Contents


```
% Zain Bhaila
% Math 401
% Homework 3
```

4.9

```
syms x;
A = [ 1 2 ; 1 2]
b = [ 4 ; 3]
M = transpose(A) * A % A^T * A
q = transpose(A) * b % A^T * b
r = inv(M) * q  % (A^T * A)^-1 * A^T * b
% since (A^T * A)^-1 * A^T * b does not have linearly independent
% columns, there are infinite solutions to the least square equation
```

```
A =

     1     2
     1     2


b =

     4
     3


M =

     2     4
     4     8


q =

     7
    14

Warning: Matrix is singular to working precision.

r =
```

```
   Inf
   Inf
```

4.10

```
A = [ 1 2 ; -1 1 ; 0 3]
b = [1 ; 3 ; 0]

% part a
c1 = A(:,1)
c2 = A(:,2)
proj = dot(c1, c2)/norm(c1)^2 * c1 % proj of c2 to c1
basis = [c1  c2 - proj] % orthogonal basis for col(A)
b1 = dot(b, basis(:,1))/norm(basis(:,1))^2 * basis(:,1)
b2 = dot(b, basis(:,2))/norm(basis(:,2))^2 * basis(:,2)
bhat = b1 + b2 % sum of the projections of b onto basis
x = linsolve(A,bhat) % Ax=b

% part b
M = transpose(A) * A % A^T * A
q = transpose(A) * b % A^T * b
x = inv(M) * q % x = M^-1 * q
% the l-s solution for both methods is the same
```

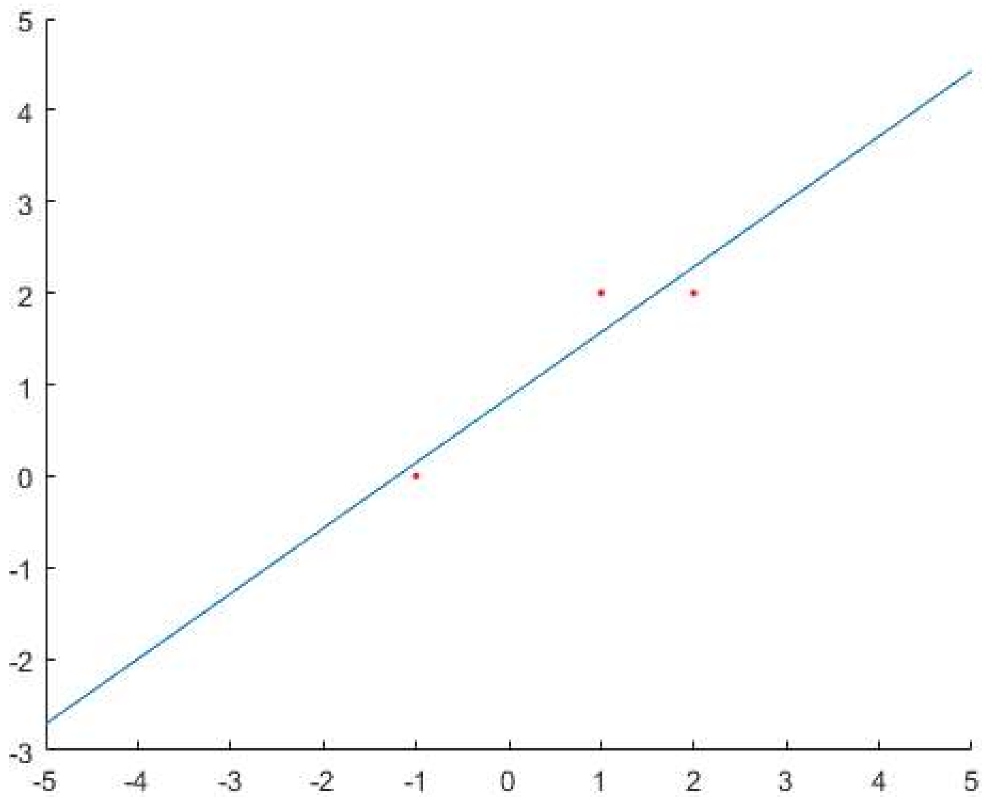
```
A =

     1     2
    -1     1
     0     3


b =

     1
     3
     0


c1 =

     1
    -1
     0


c2 =

     2
     1
     3


proj =
```

```
    0.5000
   -0.5000
         0


basis =

    1.0000    1.5000
   -1.0000    1.5000
         0    3.0000


b1 =

   -1.0000
    1.0000
         0


b2 =

    0.6667
    0.6667
    1.3333


bhat =

   -0.3333
    1.6667
    1.3333


x =

   -1.2222
    0.4444


M =

     2     1
     1    14


q =

    -2
     5


x =

   -1.2222
    0.4444
```

5.1

```
% part a
syms x;
A = [-1 1 ; 1 1 ; 2 1]
y = [ 0 ; 2 ; 2]
r = A\y % muldivide in matlab gets l-s solution for rectangular matrices
f = r(1,:) * x + r(2,:) % least squares line

% part b
hold on; % make plot use same graph
plot(-1,0, 'r.') % plot points
plot(1,2, 'r.')
plot(2,2, 'r.')
fplot(f) % plot line
% the picture minimizes the distance between the y-values of the
% line and the y-values of the original points
```

```
A =

    -1     1
     1     1
     2     1


y =

     0
     2
     2


r =

    0.7143
    0.8571


f =

(5*x)/7 + 6/7
```



5.10

```
% part a
% on attached sheet

% part b
% begins on attached sheet
syms n x;
M = [(9*n+5) (3*n+3) ; (3*n+3) (n+2)] % A^T * A
p = [(6*n+3) ; (2*n+2)] % A^T * b
xhat_n = simplify(inv(M) * p) % (A^T * A)^-1 * A^T * b, answer
y = simplify(xhat_n(1,:) * x + xhat_n(2,:)) % equation
% y = (3*n/(5*n + 1)) * x + (n+1)/(5*n + 1)
% x = ((5*n + 1)/(3*n)) * y - (n+1)/(3*n)

% part c
xhat_n = limit(xhat_n, n, inf) % limit as n approaches infinity
y = simplify(xhat_n(1,:) * x + xhat_n(2,:)) % equation

% part d
subs(y,x,3) % show the line passes through (3,2)
% this makes sense since if there are an infinite number of points
% of the value (3,2) the best fit line with the least distance
% between points must pass through all of them
```

```
M =
```

```
[ 9*n + 5, 3*n + 3]
[ 3*n + 3,   n + 2]


p =

 6*n + 3
 2*n + 2


xhat_n =

   (3*n)/(5*n + 1)
 (n + 1)/(5*n + 1)


y =

(n + 3*n*x + 1)/(5*n + 1)


xhat_n =

 3/5
 1/5


y =

(3*x)/5 + 1/5


ans =

2
```

5.11

```
% part a
A1 = [ 0 1 ; 2 1 ; 3 1 ;5 1 ;7 1 ;8 1]
b1 = [4.2;5;5.3;6.1;7.9;8.6]
r1 = inv(transpose(A1) * A1) * transpose(A1) * b1 % (A^T * A)^-1 * A^T * b
f2 = r1(1,:)*x + r1(2,:)

% part b
A2 = [0 0 1 ;4 2 1 ;9 3 1 ;25 5 1 ;49 7 1 ;64 8 1]
b2 = [4.2;5;5.3;6.1;7.9;8.6]
r2 = inv(transpose(A2) * A2) * transpose(A2) * b2 % (A^T * A)^-1 * A^T * b
f2 = r2(1,:)*x^2 + r2(2,:)*x + r2(3,:)

% part c
syms c;
c = 8.6^(1/8)
A3 = [1 (c)^0 ;1 (c)^2 ;1 (c)^3 ;1 (c)^5 ;1 (c)^7 ;1 (c)^8]
```

```
b3 = [4.2;5;5.3;6.1;7.9;8.6]
r3 = inv(transpose(A3) * A3) * transpose(A3) * b3 % (A^T * A)^-1 * A^T * b
f3 = r3(1,:) + r3(2,:) * c^x

% part d
% least squares for a
norm(A1*r1-b1)
% least squares for b
norm(A2*r2-b2)
% least squares for c
norm(A3*r3-b3)
% the quadratic fits best (part b)

% part e
y = subs(f2, x, 10) % set x = 10
% f(10) is approx 10.5824

% part f
solve(f2 == 50, x) % solve for f(x) = 50
% x is approximately 30.2010
% ignore the negative value that is output
```

```
A1 =

     0     1
     2     1
     3     1
     5     1
     7     1
     8     1


b1 =

    4.2000
    5.0000
    5.3000
    6.1000
    7.9000
    8.6000


r1 =

    0.5534
    3.8776


f2 =

(311*x)/562 + 5448/1405


A2 =
```

```
     0     0     1
     4     2     1
     9     3     1
    25     5     1
    49     7     1
    64     8     1


b2 =

    4.2000
    5.0000
    5.3000
    6.1000
    7.9000
    8.6000


r2 =

    0.0437
    0.1931
    4.2782


f2 =

(6302904058865179*x^2)/144115188075855872 + (6515*x)/33744 + 1204206405519987/281474976710656


c =

    1.3086


A3 =

    1.0000    1.0000
    1.0000    1.7125
    1.0000    2.2410
    1.0000    3.8376
    1.0000    6.5718
    1.0000    8.6000


b3 =

    4.2000
    5.0000
    5.3000
    6.1000
    7.9000
    8.6000


r3 =
```

```
    3.9039
    0.5707


f3 =

(5140745503474415*(2946741810785137/2251799813685248)^x)/9007199254740992 + 274713476997711/70368744177664


ans =

    0.7527


ans =

    0.3273


ans =

    0.4566


y =

804103301948192000599/75984732912995008512


ans =

 - (16*2^(1/2)*362473480785737621635060275393133184087^(1/2))/13292824660146662511 - 29340951572318781440/13292824660146662511
   (16*2^(1/2)*362473480785737621635060275393133184087^(1/2))/13292824660146662511 - 29340951572318781440/13292824660146662511
```

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