## Contents



```
% Zain Bhaila
% Math 240 Fall 2018 Project 4
% Section 0132
```

## Problem 0

```
format short
clock
```

```
ans =

   1.0e+03 *

    2.0180    0.0110    0.0260         0    0.0190    0.0130
```

## Problem 1

```
A = [345 -654 -928; 84 -158 -228; 66 -126 -176]

% Problem 1(a)
[P,D] = eig(A)

% Problem 1(b)
A == P * D * inv(P) %#ok

% Problem 1(c)
% lambda = 6, v = [ .957 .2051 .2051 ]
% lambda = 1, v = [ -.9456 -.291 -.1455 ]
% lambda = 4, v = [ .9577 .1596 .2394 ]
```

```
A =

   345  -654  -928
    84  -158  -228
    66  -126  -176
```

```
P =

    0.9570   -0.9456    0.9577
    0.2051   -0.2910    0.1596
    0.2051   -0.1455    0.2394


D =

    6.0000         0         0
         0    1.0000         0
         0         0    4.0000


ans =

  3×3 logical array

   0   0   0
   0   0   0
   0   0   0
```

## Problem 2

```
A = [-65 -39 -15; 100 60 23; 22 13 6]

% Problem 2(a)
A2 = A^2
A3 = A^3
A4 = A^4
A5 = A^5
A6 = A^6
A7 = A^7
A8 = A^8
% the values repeat after intervals of 4

% Problem 2(b)
[P,D] = eig(A) %#ok

% Problem 2(c)
D
D2 = D^2
D3 = D^3
D4 = D^4
% all the values have the same magnitude
% the first and third powers have the same eigenvalues
% the fourth power has one unique real eigenvalue
% the second power has all real eigenvalues

% Problem 2(d)
% A^10000001 = A = [-65 -39 -15; 100 60 23; 22 13 6]
```

A =

```
   -65   -39   -15
   100    60    23
    22    13     6
```

A2 =

```
    -5     0   -12
     6    -1    18
     2     0     5
```

A3 =

```
    61    39     3
   -94   -60    -5
   -20   -13     0
```

A4 =

```
     1     0     0
     0     1     0
     0     0     1
```

A5 =

```
   -65   -39   -15
   100    60    23
    22    13     6
```

A6 =

```
    -5     0   -12
     6    -1    18
     2     0     5
```

A7 =

```
    61    39     3
   -94   -60    -5
   -20   -13     0
```

A8 =

```
     1     0     0
     0     1     0
     0     0     1
```

P =

```
  -0.5361 + 0.0089i  -0.5361 - 0.0089i  -0.5345 + 0.0000i
   0.8250 + 0.0000i   0.8250 + 0.0000i   0.8018 + 0.0000i
   0.1787 - 0.0030i   0.1787 + 0.0030i   0.2673 + 0.0000i


D =

  -0.0000 + 1.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -0.0000 - 1.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i


D =

  -0.0000 + 1.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -0.0000 - 1.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i


D2 =

  -1.0000 - 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -1.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i


D3 =

   0.0000 - 1.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 1.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i


D4 =

   1.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   1.0000 - 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i
```

## Problem 3

```
A = [3 1 ; 0 3]

% Problem 3(a)
[P,D] = eig(A)

% Problem 3(b)
A == P * D * inv(P) %#ok

% Problem 3(c)
A_space = null(A - 3*eye(2))

% Problem 3(d)
% No, there is not a basis.
```

```
% A must have 2 distinct eignvalues in order to be
% diagonalizable. A only has one distinct eigenvalue, and
% thus is not diagonalizable.
```

```
A =

     3     1
     0     3


P =

    1.0000   -1.0000
         0    0.0000


D =

     3     0
     0     3


ans =

  2×2 logical array

   1   0
   1   1


A_space =

    -1
     0
```

## Problem 4

```
A = [ 3 0 3 ; -1 -1 -2 ; 0 1 2 ; 5 2 0]

% Problem 4(a)
dot(A(:,1),A(:,2))
dot(A(:,3),A(:,3))

% Problem 4(b)
transpose(A) * A

% Problem 4(c)
% The i,j entry of A^T * A is the dot product of the
% ith column and jth column of A.

% Problem 4(d)
% The i,j entry of A * A^T is the dot product of the
% ith row and jth row of A.
```

```
% Problem 4(e)
A * transpose(A)
dot(A(1,:),A(1,:))
dot(A(4,:),A(4,:))

% Problem 4(f)
Q = [1/sqrt(14) 1/sqrt(3) 5/sqrt(42) ;
     2/sqrt(14) 1/sqrt(3) -4/sqrt(42) ;
     3/sqrt(14) -1/sqrt(3) 1/sqrt(42)]
transpose(Q) * Q

% Problem 4(g)
% The dot products of two columns i,j where i = j
% are 1, meaning they are unit vectors.
% The dot products of two columns i,j where i =/= j
% are 0, meaning they are orthogonal.

% Problem 4(h)
% If Q^T * Q = I then I = Q * Q^T. If
% Q * Q^T = I, and the dot products of the rows
% of Q form Q * Q^T, then the rows form an orthonormal set.
```

```
A =

     3     0     3
    -1    -1    -2
     0     1     2
     5     2     0


ans =

    11


ans =

    17


ans =

    35    11    11
    11     6     4
    11     4    17


ans =

    18    -9     6    15
    -9     6    -5    -7
     6    -5     5     2
    15    -7     2    29
```

```
ans =

    18


ans =

    29


Q =

    0.2673    0.5774    0.7715
    0.5345    0.5774   -0.6172
    0.8018   -0.5774    0.1543


ans =

    1.0000    0.0000   -0.0000
    0.0000    1.0000   -0.0000
   -0.0000   -0.0000    1.0000
```

## Problem 5

Problem 5(a)

```
v1 = [ -5; -2; 13; 7; -5 ]
v2 = [ 3; 5; -3; -11; -6 ]
v3 = [ 3; 24; -9; -15; 3 ]
v4 = [ 1; -16; 3; 13; 1 ]

% Problem 5(b)
A = [v1 v2 v3 v4]
rank(A)
% A basis is a set of linearly independent vectors and
% the rank is equivalent to the dimension of the column
% space. Since the rank of of A is 4, its columns are lin indep
% meaning they form a basis for W.

% Problem 5(c)
w1 = v1
w2 = v2 - dot(w1,v2)/dot(w1,w1)*w1

% Problem 5(d)
w3 = v3 - dot(w1,v3)/dot(w1,w1)*w1 - dot(w2,v3)/dot(w2,w2)*w2
w4 = v4 - dot(w1,v4)/dot(w1,w1)*w1 - dot(w2,v4)/dot(w2,w2)*w2 - dot(w3,v4)/dot(w3,w3)*w3

% Problem 5(e)
u1 = w1/norm(w1)
u2 = w2/norm(w2)
u3 = w3/norm(w3)
u4 = w4/norm(w4)
```

```
% Problem 5(f)
Q = [u1 u2 u3 u4]
transpose(Q) * Q

% Problem 5(g)
R = transpose(Q) * A %#ok

% Problem 5(h)
[Q1, R1] = qr(A,0)
```

```
v1 =

    -5
    -2
    13
     7
    -5


v2 =

     3
     5
    -3
   -11
    -6


v3 =

     3
    24
    -9
   -15
     3


v4 =

     1
   -16
     3
    13
     1


A =

    -5     3     3     1
    -2     5    24   -16
    13    -3    -9     3
     7   -11   -15    13
    -5    -6     3     1
```

```
ans =

     4


w1 =

    -5
    -2
    13
     7
    -5


w2 =

    0.9596
    4.1838
    2.3051
   -8.1434
   -8.0404


w3 =

   -3.6347
   16.9106
    2.6476
    2.2258
    6.8704


w4 =

    2.5295
    0.6709
    0.0153
    1.4147
   -0.7774


u1 =

   -0.3032
   -0.1213
    0.7882
    0.4244
   -0.3032


u2 =

    0.0771
    0.3364
    0.1853
   -0.6547
```

```
   -0.6464


u3 =

   -0.1920
    0.8933
    0.1399
    0.1176
    0.3629


u4 =

    0.8227
    0.2182
    0.0050
    0.4601
   -0.2528


Q =

   -0.3032    0.0771   -0.1920    0.8227
   -0.1213    0.3364    0.8933    0.2182
    0.7882    0.1853    0.1399    0.0050
    0.4244   -0.6547    0.1176    0.4601
   -0.3032   -0.6464    0.3629   -0.2528


ans =

    1.0000   -0.0000    0.0000    0.0000
   -0.0000    1.0000    0.0000    0.0000
    0.0000    0.0000    1.0000    0.0000
    0.0000    0.0000    0.0000    1.0000


R =

   16.4924   -6.7304  -18.1902    9.2164
   -0.0000   12.4379   14.5180  -13.9067
    0.0000    0.0000   18.9300  -12.1741
    0.0000    0.0000    0.0000    3.0748


Q1 =

   -0.3032   -0.0771   -0.1920   -0.8227
   -0.1213   -0.3364    0.8933   -0.2182
    0.7882   -0.1853    0.1399   -0.0050
    0.4244    0.6547    0.1176   -0.4601
   -0.3032    0.6464    0.3629    0.2528


R1 =
```

```
   16.4924   -6.7304  -18.1902    9.2164
         0  -12.4379  -14.5180   13.9067
         0         0   18.9300  -12.1741
         0         0         0   -3.0748
```

## Problem 6

```
% Problem 6(a)
A = [2 3 -2 -1 2; 0 2 4 2 -2; 3 5 -2 -1 3; 5 9 -2 -1 2;
    0 -3 -6 -3 5; 1 4 4 2 0]
rank(A)

% Problem 6(b)
rref(A)
basis = [A(:,1),A(:,2),A(:,5)] % treat as a set of vectors

% Problem 6(c)
B = basis
[Q,R] = qr(B,0)
W_orth = Q

% Problem 6(d)
E = Q * transpose(Q)
v = [ 1 ; 1 ; 1 ; 1 ; 1 ; 1 ]
a1 = dot(v,E(:,1))/dot(E(:,1),E(:,1)) * E(:,1) + dot(v,E(:,2))/dot(E(:,2),E(:,2)) * E(:,2);
a2 = dot(v,E(:,3))/dot(E(:,3),E(:,3)) * E(:,3) + dot(v,E(:,4))/dot(E(:,4),E(:,4)) * E(:,4);
a3 = dot(v,E(:,5))/dot(E(:,5),E(:,5)) * E(:,5) + dot(v,E(:,6))/dot(E(:,6),E(:,6)) * E(:,6);
proj_v = a1 + a2 + a3 % it didnt all fit into one line

% Problem 6(e)
W_perp = null(transpose(B))

% Problem 6(f)
[Q2,R2] = qr(W_perp,0)
W_perp_orth = Q2

% Problem 6(g)
F = W_perp_orth * transpose(W_perp_orth)
E+F
```

```
A =

     2     3    -2    -1     2
     0     2     4     2    -2
     3     5    -2    -1     3
     5     9    -2    -1     2
     0    -3    -6    -3     5
     1     4     4     2     0


ans =

     3
```

```
ans =

     1     0    -4    -2     0
     0     1     2     1     0
     0     0     0     0     1
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0


basis =

     2     3     2
     0     2    -2
     3     5     3
     5     9     2
     0    -3     5
     1     4     0


B =

     2     3     2
     0     2    -2
     3     5     3
     5     9     2
     0    -3     5
     1     4     0


Q =

   -0.3203    0.1376   -0.0491
         0   -0.4668   -0.1143
   -0.4804    0.0898   -0.2817
   -0.8006   -0.0060    0.3298
         0    0.7002   -0.5463
   -0.1601   -0.5146   -0.7057


R =

   -6.2450  -11.2090   -3.6829
         0   -4.2847    4.9669
         0         0   -2.7866


W_orth =

   -0.3203    0.1376   -0.0491
         0   -0.4668   -0.1143
   -0.4804    0.0898   -0.2817
   -0.8006   -0.0060    0.3298
         0    0.7002   -0.5463
   -0.1601   -0.5146   -0.7057
```

E =

```
    0.1239   -0.0586    0.1800    0.2394    0.1232    0.0151
   -0.0586    0.2309   -0.0097   -0.0349   -0.2644    0.3209
    0.1800   -0.0097    0.3182    0.2912    0.2167    0.2295
    0.2394   -0.0349    0.2912    0.7498   -0.1844   -0.1014
    0.1232   -0.2644    0.2167   -0.1844    0.7887    0.0252
    0.0151    0.3209    0.2295   -0.1014    0.0252    0.7885
```

v =

```
     1
     1
     1
     1
     1
     1
```

proj_v =

```
    1.7110
    0.0909
    3.0616
    2.9282
    1.7535
    2.3865
```

W_perp =

```
   -0.9181    0.1406    0.1158
   -0.0449    0.6271   -0.6113
    0.0337   -0.5667   -0.5997
    0.3345    0.2974    0.2234
    0.1953    0.4157   -0.0204
    0.0628   -0.0680    0.4505
```

Q2 =

```
   -0.9181   -0.1406    0.1158
   -0.0449   -0.6271   -0.6113
    0.0337    0.5667   -0.5997
    0.3345   -0.2974    0.2234
    0.1953   -0.4157   -0.0204
    0.0628    0.0680    0.4505
```

R2 =

```
    1.0000    0.0000         0
         0   -1.0000    0.0000
         0         0    1.0000
```

```
W_perp_orth =

   -0.9181   -0.1406    0.1158
   -0.0449   -0.6271   -0.6113
    0.0337    0.5667   -0.5997
    0.3345   -0.2974    0.2234
    0.1953   -0.4157   -0.0204
    0.0628    0.0680    0.4505


F =

    0.8761    0.0586   -0.1800   -0.2394   -0.1232   -0.0151
    0.0586    0.7691    0.0097    0.0349    0.2644   -0.3209
   -0.1800    0.0097    0.6818   -0.2912   -0.2167   -0.2295
   -0.2394    0.0349   -0.2912    0.2502    0.1844    0.1014
   -0.1232    0.2644   -0.2167    0.1844    0.2113   -0.0252
   -0.0151   -0.3209   -0.2295    0.1014   -0.0252    0.2115


ans =

    1.0000    0.0000   -0.0000   -0.0000    0.0000    0.0000
    0.0000    1.0000   -0.0000   -0.0000    0.0000    0.0000
   -0.0000   -0.0000    1.0000         0    0.0000   -0.0000
   -0.0000   -0.0000         0    1.0000   -0.0000         0
    0.0000    0.0000    0.0000   -0.0000    1.0000    0.0000
    0.0000    0.0000   -0.0000         0    0.0000    1.0000
```

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