Maths 1 - Midterm Exam - v3

Time Allowed - 1 hour

Total Marks: 40

1. Use partial fraction decomposition to simplify the following fractions

   (a) $\frac{11x^2+11x+8}{(x^2+1)(2x+3)}$ **(3 marks)**

2. Express the following in the form $a + bi$

   (a) $(5 - 2i)(3 + i)$ **(2 marks)**

   (b) $\frac{1+2i}{4-4i}$ **(2 marks)**

3. (a) Express in polar form $(3 - 7i)$ **(2 marks)**

   (b) Express $z = 5\angle 120°$ in the form $a + bi$ **(2 marks)**

4. Differentiate with respect to x

   (a) $y = \cos 4x + 3x^3 + 3e^{3x}$ **(2 marks)**

   (b) $y = \sin(x^4)$ **(2 marks)**

   (c) $y = \frac{2x^2}{\cos x}$ **(2 marks)**

   (d) $y = x^3 \cdot e^x$ **(2 marks)**

5. (a) Find the 4th derivitive of $2e^{3x}$ **(2 marks)**

   (b) Find the stationary co-ordidnates of the function given by $y = x^3 - 3x + 2$ and determine the nature of each stationary point **(3 marks)**

6. Evaluate the following integrals

   (a) $\int (x^{\frac{1}{2}} + \sin x)\, dx$ **(2 marks)**

   (b) $\int_0^2 \sqrt{x} + e^x\, dx$ **(3 marks)**

7. Use simpson's rule with (6 intervals) to approximate the integral below to 3 decimal places

   $\int_0^\pi \sin \sqrt{2x}\, dx$ **(3 marks)**

8. Find the nth term for the following sequences
   (a) 3,11,19,27... **(2 marks)**
   (b) 3,18,108,648... **(2 marks)**

9. (a) Write the first 5 terms of the sequence described by the recurrence relation $U_{n+2} = 2U_{n+1} + U_n, U_1 = 2, U_2 = 2$ **(2 marks)**
   (b) Find the sum of the first fifteen terms of the arithmetic sequence $a_n = 3n + 2$ **(2 marks)**

Formulae

Differentiation

$\frac{d}{dx}e^x = e^x$

$\frac{d}{dx}\ln x = \frac{1}{x}$

$\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}\tan x = \sec^2 x$

Product Rule: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

Quotient Rule: $\frac{dy}{dx} = \frac{v\frac{dv}{dx} - u\frac{du}{dx}}{v^2}$

Integration

$\int e^x dx = e^x + c$

$\int \frac{1}{x}dx = \ln|x| + c$

$\int \sin x dx = -\cos x + c$

$\int \cos x dx = \sin x + c$

Simpson's Rule

Area $\approx \sum \frac{d}{3}(y_0 + y_n + 4(y_2 + y_4 ...) + 2(y_1 + y_3 + y_5 ...))$

Sequences and Series

Arithemetic Sequence: $a_n = a + (n-1)d$

Arithmetic Series: $S_n = \frac{n}{2}(2a + (n-1)d)$

Geometric Sequence: $U_n = ar^{n-1}$

Geometric Series: $S_n = \frac{a(1-r^n)}{1-r}$

Infinite Series: $S_\infty = \frac{a}{1-r}$