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% Zain Bhaila
% Math 401
% Special Assignment #2

syms a b c d e;
% transition matrix given by problem
% ab,c,d,e are the values for teh transitions from S5
M = [3/12 5/12 0/12 2/12 a ;
     5/12 7/12 2/12 2/12 b ;
     3/12 0/12 8/12 2/12 c ;
     0/12 0/12 0/12 4/12 d ;
     1/12 0/12 2/12 2/12 e ]

% combinations with repetition
% number of ways to divide 12 items into 5 groups
V = 0:12;
N = 5;
[Y{N:-1:1}] = ndgrid(1:numel(V));
I = reshape(cat(N+1, Y{:}), [], N);
P = V(I); % contains every possible permutation with repetition with
 range 0-12 of 5 selections
[h, w] = size(P) % dimensions of P, h is the number of permutations
F = []; % filter P so that total tickets is 12
for i = 1:h % for each permutation
    if P(i,1) + P(i,2) + P(i,3) + P(i, 4) + P(i,5) == 12 % if the
 total # of tickets is twelve, keep the row
        F = [F;P(i,:)]; %#ok<*AGROW>
    end
end
[h, w] = size(F) % dimensions of F, h is the number of permutations
 where the total ticket amount is 12

min = 2147483647; % store minimum variance, set to max int to start
 (2^32 - 1)
loc = 0; % row of F with minimum variance distribution of tickets
S = []; % steady state
for i = 1:h % for each permutation in F
    Q = M; % set Q to M
    Q(1,5) = F(i,1)/12; % set values of Q to reflect current ticket
 distribution
    Q(2,5) = F(i,2)/12;
    Q(3,5) = F(i,3)/12;
    Q(4,5) = F(i,4)/12;
    Q(5,5) = F(i,5)/12;
    T = Q^4; % 4 is the longest possible path between states
    if all(T, 'all') % if T has no 0 values
        [V,D] = eig(Q); % get eigenvalue/eigenvector set
        [row,col] = find(D == 1); % index of eigenvalue = 1, row will
 be the same as col
        V(:,col) = V(:,col)/sum(V(:,col)); % divide corresponding
 eigenvector by sum of its values
        m = (V(1,col) + V(2,col) + V(3,col) + V(4,col) + V(5,col))/5;
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            s = (V(1,col) - m)^2 + (V(2,col) - m)^2 + (V(3,col) - m)^2 +
    (V(4,col) - m)^2 + (V(5,col) - m)^2;
            if (s < min) % if this variance is the new minimum
                min = s; % store variance as new minimum
                loc = i; % save the row this occurs at
                S = V(:,col); % save the steady state
            end
        end
end

% output answers
min % minimum variance
F(loc,:) % ticket distribution to S1-S5 from S5 in order
S % steady state
Q = M;
Q(1,5) = F(loc,1)/12;
Q(2,5) = F(loc,2)/12;
Q(3,5) = F(loc,3)/12;
Q(4,5) = F(loc,4)/12;
Q(5,5) = F(loc,5)/12 % final transition matrix
Q^4 % proof of regularity
% since the matrix is regular, it will always converge to the same
 steady
% state corresponding to eigenvalue = 1


M =

[  1/4, 5/12,    0, 1/6, a]
[ 5/12, 7/12, 1/6, 1/6, b]
[  1/4,    0, 2/3, 1/6, c]
[    0,    0,   0, 1/3, d]
[ 1/12,    0, 1/6, 1/6, e]


h =

     371293


w =

     5


h =

      1820


w =

     5
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min =

3076/132845


ans =

    0     0     0     3     9


S =

 30/163
 48/163
 30/163
 15/163
 40/163


Q =

[  1/4, 5/12,    0, 1/6,    0]
[ 5/12, 7/12, 1/6, 1/6,    0]
[  1/4,    0, 2/3, 1/6,    0]
[    0,    0,    0, 1/3, 1/4]
[ 1/12,    0, 1/6, 1/6, 3/4]


ans =

[ 2233/10368,      155/576,  707/5184,  101/576,   337/3456]
[   403/1152,  2189/5184,      20/81, 491/1728,   481/3456]
[     89/432,   625/3456, 1315/5184,    19/96,   197/1728]
[     89/1728,   145/6912,    13/144,   31/324, 1435/6912]
[    919/5184, 2215/20736,  707/2592,    20/81, 3053/6912]
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