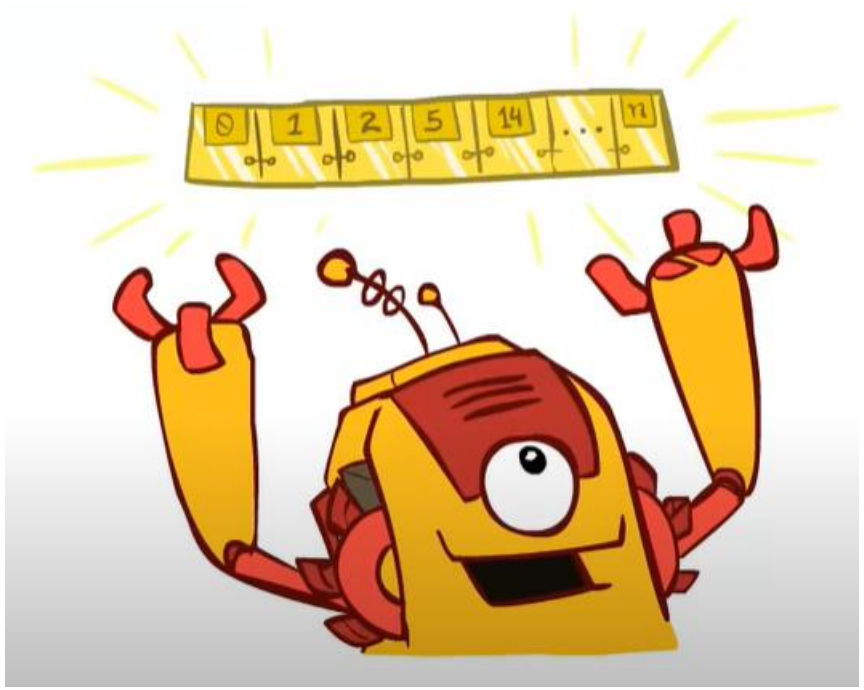
A* was created as part of the Shakey project

IEEE Milestones Award

A*

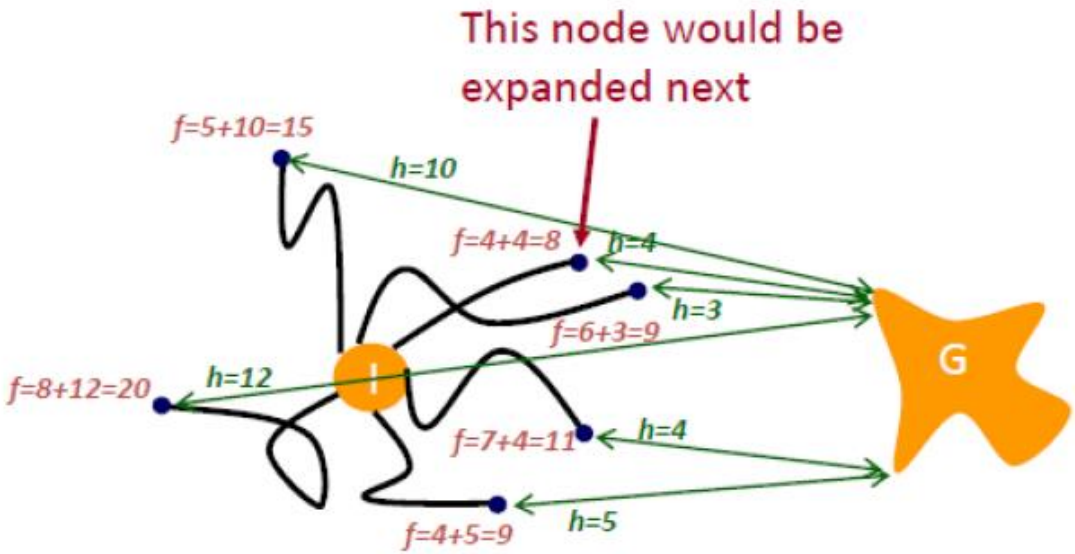
- Optimize form Best first search
- Implementation: priority queue (cost + heuristic)
- F(n) =g(n)+h(n)





A*

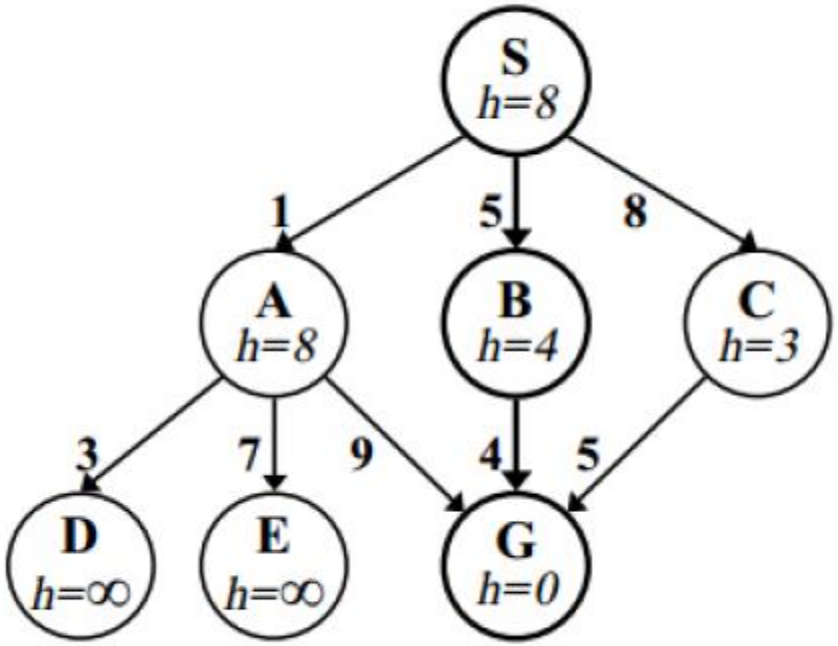
- f(*n*) = *g*(*n*) + *h*(*n*) : *Expand node that appears to* be on cheapest paths to the goal



```
A* search {

closed list = [ ]
open list = [start node]

    do {
        if open list is empty then {
            return no solution
        }
        n = heuristic best node
        if n == final node then {
            return path from start to goal node
        }
        foreach direct available node do{
            if current node not in open and not in closed list do {
                add current node to open list and calculate heuristic
                set n as his parent node
            }
            else{
                check if path from star node to current node is
                better;
                if it is better calculate heuristics and transfer
                current node from closed list to open list
                set n as his parrent node
            }
        delete n from open list
        add n to closed list
    } while (open list is not empty)

}
```

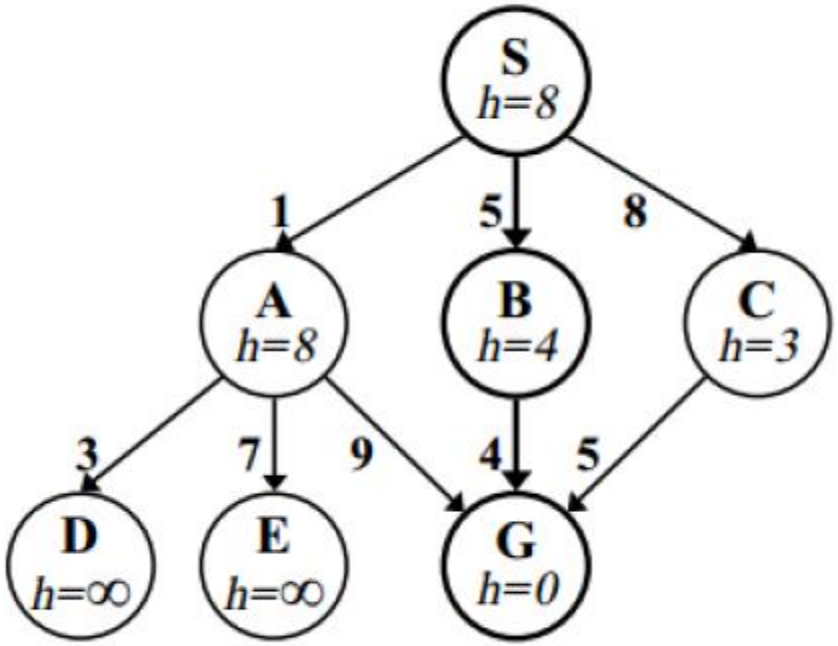
A* Pseudocode

A* (Example1-Table)

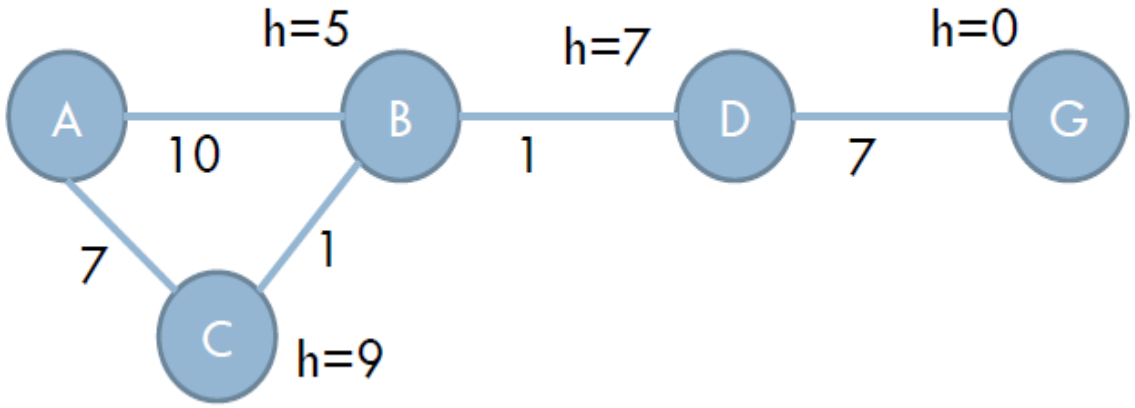


Close	Open
	S

A* (Example1-Tree)

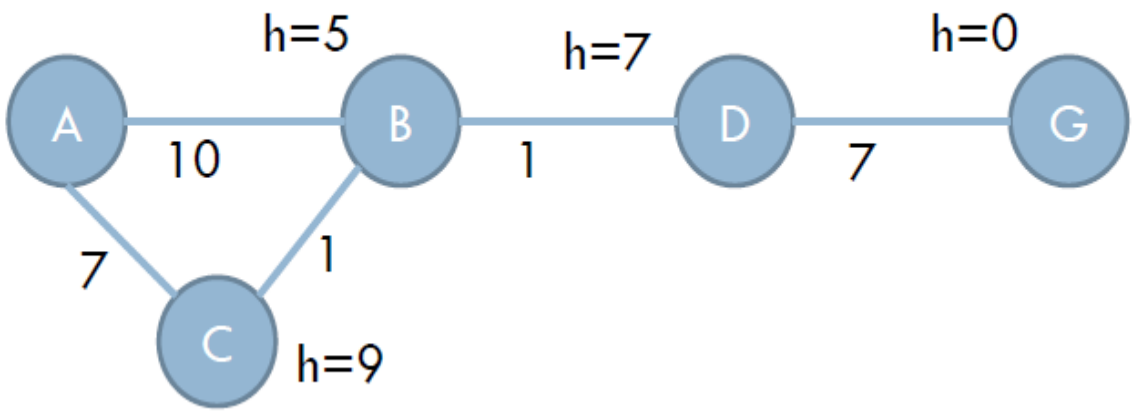


A* (Example2-Table)

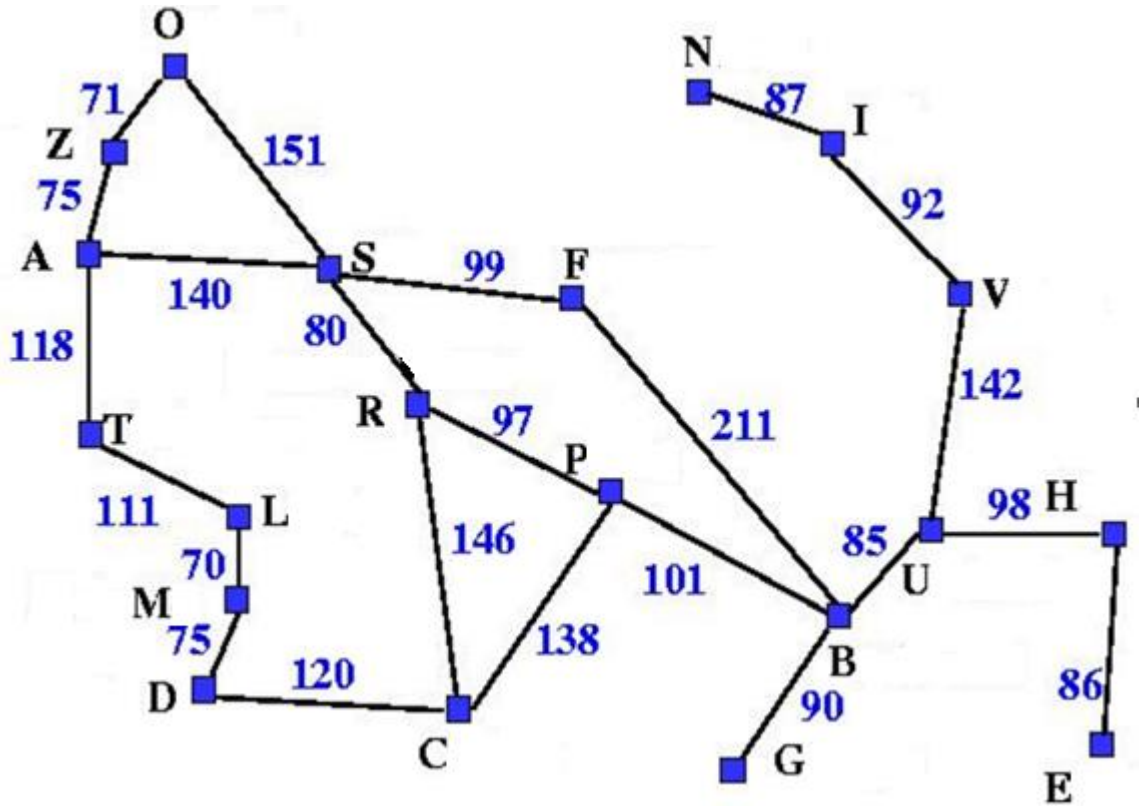


Close	Open
	A

A* (Example2-Tree)

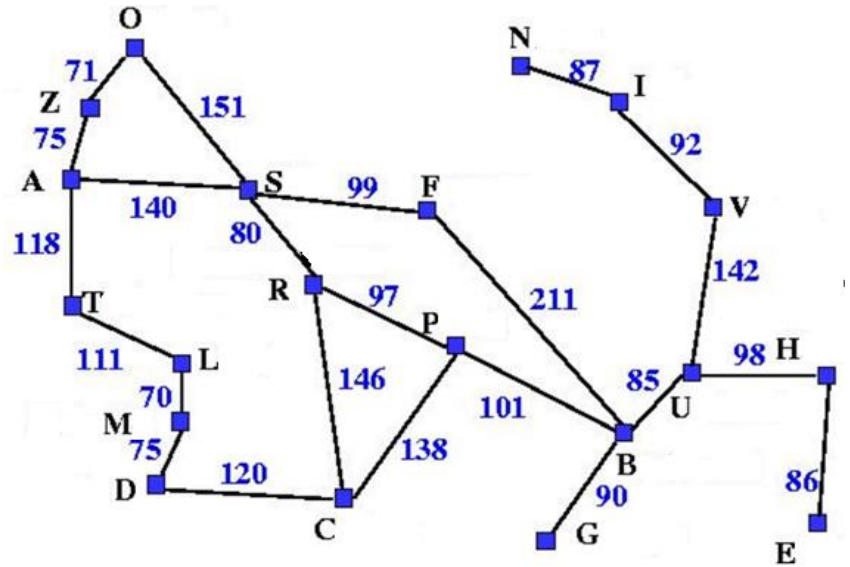


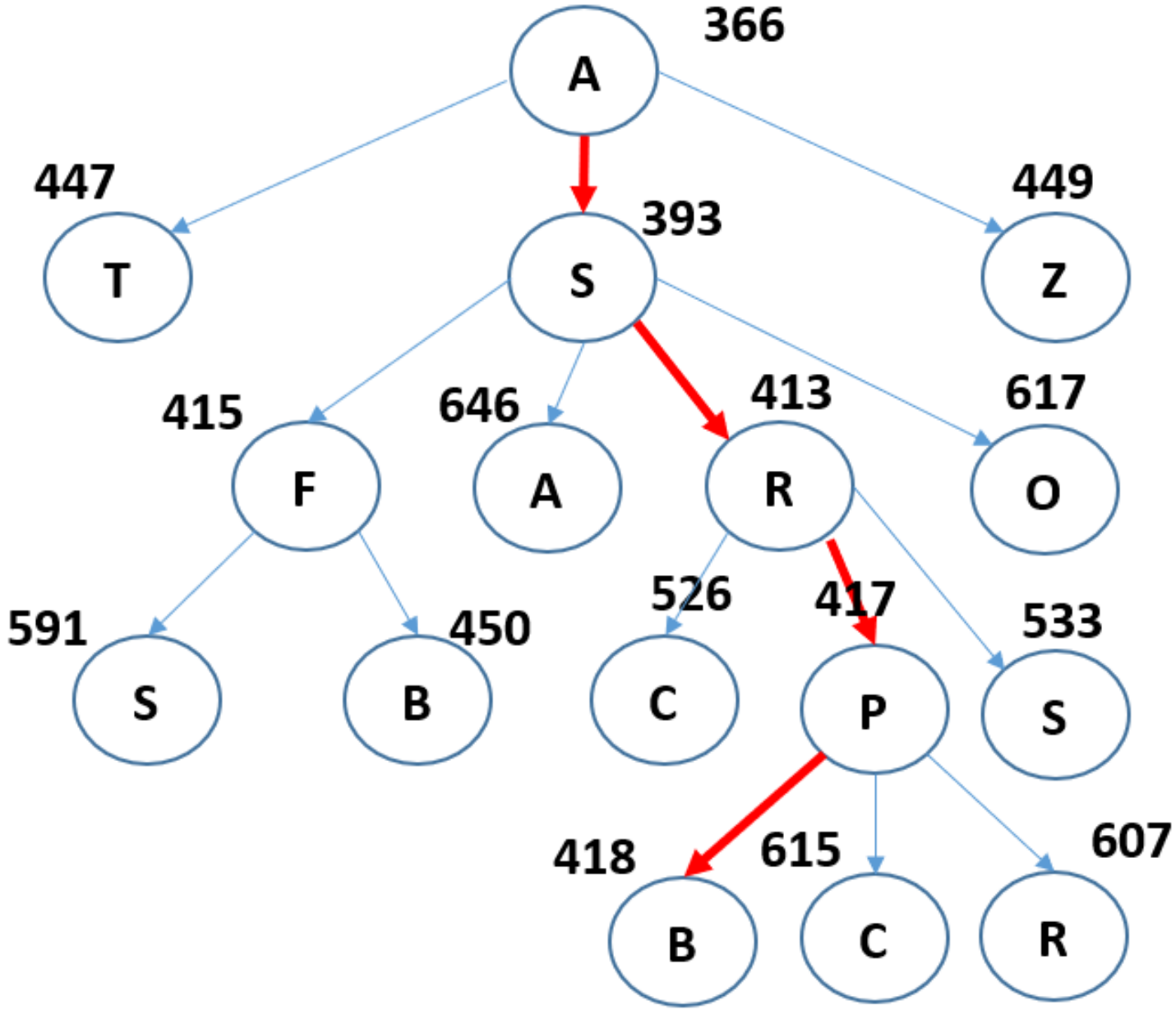
A* (Path Finding Example)



Town	h(n)
A	366
B	0
C	160
D	242
E	161
F	176
G	77
H	151
I	226
L	244
M	241
N	234
O	380
P	100
R	193
S	253
T	329
U	80
V	199
Z	374

Town	h(n)
A	366
B	0
C	160
D	242
E	161
F	176
G	77
H	151
I	226
L	244
M	241
N	234
O	380
P	100
R	193
S	253
T	329
U	80
V	199
Z	374




366
A
447
T
393
S
449
Z
415
F
646
A
413
R
617
O
591
S
450
B
526
C
417
P
533
S
418
B
615
C
607
R

A* (8 puzzle Example)

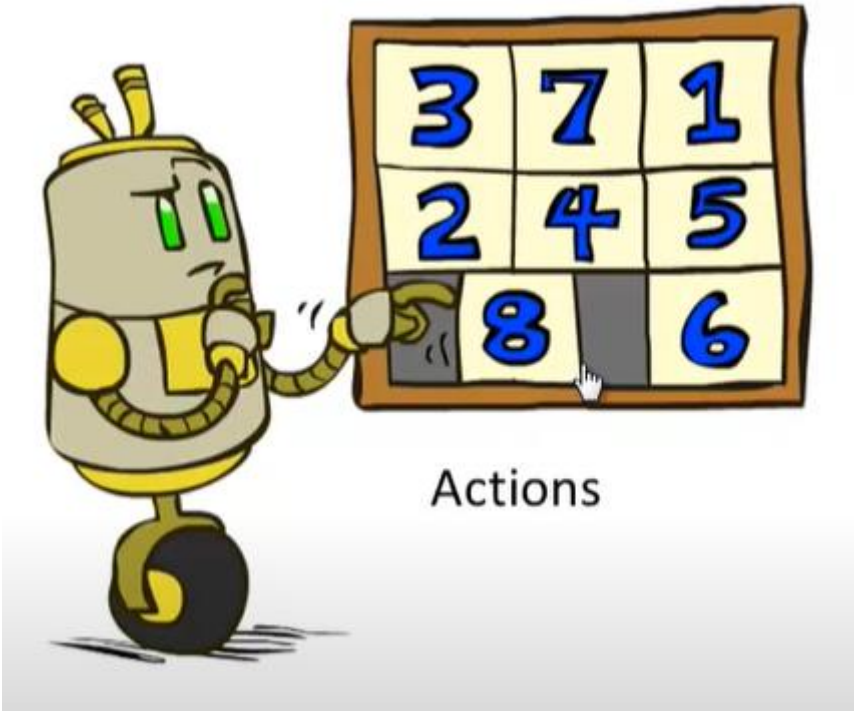
Start state

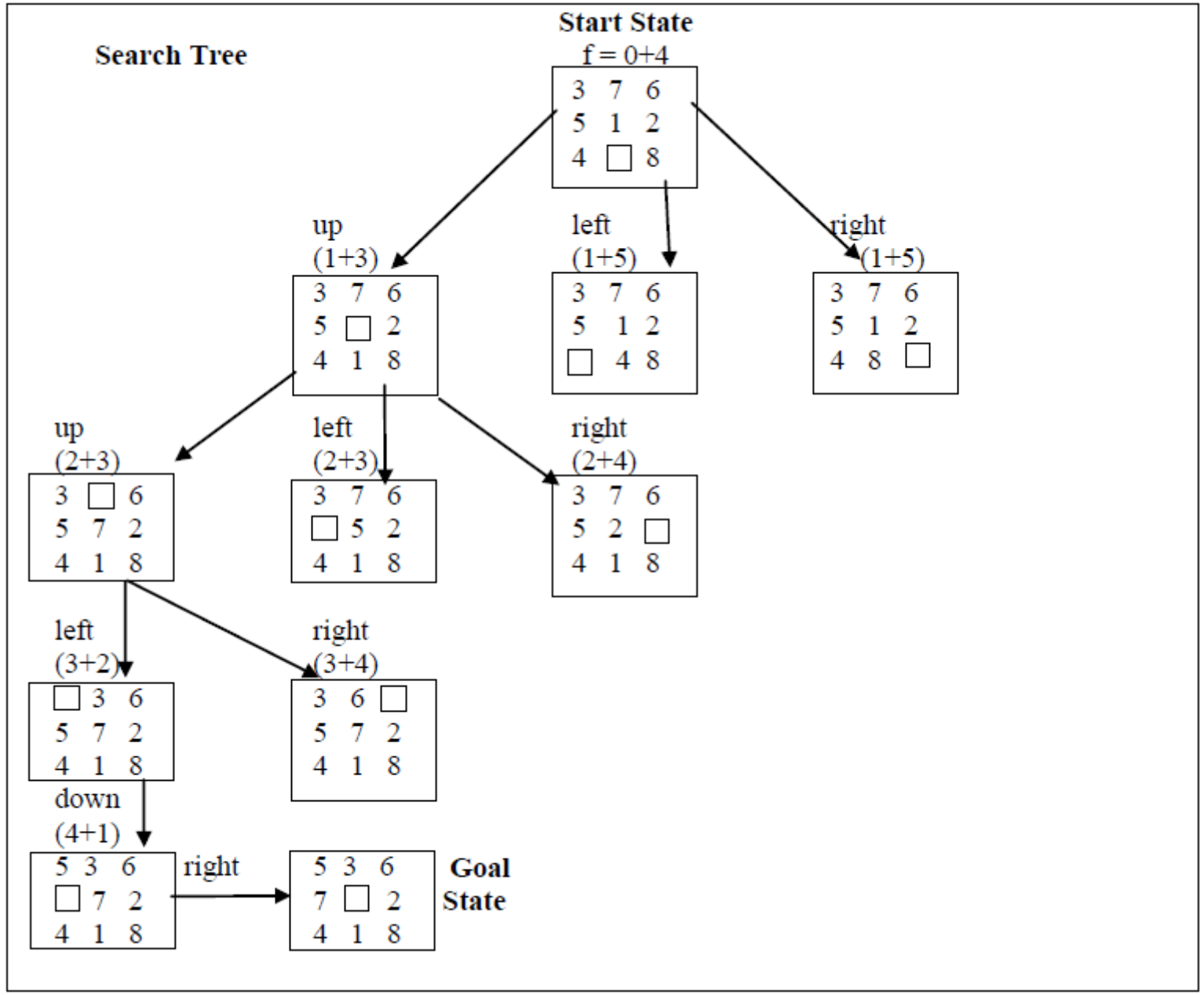
3	7	6
5	1	2
4	□	8

Goal state

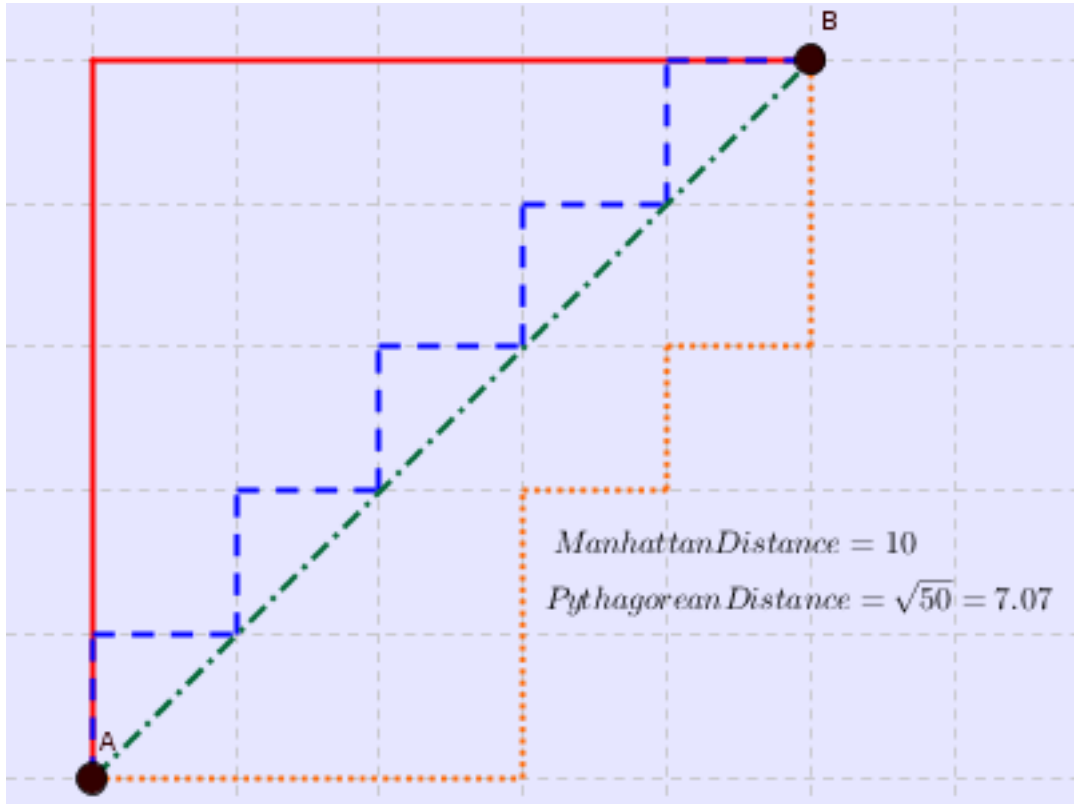
5	3	6
7	□	2
4	1	8

- The choice of evaluation function critically determines search results.
- Consider Evaluation function **f (X) = g (X) + h(X)**
  - h (X)= the number of tiles not in their goal position in a given state X
  - g(X)=depth of node X in the search tree
- For Initial node
  - **f(s)** = 4
- Apply A* algorithm to solve it.




Search Tree
Start State
f = 0+4
3 7 6
5 1 2
4 □ 8
up
(1+3)
3 7 6
5 □ 2
4 1 8
left
(1+5)
3 7 6
5 1 2
□ 4 8
right
(1+5)
3 7 6
5 1 2
4 8 □
up
(2+3)
3 □ 6
5 7 2
4 1 8
left
(2+3)
3 7 6
□ 5 2
4 1 8
right
(2+4)
3 7 6
5 2 □
4 1 8
left
(3+2)
□ 3 6
5 7 2
4 1 8
right
(3+4)
3 6 □
5 7 2
4 1 8
down
(4+1)
5 3 6
□ 7 2
4 1 8
right
5 3 6
7 □ 2
4 1 8
Goal
State

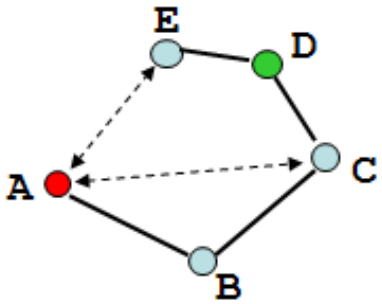
Manhattan Distance



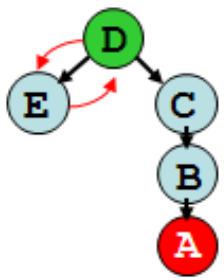
$$|x1 - x2| + |y1 - y2|$$

Straight-line distance

- The distance between two locations on a map can be known without knowing how they are linked by roads (i.e. the absolute path to the goal).



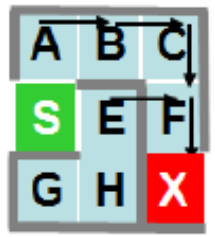
Problem Space



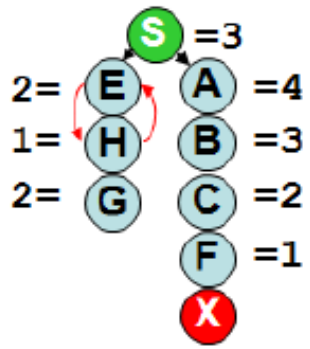
Search Tree

Manhattan Distance

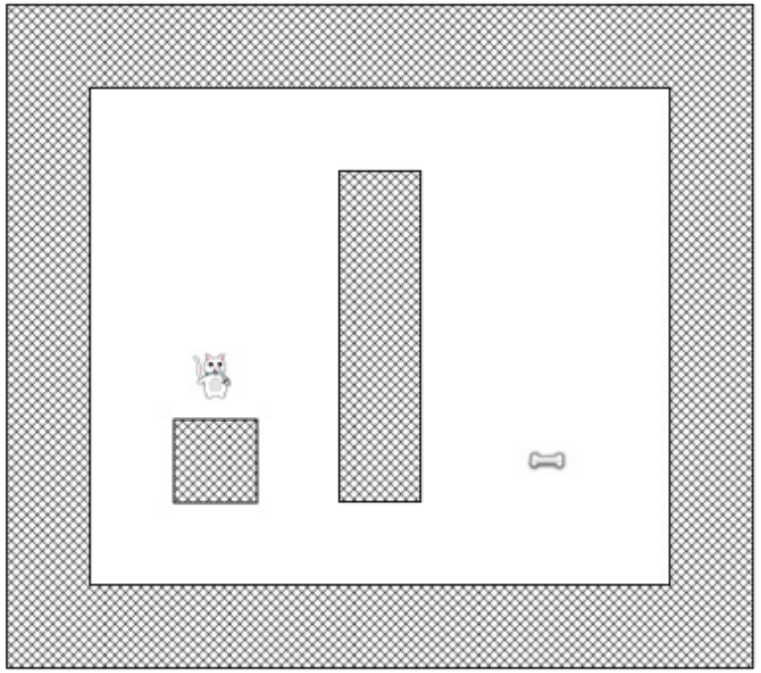
- The smallest number of vertical and horizontal moves needed to get to the goal (ignoring obstacles).

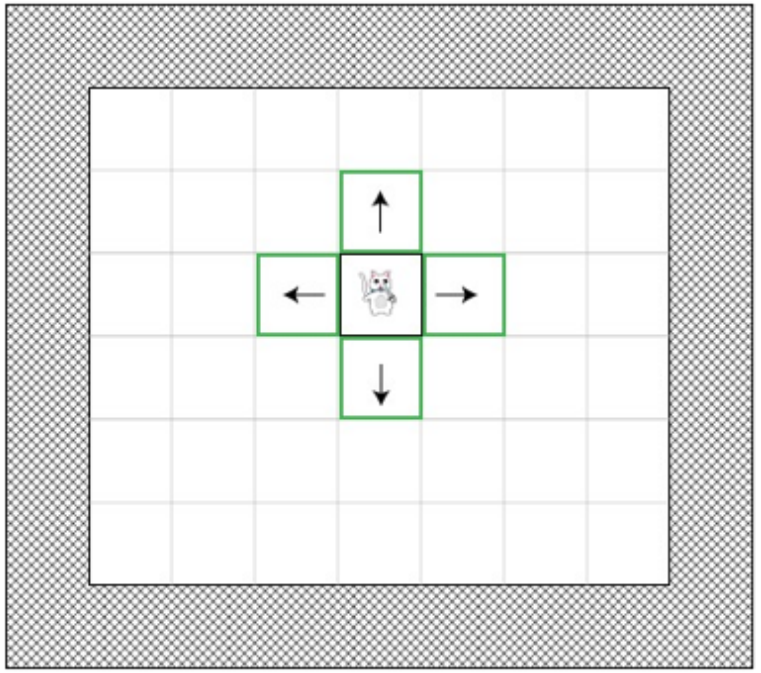


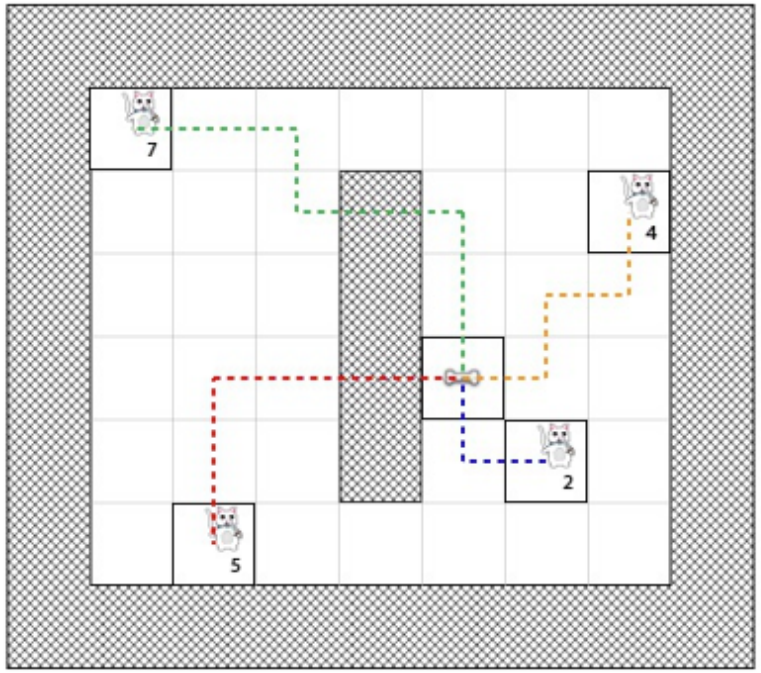
Manhattan Distance
A = 4
E = 2

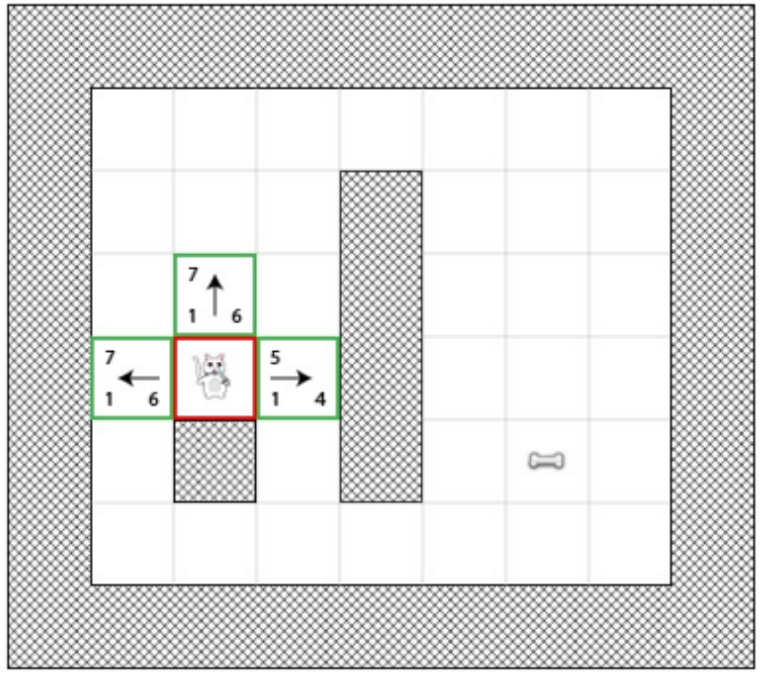


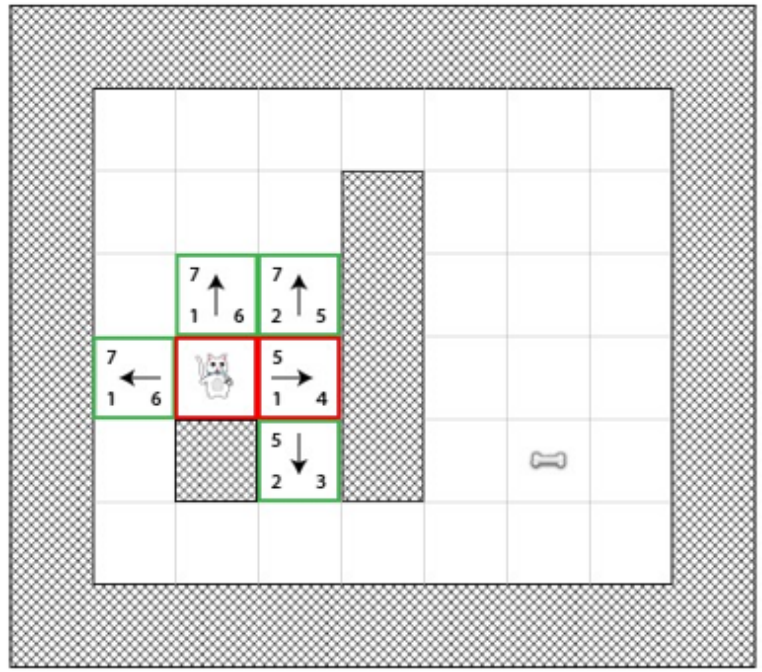
Maze World

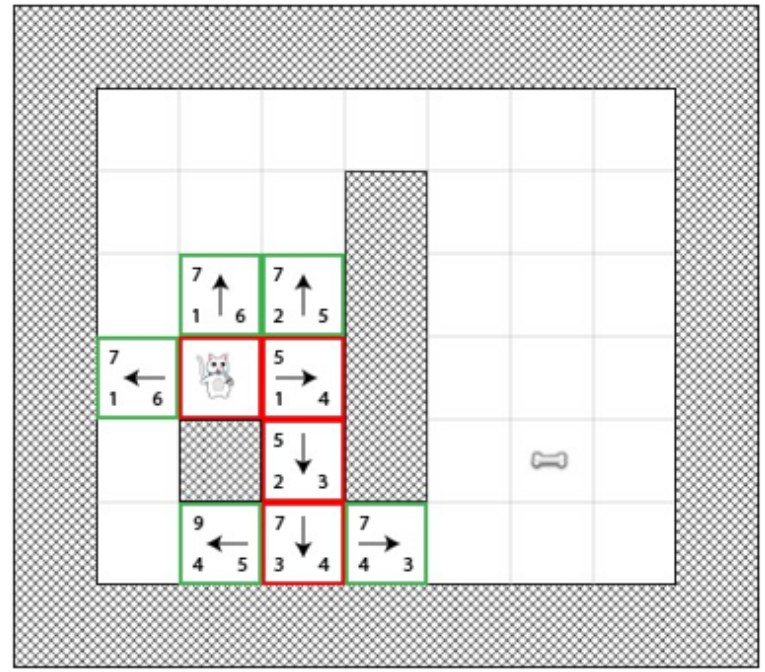


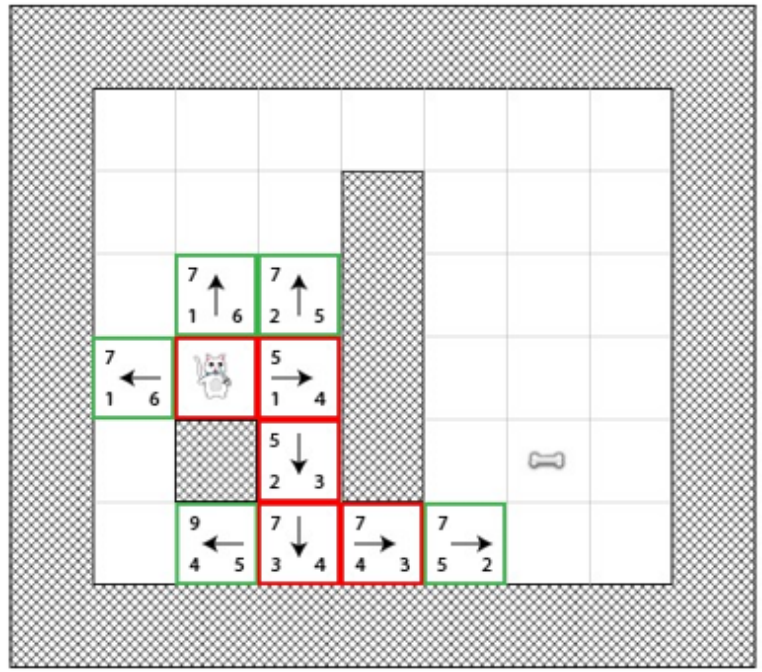


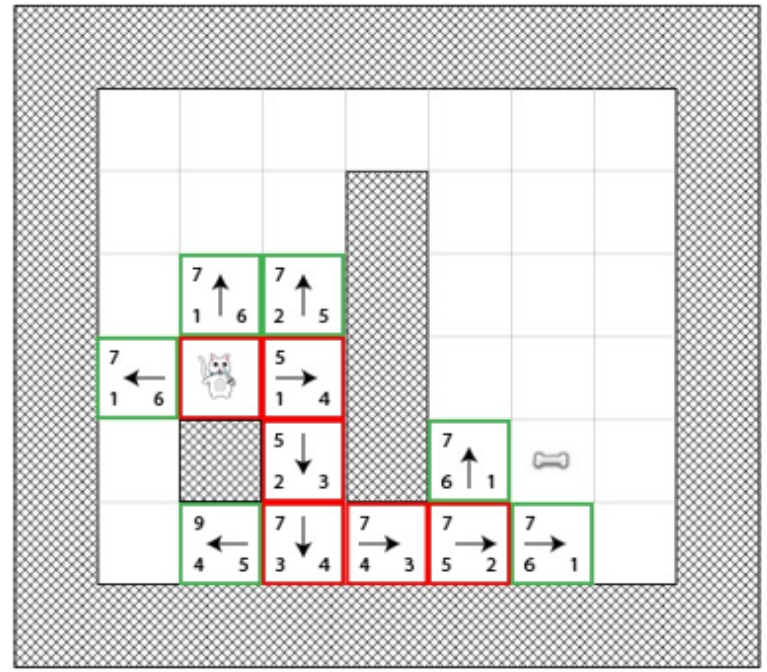





7 1 6
7 2 5
7 1 6
5 1 4
5 2 3


7 1 6
7 2 5
7 1 6
5 1 4
5 2 3
9 4 5
7 3 4
7 4 3

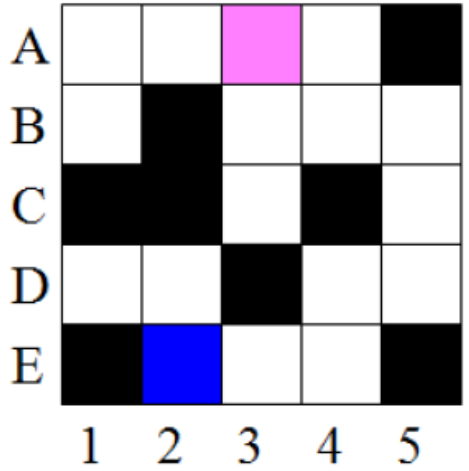

7 1 6
7 2 5
7 1 6
5 1 4
5 2 3
9 4 5
7 3 4
7 4 3
7 5 2


7 1 6
7 2 5
7 1 6
5 1 4
5 2 3
7 6 1
9 4 5
7 3 4
7 4 3
7 5 2
7 6 1

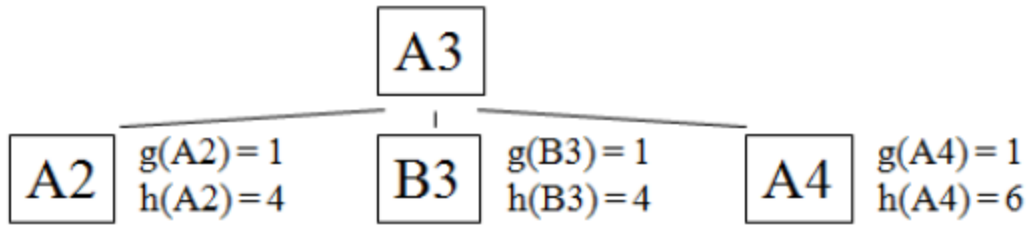
A* (Maze Example)

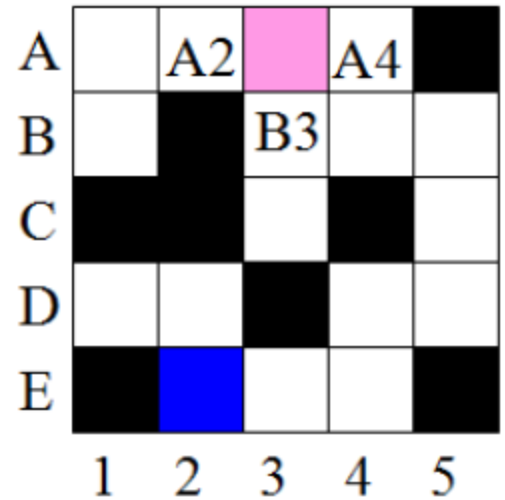
- **Problem**: To get from square **A3** to square **E2**, one step at a time, avoiding obstacles (black squares).

- **Operators**: (in order)
  - **go_left**(n)
  - **go_down**(n)
  - **go_right**(n)
- each operator costs 1.



- **Heuristic**: Manhattan distance

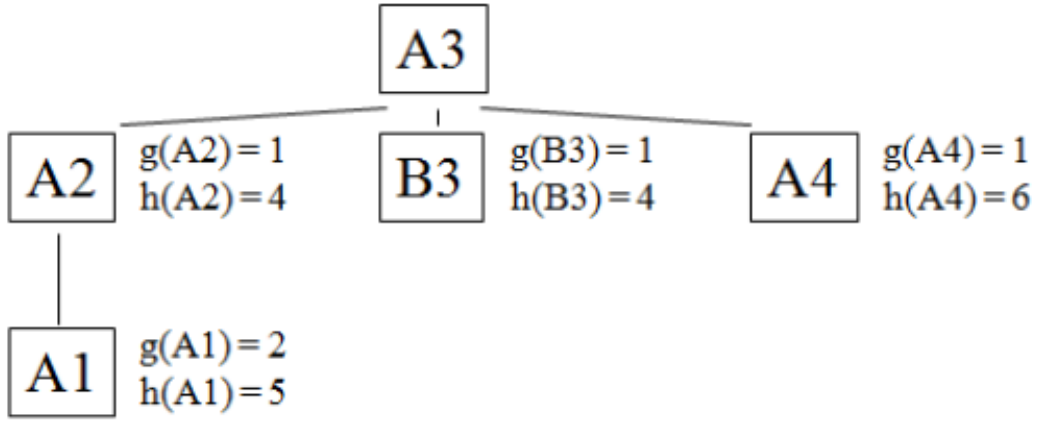


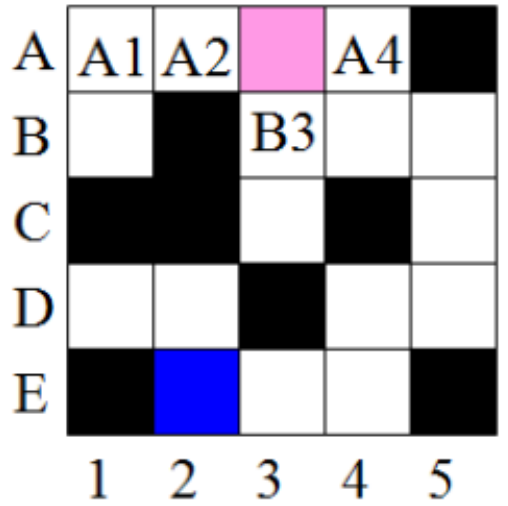


Operators: (in order)

- `go_left`(n)
- `go_down`(n)
- `go_right`(n)

each operator costs 1.

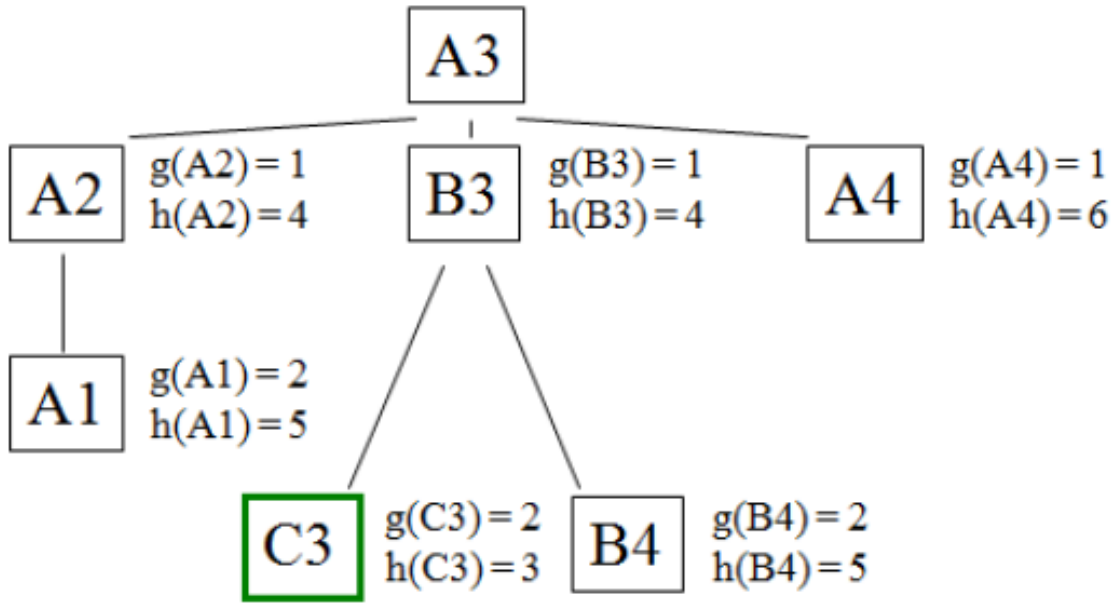


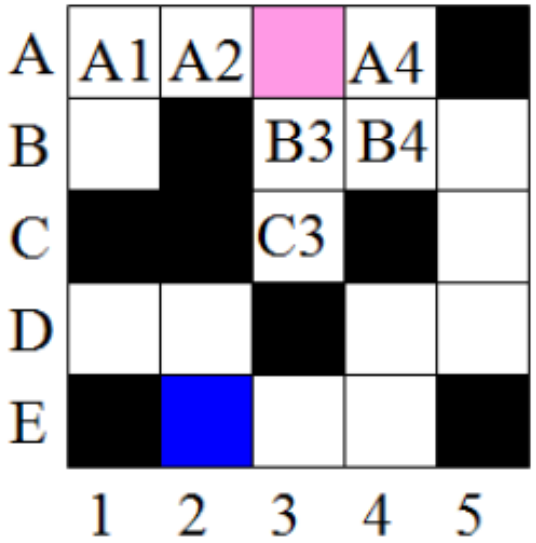


Operators: (in order)

- `go_left`(n)
- `go_down`(n)
- `go_right`(n)

each operator costs 1.



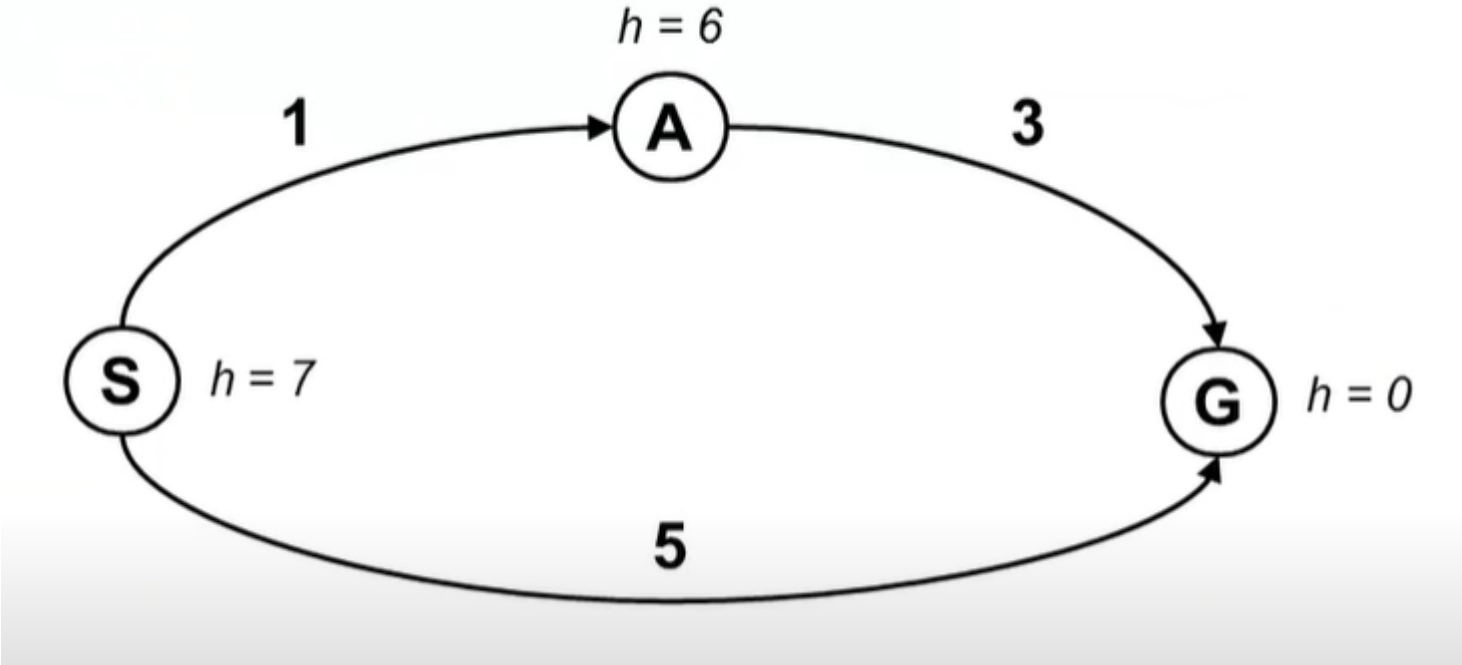


Operators: (in order)

- `go_left`(n)
- `go_down`(n)
- `go_right`(n)

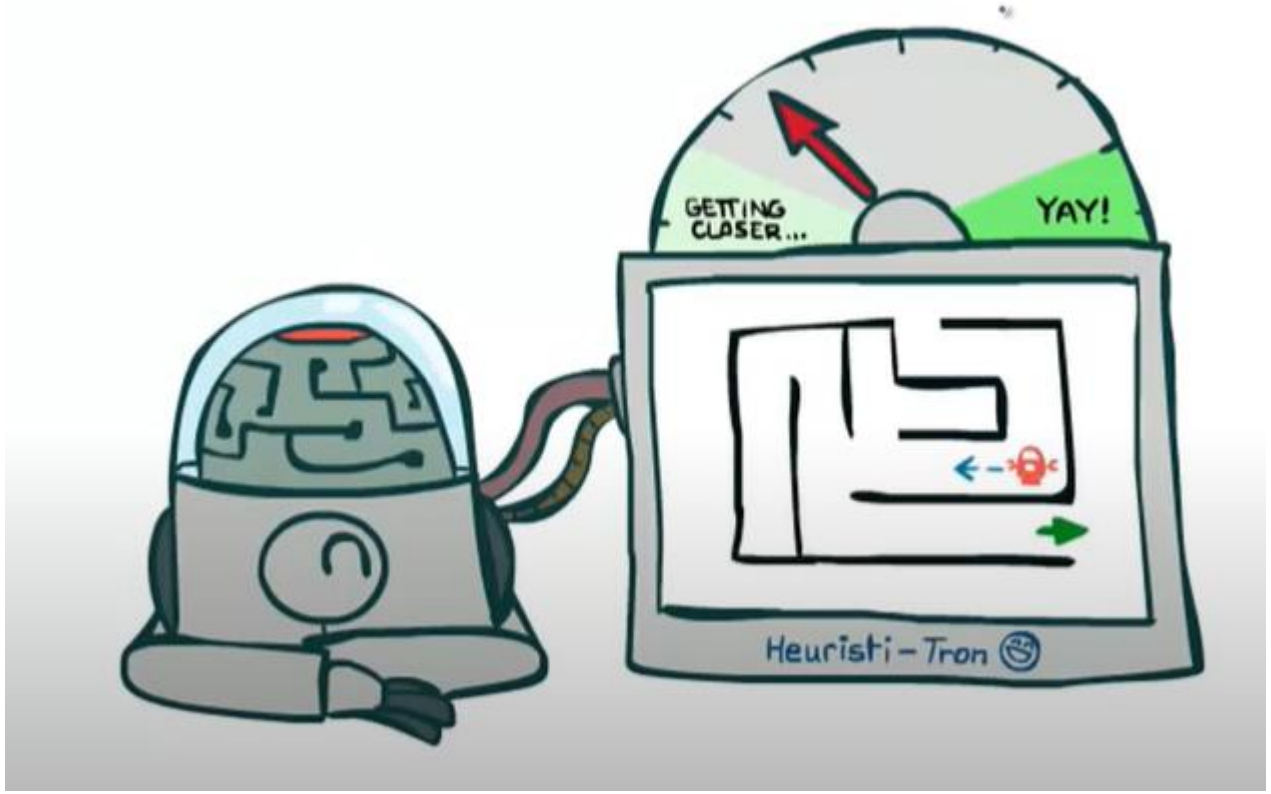
each operator costs 1.

Is A* Optimal?

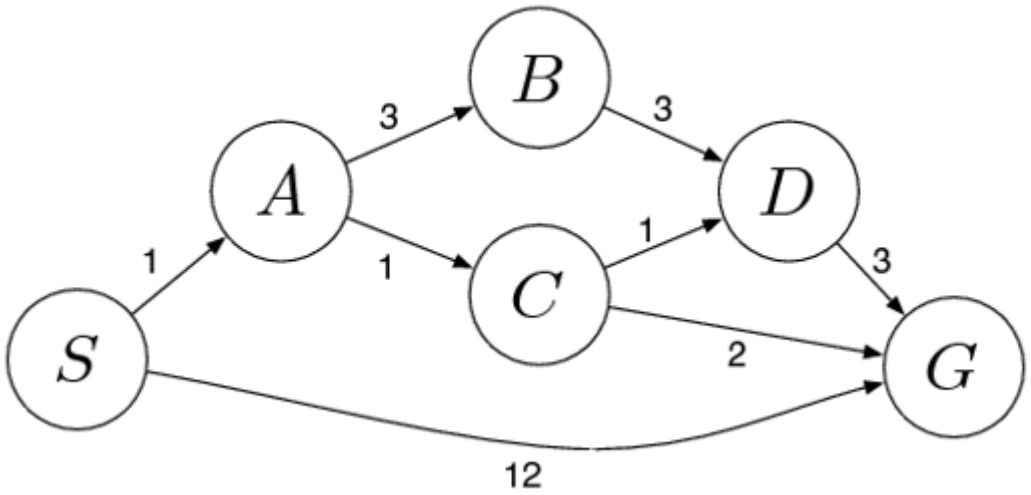


- What went wrong?

Admissible Heuristic



Admissible Heuristic (Example)

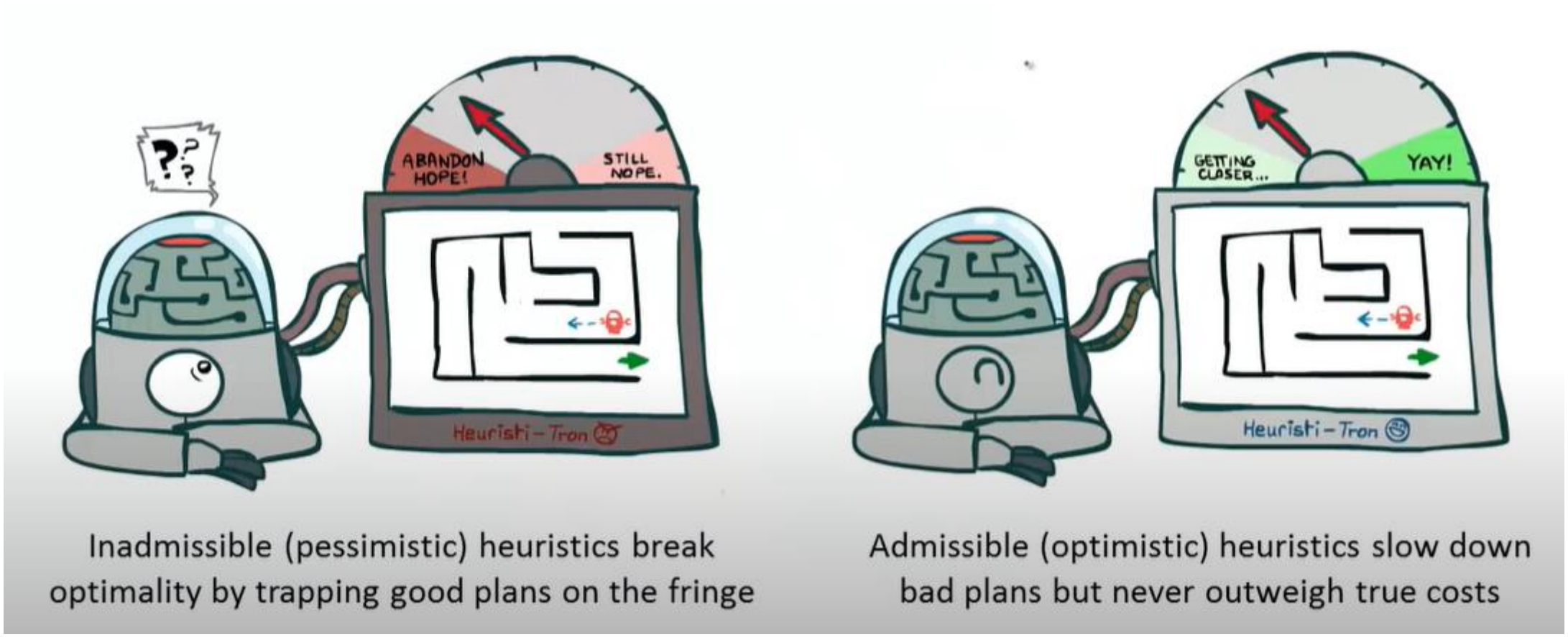


State (n)	H(n)
S	5
A	3
B	6
C	2
D	3
G	0

H(n) <= distance to goal(n)

State (n)	H(n)	distance to goal(n)
S		
A		
B		
C		
D		
G		

Admissibility

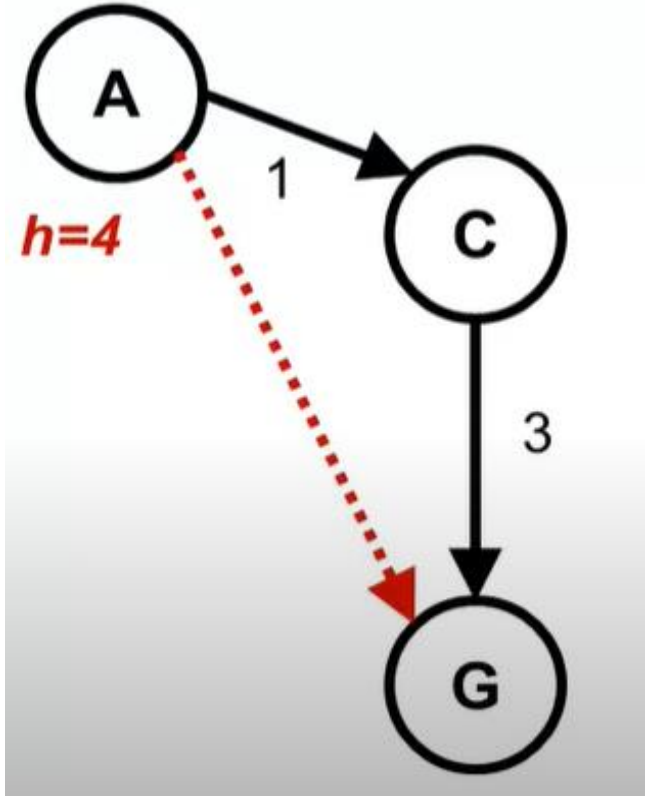


Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe

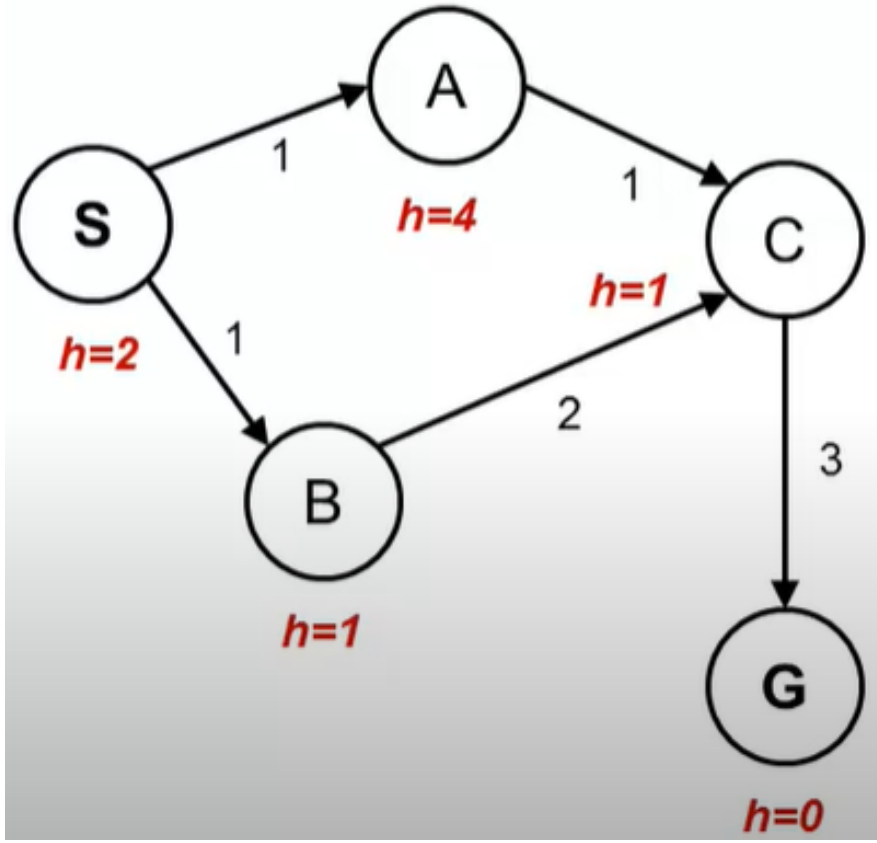
Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs

Consistency of Heuristic (Monotonicity)

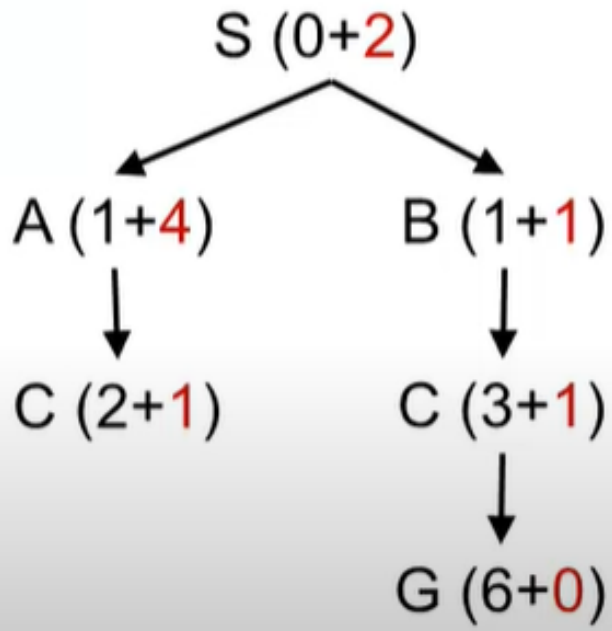
- Main idea: estimated heuristic costs <= actual costs
  - Admissibility: heuristic cost <=actual cost to goal
    - H(A) <= actual cost from A to G
  - Consistency: heuristic “arc’’ cost <= actual cost for each arc
    - H(A)-H(C) <= cost(A to C)
- Consequences of consistency
  - The f value along a path never decreases



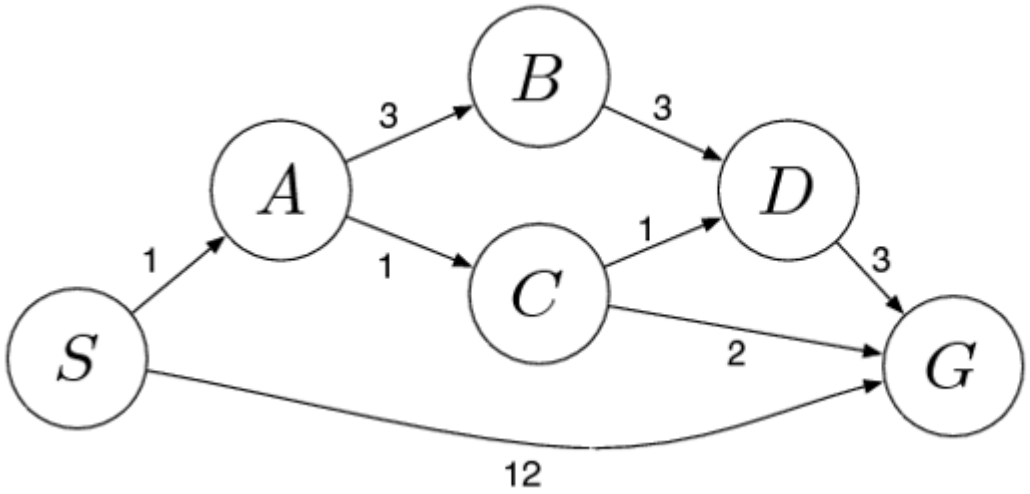
State space graph



Search tree



Monotonic Heuristic (Example)



State (n)	H(n)
S	5
A	3
B	6
C	2
D	3
G	0

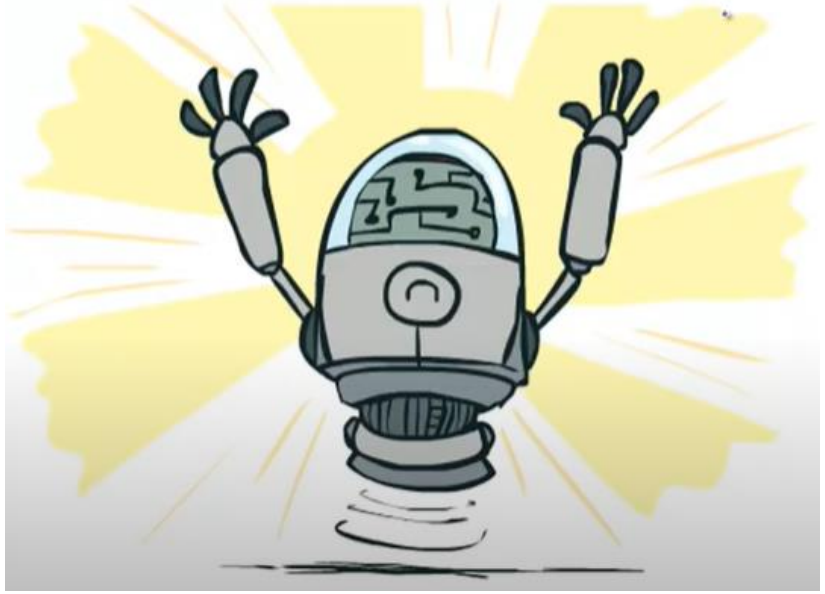
If it's monotonic then it's admissible

H(n) <= h(n')+c(n-n')

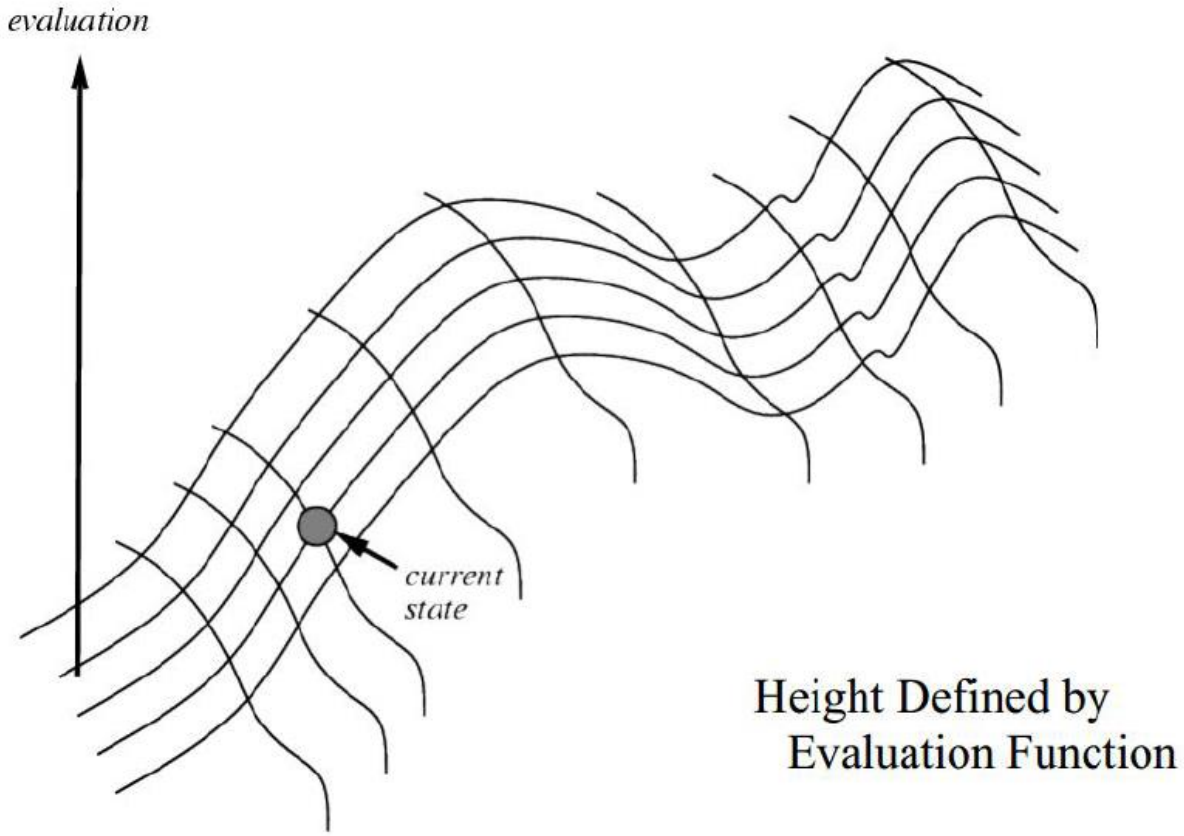
n-n'	H(n)	H(n')	C(n-n')	H(n')+c(n-n')
S-A				
S-G				
A-B				
A-C				
B-D				
C-D				
C-G				
D-G				

Optimality

- Tree Search
  - A* is optimal if heuristic is admissible
- Graph Search
  - A* is optimal if heuristic is consistent
- Consistency implies admissibility



Hill Climbing on a surface of states

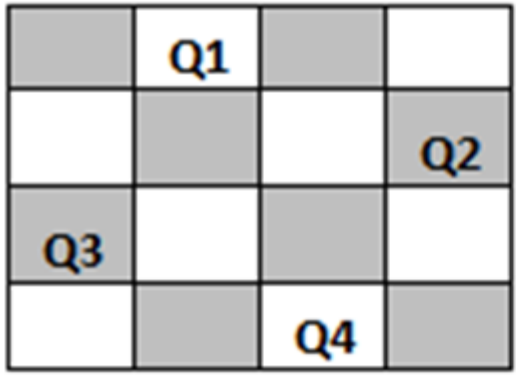


Hill Climbing (Local Search)

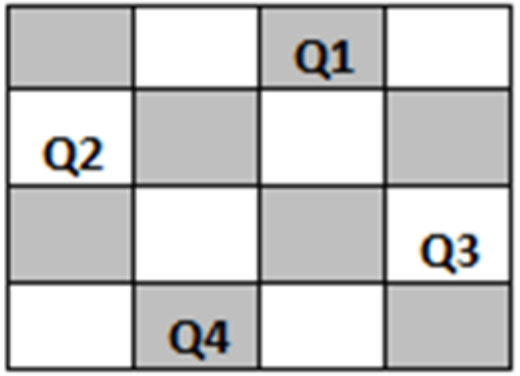
- It is an iterative algorithm that starts with an arbitrary solution to a problem, then attempts to find a better solution by incrementally changing a single element of the solution. If the change produces a better solution, an incremental change is made to the new solution, repeating until no further improvements can be found.
- In many optimization problems, the path to the goal I irrelevant; the goal state itself is the solution
- In such cases, we can use local search algorithms
- Keep single “current” state, try to improve it

N Queen Problem

- The N Queen is the problem of placing N chess queens on an N×N chessboard so that no two queens attack each other
- Example 4 Queen



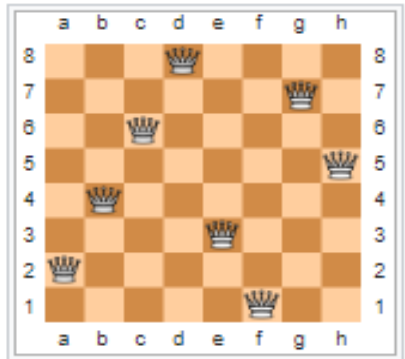
Solution 1

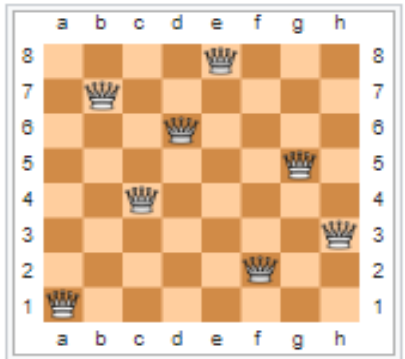


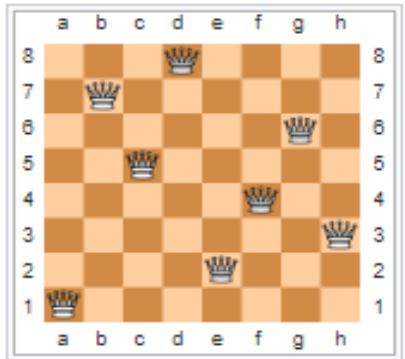
Solution 2

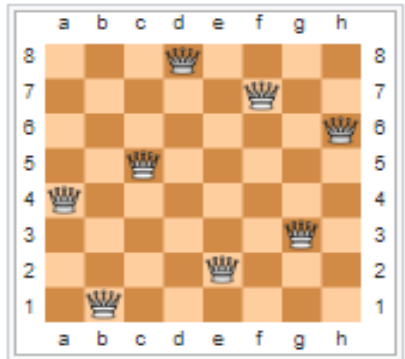
.

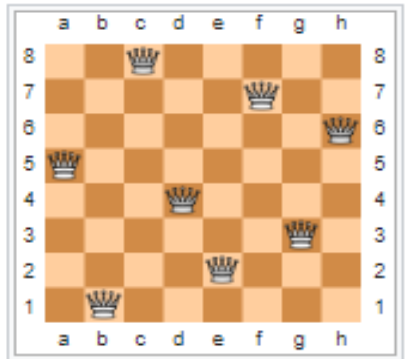
N (problem size)	*Number of solutions*	*Search space (N!)*	*Solution density* (per 10^6)
4	2	24	83,333
5	10	120	83,333
6	4	720	5,555
7	40	5,040	7,936
8	92	40,320	2,281
9	352	362,880	970
10	724	3,628,800	199
11	2,680	39,916,800	67
12	14,032	479,001,600	29

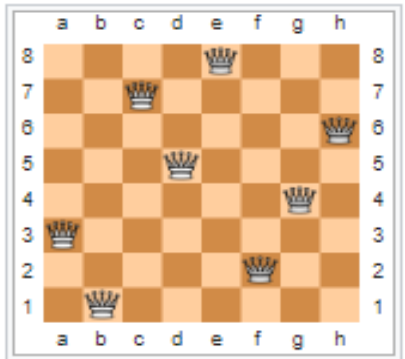

Solution 1

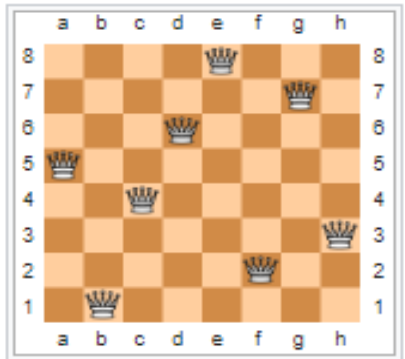

Solution 2

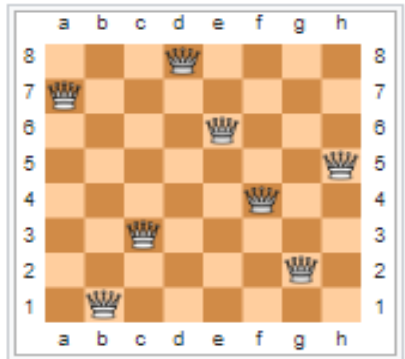

Solution 3

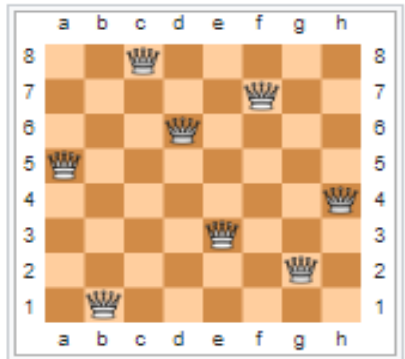

Solution 4

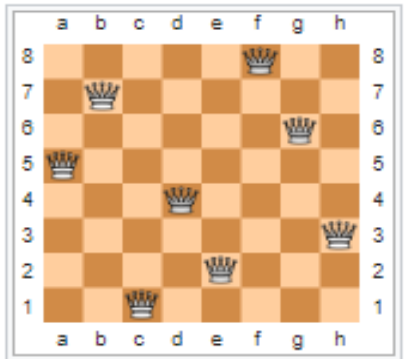

Solution 5

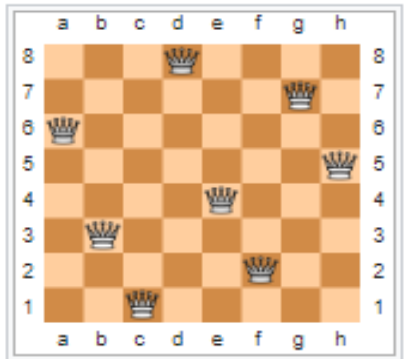

Solution 6

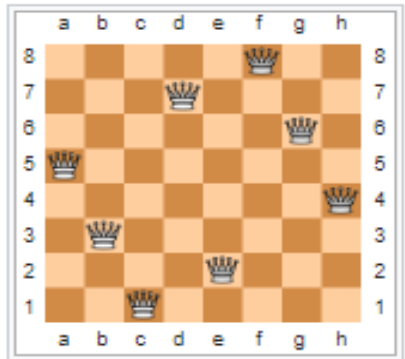

Solution 7


Solution 8


Solution 9


Solution 10


Solution 11


Solution 12

Hill Climbing

- Looks one step ahead to determine if any successor is better than the current state; if there is, move to the best successor.

Rule: If there exists a successor s for the current state n such that

  - $h(s) < h(n)$ and
  - $h(s) \leq h(t)$ for all the successors t of n,

then move from n to s. Otherwise, halt at n.

- Similar to Greedy search in that it uses h(), but does not allow backtracking or jumping to an alternative path since it doesn't "remember" where it has been.
- Appropriate for problems in which we need only a solution. The path to the solution is immaterial
  - E.g: 8- Queen problem, Graph colouring problem

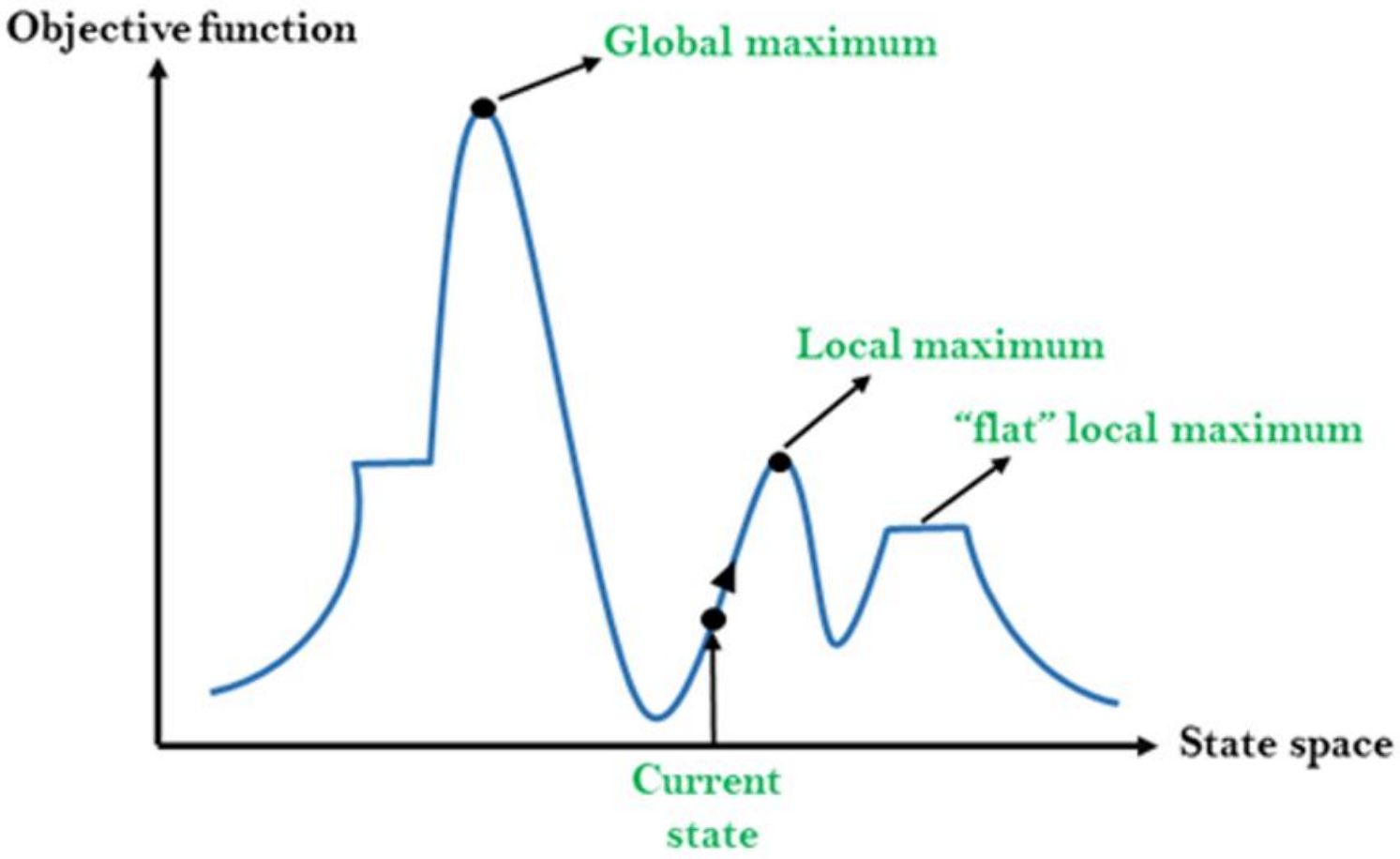
Pseudocode Hill Climbing

```
function HILL-CLIMBING(problem) returns a state that is a local maximum
   inputs: problem, a problem
   local variables: current, a node
                    neighbor, a node

   current ← MAKE-NODE(INITIAL-STATE[problem])
   loop do
      neighbor ← a highest-valued successor of current
      if VALUE[neighbor] < VALUE[current] then return STATE[current]
      current ← neighbor
   end
```

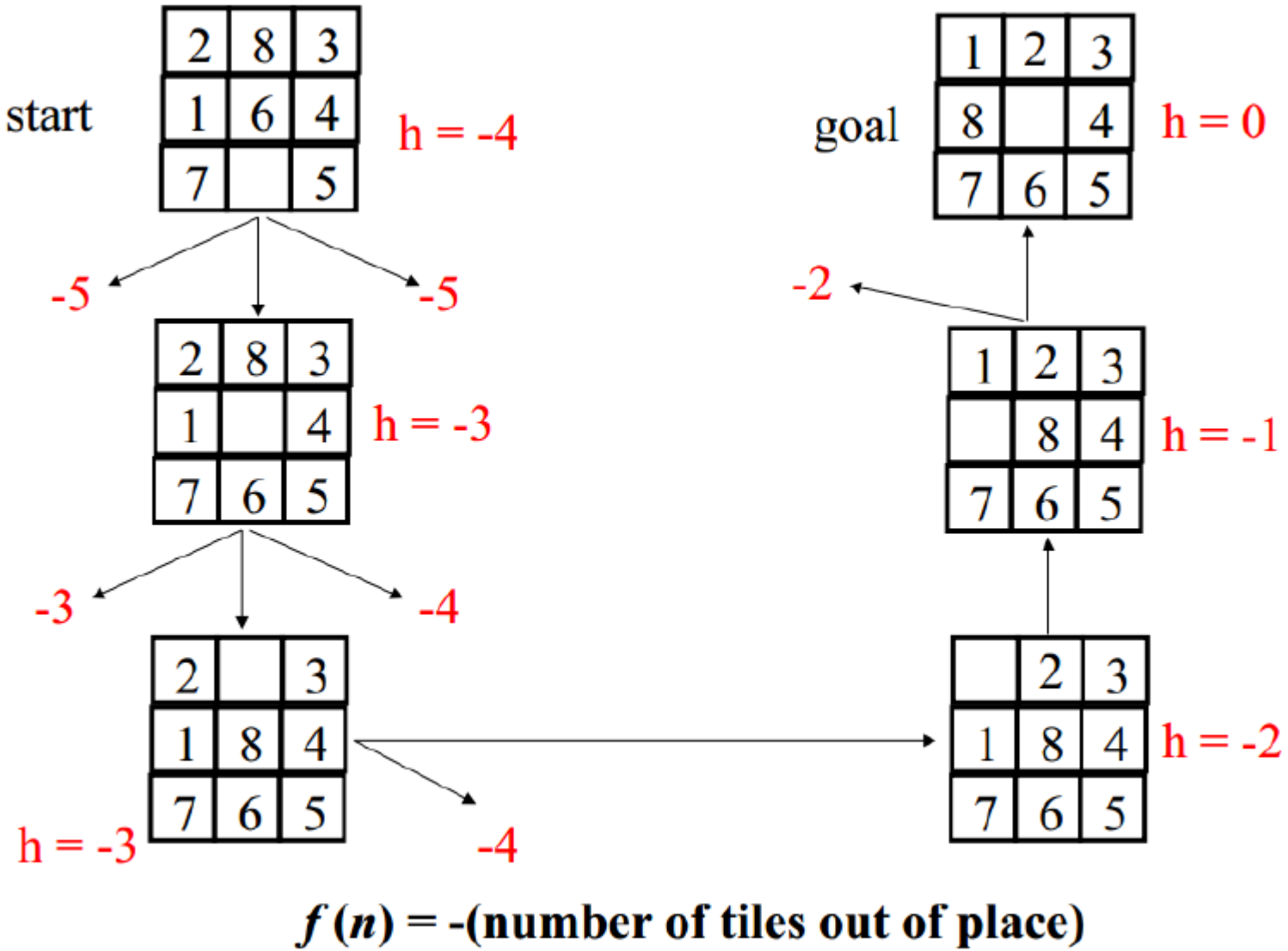
Looks at all successors

Example

Drawbacks of hill climbing

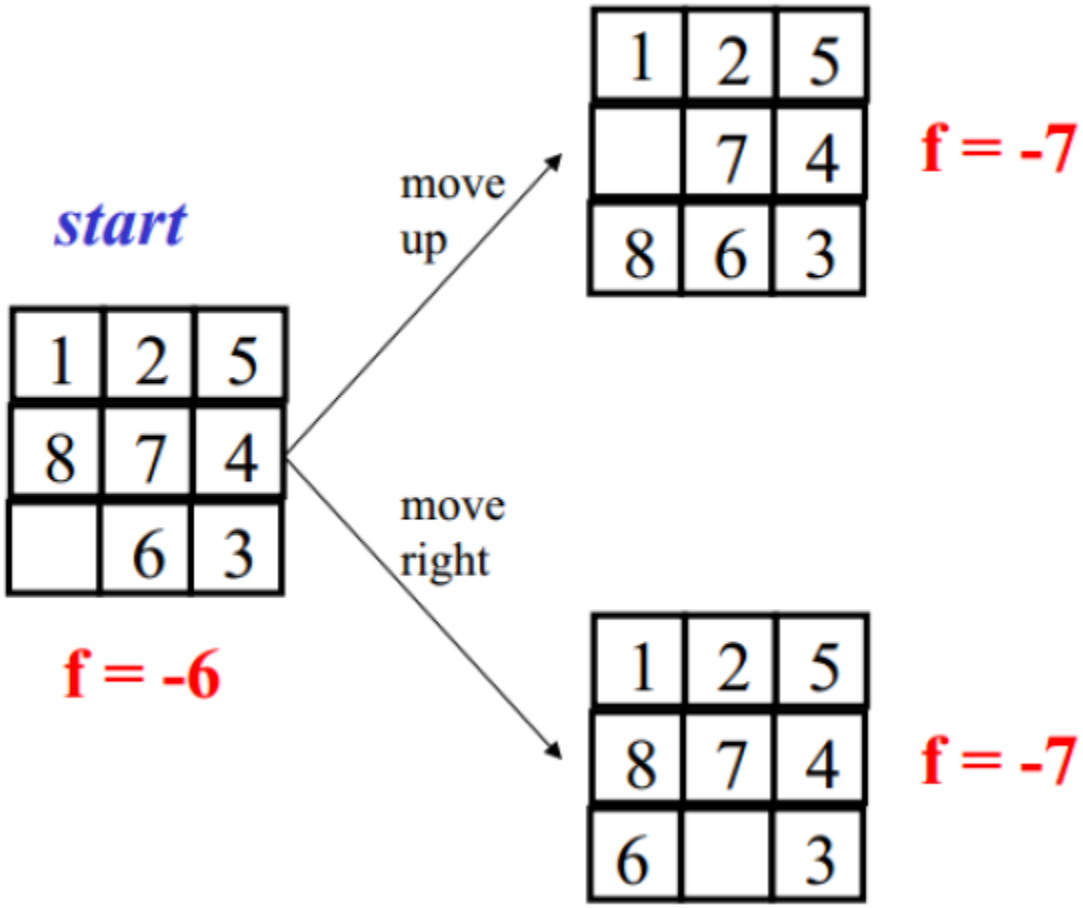


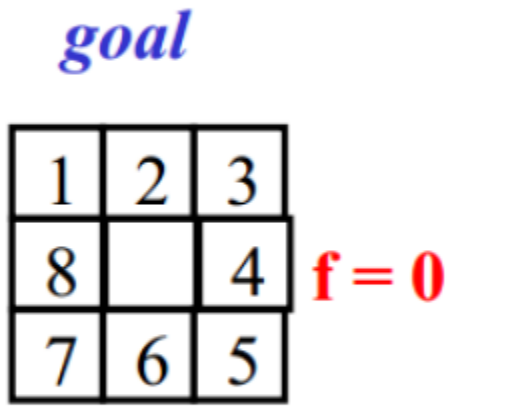
Example 1



***f* (*n*) = -(number of tiles out of place)**

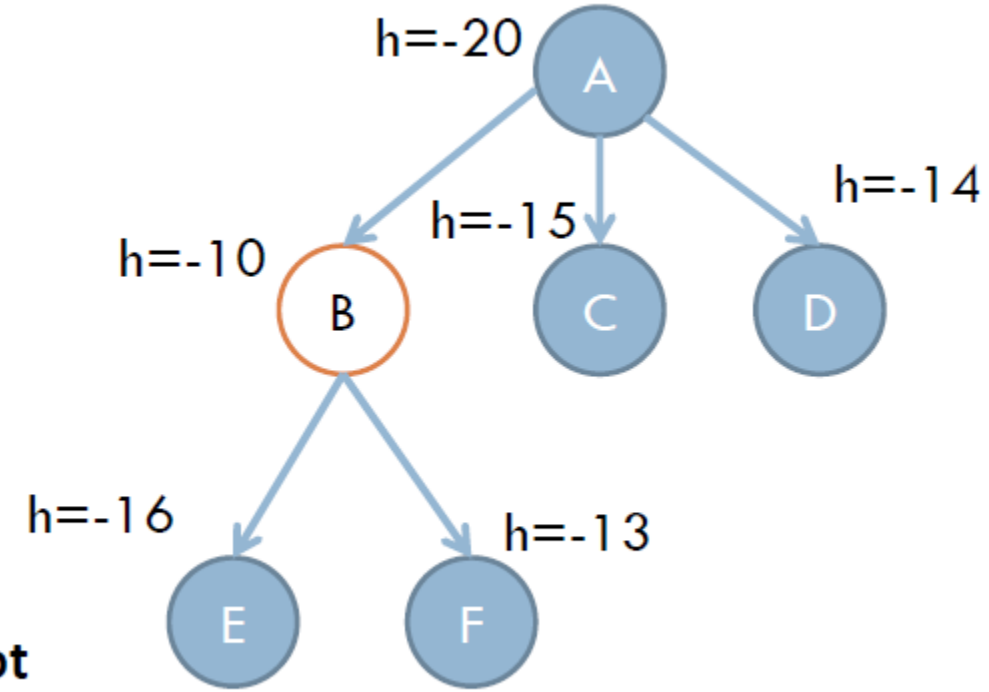
Example 2





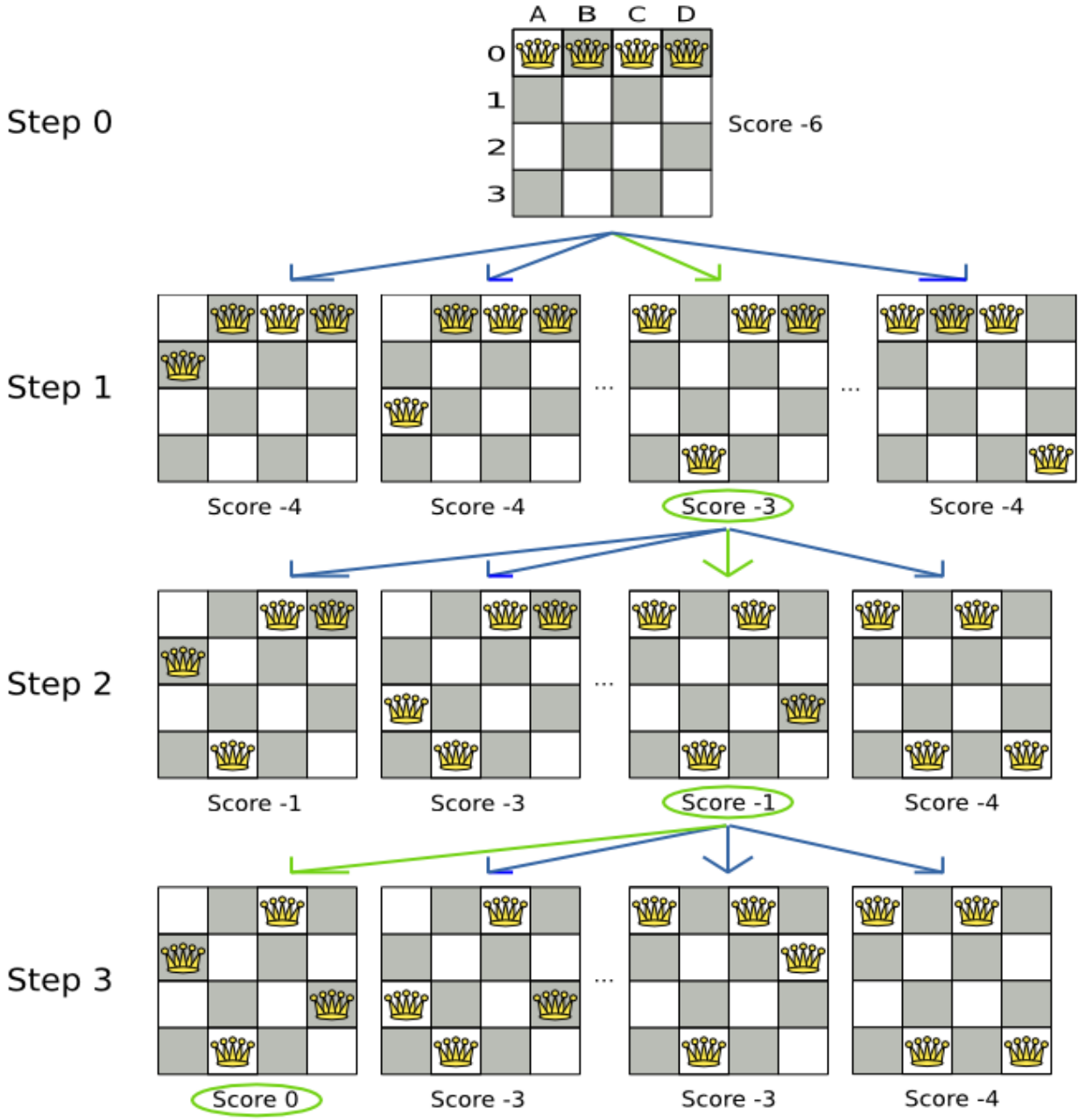
f = -(manhattan distance)

Example 3



Algorithm is not Complete

Example 4


A B C D
0
1
2
3
Step 0
Score -6
Step 1
Score -4
Score -4
Score -3
Score -4
Step 2
Score -1
Score -3
Score -1
Score -4
Step 3
Score 0
Score -3
Score -3
Score -4

Hill Climbing Analysis

- Complete ?
- Optimal ?
- Time Complexity
- Space Complexity

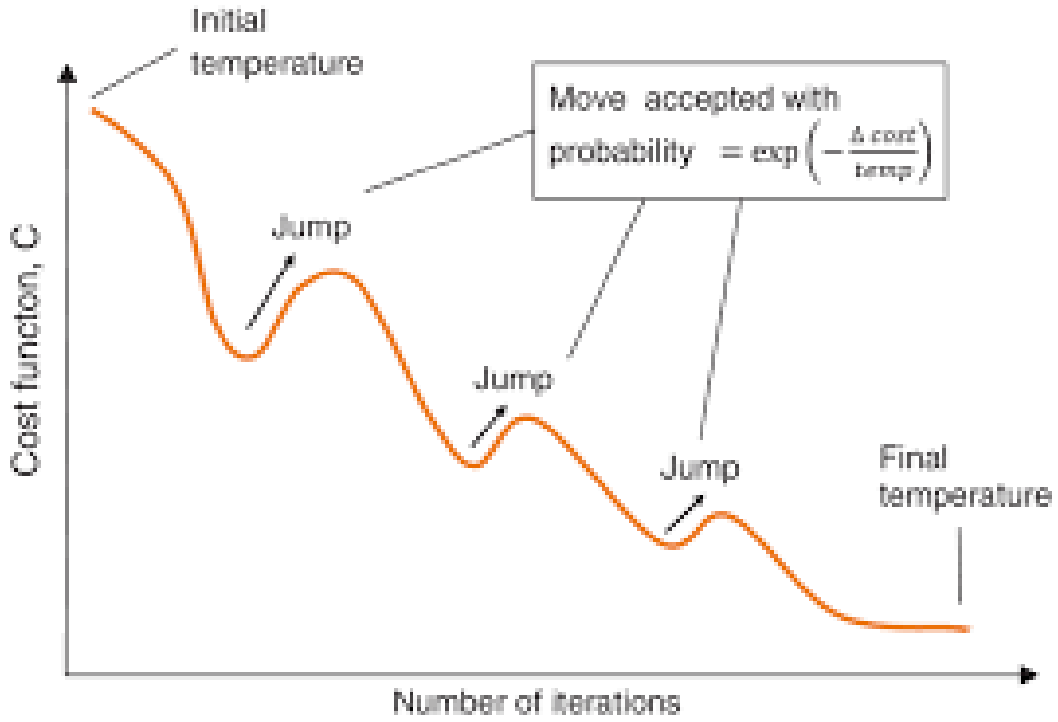
Simulated Annealing

Anneal

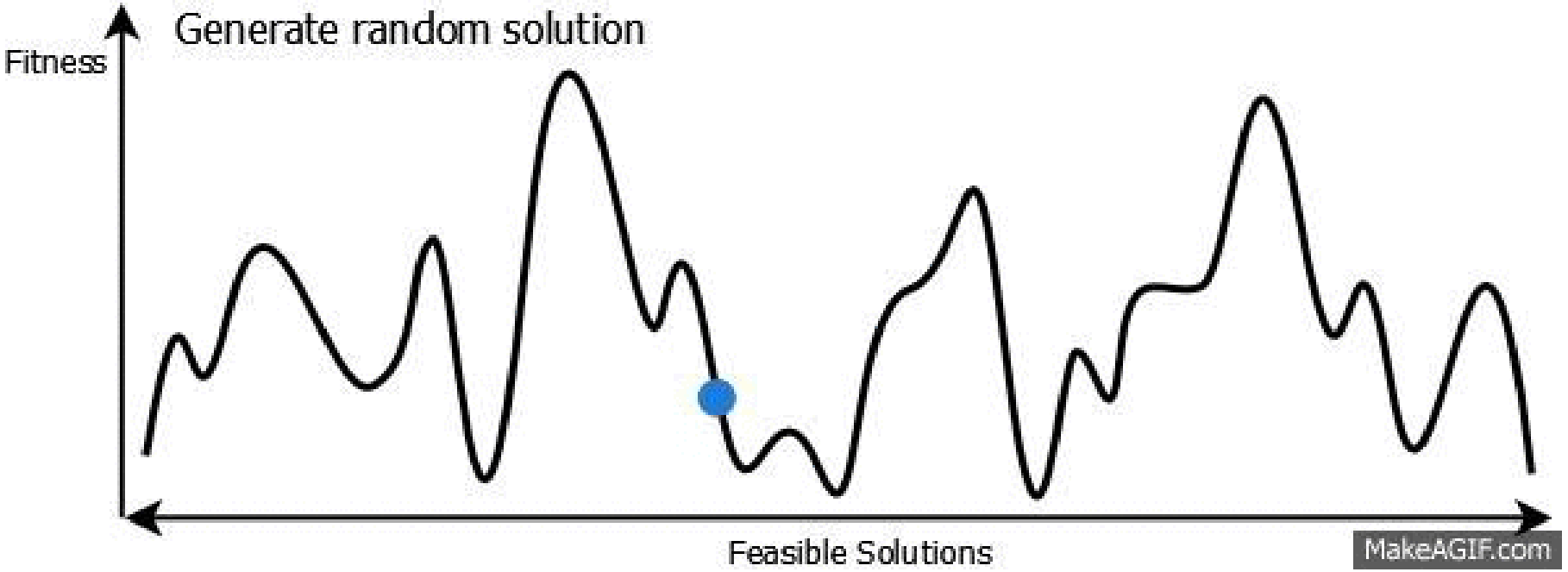
- To subject (glass or metal) to a process of heating and slow cooling in order to toughen and reduce brittleness.

Pseudocode of Simulated Annealing

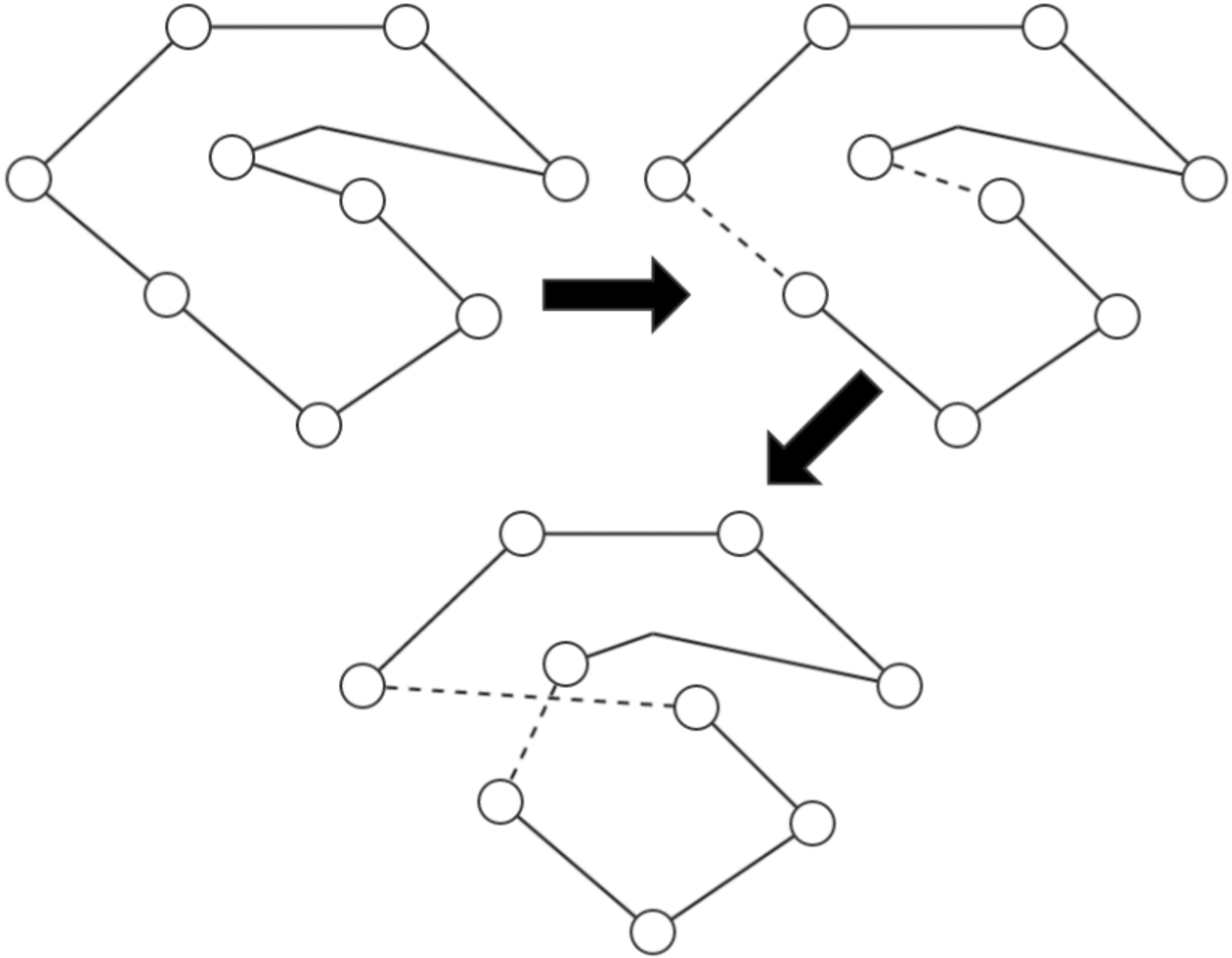
```
Procedure SIM_ANNEAL(m, iter_max, T)
BEGIN
    xcurr ← InitalConfig(m)
    xbest ← xcurr
    for i=1 to iter_max do
    BEGIN
        Tc ←Calc_Temp(i, T)
        xnext ←Random-Successor(x_curr)
        ΔE ← xnext – xcurr
        if (ΔE>0) BEGIN
            xcurr ← xnext
            if (xbest < xcurr) xbest ← xcurr
        END
        else if (e^(−ΔE/T) > Random(0, 1))
        xcurr ← xnext
    END
END
```



Simulated annealing algorithm



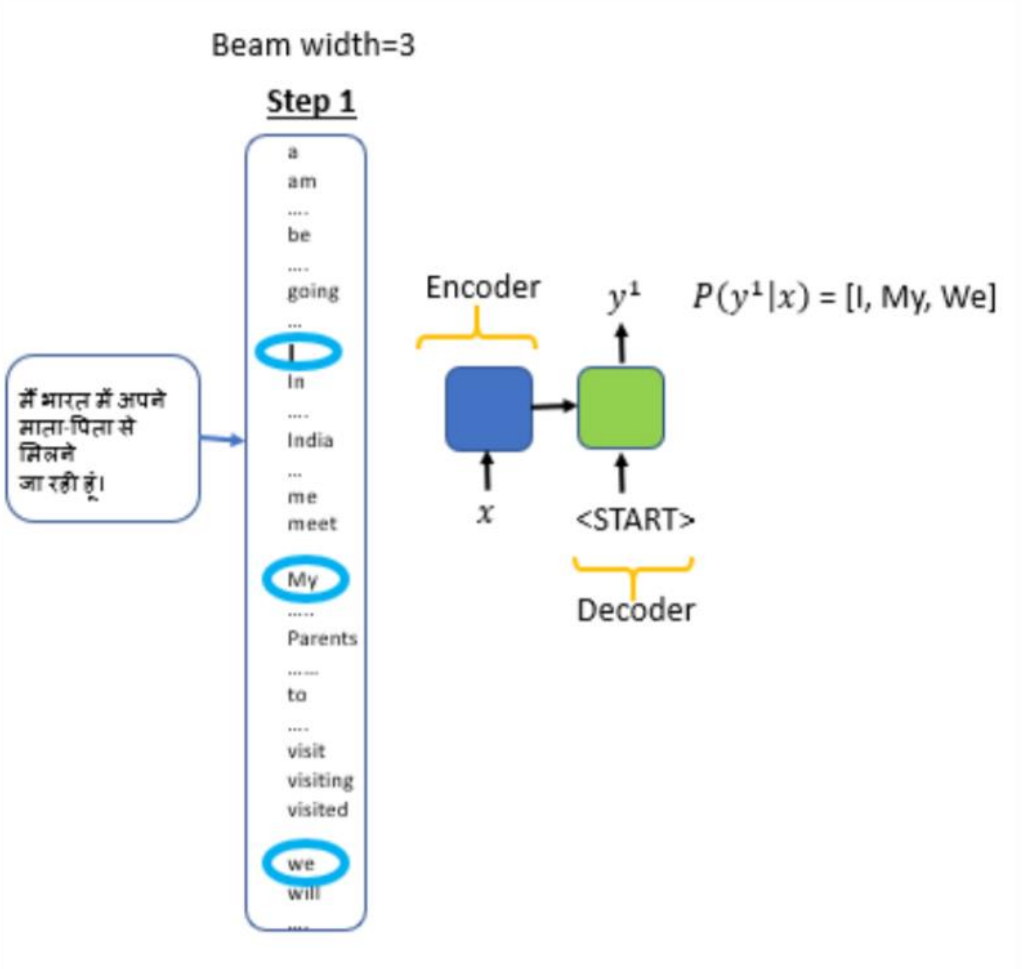
https://toddwschneider.com/posts/traveling-salesman-with-simulated-annealing-r-and-shiny/



Beam Search

- Beam search is a heuristic search algorithm that explores a graph by expanding the most promising node in a limited set.
- Beam search is an optimization of breadth-first search that reduces its memory requirements.
- A beam search is most often used to maintain tractability in large systems with insufficient amount of memory to store the entire search tree. For example, it has been used in many machine translation systems.

Machine Translation(Seq to Seq)

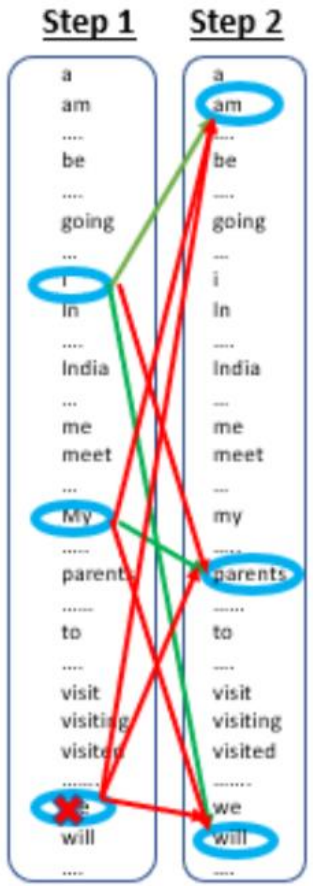


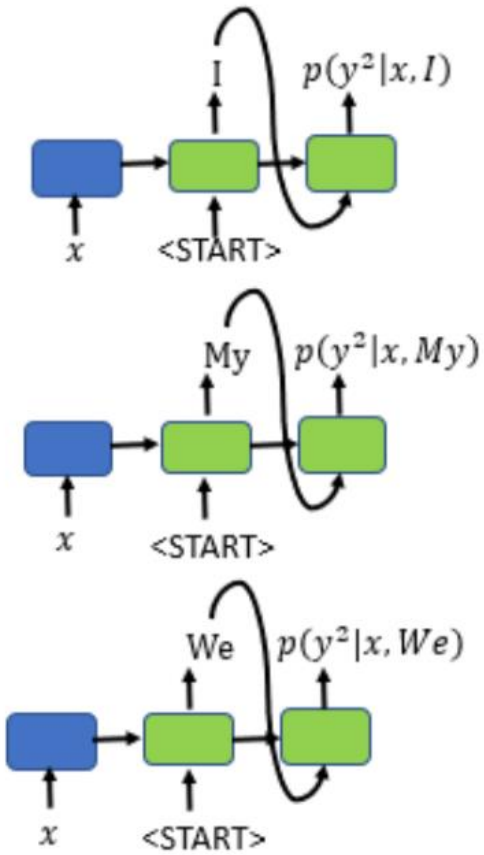
- Input the encoded input sentence to the decoder; the decoder will then apply softmax function to all the 10,000 words in the vocabulary.
- From 10,000 possibilities, we will select only the top 3 words with the highest probability.

Machine Translation(Seq to Seq)

Beam width=3

मैं भारत में अपने माता-पिता से मिलने जा रही हूं।



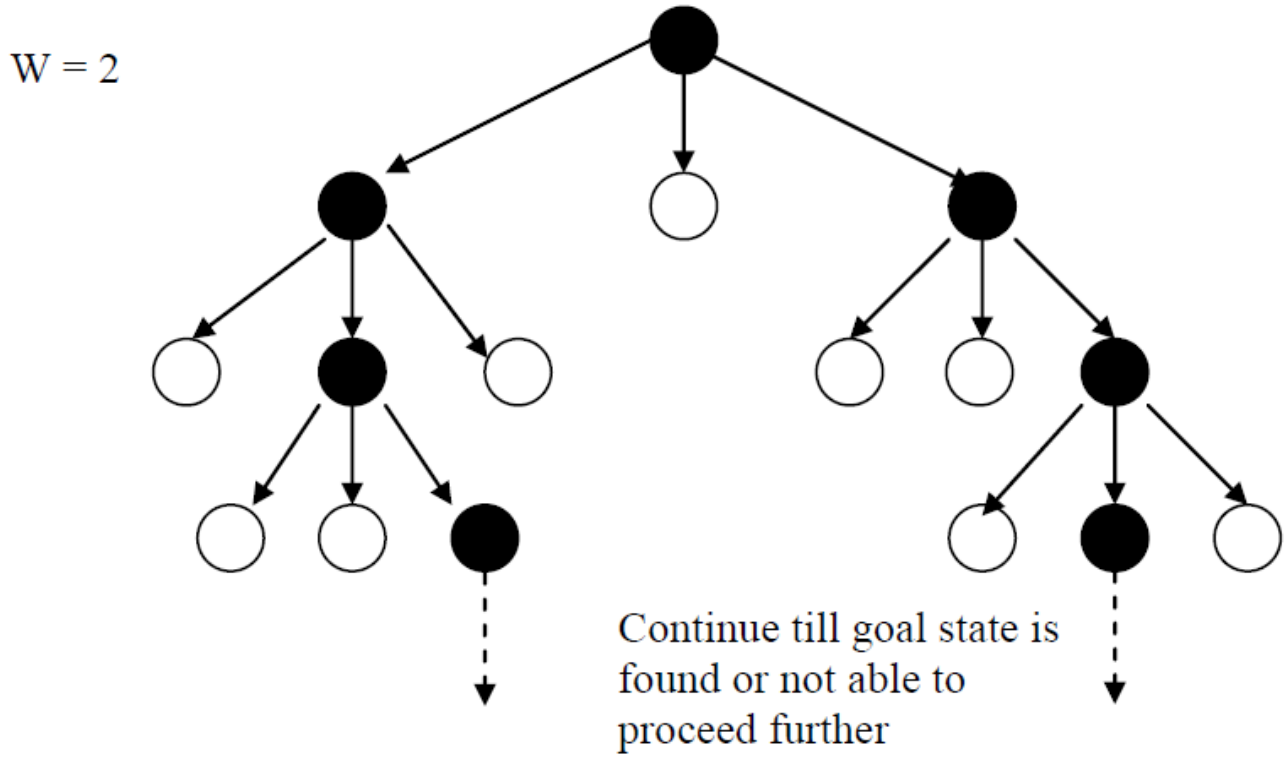


$P(y^1, y^2|x) = p(y^1|x) * p(y^2|x, y^1)$

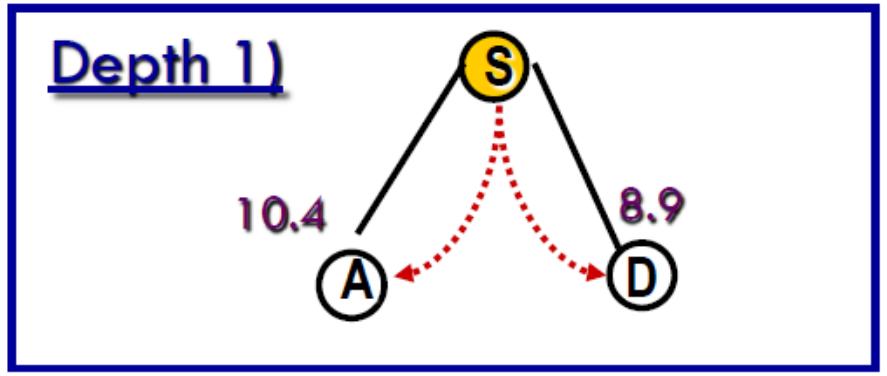
$P(y^1|x)$ = [I, My, We]

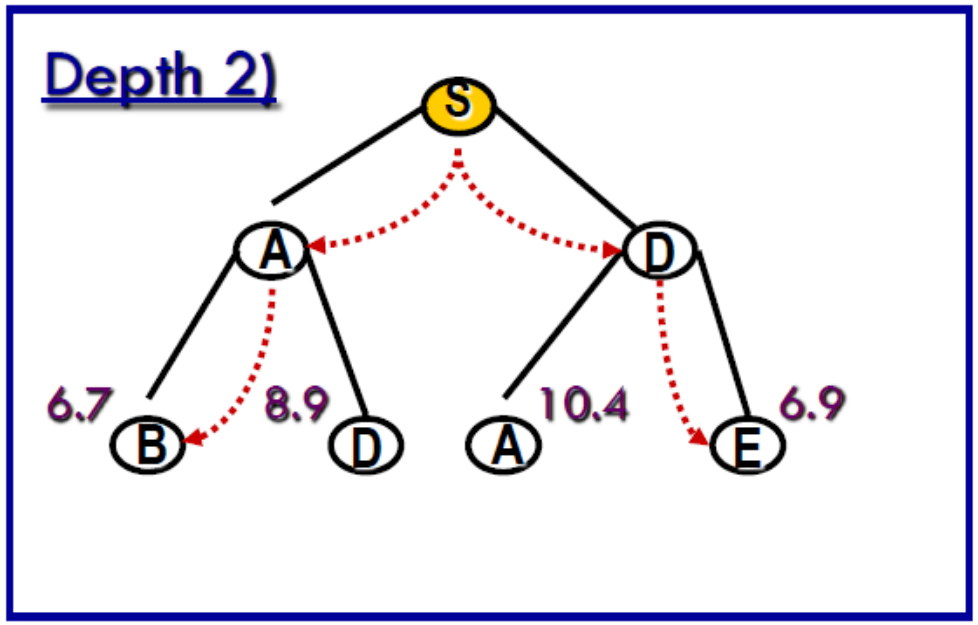
$P(y^1, y^2|x)$= [I am, I will, My parents]

Beam Search


W = 2
Continue till goal state is found or not able to proceed further

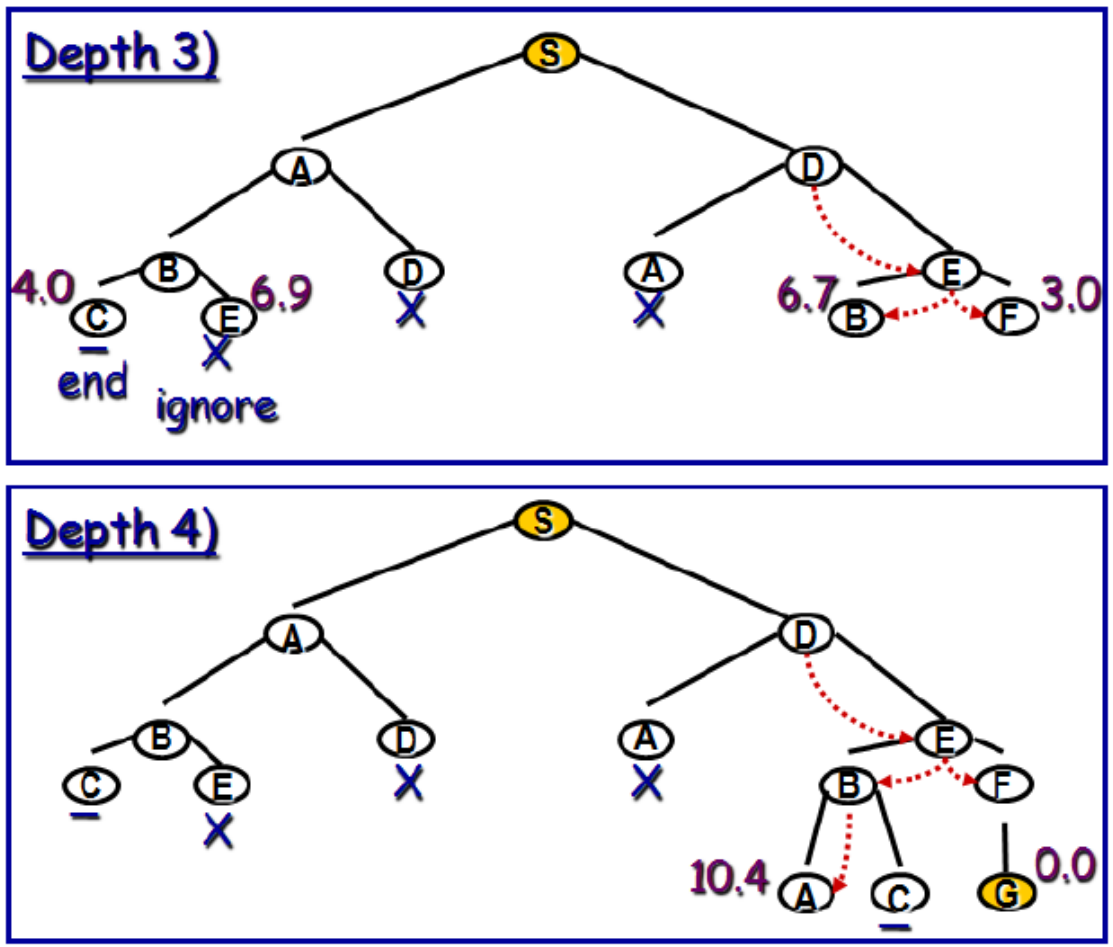
Beam Search Example





- Assume a pre-fixed WIDTH (example : 2)
- Perform breadth-first, BUT Only keep the width best new nodes depending on heuristic at each new level.

Beam Search Example



- Optimization: ignore leafs that are not goal nodes (see C)

Pseudocode of Beam Search

- Found = false;
- NODE = Root_node;
- If NODE is the goal node, then *Found = true* else find SUCCs of NODE, if any with its estimated cost and store in OPEN list;
- While (Found = false and not able to proceed further)

  {
  - Sort OPEN list;
  - Select top W elements from OPEN list and put it in W_OPEN list and empty OPEN list;
  - While (W_OPEN ≠ ϕ and Found = false)

  {
    - Get NODE from W_OPEN;
    - If NODE = Goal state then Found = true else

      {
      - Find SUCCs of NODE, if any with its estimated cost
      - store in OPEN list;

      }

  } // end while

} // end while

- If *Found = true* then return *Yes* otherwise return *No* and Stop

Beam Search Analysis

- Complete ?
- Optimal ?
- Time Complexity
- Space Complexity

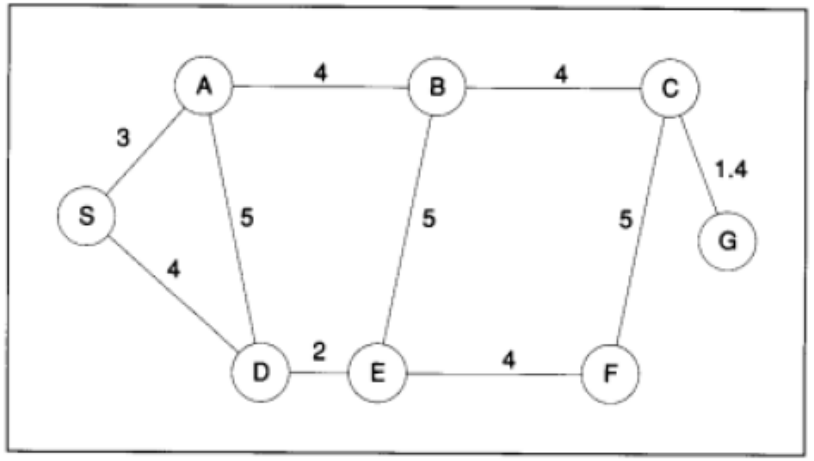
Class Exercise 1

- Consider the following graph. S denotes the starting state and G denotes the goal state. The number attached to each edge in the graph represents the COST of traversing the edge. Assume also that the ESTIMATED distances to the goal are given by the following table:

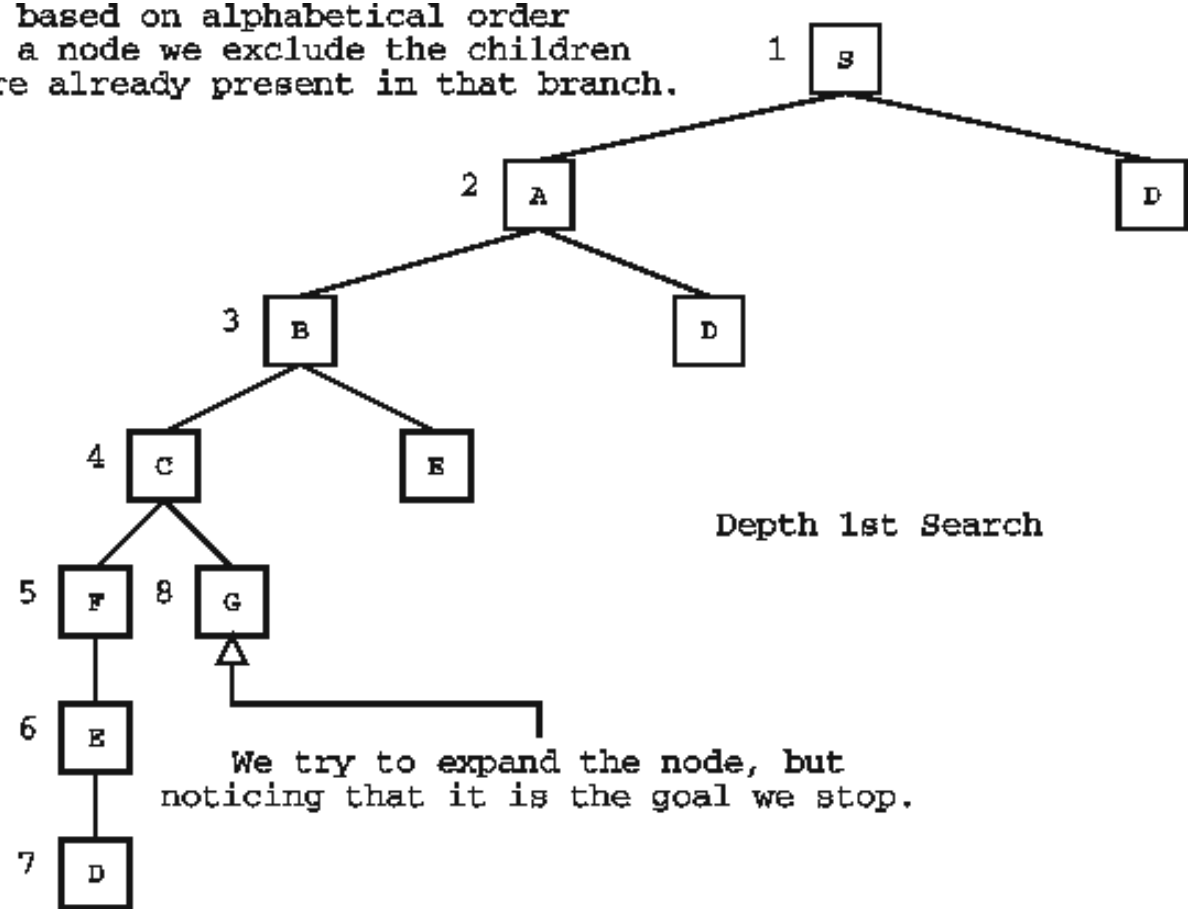
FROM	TO	ESTIMATED DISTANCE
S	G	10
A	G	8
B	G	5
C	G	1.4
D	G	9
E	G	6
F	G	2
G	G	0

Class Exercise 1

- For each of the following search methods show the search tree as it is explored/expanded by the method until it finds a path from S to G. Number the nodes in the order in which they are expanded by each method. **Show your work**.
  - Depth First Search
  - Breadth First Search
  - Uniform Cost Search
  - Best First Search
  - A*
  - Hill-climbing
  - Beam Search (m = 2)



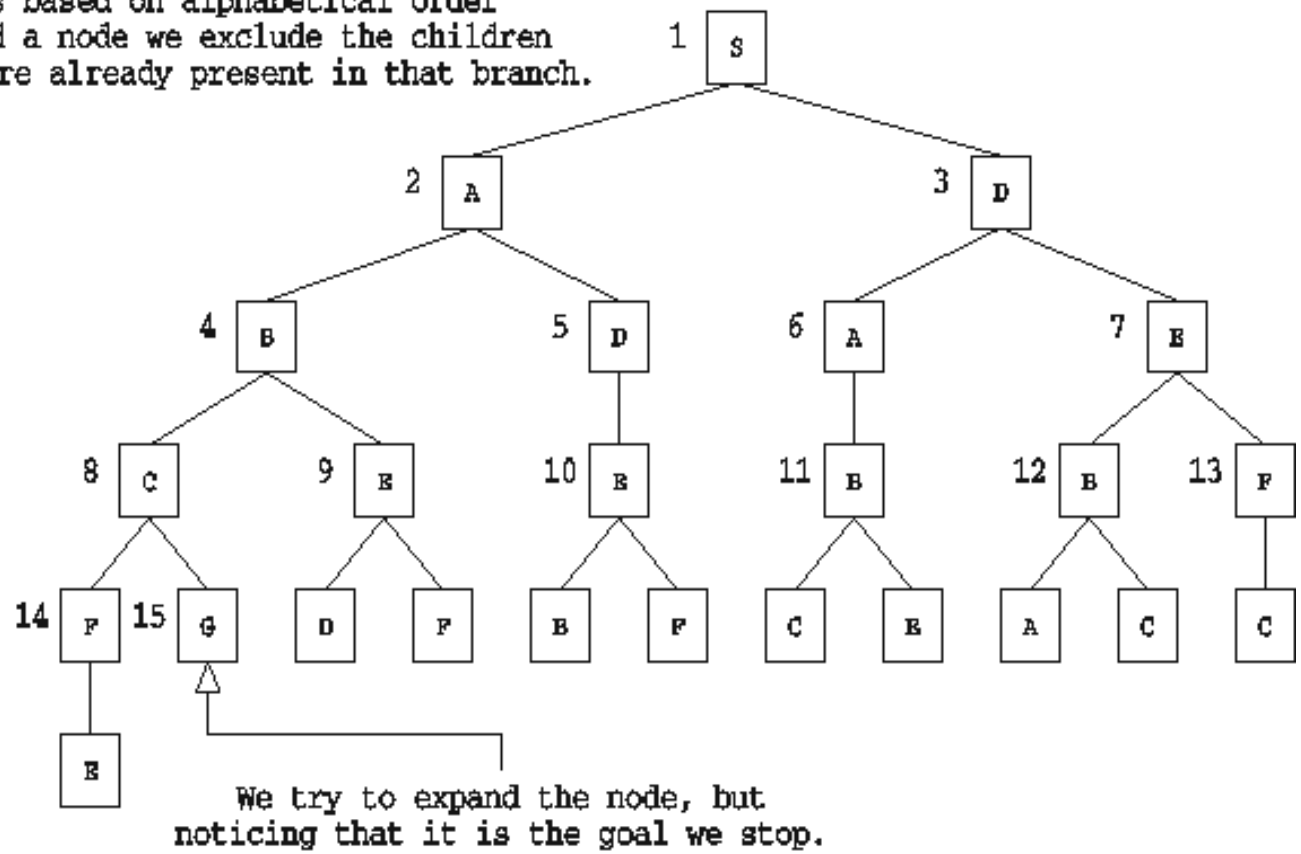
We select nodes based on alphabetical order
and when we expand a node we exclude the children
of that node which are already present in that branch.



Depth 1st Search

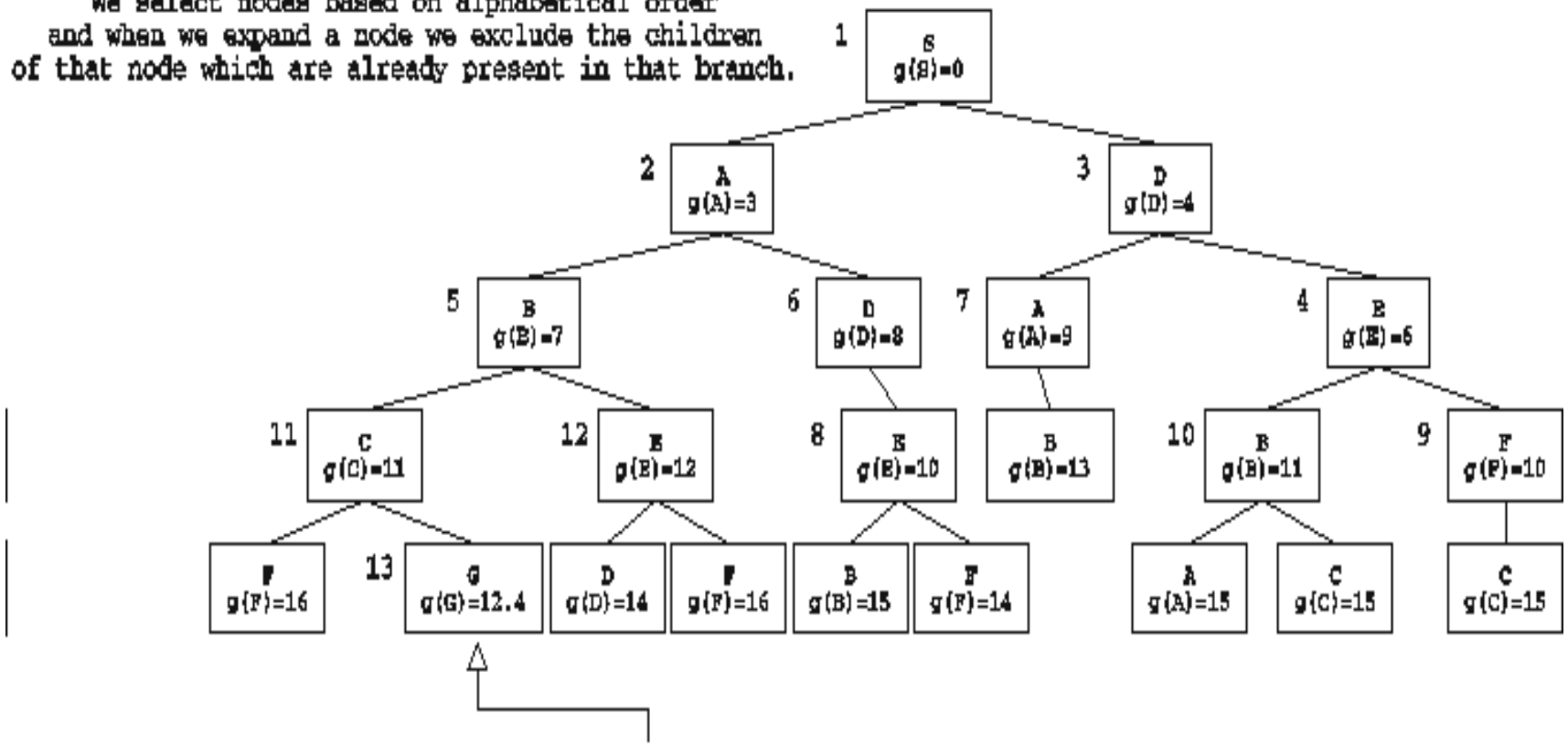
We try to expand the node, but
noticing that it is the goal we stop.

We select nodes based on alphabetical order
and when we expand a node we exclude the children
of that node which are already present in that branch.



Breadth 1st Search

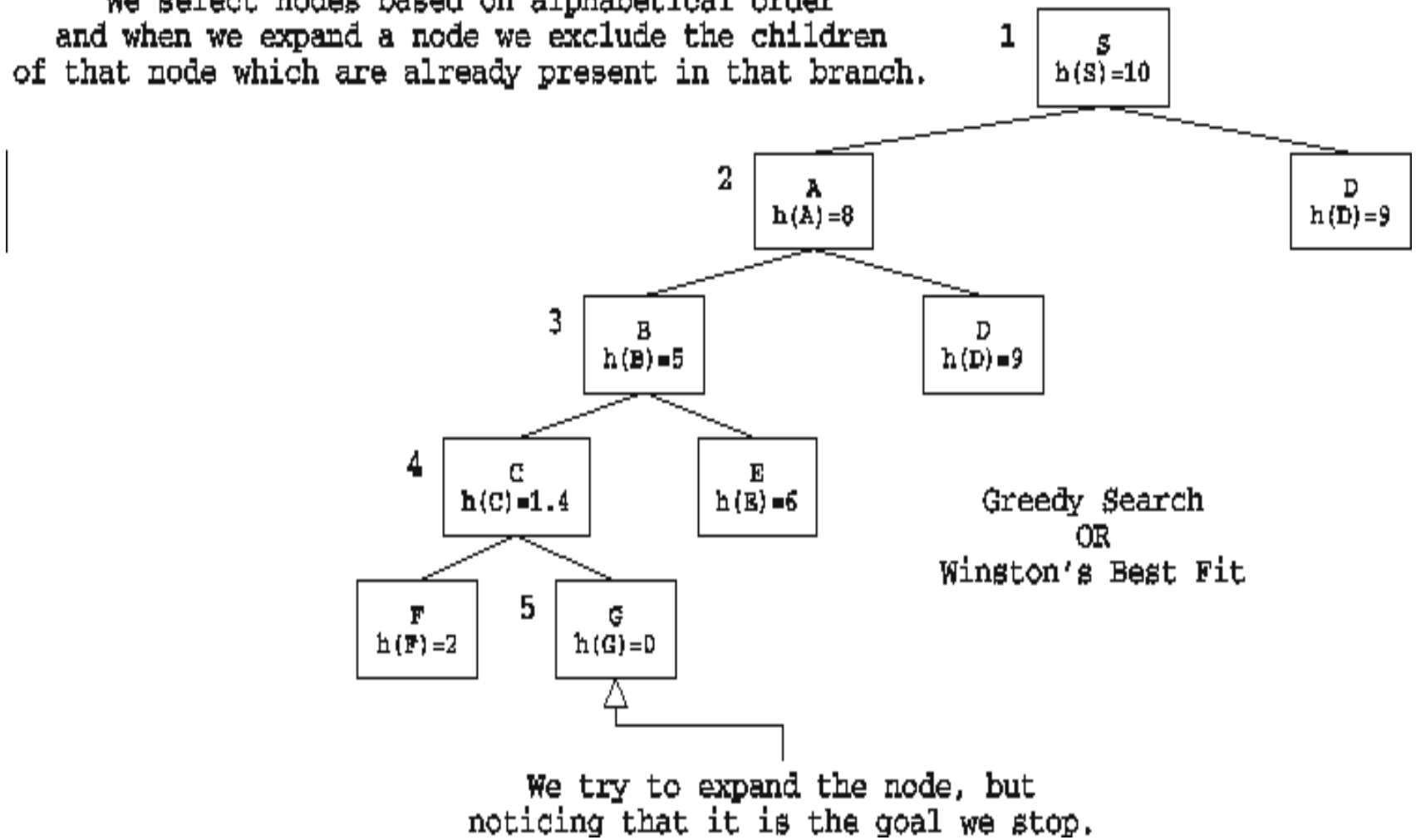
We select nodes based on alphabetical order
and when we expand a node we exclude the children
of that node which are already present in that branch.



We try to expand the node, but
noticing that it is the goal we stop.

Branch and Bound Search OR Uniform-Cost Search

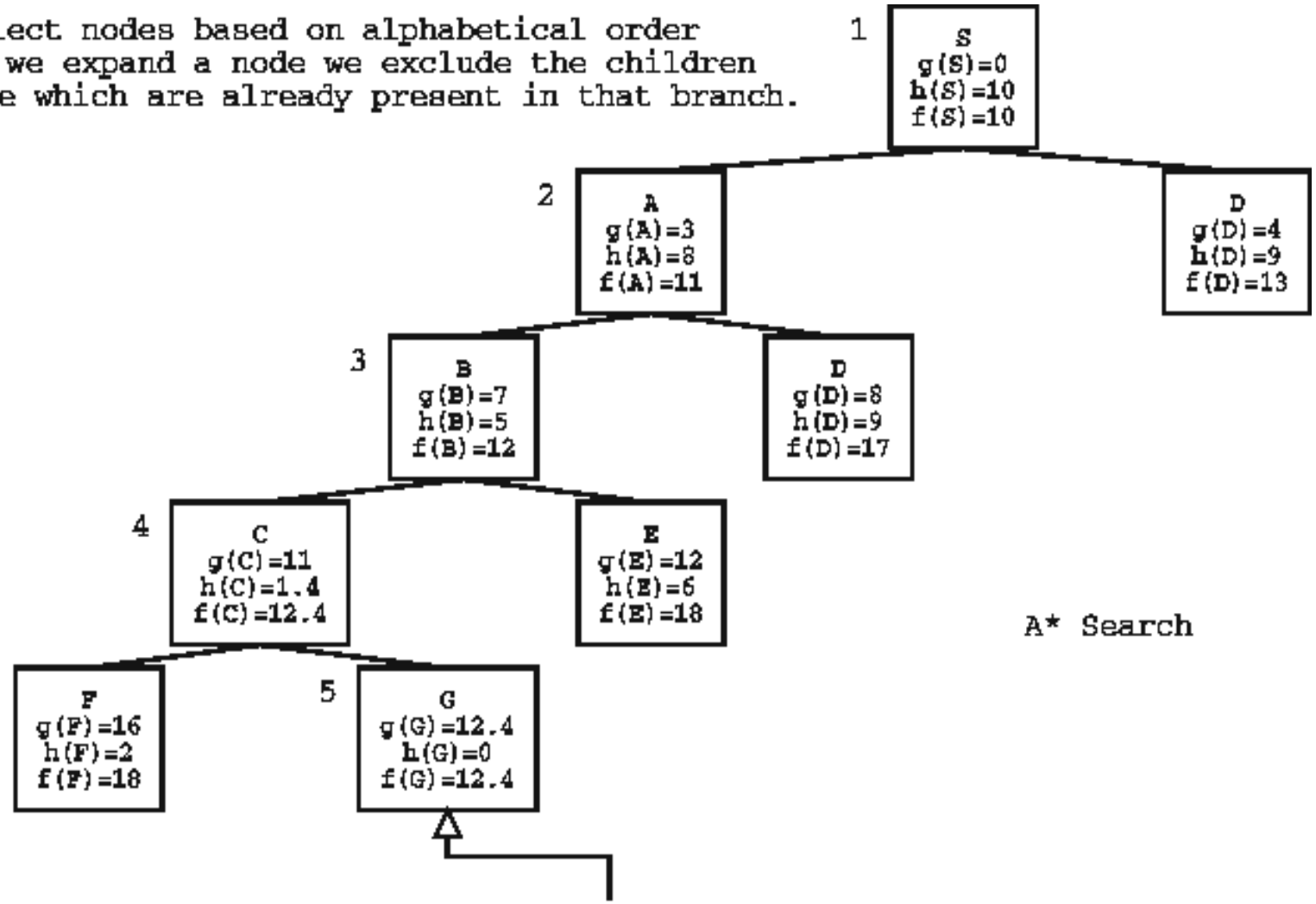
We select nodes based on alphabetical order
and when we expand a node we exclude the children
of that node which are already present in that branch.



Greedy Search
OR
Winston's Best Fit

We try to expand the node, but
noticing that it is the goal we stop.

We select nodes based on alphabetical order and when we expand a node we exclude the children of that node which are already present in that branch.

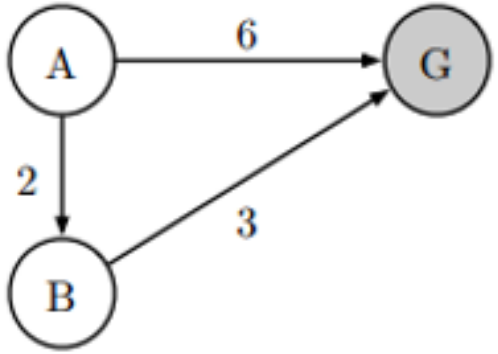


A* Search

We try to expand the node, but noticing that it is the goal we stop.

Class Exercise 2

Assume there is two heuristic function for the problem shown below.



	H1	H2
A	4	5
B	1	4
G	0	0

For each heuristic function, tell whether it is admissible and whether it is monotonic with respect to the search problem given above.

Class Exercise 3

- Assume you have the following Maze environment, and you want to help the slug to reach here food as soon as possible. Which among the following algorithm is the best to use: **Breadth-First Search, A-star,** or **Greedy search.** Explain your result by showing the sequence of tested node (In-order)?
- The slug can only move **up, down, left, or right**. Any of these actions can only be performed on condition that the resulting state remains within the maze and that the resulting state is not a black cell. Also, actions that bring you back to a previous state are not allowed. Note: Assume that ties are broken alphabetically (alphabetical order of the labels in the cells).