```
# %% [markdown]
#### Packages
# %%
# import cupy as cp
import torch
import hcp_utils as hcp # https://rmldj.github.io/hcp-utils/
from statsmodels.stats.multitest import multipletests
import matlab.engine
import os
import json
import psutil
import random
import numpy as np
import pandas as pd
import re
import psutil
import matplotlib.pyplot as plt
import seaborn as sns
from scipy.linalg import eigh, svd
from scipy.stats import norm
from sklearn.decomposition import FastICA, PCA
from sklearn.covariance import LedoitWolf
from sklearn.metrics.pairwise import cosine_similarity
from sklearn.preprocessing import normalize
from sklearn.preprocessing import StandardScaler
from sklearn.impute import SimpleImputer
from sklearn.svm import SVC, LinearSVC
from sklearn.linear model import LogisticRegression, LinearRegression, Lasso, LassoCV,
MultiTaskLasso, ElasticNet, LogisticRegressionCV
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis as LDA
from nilearn import image as nimg
from nilearn import plotting
import nibabel as nib
from pyriemann.estimation import Covariances
from pyriemann.utils.mean import mean covariance
from pyriemann.utils.tangentspace import tangent space, untangent space, log map riemann,
unupper
from pyriemann.utils.distance import distance_riemann, distance
from pyriemann.utils.base import logm, expm
from concurrent.futures import ProcessPoolExecutor, TimeoutError
from sklearn.model selection import train test split
from sklearn.metrics import accuracy score, confusion matrix
from sklearn.model selection import StratifiedKFold
# %% [markdown]
#### Options
# %%
# Define your settings
settings = {
  "phenotype": "Gender",
```

```
"percentile": 0.2,
  "outputfolder": "Gender_TangentSVMC1_logeuclid",
  "n folds": 5,
  "TanSVM_C": 1,
  "random state": 42,
  "n_filters_per_group": 1,
  "Tangent Class": True,
  "metric": "logeuclid"
# Ensure the output folder exists
outputfolder = settings["outputfolder"]
if not os.path.exists(outputfolder):
  os.makedirs(outputfolder)
# Define the path for the settings file
settings_filepath = os.path.join(outputfolder, "settings.json")
# Save the settings to a JSON file
with open(settings filepath, "w") as f:
  json.dump(settings, f, indent=4)
print(f"Settings have been saved to {settings filepath}")
# Define the output folder
phenotype = settings["phenotype"]
percentile = settings["percentile"]
n_folds = settings["n_folds"]
TanSVM_C = settings["TanSVM_C"]
random state = settings["random state"]
n_filters_per_group = settings["n_filters_per_group"]
Tangent_Class = settings["Tangent_Class"]
# Pyriemannian Mean https://github.com/pyRiemann/pyRiemann/blob/master/pyriemann/utils/
mean.py#L633 Metric for mean estimation, can be: "ale", "alm", "euclid", "harmonic", "identity",
"kullback_sym", "logdet", "logeuclid", "riemann", "wasserstein", or a callable function.
# https://link.springer.com/article/10.1007/s12021-020-09473-9 <---- best descriptions/plots
# Geometric means in a novel vector space structure on symmetric positive-definite matrices
<a href="https://epubs.siam.org/doi/abs/10.1137/050637996?journalCode=sjmael">https://epubs.siam.org/doi/abs/10.1137/050637996?journalCode=sjmael</a>
metric = settings["metric"]
def load_array_from_outputfolder(filename):
  filepath = os.path.join(outputfolder, filename)
  return np.load(filepath)
# Function to save an array to the output folder
def save array to outputfolder(filename, array):
  filepath = os.path.join(outputfolder, filename)
  np.save(filepath, array)
def save text results(text, filename="results.txt"):
  """Save text results to a file."""
  filepath = os.path.join(outputfolder, filename)
  with open(filepath, "a") as f: # Using 'a' to append results to the file
     f.write(text + "\n")
```

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# %% [markdown]
### Memory and Processor Usage/Limits Checks
# %%
# https://www.kernel.org/doc/Documentation/cgroup-v1/memory.txt
#Open terminal for job
# srun --jobid=68974 --overlap --pty /bin/bash
##SLURM RAM
!cgget -r memory.limit_in_bytes /slurm/uid_$SLURM_JOB_UID/job_$SLURM_JOB_ID
#SLURM VM
!cgget -r memory.memsw.limit_in_bytes /slurm/uid_$SLURM_JOB_UID/job_$SLURM_JOB_ID
#SLURM USAGE
!cgget -r memory.memsw.usage_in_bytes /slurm/uid_$SLURM_JOB_UID/job_$SLURM_JOB_ID
!echo "SLURM_JOB_ID: $SLURM_JOB_ID"
!echo "SLURM JOB NAME: $SLURM JOB NAME"
!echo "SLURM_JOB_NODELIST: $SLURM_JOB_NODELIST"
!echo "SLURM MEM PER NODE: $SLURM MEM PER NODE"
!echo "SLURM CPUS ON NODE: $SLURM CPUS ON NODE"
!echo "SLURM_MEM_PER_CPU: $SLURM_MEM_PER_CPU"
!free -h
import resource
# Get the soft and hard limits of virtual memory (address space)
soft, hard = resource.getrlimit(resource.RLIMIT_AS)
print(f"Soft limit: {soft / (1024 ** 3):.2f} GB")
print(f"Hard limit: {hard / (1024 ** 3):.2f} GB")
# Get the soft and hard limits of the data segment (physical memory usage)
soft, hard = resource.getrlimit(resource.RLIMIT_DATA)
print(f"Soft limit: {soft / (1024 ** 3):.2f} GB")
print(f"Hard limit: {hard / (1024 ** 3):.2f} GB")
#TORQUE Virtual Memory
#!cgget -r memory.memsw.limit_in_bytes /torque/$PBS_JOBID
# #TORQUE RAM
# !cgget -r memory.limit_in_bytes /torque/$PBS_JOBID
# #TORQUE USAGE
#!cgget -r memory.memsw.usage_in_bytes /torque/$PBS_JOBID
# print(int(os.environ['PBS NP']))
!nvidia-smi
def gpu_mem():
  # Memory usage information
```

```
print(f"Total memory available: {(torch.cuda.get_device_properties('cuda').total_memory /
1024**3):.2f} GB")
  print(f"Allocated memory: {torch.cuda.memory_allocated() / 1024**3:.2f} GB")
  print(f"Reserved memory: {torch.cuda.memory_reserved() / 1024**3:.2f} GB")
def cpu_mem():
 # Display memory information
  print(f"Total Memory: { psutil.virtual_memory().total / (1024**3):.2f} GB")
  print(f"Available Memory: { psutil.virtual memory().available / (1024**3):.2f} GB")
  print(f"Used Memory: { psutil.virtual_memory().used / (1024**3):.2f} GB")
  print(f"Memory Usage: { psutil.virtual_memory().percent}%")
gpu_mem()
cpu_mem()
# %% [markdown]
#### Select Paths, Parcellate, Standardize, and Save
# %%
def load(phenotype, percentile, folder1=0):
  Load data for a specified number of subjects and fMRI tasks, only if they have not been
parcellated.
  base_directory = "/project_cephfs/3022017.01/S1200"
  subdirectory = "MNINonLinear/Results"
  folders = [
    "rfMRI REST1 LR", "rfMRI REST1 RL", "rfMRI REST2 LR", "rfMRI REST2 RL",
    "tfMRI_EMOTION_LR", "tfMRI_EMOTION_RL", "tfMRI_GAMBLING_LR",
"tfMRI_GAMBLING_RL",
    "tfMRI_LANGUAGE_LR", "tfMRI_LANGUAGE_RL", "tfMRI_MOTOR_LR",
"tfMRI_MOTOR_RL",
    "tfMRI_RELATIONAL_LR", "tfMRI_RELATIONAL_RL", "tfMRI_SOCIAL_LR",
"tfMRI_SOCIAL_RL",
    "tfMRI WM LR", "tfMRI WM RL"
  1
  if folder1 + 1 >= len(folders):
    raise IndexError(f"Invalid folder1 index: {folder1}. Check folder list.")
  if folder1 + 2 \ge len(folders):
    raise IndexError(f"Invalid folder1 index: {folder1}. Check folder list.")
  if folder1 + 3 >= len(folders):
    raise IndexError(f"Invalid folder1 index: {folder1}. Check folder list.")
  subids = [sub for sub in os.listdir(base directory) if os.path.isdir(os.path.join(base directory,
sub))]
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```
file path restricted = r'../HCP/RESTRICTED zainsou 8 6 2024 2 11 21.csv'
file path unrestricted = r'../HCP/unrestricted zainsou 8 2 2024 6 13 22.csv'
try:
  # Load the data from CSV files
  data_r = pd.read_csv(file_path_restricted)
  data ur = pd.read csv(file path unrestricted)
  print("Files loaded successfully.")
except FileNotFoundError:
  print(f"File not found: {file_path_restricted} or {file_path_unrestricted}")
  raise
# Combine restricted and unrestricted data on Subject ID
data = pd.merge(data_r, data_ur, on='Subject', how='outer')
filtered data = data[data['Subject'].astype(str).isin(subids)]
# plt.hist(filtered_data["SSAGA_FTND_Score"], bins=10)
# plt.show()
# picvocab_low_threshold = filtered_data["PicVocab_AgeAdj"].quantile(.2)
# picvocab high threshold = filtered data["PicVocab AgeAdj"].quantile(0.8)
low_threshold = filtered_data[phenotype].quantile(percentile)
high threshold = filtered data[phenotype].guantile(1.0 - percentile)
# pmat24_low_threshold = filtered_data["PMAT24_A_CR"].quantile(.2)
# pmat24_high_threshold = filtered_data["PMAT24_A_CR"].quantile(.8)
# psqi_low_threshold = filtered_data["PSQI_Score"].quantile(.7)
# psqi_high_threshold = filtered_data["PSQI_Score"].quantile(.3)
# ssaga educ low threshold = filtered data["SSAGA Educ"].quantile(.3)
# ssaga_educ_high_threshold = filtered_data["SSAGA_Educ"].quantile(.7)
# lifesatisf low threshold = filtered data["LifeSatisf Unadj"].guantile(.2)
# lifesatisf_high_threshold = filtered_data["LifeSatisf_Unadj"].quantile(.8)
# DDisc low threshold = filtered data["DDisc AUC 200"].quantile(.2)
# DDisc high threshold = filtered data["DDisc AUC 200"].guantile(.8)
## Handedness low threshold = 0
# Handedness_high_threshold = 0
# tobacco low threshold = 4
# tobacco high threshold = 0
# # Binary variables - set the thresholds manually
# fam_hist_threshold_high = 0
# fam hist threshold low = 1
# thc threshold high = 0
# thc_threshold_low = 1
# Filtering Group 1 (high tail) and Group 2 (low tail)
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group_1 = np.array(filtered_data[
     (filtered_data[phenotype] >= high_threshold)
  ]['Subject']).astype(str)
  group_2 = np.array(filtered_data[
     (filtered_data[phenotype] <= low_threshold)
  [['Subject']).astype(str)
  # Print the number of subjects in each group
  print(f"Group 1 (high tail): {len(group_1)} subjects")
  print(f"Group 2 (low tail): {len(group_2)} subjects")
  group_1_paths = []
  for subject in group_1:
     subject_data1 = os.path.join(base_directory, subject, subdirectory, folders[folder1],
folders[folder1] + " Atlas MSMAII hp2000 clean.dtseries.nii")
     subject_data2 = os.path.join(base_directory, subject, subdirectory, folders[folder1 + 1],
folders[folder1 + 1] + "_Atlas_MSMAII_hp2000_clean.dtseries.nii")
     subject_data3 = os.path.join(base_directory, subject, subdirectory, folders[folder1 + 2],
folders[folder1 + 2] + "_Atlas_MSMAII_hp2000_clean.dtseries.nii")
     subject_data4 = os.path.join(base_directory, subject, subdirectory, folders[folder1 + 3],
folders[folder1 + 3] + " Atlas MSMAII hp2000 clean.dtseries.nii")
     if os.path.exists(subject_data1) and os.path.exists(subject_data2) and
os.path.exists(subject_data3) and os.path.exists(subject_data4):
       group_1_paths.append((subject_data1, subject_data2,subject_data3,subject_data4))
  group_2_paths = []
  for subject in group_2:
     subject_data1 = os.path.join(base_directory, subject, subdirectory, folders[folder1],
folders[folder1] + "_Atlas_MSMAII_hp2000_clean.dtseries.nii")
     subject_data2 = os.path.join(base_directory, subject, subdirectory, folders[folder1 + 1],
folders[folder1 + 1] + "_Atlas_MSMAII_hp2000_clean.dtseries.nii")
     subject_data3 = os.path.join(base_directory, subject, subdirectory, folders[folder1 + 2],
folders[folder1 + 2] + "_Atlas_MSMAII_hp2000_clean.dtseries.nii")
     subject_data4 = os.path.join(base_directory, subject, subdirectory, folders[folder1 + 3],
folders[folder1 + 3] + " Atlas MSMAII hp2000 clean.dtseries.nii")
     if os.path.exists(subject_data1) and os.path.exists(subject_data2) and
os.path.exists(subject_data3) and os.path.exists(subject_data4):
       group_2_paths.append((subject_data1, subject_data2,subject_data3,subject_data4))
  print("Length of Group 1:", len(group_1_paths))
  print("Length of Group 2:", len(group_2_paths))
  ## Determine the minimum length
  # min_length = min(len(group_1_paths), len(group_2_paths))
  # # Randomly sample from the larger group to match the size of the smaller group
  # if len(group_1_paths) > min_length:
      group_1_paths = random.sample(group_1_paths, min_length)
```

```
# elif len(group_2_paths) > min_length:
      group_2_paths = random.sample(group_2_paths, min_length)
  # Print the new sizes of both groups
  print("New Length of Group 1:", len(group_1_paths))
  print("New Length of Group 2:", len(group_2_paths))
  return group_1_paths,group_2_paths
groupA paths,groupB paths = load(phenotype=phenotype,percentile=percentile,folder1=0)
# %%
import traceback
def process subject(sub):
  try:
     concatenated data = []
     for task in sub:
       X = nib.load(task).get_fdata(dtype=np.float32)
       Xn = hcp.normalize(X-X.mean(axis=1, keepdims=True))
       concatenated_data.append(Xn)
       del X, Xn
     # Concatenate data along the first axis
     subject = np.concatenate(concatenated_data, axis=0)
     del concatenated_data # Explicitly delete the concatenated data list
     Xp = hcp.parcellate(hcp.normalize(subject - subject.mean(axis=1,keepdims=True)), hcp.mmp)
     Xp = hcp.normalize(Xp - Xp.mean(axis=1,keepdims=True))
     del subject # Explicitly delete the subject array
     return Xp
  except Exception as e:
     print(f"Error processing subject: {e}")
     traceback.print_exc() # Print the full traceback
     return None
def parcellate(group):
  try:
     with ProcessPoolExecutor(max_workers=(int(os.cpu_count()*.3))) as executor:
       # Use map to process subjects in parallel
       group_parcellated = list(executor.map(process_subject, group))
     # Filter out any None results to continue with successful parcellations
     group_parcellated = [result for result in group_parcellated if result is not None]
     return group_parcellated
  except Exception as e:
     print(f"Error in parcellation process: {e}")
     traceback.print_exc()
     return []
```

```
# Example usage
try:
  cpu_mem() # Monitor CPU and memory usage before the operation
  groupA_parcellated = parcellate(groupA_paths)
  cpu_mem() # Monitor CPU and memory usage after processing group A
  groupB parcellated = parcellate(groupB paths)
  cpu_mem() # Monitor CPU and memory usage after processing group B
except Exception as e:
  print(f"An error occurred during the outer loop: {e}")
  traceback.print_exc()
target\_shape = (4800, 379)
# Initialize lists to collect indices of mismatched arrays
mismatched indices A = []
mismatched_indices_B = []
# Create the array for group A, collecting indices of mismatches
groupA parcellated array = np.array([
  array for index, array in enumerate(groupA_parcellated)
  if array.shape == target_shape or mismatched_indices_A.append(index)
])
# Create the array for group B, collecting indices of mismatches
groupB_parcellated_array = np.array([
  array for index, array in enumerate(groupB_parcellated)
  if array.shape == target_shape or mismatched_indices_B.append(index)
1)
# Print the indices of arrays that did not match the target shape
print("Mismatched indices in group A:", mismatched_indices_A)
print("Mismatched indices in group B:", mismatched_indices_B)
groupA_paths_filtered = np.array([path for i, path in enumerate(groupA_paths) if i not in
mismatched_indices_A])
groupB_paths_filtered = np.array([path for i, path in enumerate(groupB_paths) if i not in
mismatched indices B])
print(len(groupA parcellated array))
print(len(groupB_parcellated_array))
# Save the arrays in the specified output folder
# Example usage to save the arrays
save_array_to_outputfolder("groupA_parcellated_array.npy", groupA_parcellated_array)
save_array_to_outputfolder("groupB_parcellated_array.npy", groupB_parcellated_array)
save_array_to_outputfolder("groupA_paths_filtered.npy", groupA_paths_filtered)
save_array_to_outputfolder("groupB_paths_filtered.npy", groupB_paths_filtered)
# %% [markdown]
#### Load Paths & Parcellated
# %%
groupA parcellated array = load array from outputfolder("groupA parcellated array.npy")
groupB_parcellated_array = load_array_from_outputfolder("groupB_parcellated_array.npy")
groupA_paths_filtered = load_array_from_outputfolder("groupA_paths_filtered.npy")
```

```
groupB_paths_filtered = load_array_from_outputfolder("groupB_paths_filtered.npy")
# %%
print(groupA_parcellated_array.shape)
print(groupB_parcellated_array.shape)
total_subjects = groupA_parcellated_array.shape[0] + groupB_parcellated_array.shape[0]
save_text_results(f"Group A: {groupA_parcellated_array.shape}")
save text results(f"Group B: {groupB parcellated array.shape}")
save_text_results(f"Total subjects: {total_subjects}")
# %% [markdown]
#### Test Linear Seperability of Groups Full Parcellated Tangent Covs
def preprocess(train, test=None, method="zscore"):
  Preprocesses the training (and optionally test) data using the specified method.
  Parameters:
  -----
  train: np.ndarray
     The training data array of shape (n_samples, n_features).
  test: np.ndarray, optional
     The test data array of shape (n_samples, n_features), by default None.
  method: str, optional
     The preprocessing method to apply. Supported methods are:
       - "zscore": Standardize features by removing the mean and scaling to unit variance.
       - "mean_center": Center features by subtracting the mean without scaling.
     Default is "zscore".
  Returns:
  np.ndarray or tuple of np.ndarray
     The preprocessed training data. If 'test' is provided, returns a tuple of
     (train_processed, test_processed). If `test` is None, returns only `train_processed`.
  Raises:
  ValueError:
     If an unsupported `method` is provided.
  Examples:
  >>> train_data = np.array([[1, 2], [3, 4], [5, 6]])
  >>> test_data = np.array([[7, 8], [9, 10]])
  >>> preproc(train data, test data, method="zscore")
  (array([[-1.22474487, -1.22474487],
       [0. , 0.
```

```
[ 1.22474487, 1.22474487]]),
   array([[2.44948974, 2.44948974],
       [3.67423461, 3.67423461]]))
  # Initialize variables for consistent return type
  train_processed, test_processed = None, None
  if method == "zscore":
    scaler = StandardScaler()
     train_processed = scaler.fit_transform(train)
     if test is not None:
       test_processed = scaler.transform(test)
  elif method == "mean_center":
     mean = train.mean(axis=0)
     train processed = train - mean
     if test is not None:
       test_processed = test - mean
     raise ValueError(f"Unknown method '{method}'. Supported methods are 'zscore' and
'mean center'.")
  # Return train_processed, and test_processed if test is provided; else return only
train_processed
  return (train_processed, test_processed) if test is not None else train_processed
def extract_sample_weights(train_subject_ids, y_train):
  Compute sample weights based on normalized phenotype scores for the training data,
  ensuring equal total contribution from both groups.
  file path restricted = '../HCP/RESTRICTED zainsou 8 6 2024 2 11 21.csv'
  file_path_unrestricted = '../HCP/unrestricted_zainsou_8_2_2024_6_13_22.csv'
  try:
     data_r = pd.read_csv(file_path_restricted)
     data_ur = pd.read_csv(file_path_unrestricted)
  except FileNotFoundError:
     print(f"File not found: {file_path_restricted} or {file_path_unrestricted}")
     raise
  # Combine restricted and unrestricted data on Subject ID
  data = pd.merge(data_r, data_ur, on='Subject', how='outer')
  # Convert Subject IDs to string for consistency
  data['Subject'] = data['Subject'].astype(str)
  train_subject_ids = train_subject_ids.astype(str)
  # Filter data for training subjects
```

```
train_data = data[data['Subject'].isin(train_subject_ids)]
# Ensure the order matches the training data
train_data = train_data.set_index('Subject').loc[train_subject_ids].reset_index()
# Extract individual phenotype scores
# pic vocab scores = train data['PicVocab AgeAdi']
# pmat24_scores = train_data['PMAT24_A_CR']
# Normalize each phenotype score individually between 0 and 1
# pic_vocab_min = pic_vocab_scores.min()
# pic_vocab_max = pic_vocab_scores.max()
# pic_vocab_norm = (pic_vocab_scores - pic_vocab_min) / (pic_vocab_max - pic_vocab_min)
# pmat24 min = pmat24 scores.min()
# pmat24 max = pmat24 scores.max()
# pmat24_norm = (pmat24_scores - pmat24_min) / (pmat24_max - pmat24_min)
# phenotype_scores = (pic_vocab_norm + pmat24_norm) / 2
pheno_score = train_data['CogCrystalComp_Unadj']
pheno_min = pheno_score.min()
pheno max = pheno score.max()
pheno_norm = (pheno_score - pheno_min) / (pheno_max - pheno_min)
phenotype_scores = pheno_norm
# Initialize sample weights array
sample weights = np.zeros(len(y train))
# Assign weights based on group labels
for idx, label in enumerate(y_train):
  if label == 1:
    # For Group 1: Higher scores ⇒ Higher weights
    sample_weights[idx] = phenotype_scores.iloc[idx]
  else:
    # For Group 2: Higher scores ⇒ Lower weights
    sample weights[idx] = 1 - phenotype scores.iloc[idx]
# Optional: Raise weights to a power to accentuate differences (if desired)
# sample_weights = sample_weights ** exponent # Adjust the exponent as needed
# Ensure total weights for each group are equal
group1_indices = np.where(y_train == 1)[0]
group2 indices = np.where(y train == 0)[0]
sum_group1_weights = np.sum(sample_weights[group1_indices])
sum_group2_weights = np.sum(sample_weights[group2_indices])
# Compute the scaling factor for each group
total_weight = (sum_group1_weights + sum_group2_weights) / 2
scale group1 = total weight / sum group1 weights if sum group1 weights != 0 else 1
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scale group2 = total weight / sum group2 weights if sum group2 weights != 0 else 1
  # Apply scaling factors to ensure equal total weight per group
  sample_weights[group1_indices] *= scale_group1
  sample weights[group2 indices] *= scale group2
  # Plotting to visualize the sample weights
  plt.figure(figsize=(10, 6))
  plt.scatter(phenotype scores, sample weights, c=y train, cmap='coolwarm', alpha=0.7)
  plt.xlabel('Normalized Combined Phenotype Score')
  plt.ylabel('Sample Weight')
  plt.title('Sample Weights After Scaling')
  plt.colorbar(label='Group Label')
  plt.show()
  return sample_weights
def tangent_transform(group1_train_cov, group1_test_cov, group2_train_cov, group2_test_cov,
  # Compute the Fréchet mean using only the training data from both groups
  Frechet Mean = mean covariance(
    np.concatenate((group1_train_cov, group2_train_cov)),
    metric=metric
  )
  # Perform tangent space projection
  train_1 = tangent_space(group1_train_cov, Frechet_Mean, metric=metric)
  train_2 = tangent_space(group2_train_cov, Frechet_Mean, metric=metric)
  test 1 = tangent space(group1 test cov, Frechet Mean, metric=metric)
  test 2 = tangent space(group2 test cov, Frechet Mean, metric=metric)
  return train_1, train_2, test_1, test_2
def test_classifiers(train_1, test_1, train_2, test_2, sample_weights_train=None):
  # Dictionary with keys for each classifier
  clf dict = {
    "SVM (C=1)": SVC(kernel='linear', C=1, class weight='balanced'),
    "SVM (C=0.1)": SVC(kernel='linear', C=0.1, class_weight='balanced'),
    "SVM (C=0.01)": SVC(kernel='linear', C=0.01, class_weight='balanced'),
    "L2 SVM (C=1)": LinearSVC(penalty='l2',loss='squared_hinge',C=1,class_weight='balanced'),
    "L2 SVM (C=0.1)":
LinearSVC(penalty='12',loss='squared hinge',C=.1,class weight='balanced'),
    "L2 SVM (C=0.01)":
LinearSVC(penalty='l2',loss='squared hinge',C=.01,class weight='balanced'),
    "L2 SVM Hinge (C=1)": LinearSVC(penalty='l2',loss='hinge',C=1,class_weight='balanced'),
    "L2 SVM Hinge (C=0.1)": LinearSVC(penalty='l2',loss='hinge',C=.1,class weight='balanced'),
    "L2 SVM Hinge (C=0.01)":
LinearSVC(penalty='12',loss='hinge',C=.01,class weight='balanced'),
    "L1 SVM (C=1)":
LinearSVC(penalty="11',loss='squared hinge',dual=False,C=1,class weight='balanced'),
     # "L1 SVM (C=0.1)":
LinearSVC(penalty='l1',loss='squared_hinge',dual=False,C=.1,class_weight='balanced'),
```

```
# "L1 SVM (C=0.01)":
LinearSVC(penalty='I1',loss='squared hinge',dual=False,C=.01,class weight='balanced'),
     "LDA": LDA(),
     "Logistic Regression (default)": LogisticRegression(),
     "Logistic Regression (I2)": LogisticRegression(penalty='I2', class_weight='balanced'),
     "Logistic Regression (I1)": LogisticRegression(penalty='I1', solver='liblinear',
class weight='balanced').
  # "Logistic Regression (elasticnet)": LogisticRegression(penalty='elasticnet', solver='saga',
11 ratio=0.2, class weight='balanced')
   # Combine training data and labels
  X_train = np.concatenate((train_1, train_2))
  y_train = np.concatenate((np.ones(len(train_1)), np.zeros(len(train_2))))
  # Combine test data and labels
  X_test = np.concatenate((test_1, test_2))
  y_test = np.concatenate((np.ones(len(test_1)), np.zeros(len(test_2))))
  X_train, X_test = preprocess(X_train, X_test)
  # Ensure sample weights train aligns with X train and y train
  if sample weights train is not None:
     assert len(sample_weights_train) == len(y_train), "Sample weights length mismatch."
  # Calculate the distance between the two class means
  # mean_group1_train = np.mean(group1_test, axis=0)
  # mean_group2_train = np.mean(group2_test, axis=0)
  # distance_vars = np.linalg.norm(mean_group1_train - mean_group2_train)
  # Initialize a dictionary to store the metrics
  metrics_dict = {}
  # Iterate through each classifier and calculate accuracy
  for key, clf in clf_dict.items():
     clf.fit(X train, y train)
     # clf.fit(X_train, y_train, sample_weight=sample_weights_train)
     predictions = clf.predict(X_test)
     accuracy = accuracy_score(y_test, predictions)
     correct_predictions = np.sum(predictions == y_test)
     total_predictions = len(y_test)
     # Confusion matrix to get per-class accuracy
     cm = confusion_matrix(y_test, predictions, labels=[1, 0])
     per_class_correct = np.diag(cm)
     per_class_total = np.sum(cm, axis=1)
     per_class_accuracy = per_class_correct / per_class_total
     metrics dict[key] = {
       'accuracy': accuracy,
       'correct_predictions': correct_predictions,
```

```
'total_predictions': total_predictions,
       'per_class_correct': per_class_correct,
       'per_class_total': per_class_total,
       'per_class_accuracy': per_class_accuracy
    }
  # if test_data.shape[1] == 2:
     # Plot when n=1
      plt.figure(figsize=(8, 6))
      plt.scatter(group1_test[:, 0], group1_test[:, 1], label='Group 1 Log Variance (Test)',
color='blue')
      plt.scatter(group2_test[:, 0], group2_test[:, 1], label='Group 2 Log Variance (Test)',
color='red')
       # Plot the line connecting the two means
       plt.plot([mean_group1_train[0], mean_group2_train[0]], [mean_group1_train[1],
mean_group2_train[1]], 'k--', label=f'Mean Distance: {distance_vars:.2f}')
      # Decision boundary
  #
      x_values = np.array([data[:, 0].min(), data[:, 0].max()])
      y_values = -(max_clf.intercept_ + max_clf.coef_[0][0] * x_values) / max_clf.coef_[0][1]
      plt.plot(x_values, y_values, 'g-', label='Decision Boundary')
  #
       # Display plot
       plt.xlabel('Log Variance Feature 1')
      plt.ylabel('Log Variance Feature 2')
      plt.title(f'Log Variance Comparison and {max_key} Decision Boundary')
  #
      # Display classification accuracy on the plot
       plt.text(0.05, 0.95, f'Accuracy: {accuracy:.2f}', transform=plt.gca().transAxes, fontsize=12,
      verticalalignment='top', bbox=dict(boxstyle='round,pad=0.3', edgecolor='black',
facecolor='lightgrey'))
       plt.legend()
  #
      plt.grid(True)
      plt.show()
  return metrics_dict
def test_linear_sep(groupA_parcellated_array, groupB_parcellated_array,groupA_paths,
groupB_paths,metric='riemann',n_splits=10):
  cov_est = Covariances(estimator='lwf')
  groupA_parcellated_covs = cov_est.transform(np.transpose(groupA_parcellated_array, (0, 2, 1)))
  groupB_parcellated_covs = cov_est.transform(np.transpose(groupB_parcellated_array, (0, 2, 1)))
  data = np.concatenate((groupA_parcellated_covs, groupB_parcellated_covs))
  labels = np.concatenate((np.ones(len(groupA_parcellated_covs)),
np.zeros(len(groupB_parcellated_covs))))
  all_paths = np.concatenate((groupA_paths, groupB_paths))
  subject_ids = np.array([re.search(r'/(\d+)/', path[0]).group(1) for path in all_paths])
  skf = StratifiedKFold(n_splits=n_splits, shuffle=True, random_state=42)
```

```
metrics_dict = {}
for fold, (train_index, test_index) in enumerate(skf.split(data, labels)):
  print(f"\nProcessing Fold {fold + 1}/{n_splits}")
  data_train, data_test = data[train_index], data[test_index]
  labels train, labels test = labels[train index], labels[test index]
  group1 train cov = data train[labels train == 1]
  group2_train_cov = data_train[labels_train == 0]
  group1_test_cov = data_test[labels_test == 1]
  group2_test_cov = data_test[labels_test == 0]
  # train_subject_ids = subject_ids[train_index]
  # # Compute sample weights for the training data
  # sample_weights_train = extract_sample_weights(train_subject_ids, labels_train)
  # Perform tangent space transformation
  train_1, train_2, test_1, test_2 = tangent_transform(
     group1_train_cov, group1_test_cov,
     group2_train_cov, group2_test_cov,
     metric=metric
  )
  fold_metrics = test_classifiers(train_1, test_1, train_2, test_2, sample_weights_train=None)
  # Initialize metrics_dict keys on first fold
  if fold == 0:
     for clf name in fold metrics.keys():
       metrics dict[clf name] = {
          'correct_predictions': 0,
          'total_predictions': 0,
          'accuracies': [],
          'per_class_correct': np.array([0, 0]),
          'per_class_total': np.array([0, 0])
       }
  # Aggregate metrics
  for clf_name, metrics in fold_metrics.items():
     # Update total correct predictions and total samples
     metrics_dict[clf_name]['correct_predictions'] += metrics['correct_predictions']
     metrics_dict[clf_name]['total_predictions'] += metrics['total_predictions']
     # Store accuracies for mean and std calculation
     metrics_dict[clf_name]['accuracies'].append(metrics['accuracy'])
     # Update per-class correct and total counts
     metrics dict[clf name]['per class correct'] += metrics['per class correct']
     metrics_dict[clf_name]['per_class_total'] += metrics['per_class_total']
     # Print per-fold metrics
     print(f"Classifier: {clf name}")
     print(f" Fold Accuracy: {metrics['accuracy'] * 100:.2f}%")
     print(f" Correct Predictions: {metrics['correct_predictions']}/{metrics['total_predictions']}")
```

```
print(f" Per-Class Accuracy: {metrics['per_class_accuracy'] * 100}")
  # After all folds, compute overall metrics
  print("\nOverall Metrics Across All Folds:")
  for clf_name, clf_metrics in metrics_dict.items():
    overall_accuracy = clf_metrics['correct_predictions'] / clf_metrics['total_predictions']
    mean accuracy = np.mean(clf metrics['accuracies'])
    std_accuracy = np.std(clf_metrics['accuracies'])
    per class accuracy = clf metrics['per class correct'] / clf metrics['per class total']
    print(f"\nClassifier: {clf_name}")
    print(f" Total Accuracy: {overall_accuracy * 100:.2f}%")
    print(f" Average Fold Accuracy: {mean_accuracy * 100:.2f}%")
    print(f" Fold Accuracy Std Dev: {std_accuracy * 100:.2f}%")
    print(f" Per-Class Accuracy: {per_class_accuracy * 100}")
  return metrics_dict
# %%
full_cov_cross_val = test_linear_sep(groupA_parcellated_array,
groupB_parcellated_array,groupA_paths_filtered, groupB_paths_filtered, metric=metric,n_splits=10)
save text results(f"Tangent Space Classification Accuracy Results: {full cov cross val}")
# %% [markdown]
#### FKT Functions and Cross Validate Filters
# %%
# def tangent_LDA(group1_covs=None, group2_covs=None,
Frechet Mean=None,tangent projected 1=None, tangent projected 2=None, tangent calc=True,
metric="riemann",k=1, visualize=False):
    if tangent_calc:
#
      all_covs = np.concatenate((group1_covs, group2_covs))
#
      Frechet_Mean = mean_covariance(all_covs, metric=metric)
#
      tangent_projected_1 = tangent_space(group1_covs, Frechet_Mean, metric=metric)
      tangent projected 2 = tangent space(group2 covs, Frechet Mean, metric=metric)
   # Initialize the Covariances estimator
   cov_estimator = Covariances(estimator='lwf')
    tangent_projected_mean = np.mean(np.concatenate((tangent_projected_1,
tangent_projected_2)),axis=0,keepdims=True)
    tangent_1_mean = np.mean(tangent_projected_1,axis=0,keepdims=True) -
tangent projected mean
    tangent_2_mean = np.mean(tangent_projected_2,axis=0,keepdims=True) -
tangent projected mean
   tangent_between_scatter = tangent_1_mean.T@tangent_1_mean +
tangent 2 mean.T@tangent 2 mean
# # Estimate the covariance matrices
# cov_tangent_projected_1 =
cov_estimator.transform(np.transpose(tangent_projected_1[np.newaxis,:,:],(0,2,1)))[0]
```

```
cov_tangent_projected_2 =
cov_estimator.transform(np.transpose(tangent_projected_2[np.newaxis,:,:],(0,2,1)))[0]
    # Convert to PyTorch tensors and move to GPU
    device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
    print(("cuda" if torch.cuda.is available() else "cpu"))
   gpu_mem()
   sum = torch.tensor(cov tangent projected 1, dtype=torch.float32, device="cuda") +
torch.tensor(cov_tangent_projected_2, dtype=torch.float32, device="cuda")
# # cov_tangent_projected_1 = torch.tensor(cov_tangent_projected_1, dtype=torch.float32,
device=device)
   # gpu_mem()
   # cov_tangent_projected_2 = torch.tensor(cov_tangent_projected_2, dtype=torch.float32,
device=device)
# # gpu_mem()
   gpu_mem()
   tangent_between_scatter = torch.tensor(tangent_between_scatter, dtype=torch.float32,
device=device)
   gpu mem()
   # For filtersA
    _, eigvecs = torch.lobpcg(tangent_between_scatter, B=sum, k=k, largest=True)
    del sum, tangent_between_scatter
   torch.cuda.empty_cache()
   gpu_mem()
   eigvecs_array = eigvecs.clone().cpu().numpy()
   filters = untangent_space(eigvecs_array.T, Frechet_Mean)
    fkt_riem_eigs, filtersA, filtersB = FKT(filters[0,:,:], Frechet_Mean, mean=metric,
average=False, visualize=visualize)
   return fkt_riem_eigs, filters, filtersA, filtersB
def tangent haufe(data, filters, method="basic", alpha=1, beta=0, l1 ratio=0.5, lambda1=.01,
lambda2=.01):
  S = (data @ filters)
  if method == "basic":
    proj = (np.linalg.pinv(S)@ data)
  elif method == "covs":
    cov_est_scm = Covariances(estimator='scm')
    s\_cov = cov\_est\_scm.transform(S.T[np.newaxis,:,:])[0,:,:]
    data_cov = cov_est_scm.transform(data.T[np.newaxis,:,:])[0,:,:]
    proj = (data_cov @ filters @ np.linalg.inv(s_cov)).T
  elif method == "linreg":
    reg = LinearRegression()
    reg.fit(S, data)
    proj = reg.coef_.T
  elif method == "grouplassolinreg":
```

```
reg = MultiTaskLasso(alpha=alpha) # Using 5-fold cross-validation
     reg.fit(S, data)
     proj = reg.coef_.T
  elif method == "lassolinreg":
     reg = Lasso(alpha=alpha) # Using 5-fold cross-validation
     reg.fit(S, data)
     proj = req.coef .T
  elif method == "elasticlinreg":
     reg = ElasticNet(alpha=alpha, I1 ratio=I1 ratio)
     reg.fit(S, data)
     proj = reg.coef_.T
  elif method == "growl":
     # Proximal Operator for GrOWL targeting columns
    def prox_growl(V, lambda1, lambda2, tau):
       p, r = V.shape
       norms = np.linalg.norm(V, axis=0) # Norms of columns
       indices = np.argsort(-norms) # Sort indices by descending norms
       weights = lambda1 + lambda2 * np.linspace(1, 0, r) # Weights decrease
       V_new = np.zeros_like(V)
       for i in range(r):
         idx = indices[i]
         if norms[idx] > weights[i] * tau:
            V_{new[:, idx]} = (1 - tau * weights[i] / norms[idx]) * V[:, idx]
       return V_new
     # Initialization
     B = np.zeros((filters.shape[1], data.shape[1]))
     # Optimization Loop
     max iter = 100
     learning_rate = 0.01
     for _ in range(max_iter):
       gradient = S.T @ (S @ B - data)
       B -= learning_rate * gradient
       B = prox growl(B, lambda1, lambda2, tau=learning rate)
       if np.linalg.norm(gradient) < 1e-1:
         break
     proj = B.T
  return proj
def tangent_CSP(group1_covs=None, group2_covs=None,
Frechet Mean=None,tangent projected 1=None, tangent projected 2=None, tangent calc=True,
metric="riemann",k=1, visualize=False):
  if tangent calc:
     all_covs = np.concatenate((group1_covs, group2_covs))
     Frechet Mean = mean covariance(all covs, metric=metric)
     tangent_projected_1 = tangent_space(group1_covs, Frechet_Mean, metric=metric)
     tangent projected 2 = tangent space(group2 covs, Frechet Mean, metric=metric)
  device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
```

```
print(("cuda" if torch.cuda.is_available() else "cpu"))
  tangent_projected_1_gpu = torch.tensor(tangent_projected_1, dtype=torch.float32,
device=device)
  cov_tangent_projected_1 = (tangent_projected_1_gpu.T@tangent_projected_1_gpu)/
tangent_projected_1_gpu.shape[0]
  del tangent projected 1 apu
  torch.cuda.empty_cache()
  tangent_projected_2_gpu = torch.tensor(tangent_projected_2, dtype=torch.float32,
device=device)
  cov_tangent_projected_2 = (tangent_projected_2_gpu.T@tangent_projected_2_gpu)/
tangent_projected_2_gpu.shape[0]
  del tangent_projected_2_gpu
  torch.cuda.empty_cache()
  sum = cov_tangent_projected_1 + cov_tangent_projected_2
  # Group 1 Filters
  _, tangent_eigvecs1 = torch.lobpcg(cov_tangent_projected_1, B=sum, method='ortho', k=k,
largest=True)
  # Convert tangent eigvecs1 back to NumPy and CPU
  tangent_eigvecs1_np = tangent_eigvecs1.cpu().numpy()
  del cov_tangent_projected_1, tangent_eigvecs1
  torch.cuda.empty_cache()
  filters1 = []
  for i in range(0,k):
     filters untangent 1 i = untangent space(tangent eigvecs1 np.T, Frechet Mean)[i,:,:]
     _, filters_1_i = eigh(filters_untangent_1_i, Frechet_Mean)
     for j in range(0,k):
       filters1.append(filters_1_i[:,-j])
  filters1 = np.array(filters1).T
  # Group 2 Filters
  _, tangent_eigvecs2 = torch.lobpcg(cov_tangent_projected_2, B=sum,method='ortho', k=k,
largest=True)
  tangent_eigvecs2_np = tangent_eigvecs2.cpu().numpy()
  del sum,cov_tangent_projected_2, tangent_eigvecs2
  torch.cuda.empty_cache()
  filters2 = []
  for i in range(0,k):
     filters untangent 2 i = untangent space(tangent eigvecs2 np.T, Frechet Mean)[i,...]
     _, filters_2_i = eigh(filters_untangent_2_i, Frechet_Mean)
     for j in range(0,k):
       filters2.append(filters_2_i[:,-j])
  filters2 = np.array(filters2).T
  filters = np.concatenate((filters2[:, ::-1], filters1), axis=1)
  print(filters.shape, filters1.shape, filters2.shape)
  return None, filters, filters1, filters2
```

```
def FKT(groupA_cov_matrices, groupB_cov_matrices, mean="riemann", average=True,
visualize=True, n=0):
  # Eigenvalues in ascending order from scipy eigh https://docs.scipy.org/doc/scipy/reference/
generated/scipy.linalg.eigh.html
  if average:
    groupA cov = mean covariance(groupA cov matrices, metric=mean)
    groupB_cov = mean_covariance(groupB_cov_matrices, metric=mean)
    # eigs, filters = eigh(groupA cov, groupA cov + groupB cov +
gamma*np.identity(groupB_cov.shape[0]),eigvals_only=False)
    groupA_cov = groupA_cov_matrices
    groupB_cov = groupB_cov_matrices
  if n > 0:
    eigsA, filtersA = eigh(groupA_cov, groupA_cov +
groupB cov,eigvals only=False, subset by index=[groupA cov.shape[0]-n,
groupA_cov.shape[0]-1])
     eigsB, filtersB = eigh(groupB_cov, groupA_cov +
groupB_cov,eigvals_only=False,subset_by_index=[groupB_cov.shape[0]-n,
groupB_cov.shape[0]-1])
  else:
     eigsA, filtersA = eigh(groupA cov, groupA cov +
groupB_cov,eigvals_only=False,subset_by_value=[0.5,np.inf])
    eigsB, filtersB = eigh(groupB_cov, groupA_cov +
groupB_cov,eigvals_only=False,subset_by_value=[0.5,np.inf])
  eigs = np.concatenate((eigsB[::-1], eigsA))
  filters = np.concatenate((filtersB[:, ::-1], filtersA), axis=1)
  fkt_riem_eigs = np.abs(np.log(eigs/(1-eigs)))**2
  if visualize:
    plt.figure(figsize=(10, 5))
    plt.scatter(range(0,fkt_riem_eigs.shape[0]),fkt_riem_eigs)
    plt.show()
  return fkt riem eigs, filters, filtersA, filtersB
def tangent classifier(group1 covs=None, group2 covs=None, Frechet Mean=None,
tangent_projected_1=None, tangent_projected_2=None, TSVM=True, TLDA=False,
tangent_calc=True,metric="riemann",visualize=False,n=0):
  if tangent_calc:
    all_covs = np.concatenate((group1_covs, group2_covs))
    Frechet Mean = mean covariance(all covs, metric=metric)
    tangent_projected_1 = tangent_space(group1_covs, Frechet_Mean, metric=metric)
    tangent_projected_2 = tangent_space(group2_covs, Frechet_Mean, metric=metric)
  # Create labels for each group
  labels_1 = np.ones(len(tangent_projected_1)) # Labels for group 1
  labels_2 = np.zeros(len(tangent_projected_2)) # Labels for group 2
  data = np.concatenate((tangent_projected_1, tangent_projected_2))
  data = preprocess(data)
  labels = np.concatenate((labels_1, labels_2))
```

```
# Create SVM classifier (adjust kernel and parameters as needed)
    # C_values = [0.001, 0.01, 0.1, 1, 10]
    clf = LinearSVC(penalty='l1',loss='squared_hinge',dual=False,C=1,class_weight='balanced')
    # clf = LogisticRegression(class_weight='balanced')
    # clf = LinearSVC(penalty='l2',loss='hinge',C=.01,class_weight='balanced')
    # clf = LogisticRegression(penalty='elasticnet', C=1, solver='saga', 11 ratio=0.1)
    # clf = SVC(kernel='linear', C=1, class_weight='balanced')
    # clf = LogisticRegression()
    # Train the classifier
    clf.fit(data, labels)
    # coef = clf.coef
    # coef = normalize(clf.coef_, axis=1)
    coef = tangent_haufe(data, clf.coef_.T,method="basic")
    filters SVM = untangent space(coef, Frechet Mean)[0,:,:]
    fkt_riem_eigs_tangent_SVM, fkt_filters_tangent_SVM = eigh(filters_SVM, Frechet_Mean)
    # If test data is provided, project the test data to tangent space
    # tangent_projected_1_discrim_reconstruction =
np.mean(tangent_projected_1@np.linalg.pinv(clf.coef_)@clf.coef_,axis=0)
    # print(tangent projected 1 discrim reconstruction.shape)
    # tangent_projected_2_discrim_reconstruction =
np.mean(tangent_projected_2@np.linalg.pinv(clf.coef_)@clf.coef_,axis=0)
    # print(tangent_projected_2_discrim_reconstruction.shape)
    # group1_discrim_mean = untangent_space(tangent_projected_1_discrim_reconstruction,
Frechet_Mean)
    # print(group1_discrim_mean.shape)
    # group2 discrim mean = untangent space(tangent projected 2 discrim reconstruction,
Frechet Mean)
    # print(group2_discrim_mean.shape)
    # fkt_riem_eigs_tangent_SVM, fkt_filters_tangent_SVM, filtersA, filtersB =
FKT(group1_discrim_mean, group2_discrim_mean, mean=metric, average=False,
visualize=visualize, n=n)
    # Return accuracy along with filters
    # fkt_riem_eigs_tangent_SVM, fkt_filters_tangent_SVM, filtersA, filtersB =
FKT(filters_SVM[0, :, :], Frechet_Mean, mean=metric, average=False, visualize=visualize, n=n)
    # eigs1, filters1 = eigh(filters_SVM, mean_covariance(group2_covs,
metric=metric) ,eigvals_only=False,subset_by_value=[0.5,np.inf])
    # eigs2, filters2 = eigh(filters_SVM, mean_covariance(group1_covs,
metric=metric) ,eigvals_only=False,subset_by_value=[0.5,np.inf])
    # eigs = np.concatenate((eigs2[::-1], eigs1))
    # fkt_filters_tangent_SVM = np.concatenate((filters2[:, ::-1], filters1), axis=1)
    # fkt_riem_eigs_tangent_SVM = np.abs(np.log(eigs))**2
    if visualize:
plt.scatter(range(0,fkt_riem_eigs_tangent_SVM.shape[0]),np.abs(np.log(fkt_riem_eigs_tangent_SV
M)))
       plt.show()
```

if TSVM:

```
return fkt_riem_eigs_tangent_SVM, fkt_filters_tangent_SVM, _, _
  if TLDA:
     # Create LDA classifier
     Ida = LDA()
     # Train the classifier
     Ida.fit(data, labels)
     # Get the coefficients from LDA
     normalized coef = normalize(lda.coef , axis=1)
     filters_LDA = untangent_space(normalized_coef, Frechet_Mean)
     fkt_filters_tangent_LDA, fkt_riem_eigs_tangent_LDA, filtersA, filtersB = FKT(filters_LDA[0,:,:],
Frechet_Mean, mean=metric, average=False, visualize=visualize,n=n)
     return fkt_filters_tangent_LDA, fkt_riem_eigs_tangent_LDA, filtersA, filtersB
def tangent_classifier_multi(group1_covs=None, group2_covs=None, Frechet_Mean=None,
tangent projected 1=None, tangent projected 2=None, TSVM=True, TLDA=False,
tangent_calc=True,metric="riemann",visualize=False,k=1):
  if tangent calc:
     all_covs = np.concatenate((group1_covs, group2_covs))
     Frechet Mean = mean covariance(all covs, metric=metric)
     tangent_projected_1 = tangent_space(group1_covs, Frechet_Mean, metric=metric)
     tangent projected 2 = tangent space(group2 covs, Frechet Mean, metric=metric)
  # Create labels for each group
  labels_1 = np.ones(len(tangent_projected_1)) # Labels for group 1
  labels_2 = np.zeros(len(tangent_projected_2)) # Labels for group 2
  data = np.concatenate((tangent_projected_1, tangent_projected_2))
  labels = np.concatenate((labels_1, labels_2))
  tan vecs = None
  for i in range(k):
     clf = SVC(kernel='linear', C=1, class_weight='balanced')
     # Collect classifier coefficients (clf.coef_ for linear SVM)
     if i == 0:
       clf.fit(data, labels)
       tan vecs = clf.coef
     else:
       residual proj = data - (data@np.linalg.pinv(tan vecs))@tan vecs
       clf.fit(residual_proj, labels)
       tan_vecs = np.concatenate((tan_vecs, clf.coef_), axis=0) # Concatenate along the row axis
     tan_vecs = np.array(tan_vecs)
  untan_vecs = untangent_space(tan_vecs, Frechet_Mean)
  filters1 = []
  filters2 = []
  for i in range(0,k):
     _, filters_iter = eigh(untan_vecs[i,:,:], Frechet_Mean)
     filters1.append(filters_iter[:,-1:])
     filters2.append(filters iter[:,0:1])
  # filters1 = np.array(filters1).T
  # filters2 = np.array(filters2).T
```

# Convert lists to NumPy arrays for proper shape handling and transpose if needed

```
filters1 = np.hstack(filters1) # Stack horizontally
  filters2 = np.hstack(filters2) # Stack horizontally
  print("Filters1 shape (largest eigenvectors):", filters1.shape)
  print("Filters2 shape (smallest eigenvectors):", filters2.shape)
  filters = np.concatenate((filters2, filters1[:, ::-1]), axis=1)
  return None, filters, filters1, filters2
# %%
def feature_generation(group1_train,group1_test, group2_train, group2_test, filters,method='log-
var',metric='riemann'):
  group1 train transformed = group1 train @ filters
  group2 train transformed = group2 train @ filters
  group1_test_transformed = group1_test @ filters
  group2_test_transformed = group2_test @ filters
  if method == 'log-var':
     train_1 = np.log(np.var(group1_train_transformed, axis=1))
     train 2 = np.log(np.var(group2 train transformed, axis=1))
     test_1 = np.log(np.var(group1_test_transformed, axis=1))
     test_2 = np.log(np.var(group2_test_transformed, axis=1))
  elif method == 'log-cov':
     cov_est = Covariances(estimator='lwf')
     train_1_cov = cov_est.transform(np.transpose(group1_train_transformed, (0, 2, 1)))
     train_2_cov = cov_est.transform(np.transpose(group2_train_transformed, (0, 2, 1)))
     test_1_cov = cov_est.transform(np.transpose(group1_test_transformed, (0, 2, 1)))
     test_2_cov = cov_est.transform(np.transpose(group2_test_transformed, (0, 2, 1)))
     train_1, train_2, test_1, test_2 = tangent_transform(train_1_cov, test_1_cov, train_2_cov,
test_2_cov, metric)
  return train 1, test 1, train 2, test 2
def validate_parcellated_filters(groupA_parcellated_array, groupB_parcellated_array,
metric='riemann',method='log-var',n_splits=10,filters_per_group=1,Tangent_Class=True):
  cov_est = Covariances(estimator='oas')
  groupA_parcellated_covs = cov_est.transform(np.transpose(groupA_parcellated_array, (0, 2, 1)))
  groupB_parcellated_covs = cov_est.transform(np.transpose(groupB_parcellated_array, (0, 2, 1)))
  covs = np.concatenate((groupA_parcellated_covs, groupB_parcellated_covs))
  data = np.concatenate((groupA_parcellated_array, groupB_parcellated_array))
  labels = np.concatenate((np.ones(len(groupA_parcellated_covs)),
np.zeros(len(groupB_parcellated_covs))))
  skf = StratifiedKFold(n_splits=n_splits, shuffle=True, random_state=42)
  metrics_dict = {}
```

```
for fold, (train index, test index) in enumerate(skf.split(data, labels)):
     print(f"\nProcessing Fold {fold + 1}/{n_splits}")
     data_train, data_test = data[train_index], data[test_index]
     covs_train, covs_test = covs[train_index], covs[test_index]
     labels_train, labels_test = labels[train_index], labels[test_index]
     data1 train = data train[labels train == 1]
     data2_train = data_train[labels_train == 0]
     data1 test = data test[labels test == 1]
     data2_test = data_test[labels_test == 0]
     cov1_train = covs_train[labels_train == 1]
     cov2_train = covs_train[labels_train == 0]
     if Tangent Class:
        _, filters, _, _ = tangent_classifier(cov1_train, cov2_train, TSVM=True, TLDA=False,
tangent_calc=True, metric=metric, visualize=False, n=0)
        _, filters, _, _ = FKT(cov1_train, cov2_train, mean=metric, average=True, visualize=False,
n=0)
     selected filters = np.concatenate((filters[:, :filters per group], filters[:, -filters per group:]),
axis=1)
     train_1, test_1, train_2, test_2 = feature_generation(data1_train, data1_test, data2_train,
data2_test, selected_filters,method=method,metric=metric)
     fold_metrics = test_classifiers(train_1, test_1, train_2, test_2, sample_weights_train=None)
     if fold == 0:
       for clf name in fold metrics.keys():
          metrics_dict[clf_name] = {
             'correct_predictions': 0,
             'total_predictions': 0,
             'accuracies': [],
             'per class correct': np.array([0, 0]).
             'per_class_total': np.array([0, 0])
          }
     # Aggregate metrics
     for clf_name, metrics in fold_metrics.items():
       # Update total correct predictions and total samples
       metrics_dict[clf_name]['correct_predictions'] += metrics['correct_predictions']
       metrics_dict[clf_name]['total_predictions'] += metrics['total_predictions']
       # Store accuracies for mean and std calculation
       metrics_dict[clf_name]['accuracies'].append(metrics['accuracy'])
       # Update per-class correct and total counts
       metrics_dict[clf_name]['per_class_correct'] += metrics['per_class_correct']
       metrics_dict[clf_name]['per_class_total'] += metrics['per_class_total']
       # Print per-fold metrics
       print(f"Classifier: {clf_name}")
       print(f" Fold Accuracy: {metrics['accuracy'] * 100:.2f}%")
```

```
print(f" Correct Predictions: {metrics['correct_predictions']}/{metrics['total_predictions']}")
       print(f" Per-Class Accuracy: {metrics['per_class_accuracy'] * 100}")
  # After all folds, compute overall metrics
  print("\nOverall Metrics Across All Folds:")
  for clf_name, clf_metrics in metrics_dict.items():
     overall accuracy = clf metrics['correct predictions'] / clf metrics['total predictions']
     mean_accuracy = np.mean(clf_metrics['accuracies'])
     std accuracy = np.std(clf metrics['accuracies'])
     per_class_accuracy = clf_metrics['per_class_correct'] / clf_metrics['per_class_total']
     print(f"\nClassifier: {clf_name}")
     print(f" Total Accuracy: {overall_accuracy * 100:.2f}%")
     print(f" Average Fold Accuracy: {mean_accuracy * 100:.2f}%")
     print(f" Fold Accuracy Std Dev: {std_accuracy * 100:.2f}%")
     print(f" Per-Class Accuracy: {per_class_accuracy * 100}")
  return metrics dict
# %%
filters cross val log var = validate parcellated filters(groupA parcellated array,
groupB_parcellated_array, metric=metric,method='log-
var',n_splits=10,filters_per_group=3,Tangent_Class=Tangent_Class)
save_text_results(f"Parcellated Log Var Filter Classification Accuracy Results:
{filters_cross_val_log_var}")
filters_cross_val_log_cov = validate_parcellated_filters(groupA_parcellated_array,
groupB_parcellated_array, metric=metric,method='log-
cov',n_splits=10,filters_per_group=3,Tangent_Class=Tangent_Class)
save_text_results(f"Parcellated Log Cov Filter Classification Accuracy Results:
{filters_cross_val_log_cov}")
# %% [markdown]
#### Regression
# %%
from sklearn.svm import LinearSVR
from sklearn.linear_model import LassoCV, LinearRegression, Lasso, Ridge
from sklearn.model_selection import KFold
from sklearn.model selection import StratifiedKFold
from sklearn.preprocessing import KBinsDiscretizer
from scipy.stats import pearsonr
def tangent_haufe(data, filters, method="basic", alpha=1, beta=0, l1_ratio=0.5, lambda1=.01,
lambda2=.01):
  S = (data @ filters)
  if method == "basic":
     proj = (np.linalg.pinv(S)@ data)
  elif method == "covs":
     cov_est_scm = Covariances(estimator='scm')
```

```
s_{cov} = cov_{est_{scm.transform}(S.T[np.newaxis,:,:])[0,:,:]}
     data_cov = cov_est_scm.transform(data.T[np.newaxis,:,:])[0,:,:]
     proj = (data_cov @ filters @ np.linalg.inv(s_cov)).T
  elif method == "linreg":
     reg = LinearRegression()
     reg.fit(S, data)
     proj = req.coef .T
  elif method == "grouplassolinreg":
     reg = MultiTaskLasso(alpha=alpha) # Using 5-fold cross-validation
     reg.fit(S, data)
     proj = reg.coef_.T
  elif method == "lassolinreg":
     reg = Lasso(alpha=alpha) # Using 5-fold cross-validation
     req.fit(S, data)
     proj = reg.coef_.T
  elif method == "elasticlinreg":
     reg = ElasticNet(alpha=alpha, I1_ratio=I1_ratio)
     reg.fit(S, data)
     proj = reg.coef_.T
  elif method == "growl":
     # Proximal Operator for GrOWL targeting columns
     def prox growl(V, lambda1, lambda2, tau):
       p, r = V.shape
       norms = np.linalg.norm(V, axis=0) # Norms of columns
       indices = np.argsort(-norms) # Sort indices by descending norms
       weights = lambda1 + lambda2 * np.linspace(1, 0, r) # Weights decrease
       V_new = np.zeros_like(V)
       for i in range(r):
          idx = indices[i]
          if norms[idx] > weights[i] * tau:
            V_{new[:, idx]} = (1 - tau * weights[i] / norms[idx]) * V[:, idx]
       return V_new
     # Initialization
     B = np.zeros((filters.shape[1], data.shape[1]))
     # Optimization Loop
     max_iter = 100
     learning_rate = 0.01
     for _ in range(max_iter):
       gradient = S.T @ (S @ B - data)
       B -= learning rate * gradient
       B = prox_growl(B, lambda1, lambda2, tau=learning_rate)
       if np.linalg.norm(gradient) < 1e-1:
          break
     proj = B.T
  return proj
# Combine group A and B data and paths
def combine_groups(groupA_parcellated_array, groupB_parcellated_array, groupA_paths_filtered,
```

```
groupB_paths_filtered):
  # Combine data arrays
  combined_data = np.concatenate((groupA_parcellated_array, groupB_parcellated_array),
axis=0)
  # Combine paths arrays
  combined_paths = np.concatenate((groupA_paths_filtered, groupB_paths_filtered), axis=0)
  return combined data, combined paths
# Extract subject IDs from the combined paths
def extract_subject_ids(combined_paths):
  subject_ids = np.array([re.search(r'/(\d+)/', path[0]).group(1) for path in combined_paths])
  return np.array(subject_ids)
def extract phenotype(subids,phenotype):
  file_path_restricted = '../HCP/RESTRICTED_zainsou_8_6_2024_2_11_21.csv'
  file_path_unrestricted = '../HCP/unrestricted_zainsou_8_2_2024_6_13_22.csv'
     data_r = pd.read_csv(file_path_restricted)
     data ur = pd.read csv(file path unrestricted)
  except FileNotFoundError:
     print(f"File not found: {file_path_restricted} or {file_path_unrestricted}")
     raise
  # Combine restricted and unrestricted data on Subject ID
  data = pd.merge(data_r, data_ur, on='Subject', how='outer')
  # Convert Subject IDs to string for consistency
  data['Subject'] = data['Subject'].astype(str)
  subids = subids.astype(str)
  # Filter data for training subjects
  train data = data[data['Subject'].isin(subids)]
  # Ensure the order matches the training data
  train data = train data.set index('Subject').loc[subids].reset index()
  pheno_score = train_data[phenotype]
  return pheno_score
# Regress out age from predictors
def regress_out_age(predictor, age):
  reg = Ridge(alpha=1)
  # reg = LinearRegression()
  reg.fit(age.reshape(-1, 1), predictor) # Age is the independent variable
  # reg = LinearSVR(C=1,fit intercept=False)
  # reg.fit(predictor, age) # Age is the independent variable
  return reg
```

```
def preproc(train, test,method="zscore"):
  if method == "zscore":
     scaler = StandardScaler()
     train_zscore = scaler.fit_transform(train)
     test zscore = scaler.transform(test)
  else:
     mean = train.mean(axis=0)
     train_zscore = train - mean
     test zscore = test - mean
  return train_zscore, test_zscore
# Helper function to plot regression results
def plot_predictions(true_values, predicted_values, title, train_or_test):
  plt.scatter(true_values, predicted_values, label=f'{train_or_test} Predictions', color='blue',
alpha=0.6)
  plt.xlabel('True Values')
  plt.ylabel('Predictions')
  plt.title(title)
  plt.legend()
  plt.show()
# Helper function to plot residuals vs true values
def plot_residuals(true_values, predicted_values, title, train_or_test):
  residuals = true_values - predicted_values
  plt.scatter(true_values, residuals, label=f'{train_or_test} Residuals', color='red', alpha=0.6)
  plt.axhline(y=0, color='black', linestyle='--') # Add a horizontal line at y=0 for reference
  plt.xlabel('True Values')
  plt.ylabel('Residuals')
  plt.title(title)
  plt.legend()
  plt.show()
# Tangent space regression with centering
def tan regression(data, values, age, metric="riemann", pre="znorm", n splits=5):
  # Binning the continuous target variable for stratified splits
  n bins = 10 # Adjust this based on your target distribution
  binner = KBinsDiscretizer(n_bins=n_bins, encode='ordinal', strategy='uniform')
  values_binned = binner.fit_transform(values.reshape(-1, 1))
  cov_est = Covariances(estimator='lwf')
  # cov est = Covariances(estimator='corr')
  covs = cov_est.transform(np.transpose(data, (0, 2, 1)))
  # Initialize Stratified K-Fold cross-validation
  kf = StratifiedKFold(n_splits=n_splits, shuffle=True, random_state=10)
  fold scores = []
  fold_corrs = []
  filter fold scores = []
  filter_fold_corrs = []
```

```
count = 0
  # Perform Stratified K-Fold Cross-Validation
  for train index, test index in kf.split(data, values binned):
     print("Fold", count)
     count += 1
     train, test = data[train_index], data[test_index]
     values_train, values_test = values[train_index], values[test_index]
     values train, values test = preproc(values train[:,np.newaxis], values test[:,np.newaxis],
method="center")
     values_train = values_train[:,0]
     values_test = values_test[:,0]
     # Plot the histogram of training data and calculated weights
     plt.figure(figsize=(10, 6))
     # Plot histogram of training target values
     plt.hist(values_train, bins=n_bins, alpha=0.7, color='blue', edgecolor='black', label='Train Data')
     plt.title(f'Train Data Histogram and Weights - Fold {count}')
     plt.xlabel('Target Value')
     plt.ylabel('Frequency / Weight')
     plt.legend()
     plt.show()
     age_train, age_test = age[train_index], age[test_index]
     age_train, age_test = preproc(age_train[:,np.newaxis], age_test[:,np.newaxis],
method="center")
     age_train = age_train[:,0]
     age_test = age_test[:,0]
     train_covs, test_covs = covs[train_index], covs[test_index]
     train_mean = mean_covariance(train_covs, metric=metric)
     tan_train = tangent_space(train_covs,train_mean,metric=metric)
     tan_test = tangent_space(test_covs,train_mean,metric=metric)
     if pre == "center":
       # Mean center the tangent space data
       tan_mean = np.mean(tan_train, axis=0)
       tan_train_centered = tan_train - tan_mean
       tan_test_centered = tan_test - tan_mean
     elif pre == "znorm":
       tan_train_centered, tan_test_centered = preproc(tan_train,tan_test)
     else:
       tan train centered = tan train
       tan_test_centered = tan_test
     # Regress out age using only the training data
     age_reg = regress_out_age(tan_train_centered, age_train)
     # tan train centered = tan train centered -
(tan_train_centered@np.linalg.pinv(age_reg.coef_[np.newaxis,:]))@age_reg.coef_[np.newaxis,:]
     # tan_test_centered = tan_test_centered -
```

```
(tan_test_centered@np.linalg.pinv(age_reg.coef_[np.newaxis,:]))@age_reg.coef_[np.newaxis,:]
     tan_train_centered = tan_train_centered - age_reg.predict(age_train.reshape(-1, 1))
     tan_test_centered = tan_test_centered - age_reg.predict(age_test.reshape(-1, 1))
     # Choose the regression model
     # reg_model = LinearSVR(C=1,fit_intercept=False)
     # reg model = LinearSVR(C=1)
     # reg_model = Lasso(alpha=0.001,fit_intercept=False)
     reg model = Ridge(alpha=1)
     # reg_model = LinearRegression(fit_intercept=False)
     # reg_model = LassoCV(cv=5) # Use 5-fold cross-validation within each fold to tune Lasso
     reg_model.fit(tan_train_centered, values_train)
     # Evaluate the model on the test set
     test_score = reg_model.score(tan_test_centered, values_test)
     predictions = reg model.predict(tan test centered)
     fold_corr, _ = pearsonr(values_test, predictions)
     fold scores.append(test_score)
     fold_corrs.append(fold_corr)
     print(f"Fold Test R2 Score: {test score}")
     print(f"Fold Test R Score: {fold corr}")
     # Plot predictions vs true values for test fold
     plot_predictions(values_test, predictions,f'Test Predictions vs True Values - Fold R2:
{test_score:.2f}', 'Test')
     # Now plot for train fold
     predictions_train = reg_model.predict(tan_train_centered)
     plot predictions(values train, predictions train, f'Train Predictions vs True Values - Fold R2:
{test score:.2f}', 'Train')
     # Plot residuals vs true values for test fold
     plot_residuals(values_test, predictions, f'Test Residuals vs True Values - Fold R2:
{test_score:.2f}', 'Test')
     # Plot residuals vs true values for train fold
     plot_residuals(values_train, predictions_train, f'Train Residuals vs True Values - Fold R2:
{test score:.2f}', 'Train')
     # (tan_train@np.linalg.pinv(reg_model.coef_))@reg_model.coef_
     # hauf_coef = FKT_proj(tan_train,reg_model.coef_[:,np.newaxis],method="basic")
     # hauf_coef = FKT_proj(tan_train,reg_model.coef_[:,np.newaxis],method="covs")
     hauf_coef = FKT_proj(tan_train_centered,reg_model.coef_[:,np.newaxis],method="linreg")
     weights_matrix = untangent_space(hauf_coef.T[:,0],train_mean,metric=metric)
     # weights_matrix = untangent_space(reg_model.coef_,train_mean,metric=metric)
     eigs, filters_all = eigh(weights_matrix,train_mean)
     # plt.scatter(range(0,eigs.shape[0]),np.abs(eigs))
     # plt.show()
     inds = np.argsort(np.abs(eigs))[-4:]
     filters = filters all[:,inds]
     train_transformed = train @ filters
```

```
test transformed = test @ filters
    train_transformed_cov = cov_est.transform(np.transpose(train_transformed, (0, 2, 1)))
    test_transformed_cov = cov_est.transform(np.transpose(test_transformed, (0, 2, 1)))
    reduced_mean = mean_covariance(train_transformed_cov, metric=metric)
    tangent transform train = tangent space(train transformed cov, reduced mean,
metric=metric)
    tangent transform test = tangent space(test transformed cov, reduced mean,
metric=metric)
    if pre == "center":
       tangent_transform_mean = np.mean(tangent_transform_train, axis=0)
       tangent_transform_train_centered = tangent_transform_train - tangent_transform_mean
       tangent_transform_test_centered = tangent_transform_test - tangent_transform_mean
    elif pre == "znorm":
       tangent_transform_train_centered, tangent_transform_test_centered =
preproc(tangent_transform_train,tangent_transform_test)
    else:
       tangent transform train centered = tangent transform train
       tangent_transform_test_centered = tangent_transform_test
    # Regress out age using only the training data
    age_reg_transformed = regress_out_age(tangent_transform_train_centered, age_train)
     # tangent_transform_train_centered = tangent_transform_train_centered -
(tangent_transform_train_centered@np.linalg.pinv(age_reg_transformed.coef_[np.newaxis,:]))@ag
e_reg_transformed.coef_[np.newaxis,:]
     # tangent_transform_test_centered = tangent_transform_test_centered -
(tangent_transform_test_centered@np.linalg.pinv(age_reg_transformed.coef_[np.newaxis,:]))@age
reg_transformed.coef [np.newaxis,:]
    tangent_transform_train_centered = tangent_transform_train_centered -
age_reg_transformed.predict(age_train.reshape(-1, 1))
    tangent transform test centered = tangent transform test centered -
age_reg_transformed.predict(age_test.reshape(-1, 1))
    # reg model reduced = LinearSVR(C=1,fit intercept=False)
    reg model reduced = Ridge(alpha=10)
    reg_model_reduced.fit(tangent_transform_train_centered, values_train)
    test_score_reduced = reg_model_reduced.score(tangent_transform_test_centered,
values_test)
    predictions_reduced = reg_model_reduced.predict(tangent_transform_test_centered)
    test_corr_reduced, _ = pearsonr(values_test, predictions_reduced)
    filter fold scores.append(test score reduced)
    filter_fold_corrs.append(test_corr_reduced)
    print(f"Fold Reduced Test R2 Score: {test_score_reduced}")
    print(f"Fold Reduced Test R Score: {test_corr_reduced}")
     # Plot for filter fold
    # Plot for filter fold
     plot_predictions(values_test, predictions_reduced, f'Filter Fold Predictions vs True Values -
Filter Fold R<sup>2</sup>: {test_score_reduced:.2f}', 'Filter Test')
```

```
# Now plot for train fold
     predictions_reduced_train = reg_model_reduced.predict(tangent_transform_train_centered)
     plot_predictions(values_train, predictions_reduced_train, f'Train Filter Predictions vs True
Values - Fold R<sup>2</sup>: {test_score:.2f}', 'Filter Train')
     # Plot residuals vs true values for test fold
     plot_residuals(values_test, predictions_reduced, f'Test Filter Residuals vs True Values - Fold
R<sup>2</sup>: {test_score:.2f}', 'Test')
     # Plot residuals vs true values for train fold
     plot_residuals(values_train, predictions_reduced_train, f'Train Filter Residuals vs True Values -
Fold R<sup>2</sup>: {test_score:.2f}', 'Train')
  # Output the average R2 score across all folds
  mean_score = np.mean(fold_scores)
  mean_corr = np.mean(fold_corrs)
  mean filter score = np.mean(filter fold scores)
  mean_filter_corr = np.mean(filter_fold_corrs)
  print(f"Mean R² Score across {n_splits} folds: {mean_score}")
  print(f"Mean R Score across {n_splits} folds: {mean_corr}")
  print(f"Mean R2 Filter Score across {n_splits} folds: {mean_filter_score}")
  print(f"Mean R Filter Score across {n_splits} folds: {mean_filter_corr}")
  return mean_score, mean_corr, mean_filter_score, mean_filter_corr, fold_scores, fold_corrs,
filter_fold_scores, filter_fold_corrs
combined_data, combined_paths = combine_groups(groupA_parcellated_array,
groupB_parcellated_array, groupA_paths_filtered, groupB_paths_filtered)
subject_ids = extract_subject_ids(combined_paths)
scores = np.array(extract_phenotype(subject_ids,phenotype))
age = np.array(extract_phenotype(subject_ids, 'Age_in_Yrs'))
# %%
mean_score, mean_corr, mean_filter_score, mean_filter_corr, fold_scores, fold_corrs,
filter_fold_scores, filter_fold_corrs = tan_regression(combined_data,scores,age,
metric=metric,pre="center",n_splits=5)
# %%
# from joblib import Parallel, delayed
# from sklearn.utils import shuffle
# def permutation_test(data, scores, age, metric="riemann", pre="znorm", n_splits=10,
n_permutations=1000, n_jobs=1):
    # Lists to store results for all permutations
    perm_mean_scores = []
    perm mean corrs = []
    perm_mean_filter_scores = []
    perm_mean_filter_corrs = []
```

```
perm_fold_scores = [] # To store fold scores for all permutations
    perm_fold_corrs = []
    perm_filter_fold_scores = []
    perm_filter_fold_corrs = []
    # Function to run one permutation
    def run_permutation(i):
#
      # Shuffle the scores
#
      shuffled_scores = shuffle(scores, random_state=i)
#
      # Fit the model with shuffled data
      perm_mean_score, perm_mean_corr, perm_mean_filter_score, perm_mean_filter_corr,
fold scores, fold_corrs, filter_fold_scores, filter_fold_corrs = tan_regression(data, shuffled_scores,
age, metric, pre, n_splits)
#
      # Append the mean results to the lists
#
      perm_mean_scores.append(perm_mean_score)
#
      perm_mean_corrs.append(perm_mean_corr)
#
      perm mean filter scores.append(perm mean filter score)
#
      perm_mean_filter_corrs.append(perm_mean_filter_corr)
#
      # Append the fold-specific results to the lists
#
      perm_fold_scores.append(fold_scores)
#
      perm_fold_corrs.append(fold_corrs)
#
      perm_filter_fold_scores.append(filter_fold_scores)
      perm_filter_fold_corrs.append(filter_fold_corrs)
    # Run the permutations in parallel
   Parallel(n_jobs=n_jobs)(delayed(run_permutation)(i) for i in range(n_permutations))
    # Return all the stored results
    return perm_mean_scores, perm_mean_corrs, perm_mean_filter_scores,
perm_mean_filter_corrs, perm_fold_scores, perm_fold_corrs, perm_filter_fold_scores,
perm_filter_fold_corrs
## Example of running the permutation test
# n_permutations = 6 # Set the number of permutations you want
# results = permutation_test(combined_data, scores, age, metric=metric, pre="center", n_splits=10,
n_permutations=n_permutations, n_jobs=-1)
## Extract individual results
# perm_mean_scores, perm_mean_filter_scores, perm_mean_filter_corrs,
perm_fold_scores, perm_fold_corrs, perm_filter_fold_scores, perm_filter_fold_corrs = results
# # Save the permutation test results as numpy arrays
# save_array_to_outputfolder("perm_mean_scores.npy", np.array(perm_mean_scores))
# save_array_to_outputfolder("perm_mean_corrs.npy", np.array(perm_mean_corrs))
# save_array_to_outputfolder("perm_mean_filter_scores.npy", np.array(perm_mean_filter_scores))
# save_array_to_outputfolder("perm_mean_filter_corrs.npy", np.array(perm_mean_filter_corrs))
# save_array_to_outputfolder("perm_fold_scores.npy", np.array(perm_fold_scores))
# save_array_to_outputfolder("perm_fold_corrs.npy", np.array(perm_fold_corrs))
```

```
# save array_to_outputfolder("perm_filter_fold_scores.npy", np.array(perm_filter_fold_scores))
# save_array_to_outputfolder("perm_filter_fold_corrs.npy", np.array(perm_filter_fold_corrs))
# print("All arrays have been saved to the output folder.")
# %%
# Load the arrays
perm_mean_scores = load_array_from_outputfolder("perm_mean_scores.npy")
perm mean corrs = load array from outputfolder("perm mean corrs.npy")
perm_mean_filter_scores = load_array_from_outputfolder("perm_mean_filter_scores.npy")
perm_mean_filter_corrs = load_array_from_outputfolder("perm_mean_filter_corrs.npy")
perm_fold_scores = load_array_from_outputfolder("perm_fold_scores.npy")
perm_fold_corrs = load_array_from_outputfolder("perm_fold_corrs.npy")
perm_filter_fold_scores = load_array_from_outputfolder("perm_filter_fold_scores.npy")
perm filter fold corrs = load array from outputfolder("perm filter fold corrs.npy")
# Plot the distributions
def plot_distribution(data, title, xlabel, ylabel):
  plt.figure(figsize=(8, 6))
  plt.hist(data, bins=30, edgecolor='black', alpha=0.7)
  plt.title(title)
  plt.xlabel(xlabel)
  plt.ylabel(ylabel)
  plt.grid(True)
  plt.show()
# Plot mean scores distribution
plot_distribution(perm_mean_scores, "Distribution of Permutation Mean Scores", "Mean Score",
"Frequency")
# Plot mean correlations distribution
plot_distribution(perm_mean_corrs, "Distribution of Permutation Mean Correlations", "Mean
Correlation", "Frequency")
# Plot mean filter scores distribution
plot_distribution(perm_mean_filter_scores, "Distribution of Permutation Mean Filter Scores", "Mean
Filter Score", "Frequency")
# Plot mean filter correlations distribution
plot distribution(perm mean filter corrs, "Distribution of Permutation Mean Filter Correlations",
"Mean Filter Correlation", "Frequency")
print(perm_fold_scores.shape)
# Plot mean scores distribution
plot distribution(perm fold scores, "Distribution of Permutation Scores", "Score", "Frequency")
# Plot mean correlations distribution
plot_distribution(perm_fold_corrs, "Distribution of Permutation Correlations", "Correlation",
"Frequency")
# Plot mean filter scores distribution
plot_distribution(perm_filter_fold_scores, "Distribution of Permutation Filter Scores", "Filter Scores",
"Frequency")
# Plot mean filter correlations distribution
plot distribution(perm filter fold corrs, "Distribution of Permutation Filter Correlations", "Filter
Correlation", "Frequency")
```

```
# %% [markdown]
#### Create 1 Fold and Filters for Rest of Pipeline
data = np.concatenate((groupA_parcellated_array, groupB_parcellated_array))
labels = np.concatenate((np.ones(len(groupA_parcellated_array)),
np.zeros(len(groupB parcellated array))))
paths = np.concatenate((groupA_paths_filtered, groupB_paths_filtered))
skf = StratifiedKFold(n splits=n folds, shuffle=True, random state=random state)
splits = list(skf.split(data, labels))
# %%
fold = 0
fold_outputfolder = f"fold_{fold}"
if not os.path.exists(os.path.join(outputfolder, f"fold_{fold}")):
  os.makedirs(os.path.join(outputfolder, f"fold {fold}"))
if not os.path.exists(os.path.join(outputfolder, f"fold_{fold}", "results")):
  os.makedirs(os.path.join(outputfolder, f"fold_{fold}", "results"))
train_parcellated, test_parcellated = data[splits[fold][0]], data[splits[fold][1]]
train_labels, test_labels = labels[splits[fold][0]], labels[splits[fold][1]]
train paths, test paths = paths[splits[fold][0]], paths[splits[fold][1]]
groupA_train_parcellated = train_parcellated[train_labels == 1]
groupA_test_parcellated = test_parcellated[test_labels == 1]
groupA_train_paths = train_paths[train_labels == 1]
groupA_test_paths = test_paths[test_labels == 1]
groupB train parcellated = train parcellated[train labels == 0]
groupB test parcellated = test parcellated[test labels == 0]
groupB_train_paths = train_paths[train_labels == 0]
groupB_test_paths = test_paths[test_labels == 0]
# %%
# https://pyriemann.readthedocs.io/en/latest/auto_examples/signal/plot_covariance_estimation.html
cov est = Covariances(estimator='lwf')
groupA train parcellated covs = cov est.transform(np.transpose(groupA train parcellated, (0, 2,
1)))
groupB_train_parcellated_covs = cov_est.transform(np.transpose(groupB_train_parcellated, (0, 2,
1)))
# %%
if Tangent Class:
  fkt_riem_eigs, filters, _, _ = tangent_classifier(groupA_train_parcellated_covs,
groupB_train_parcellated_covs, TSVM=True, TLDA=False, tangent_calc=True,
metric=metric, visualize=True, n=0)
else:
  fkt riem eigs, filters, filtersA, filtersB = FKT(groupA train parcellated covs,
groupB_train_parcellated_covs, mean=metric, average=True, visualize=True, n=0)
n filters per group
filtersA = filters[:, -n_filters_per_group:]
```

```
filtersB = filters[:, :n_filters_per_group]
filters = np.concatenate((filtersB, filtersA), axis=1)
# %%
_, filters, _, _ = tangent_classifier_multi(groupA_train_parcellated_covs,
groupB_train_parcellated_covs, TSVM=True, TLDA=False, tangent_calc=True,
metric=metric, visualize=False, k=3)
# %%
def test_filters(group1_train, group1_test, group_2_train,group2_test, filters, metric="riemann",
method='log-cov'):
  train_1, test_1, train_2, test_2 = feature_generation(group1_train, group1_test, group_2_train,
group2_test, filters,method=method,metric=metric)
  fold_metrics = test_classifiers(train_1, test_1, train_2, test_2, sample_weights_train=None)
  return fold metrics
print(test_filters(groupA_train_parcellated, groupA_test_parcellated, groupB_train_parcellated,
groupB_test_parcellated, filters, metric=metric,method='log-cov'))
print(test_filters(groupA_train_parcellated, groupA_test_parcellated, groupB_train_parcellated,
groupB_test_parcellated, filters, metric=metric,method='log-var'))
# %%
def test_visualize_variance(train_1, train_2, two_filter):
  train_1_transform = np.var(train_1@two_filter,axis=1)
  train_2_transform = np.var(train_2@two_filter,axis=1)
  # Plot when n=1
  plt.figure(figsize=(8, 6))
  plt.scatter(train_1_transform[:, 0], train_1_transform[:, 1], label='Group A Variance', color='blue')
  plt.scatter(train_2_transform[:, 0], train_2_transform[:, 1], label='Group B Variance', color='red')
  # Display plot
  plt.xlabel('Variance Feature B')
  plt.ylabel('Variance Feature A')
  plt.title(f'Variance Comparison')
  plt.legend()
  plt.grid(True)
  plt.show()
for i in range(0,3):
  test_visualize_variance(groupA_train_parcellated, groupB_train_parcellated,filters[:,[i,-(i+1)]])
# %%
def test_visualize_variance(train_1, train_2, two_filter):
  import matplotlib.gridspec as gridspec
  # Project data onto filters
  train_1_proj = np.var(train_1@two_filter,axis=1)
  train 2 proj = np.var(train 2@two filter,axis=1)
  # Create figure and gridspec layout
```

```
fig = plt.figure(figsize=(8, 8))
  gs = gridspec.GridSpec(4, 4)
  # Define the axes
  ax_scatter = fig.add_subplot(gs[1:4, 0:3])
  ax_hist_x = fig.add_subplot(gs[0, 0:3], sharex=ax_scatter)
  ax_hist_y = fig.add_subplot(gs[1:4, 3], sharey=ax_scatter)
  # Scatter plot
  ax_scatter.scatter(train_1_proj[:, 0], train_1_proj[:, 1], label='Group A', color='blue', alpha=0.5)
  ax_scatter.scatter(train_2_proj[:, 0], train_2_proj[:, 1], label='Group B', color='red', alpha=0.5)
  ax_scatter.set_xlabel('Projection onto Filter B')
  ax_scatter.set_ylabel('Projection onto Filter A')
  ax scatter.legend()
  ax_scatter.grid(True)
  # Histograms
  bins = 30
  # Histograms for X axis (top)
  ax_hist_x.hist(train_1_proj[:, 0], bins=bins, color='blue', alpha=0.5, density=True, label='Group A')
  ax_hist_x.hist(train_2_proj[:, 0], bins=bins, color='red', alpha=0.5, density=True, label='Group B')
  ax_hist_x.set_ylabel('Density')
  ax_hist_x.legend()
  ax_hist_x.grid(True)
  # Histograms for Y axis (right)
  ax_hist_y.hist(train_1_proj[:, 1], bins=bins, orientation='horizontal', color='blue', alpha=0.5,
density=True)
  ax_hist_y.hist(train_2_proj[:, 1], bins=bins, orientation='horizontal', color='red', alpha=0.5,
density=True)
  ax_hist_y.set_xlabel('Density')
  ax_hist_y.grid(True)
  # Hide tick labels on histograms to avoid clutter
  plt.setp(ax_hist_x.get_xticklabels(), visible=False)
  plt.setp(ax_hist_y.get_yticklabels(), visible=False)
  # Adjust layout
  plt.tight_layout()
  plt.show()
for i in range(0,3):
  test_visualize_variance(groupA_train_parcellated, groupB_train_parcellated,filters[:,[i,-(i+1)]])
# %%
# plotting.view_surf(hcp.mesh.inflated, hcp.cortex_data(hcp.unparcellate(filters[:,0], hcp.mmp)),
threshold=np.percentile(np.abs(filters[:,0]), 95), bg_map=hcp.mesh.sulc)
# %% [markdown]
#### Calculate MIGP
```

```
# %%
def migp(subs, batch_size=2, m=4800):
  W_gpu = None
  for batch_start in range(0, len(subs), batch_size):
    # Select the current batch of subjects
    batch subs = subs[batch start:batch start + batch size]
    batch_paths = [path for sublist in batch_subs for path in sublist]
    concatenated_data = []
    for task in batch_paths:
       X = nib.load(task).get_fdata()
       Xn = hcp.normalize(X-X.mean(axis=1, keepdims=True))
       # print(Xn.mean(axis=0).mean())
       # print(Xn.std(axis=0).mean())
       concatenated_data.append(Xn)
       del X, Xn
    try:
       # Concatenate data along the first axis using numpy
       batch = np.concatenate(concatenated data, axis=0)
       batch = hcp.normalize(batch - batch.mean(axis=1,keepdims=True))
       del concatenated_data
       with torch.no_grad():
         # Convert to torch tensor and move to GPU
         batch_gpu = torch.tensor(batch, dtype=torch.float32, device="cuda")
         del batch
         if torch.isnan(batch_gpu).any():
            print("NaNs detected in the batch data. Aborting SVD operation.")
            del batch_gpu
            torch.cuda.empty_cache()
            return None
         if W apu is None:
            combined_data_gpu = batch_gpu
         else:
            combined_data_gpu = torch.cat([W_gpu, batch_gpu], dim=0)
         del batch_gpu
         torch.cuda.empty_cache()
         ## Calculate size in GB
         # size_in_gb = combined_data_gpu.element_size() * combined_data_gpu.nelement() /
(1024**3)
         # print(f"Size of the array: {size_in_gb:.2f} GB")
         # cpu_mem()
         # gpu_mem()
         # Perform SVD on the GPU
         # Check for NaNs in the data
         # _, S_gpu, Vh_gpu = torch.linalg.svd(combined_data_gpu, full_matrices=False)
         _, Q = torch.linalg.eigh(combined_data_gpu@combined_data_gpu.T)
```

```
# cpu_mem()
         # gpu_mem()
         # Compute the updated W on the GPU
         # W_gpu = torch.diag(S_gpu[:m]) @ Vh_gpu[:m, :]
         # Returned in Ascending order
         W_gpu = Q[:, -m:].T@combined_data_gpu
         del Q, combined_data_gpu # Free up GPU memory
         torch.cuda.empty_cache()
         print(batch start, "done")
    except Exception as e:
       print(f"Failed during GPU processing: {e}")
       if "combined_data_gpu" in locals():
         del combined_data_gpu
       if "Q" in locals():
         del Q
       if "W gpu" in locals():
         del W_gpu
       torch.cuda.empty_cache()
       return None
  # Transfer W back to CPU only at the end
  W = W gpu.cpu().numpy()
  del W_gpu # Free up GPU memory
  return W
# %%
reducedsubsA = migp((groupA_train_paths))
save array to outputfolder(os.path.join(fold outputfolder,'reducedsubsA.npy'), reducedsubsA)
# %%
reducedsubsB = migp((groupB_train_paths))
save_array_to_outputfolder(os.path.join(fold_outputfolder,'reducedsubsB.npy'), reducedsubsB)
# %% [markdown]
#### Load MIGP
# %%
if "reducedsubsA" in locals():
  del reducedsubsA
if "reducedsubsB" in locals():
  del reducedsubsB
reducedsubsA loaded =
load array from outputfolder(os.path.join(fold outputfolder,'reducedsubsA.npy'))
reducedsubsB_loaded =
load_array_from_outputfolder(os.path.join(fold_outputfolder,'reducedsubsB.npy'))
# %%
if torch.cuda.is_available():
  device = torch.device("cuda")
else:
  device = torch.device("cpu")
```

```
reducedsubs combined gpu =
torch.tensor(np.concatenate((reducedsubsA_loaded,reducedsubsB_loaded),axis=0),dtype=torch.flo
at32,device=device)
# Returned in Descending Order
Urc,__, = torch.linalg.svd(reducedsubs_combined_gpu, full_matrices=False)
reducedsubs_combined = (Urc[:,:4800].T@reducedsubs_combined_gpu).cpu().numpy()
del Urc, reducedsubs_combined_gpu
reducedsubs_gpu = torch.tensor(reducedsubs_combined, dtype=torch.float32, device=device)
U,__, = torch.linalg.svd(reducedsubs_gpu, full_matrices=False)
reducedsubs= (U[:,:1000].T@reducedsubs_gpu).cpu().numpy()
del U, reducedsubs_gpu
# %% [markdown]
#### Run Haufe Transform
# %% [markdown]
# 1. Should I pinv(F) or just multiply by F
# %%
def process subject haufe(sub,pinv TF):
    concatenated_data = []
    for task in sub:
       X = nib.load(task).get_fdata(dtype=np.float32)
       Xn = hcp.normalize(X-X.mean(axis=1, keepdims=True))
       concatenated_data.append(Xn)
       del X, Xn
    # Concatenate data along the first axis
    subject = np.concatenate(concatenated_data, axis=0)
    del concatenated_data # Explicitly delete the concatenated data list
    Xp = hcp.normalize(subject - subject.mean(axis=1, keepdims=True))
    del subject
    Xpf = pinv TF@Xp
    del Xp
    return Xpf
  except Exception as e:
    print(f"Error processing subject: {e}")
    traceback.print_exc() # Print the full traceback
    return None
def haufe_transform(F, parcellated,paths):
  # Ensure the tensors are on the correct device
  pinv TF = np.linalg.pinv(parcellated.reshape(-1,parcellated.shape[-1]) @ np.linalg.pinv(F.T))
```

```
# pinv TF_list = pinv_TF.reshape(len(paths),F.shape[1],pinv_TF.shape[0])
  pinv_TF_list = (np.array_split(pinv_TF, len(paths), axis=1))
  with ProcessPoolExecutor(max_workers=(int(os.cpu_count()*.5))) as executor:
     # Use map to process subjects in parallel
     blocks = np.array(list(executor.map(process_subject_haufe, paths,pinv_TF_list)))
     print(blocks.shape)
     return (blocks.sum(axis=0))
# %%
# filtersA_transform = haufe_transform(filtersA[:,-
n_filters_per_group:],groupA_train_parcellated,groupA_train_paths)
# save_array_to_outputfolder("filtersA_transform.npy", filtersA_transform)
filtersA_transform = haufe_transform(filtersA,groupA_train_parcellated,groupA_train_paths)
save array to outputfolder(os.path.join(fold outputfolder,"filtersA transform.npy"),
filtersA_transform)
# %%
# filtersB_transform = haufe_transform(filtersB[:,-
n_filters_per_group:],groupB_train_parcellated,groupB_train_paths)
# save array to outputfolder("filtersB transform.npy", filtersB transform)
filtersB_transform = haufe_transform(filtersB,groupB_train_parcellated,groupB_train_paths)
save_array_to_outputfolder(os.path.join(fold_outputfolder,"filtersB_transform.npy"),
filtersB_transform)
# %% [markdown]
#### Load Haufe Transform
# %%
filtersA_transform =
load array from outputfolder(os.path.join(fold outputfolder, 'filtersA transform.npy'))
filtersB_transform =
load array from outputfolder(os.path.join(fold outputfolder, filtersB transform.npy))
# %% [markdown]
#### Orthonormalize Filters
# %%
def orthonormalize_filters(W1, W2):
  # Stack the two filters into a single matrix
  W = np.concatenate((W1, W2)).T # shape: (features x 2)
  print(W.shape)
  # Perform QR decomposition to orthonormalize the filters
  Q_{,} = np.linalg.qr(W)
  print(Q.shape)
  # Verify that the inner product between the two orthonormalized vectors is 0 (orthogonality)
  print(f'Inner product between Q[:, 0] and Q[:, 1]: {np.dot(Q[:, 0].T, Q[:, 1])} (should be 0)')
```

```
# Verify that the inner product within each vector is 1 (normalization)
  print(f'Norm of Q[:, 0]: {np.dot(Q[:, 0].T, Q[:, 0])} (should be 1)')
  print(f'Norm of Q[:, 1]: {np.dot(Q[:, 1].T, Q[:, 1])} (should be 1)')
  return Q
# Example usage
filters = orthonormalize filters(filtersA transform, filtersB transform)
# %%
plotting.view_surf(hcp.mesh.inflated, hcp.cortex_data(filters[:,0]),
threshold=np.percentile(np.abs(filters[:,0]), 90), bg_map=hcp.mesh.sulc)
# %%
plotting.view surf(hcp.mesh.inflated, hcp.cortex data(filters[:,3]),
threshold=np.percentile(np.abs(filters[:,3]), 90), bg_map=hcp.mesh.sulc)
# %% [markdown]
# ### PPCA
# %%
def call_pca_dim(Data=None,eigs=None,N=None):
 # Start MATLAB engine
  eng = matlab.engine.start_matlab()
  # Add the path to the MATLAB function
  eng.addpath("/project/3022057.01/IFA/melodic", nargout=0)
  if Data is not None:
   # Call the MATLAB function
   prob = eng.pca_dim(matlab.double(Data))
   eig_vectors = np.array(prob['E'])
   prob = eng.pca_dim_eigs(matlab.double(eigs.tolist()), matlab.double([N]))
  # Extract and convert each variable
  lap = np.array(prob['lap']).flatten().reshape(-1, 1)
  bic = np.array(prob['bic']).flatten().reshape(-1, 1)
  rrn = np.array(prob['rrn']).flatten().reshape(-1, 1)
  AIC = np.array(prob['AIC']).flatten().reshape(-1, 1)
  MDL = np.array(prob['MDL']).flatten().reshape(-1, 1)
  eig = np.array(prob['eig']).flatten()
  orig_eig = np.array(prob['orig_eig']).flatten()
  leig = np.array(prob['leig']).flatten()
  # Stop MATLAB engine
  eng.eval('clearvars', nargout=0)
  eng.quit()
  plt.figure(figsize=(10, 6))
  plt.scatter(np.arange(len(eig)),eig,label="Adjusted Eigenspectrum")
```

```
plt.scatter(np.arange(len(orig_eig)),orig_eig,label="Eigenspectrum")
  plt.xlabel('Index')
  plt.ylabel('Eigenvalue')
  plt.legend()
  plt.title('Scree Plot')
  plt.show()
  # Use SimpleImputer to handle any missing values
  imputer = SimpleImputer(strategy='mean')
  lap = imputer.fit_transform(lap)
  bic = imputer.fit_transform(bic)
  rrn = imputer.fit_transform(rrn)
  AIC = imputer.fit transform(AIC)
  MDL = imputer.fit_transform(MDL)
  # Use StandardScaler to standardize the data
  scaler = StandardScaler()
  lap_std = scaler.fit_transform(lap)
  bic std = scaler.fit transform(bic)
  rrn_std = scaler.fit_transform(rrn)
  AIC std = scaler.fit transform(AIC)
  MDL_std = scaler.fit_transform(MDL)
  # Plot the results
  plt.figure(figsize=(10, 6))
  plt.scatter(np.arange(len(lap_std)), lap_std, label='Laplacian')
  plt.scatter(np.arange(len(bic_std)), bic_std, label='BIC')
  plt.scatter(np.arange(len(rrn_std)), rrn_std, label='RRN')
  plt.scatter(np.arange(len(AIC_std)), AIC_std, label='AIC')
  plt.scatter(np.arange(len(MDL_std)), MDL_std, label='MDL')
  plt.xlabel('Index')
  plt.ylabel('Standardized Value')
  plt.legend()
  plt.title('Scatter Plot of Standardized Eigenvalues and Model Order Selection Values')
  plt.show()
  return np.argmax(rrn_std)+1
def get_n_and_some(data):
  # Check the shape of the data and determine the axis for mean subtraction
  # Move data to GPU if available
  device = torch.device('cuda' if torch.cuda.is_available() else 'cpu')
  data_gpu = data.to(device, dtype=torch.float32)
  groupN = data_gpu.shape[1] - 1
  # Subtract the mean along the specified axis
  data centered = data gpu - torch.mean(data gpu, dim=1, keepdim=True)
  del data_gpu # Free up GPU memory
  torch.cuda.empty_cache()
```

```
# Perform SVD decomposition
  _, d, v = torch.svd(data_centered)
  del data_centered # Free up GPU memory
  torch.cuda.empty_cache()
  # Convert singular values to eigenvalues
  e = (d ** 2) / groupN
  # Move eigenvalues to CPU and convert to NumPy array
  e_np = e.cpu().numpy()
  del e, d # Free up GPU memory
  torch.cuda.empty_cache()
  # Determine the number of components
  n_components = torch.tensor(call_pca_dim(eigs=e_np,
N=groupN),device=device,dtype=torch.int32)
  return n_components, v.T
def PPCA(data, filters=None, threshold=1.6, niters=10, n=-1):
  n_{components} = -1
  n prev = -2
  i = 0
  # Move data to GPU if available
  device = torch.device('cuda' if torch.cuda.is_available() else 'cpu')
  data_gpu = torch.tensor(data,device=device, dtype=torch.float32)
  while n_components != n_prev and i < niters:
    n_prev = n_components
    if filters is not None:
       basis_gpu = torch.tensor(filters.T,device=device, dtype=torch.float32)
       n_components, vt = get_n_and_some(data_gpu)
       if n \le 0:
         basis_gpu = vt[:n_components, :]
       else:
         print(n)
         basis_gpu = vt[:n, :]
       del vt
       torch.cuda.empty_cache()
    print(n_prev, n_components)
    # Estimate noise and residual standard deviation
    est_noise = data_gpu - (data_gpu @ torch.linalg.pinv(basis_gpu)) @ basis_gpu
    est_residual_std = torch.std(est_noise,dim=0,correction=torch.linalg.matrix_rank(basis_qpu))
    del est noise
    torch.cuda.empty_cache()
    # Normalize the data
    data_gpu = (data_gpu / est_residual_std)
```

```
i += 1
  data = data_gpu.cpu().numpy()
  basis = basis_gpu.cpu().numpy()
  # del data_gpu, basis_gpu, est_residual_std
  del data_gpu, basis_gpu
  torch.cuda.empty cache()
  return data, basis
# %%
subs_data_VN, vt = PPCA(reducedsubs.copy(), threshold=0.0, niters=1)
save_array_to_outputfolder(os.path.join(fold_outputfolder,"subs_data_VN.npy"), subs_data_VN)
save_array_to_outputfolder(os.path.join(fold_outputfolder,"vt.npy"), vt)
# %% [markdown]
#### Load PPCA Results
# %%
subs_data_VN = load_array_from_outputfolder(os.path.join(fold_outputfolder,"subs_data_VN.npy"))
vt = load_array_from_outputfolder(os.path.join(fold_outputfolder,"vt.npy"))
# %% [markdown]
#### Combine Basis
# %%
# Columns are samples i.e. XXT is the covariance matrix formed
def whiten(X,n_components, method="SVD", visualize=False):
  # -1 to account for demean
  n samples = X.shape[-1]-1
  X mean = X.mean(axis=-1)
  X -= X_mean[:, np.newaxis]
  if method == "SVD":
    u, d = svd(X, full_matrices=False, check_finite=False)[:2]
    # Give consistent eigenvectors for both svd solvers
    \# u *= np.sign(u[0])
    K = (u / d).T[:n\_components] # see (6.33) p.140
    del u. d
    whitening_matrix = np.sqrt(n_samples)*K
  elif method == "Cholesky":
  # Does not Orthogonalize, just has unit covariance
    # Step 2: Perform Cholesky decomposition
    L = np.linalg.cholesky(np.cov(X,ddof=1))
    # Step 3:
    whitening_matrix = np.linalg.inv(L)
  elif method == "InvCov":
    # Calculate the covariance matrix of the centered data
    cov matrix = np.cov(X)
    # Perform eigenvalue decomposition of the covariance matrix
    eigvals, eigvecs = np.linalg.eigh(cov matrix)
    # Calculate the whitening matrix
    D_inv_sqrt = np.diag(1.0 / np.sqrt(eigvals))
```

```
whitening_matrix = eigvecs @ D_inv_sqrt @ eigvecs.T
  whitened_data = whitening_matrix@X
  return whitened data, whitening matrix
# Combine Basis
combined_spatial = np.vstack((vt,filters.T))
# Whiten
whitened_basis, whitening_matrix_pre =
whiten(combined_spatial,n_components=combined_spatial.shape[0],method="InvCov",visualize=Tr
subs_data_com_VN, _ = PPCA(reducedsubs_combined.copy(), filters=whitened_basis.T,
threshold=0.0, niters=1)
# tempbasis =
np.linalg.pinv(subs_data_com_VN@np.linalg.pinv(whitened_basis))@subs_data_com_VN
# whitened_basis, _ =
whiten(tempbasis,n_components=tempbasis.shape[0],method="InvCov",visualize=True)
# for i in range(0,3):
    # Readjust the MiGP data based on the new basis
    subs_data_com_VN, _ = PPCA(subs_data_com_VN.copy(), filters=whitened_basis.T,
threshold=0.0, niters=1)
    # Recalculate the basis via Haufe transform based on adjusted MIGP data
    tempbasis =
np.linalq.pinv(subs data com VN@np.linalq.pinv(whitened basis))@subs data com VN
    # Rewhiten the basis
    whitened_basis, whitening_matrix =
whiten(tempbasis,n_components=combined_spatial.shape[0],method="InvCov",visualize=True)
# %% [markdown]
# ### ICA
# %%
def plot_scree(data, n_components=None):
  Perform PCA on the provided dataset and plot a scree plot.
  Parameters:
  - data: np.array or pd.DataFrame, the dataset to perform PCA on.
  - n_components: int or None, the number of principal components to compute.
           If None, all components are computed.
  Returns:
  - pca: PCA object after fitting to the data.
  ## Standardize the data
```

```
## Initialize PCA
  # pca = PCA()
  ## Fit PCA on the data
  # pca.fit(data)
  # # Calculate explained variance ratio
  # explained_variance_ratio = pca.explained_variance_ratio_
  _, S, _ = np.linalg.svd(data, full_matrices=False)
  e = (S ** 2) / (data.shape[-1]-1)
  # Create the scree plot
  print(e)
  plt.figure(figsize=(10, 6))
  plt.plot(range(1, len(S) + 1), e, marker='o', linestyle='--')
  plt.title('Scree Plot')
  plt.xlabel('Principal Component')
  plt.ylabel('Explained Variance Ratio')
  plt.xticks(range(1, len(S) + 1))
  plt.grid()
  plt.show()
Atemp = np.linalg.pinv(subs data com VN@np.linalg.pinv(whitened basis))
plot scree(Atemp@subs data com VN)
# %%
def ICA(data,whitened_data):
  ica = FastICA(whiten=False)
  # Takes in array-like of shape (n_samples, n_features) and returns ndarray of shape (n_samples,
n components)
  IFA_components = ica.fit_transform(whitened_data.T).T
  A = data@np.linalg.pinv(IFA_components)
  W = np.linalg.pinv(A)
  print("The combined unmixing matrix correctly calculates the components: ",
np.allclose(W@data, IFA_components))
  print("The combined mixing matrix correctly reconstructs the low rank data demean: ",
np.allclose(A@IFA_components, A@(W@data)))
  fig, axes = plt.subplots(1, 2, figsize=(14, 6))
  # Heat map for the combined unmixing matrix
  sns.heatmap(W@data, cmap='viridis', ax=axes[0])
  axes[0].set title('Combined Unmixing Matrix (W @ data)')
  axes[0].set xlabel('Components')
  axes[0].set_ylabel('Samples')
  # Heat map for the IFA components
  sns.heatmap(IFA_components, cmap='viridis', ax=axes[1])
  axes[1].set_title('IFA Components')
  axes[1].set_xlabel('Components')
  axes[1].set_ylabel('Samples')
  # Adjust layout
```

```
plt.tight_layout()
  plt.show()
  return IFA_components, A, W
# %%
raw components combined, A combined, W combined =
ICA(subs_data_com_VN,whitened_basis)
# %%
vtwhiten, = whiten(vt,n_components=vt.shape[0],method="SVD")
subs_data_VN, _ = PPCA(reducedsubs_combined.copy(), filters=vtwhiten.T, threshold=0.0,
raw_components_major, A_major, W_major = ICA(subs_data_VN,vtwhiten)
# %%
subs_data_VN_more, vtmore = PPCA(reducedsubs.copy(), threshold=0.0,
niters=1,n=vt.shape[0]+filters.shape[1])
vtmorewhiten,_ = whiten(vtmore,n_components=vtmore.shape[0],method="SVD")
subs_data_VN_more, _ = PPCA(reducedsubs_combined.copy(), filters=vtmorewhiten.T,
threshold=0.0, niters=1)
raw_components_major_more, A_major_more, W_major_more =
ICA(subs_data_VN_more,vtmorewhiten)
# %%
save_array_to_outputfolder(os.path.join(fold_outputfolder,"raw_components_combined.npy"),
raw components combined)
save array to outputfolder(os.path.join(fold outputfolder,"A combined.npy"), A combined)
save array to outputfolder(os.path.join(fold outputfolder,"W combined.npy"), W combined)
save_array_to_outputfolder(os.path.join(fold_outputfolder,"raw_components_major.npy"),
raw_components_major)
save array to outputfolder(os.path.join(fold outputfolder,"A major.npy"), A major)
save_array_to_outputfolder(os.path.join(fold_outputfolder,"W_major.npy"), W_major)
save_array_to_outputfolder(os.path.join(fold_outputfolder,"raw_components_major_more.npy"),
raw components major more)
save array to outputfolder(os.path.join(fold outputfolder,"A major more.npy"), A major more)
save_array_to_outputfolder(os.path.join(fold_outputfolder,"W_major_more.npy"), W_major_more)
# %% [markdown]
#### Load ICA
# %%
raw components combined =
load_array_from_outputfolder(os.path.join(fold_outputfolder,"raw_components_combined.npy"))
A_combined = load_array_from_outputfolder(os.path.join(fold_outputfolder,"A_combined.npy"))
W_combined = load_array_from_outputfolder(os.path.join(fold_outputfolder,"W_combined.npy"))
raw components major =
load_array_from_outputfolder(os.path.join(fold_outputfolder,"raw_components_major.npy"))
A major = load array from outputfolder(os.path.join(fold outputfolder,"A major.npy"))
W_major = load_array_from_outputfolder(os.path.join(fold_outputfolder,"W_major.npy"))
raw_components_major_more =
```

```
load array from outputfolder(os.path.join(fold outputfolder,"raw components major more.npy"))
A_major_more = load_array_from_outputfolder(os.path.join(fold_outputfolder,"A_major_more.npy"))
W_major_more =
load_array_from_outputfolder(os.path.join(fold_outputfolder,"W_major_more.npy"))
# %% [markdown]
#### Threshold
# %%
def noise_projection(W,data, visualize=True):
  Signals = np.linalg.pinv(W)@(W@data)
  Residuals = data - Signals
  residual std = np.std(Residuals,axis=0,ddof=np.linalg.matrix rank(W))
  # Trace of I-pinv(W)(W) is equal to the nullity (n-m gvien n > m) of the reconstructed matrix
  # trace = data.shape[0] - np.linalg.matrix rank(W)
  # residual_std2 = (np.einsum('ij,ij->j', Residuals, Residuals)/(trace))**.5
  if visualize:
     n=1000
     plt.figure()
     plt.plot(Signals[:n,0:1])
     plt.plot(Residuals[:n,0:1])
     # plt.plot(data[:n,0:1])
     # plt.plot(data[:n,0:1] - (Signals[:n,0:1]+Residuals[:n,0:1]))
     plt.legend(['Signal','Noise', 'Data','Reconstruction Error'])
     plt.title("Calculations based on pinv(W)W Projection Matrix")
     plt.show()
     plt.scatter(range(0,residual_std.shape[0]), residual_std)
     plt.title("Noise std Per Voxel based on pinv(W)W Projection Matrix")
     plt.show()
  return residual_std
def threshold and visualize(data, W, components, visualize=False):
  voxel_noise = noise_projection(W,data)[:, np.newaxis]
  z_scores_array = np.zeros_like(components)
  z_scores = np.zeros_like(components)
  # Process each filter individually
  for i in range(components.shape[1]):
     z_score = ((components[:, i:i+1]))/voxel_noise
     \# P(Z < -z \setminus GDF)(z)) + (1 - \ker\{GDF\}(z)) = 2 \times (1 - \ker\{GDF\}(z))
(z))
     p values = 2 * (1 - norm.cdf(np.abs(z score)))
     # Apply multiple comparisons correction for the current filter https://www.statsmodels.org/dev/
generated/statsmodels.stats.multitest.multipletests.html
     reject, pvals_corrected, _, _ = multipletests(p_values.flatten(), alpha=0.05, method='fdr_bh')
     masked_comp = z_score*(reject[:,np.newaxis])
```

```
# print(masked_comp, reject[:,np.newaxis],z_score)
     z_scores_array[:, i:i+1] = masked_comp
     z_scores[:,i:i+1] = z_score
    # Skip the iteration if there are no significant values
     if not np.any(reject) and visualize:
       print(f'Component {i} did not contain any significant values')
       plt.figure()
       plt.hist(z score, bins=30, color='blue', alpha=0.7)
       plt.title(f"Histogram for Filter {i} NO SIGNIFICANT VALUES")
       plt.xlabel('Value')
       plt.ylabel('Frequency')
       plt.show()
     else:
       if visualize:
          # Create a figure and axes for subplots (1 row of 2 plots per filter)
          fig, axes = plt.subplots(1, 2, figsize=(18, 10))
          ax_hist1 = axes[0]
          ax_{img} = axes[1]
          # Plot the histogram of the current filter
          ax_hist1.hist(z_score, bins=30, color='blue', alpha=0.7)
          ax_hist1.set_title(f"Histogram for Filter {i}")
          ax_hist1.set_xlabel('Value')
          ax_hist1.set_ylabel('Frequency')
  fig, axes = plt.subplots(1, 2, figsize=(14, 6))
  # Heat map for the combined unmixing matrix
  sns.heatmap(z_scores, cmap='viridis', ax=axes[0])
  axes[0].set_title('z_score')
  axes[0].set_xlabel('Components')
  axes[0].set_ylabel('Samples')
  # Heat map for the IFA components
  sns.heatmap(z_scores_array, cmap='viridis', ax=axes[1])
  axes[1].set title('z score thresh')
  axes[1].set_xlabel('Components')
  axes[1].set_ylabel('Samples')
  # Adjust layout
  plt.tight_layout()
  plt.show()
  return z_scores, z_scores_array
# %%
z scores unthresh, z scores thresh = threshold and visualize(subs data com VN,
W_combined, raw_components_combined.T, visualize=False)
# %%
z_scores_unthresh_major, z_scores_thresh_major = threshold_and_visualize(subs_data_VN,
```

```
W_major, raw_components_major.T, visualize=False)
# %%
z_scores_unthresh_major_more, z_scores_thresh_major_more =
threshold_and_visualize(subs_data_VN_more, W_major_more, raw_components_major_more.T,
visualize=False)
# %%
save array to outputfolder(os.path.join(fold outputfolder,"z scores unthresh.npy"),
z_scores_unthresh)
save_array_to_outputfolder(os.path.join(fold_outputfolder,"z_scores_thresh.npy"), z_scores_thresh)
save_array_to_outputfolder(os.path.join(fold_outputfolder,"z_scores_unthresh_major.npy"),
z_scores_unthresh_major)
save_array_to_outputfolder(os.path.join(fold_outputfolder,"z_scores_thresh_major.npy"),
z_scores_thresh_major)
save_array_to_outputfolder(os.path.join(fold_outputfolder,"z_scores_unthresh_major_more.npy"),
z_scores_unthresh_major_more)
save_array_to_outputfolder(os.path.join(fold_outputfolder,"z_scores_thresh_major_more.npy"),
z_scores_thresh_major_more)
# %% [markdown]
#### Load Z maps (Thresholded and Unthresholded)
# %%
z_scores_unthresh =
load_array_from_outputfolder(os.path.join(fold_outputfolder,"z_scores_unthresh.npy"))
z_scores_thresh =
load_array_from_outputfolder(os.path.join(fold_outputfolder,"z_scores_thresh.npy"))
z scores unthresh major =
load_array_from_outputfolder(os.path.join(fold_outputfolder,"z_scores_unthresh_major.npy"))
z_scores_thresh_major =
load_array_from_outputfolder(os.path.join(fold_outputfolder,"z_scores_thresh_major.npy"))
z_scores_unthresh_major_more =
load_array_from_outputfolder(os.path.join(fold_outputfolder,"z_scores_unthresh_major_more.npy")
)
z_scores_thresh_major_more =
load array from outputfolder(os.path.join(fold outputfolder,"z scores thresh major more.npy"))
# %%
# plotting.view_surf(hcp.mesh.inflated, hcp.cortex_data(z_scores_unthresh[:,20]), threshold=0,
bg_map=hcp.mesh.sulc)
# %% [markdown]
#### Run and Save Netmats + Dual Regression
# %%
from functools import partial
# https://www.frontiersin.org/journals/neuroscience/articles/10.3389/fnins.2017.00115/full
def calculate_netmat_and_spatial_map(Xn, z_maps):
```

```
Calculate the network matrix (netmat) and spatial map for a given subject and z_maps.
```

```
Parameters:
  Xn (array): Time x Grayordinates normalized data matrix (Time x V)
  z_maps (array): Grayordinates x Components map (V x C)
  Returns:
  netmat (array): Components x Components network matrix (C x C)
  spatial map (array): Components x Grayordinates matrix (C x V)
  # Time x Components
  # Demean the regressors (z_maps)
  z_maps_demeaned = z_maps - z_maps.mean(axis=0, keepdims=True) # Demean the columns
of z_maps (V x C)
  # Time x Components
  A = (Xn @ np.linalg.pinv(z_maps_demeaned.T)) # A is Time x Components (T x C)
  # Normalized Time x Components matrix
  An = hcp.normalize(A) # An is Time x Components (T x C)
  del A
  # Components x Components network matrix
  netmat = (An.T @ An) / (Xn.shape[0] - 1) # Netmat is Components x Components (C x C)
  # Components x Grayordinates spatial map
  spatial_map = np.linalg.pinv(An) @ Xn # Spatial map is Components x Grayordinates (C x V)
  return An, netmat, spatial_map
def dual_regress_sub(sub_path, z_maps_1, z_maps_2):
  try:
    concatenated_data = []
    for task in sub path:
       # Load and preprocess each task
       X = nib.load(task).get fdata(dtype=np.float32) # Grayordinates x Time (V x T)
       Xn = hcp.normalize(X - X.mean(axis=1, keepdims=True)) # Normalizing (V x T)
       concatenated_data.append(Xn)
       del X, Xn
    # Concatenate data along the first axis (all tasks into one big matrix)
    subject = np.concatenate(concatenated_data, axis=0) # Time x Grayordinates (T x V)
    del concatenated data
    # Normalize the concatenated data
    Xn = hcp.normalize(subject - subject.mean(axis=1,keepdims=True)) # Time x Grayordinates
normalized data (T x V)
    del subject
    # Calculate netmat and spatial map for the first set of z_maps
    An_1, netmat_1, spatial_map_1 = calculate_netmat_and_spatial_map(Xn, z_maps_1)
```

```
# Calculate netmat and spatial map for the second set of z maps
    An_2, netmat_2, spatial_map_2 = calculate_netmat_and_spatial_map(Xn, z_maps_2)
    return (An_1, netmat_1, spatial_map_1), (An_2, netmat_2, spatial_map_2)
  except Exception as e:
     print(f"Error processing subject: {e}")
    return None, None
def dual_regress(group_paths, z_maps_1, z_maps_2):
  # Use partial to avoid duplicating z_maps in memory
  with ProcessPoolExecutor(max_workers=int(os.cpu_count() * 0.7)) as executor:
    # Create a partial function that "binds" the z_maps_1 and z_maps_2 without duplicating them
    partial_func = partial(dual_regress_sub, z_maps_1=z_maps_1, z_maps_2=z_maps_2)
    # Pass the subject paths to the executor without copying z_maps
    results = list(executor.map(partial_func, group_paths))
    # Separate the results for the two bases, collecting An, netmat, and spatial map
    An_1, netmats_1, spatial_maps_1 = zip(*[(res[0][0], res[0][1], res[0][2])) for res in results if
res[0] is not None])
    An_2, netmats_2, spatial_maps_2 = zip(*[(res[1][0], res[1][1], res[1][2])) for res in results if
res[1] is not None])
     return (np.array(An_1), np.array(netmats_1), np.array(spatial_maps_1)), (np.array(An_2),
np.array(netmats_2), np.array(spatial_maps_2))
# Save function for An, netmats, and spatial maps
def save_numpy_arrays(output_prefix, An_1, netmats_1, spatial_maps_1, An_2, netmats_2,
spatial_maps_2):
  Saves the An arrays, netmats, and spatial maps to disk using np.save.
  Parameters:
  output prefix (str): Prefix for the output files.
  An 1 (np.array): Time x Components matrix for z maps 1.
  netmats_1 (np.array): Network matrices for z_maps_1.
  spatial_maps_1 (np.array): Spatial maps for z_maps_1.
  An_2 (np.array): Time x Components matrix for z_maps_2.
  netmats_2 (np.array): Network matrices for z_maps_2.
  spatial_maps_2 (np.array): Spatial maps for z_maps_2.
  save_array_to_outputfolder(f"{output_prefix}_An_1.npy", An_1)
  save_array_to_outputfolder(f"{output_prefix}_netmats_1.npy", netmats_1)
  save_array_to_outputfolder(f"{output_prefix}_spatial_maps_1.npy", spatial_maps_1)
  save_array_to_outputfolder(f"{output_prefix}_An_2.npy", An_2)
  save_array_to_outputfolder(f"{output_prefix}_netmats_2.npy", netmats_2)
  save_array_to_outputfolder(f"{output_prefix}_spatial_maps_2.npy", spatial_maps_2)
```

```
# For Group A - Training Set
(groupA_An_1_train, groupA_netmats_1_train, groupA_spatial_maps_1_train),
(groupA_An_2_train, groupA_netmats_2_train, groupA_spatial_maps_2_train) =
dual_regress(groupA_train_paths, z_scores_unthresh, z_scores_unthresh_major_more)
save_numpy_arrays("groupA_train", groupA_An_1_train, groupA_netmats_1_train,
groupA_spatial_maps_1_train, groupA_An_2_train, groupA_netmats_2_train,
groupA_spatial_maps_2_train)
# %%
# For Group A - Test Set
(groupA_An_1_test, groupA_netmats_1_test, groupA_spatial_maps_1_test), (groupA_An_2_test,
groupA_netmats_2_test, groupA_spatial_maps_2_test) = dual_regress(groupA_test_paths,
z_scores_unthresh, z_scores_unthresh_major_more)
save_numpy_arrays("groupA_test", groupA_An_1_test, groupA_netmats_1_test,
groupA_spatial_maps_1_test, groupA_An_2_test, groupA_netmats_2_test,
groupA_spatial_maps_2_test)
# %%
# For Group B - Training Set
(groupB_An_1_train, groupB_netmats_1_train, groupB_spatial_maps_1_train),
(groupB_An_2_train, groupB_netmats_2_train, groupB_spatial_maps_2_train) =
dual_regress(groupB_train_paths, z_scores_unthresh, z_scores_unthresh_major_more)
save_numpy_arrays("groupB_train", groupB_An_1_train, groupB_netmats_1_train,
groupB_spatial_maps_1_train, groupB_An_2_train, groupB_netmats_2_train,
groupB_spatial_maps_2_train)
# %%
# For Group B - Test Set
(groupB_An_1_test, groupB_netmats_1_test, groupB_spatial_maps_1_test), (groupB_An_2_test,
groupB_netmats_2_test, groupB_spatial_maps_2_test) = dual_regress(groupB_test_paths,
z_scores_unthresh, z_scores_unthresh_major_more)
save_numpy_arrays("groupB_test", groupB_An_1_test, groupB_netmats_1_test,
groupB_spatial_maps_1_test, groupB_An_2_test, groupB_netmats_2_test,
groupB_spatial_maps_2_test)
# %%
# For Group A - Training Set
(groupA_An_1_train, groupA_netmats_1_train, groupA_spatial_maps_1_train),
(groupA_An_2_train, groupA_netmats_2_train, groupA_spatial_maps_2_train) =
dual_regress(groupA_train_paths, z_scores_unthresh, z_scores_unthresh_major_more)
save_numpy_arrays(os.path.join(fold_outputfolder,"groupA_train"), groupA_An_1_train,
groupA_netmats_1_train, groupA_spatial_maps_1_train, groupA_An_2_train,
groupA_netmats_2_train, groupA_spatial_maps_2_train)
# For Group A - Test Set
(groupA_An_1_test, groupA_netmats_1_test, groupA_spatial_maps_1_test), (groupA_An_2_test,
groupA_netmats_2_test, groupA_spatial_maps_2_test) = dual_regress(groupA_test_paths,
z_scores_unthresh, z_scores_unthresh_major_more)
save_numpy_arrays(os.path.join(fold_outputfolder,"groupA_test"), groupA_An_1_test,
groupA_netmats_1_test, groupA_spatial_maps_1_test, groupA_An_2_test,
groupA_netmats_2_test, groupA_spatial_maps_2_test)
# For Group B - Training Set
(groupB_An_1_train, groupB_netmats_1_train, groupB_spatial_maps_1_train),
```

```
(groupB_An_2_train, groupB_netmats_2_train, groupB_spatial_maps_2_train) =
dual_regress(groupB_train_paths, z_scores_unthresh, z_scores_unthresh_major_more)
save_numpy_arrays(os.path.join(fold_outputfolder,"groupB_train"), groupB_An_1_train,
groupB_netmats_1_train, groupB_spatial_maps_1_train, groupB_An_2_train,
groupB_netmats_2_train, groupB_spatial_maps_2_train)
# For Group B - Test Set
(groupB An 1 test, groupB netmats 1 test, groupB spatial maps 1 test), (groupB An 2 test,
groupB_netmats_2_test, groupB_spatial_maps_2_test) = dual_regress(groupB_test_paths,
z scores unthresh, z scores unthresh major more)
save_numpy_arrays(os.path.join(fold_outputfolder,"groupB_test"), groupB_An_1_test,
groupB_netmats_1_test, groupB_spatial_maps_1_test, groupB_An_2_test,
groupB_netmats_2_test, groupB_spatial_maps_2_test)
# %% [markdown]
#### Load Netmats + Dual Regression
# %%
# Load function for An, netmats, and spatial maps
def load_numpy_arrays(input_prefix):
  Loads the An arrays, netmats, and spatial maps from disk using load_array_from_outputfolder.
  Parameters:
  input_prefix (str): Prefix for the input files.
  Returns:
  tuple: Six numpy arrays (An_1, netmats_1, spatial_maps_1, An_2, netmats_2, spatial_maps_2).
  An 1 = load array from outputfolder(f"{input prefix} An 1.npy")
  netmats_1 = load_array_from_outputfolder(f"{input_prefix}_netmats_1.npy")
  spatial_maps_1 = load_array_from_outputfolder(f"\{input_prefix\}_spatial_maps_1.npy")
  An_2 = load_array_from_outputfolder(f"{input_prefix}_An_2.npy")
  netmats 2 = load array from outputfolder(f"{input prefix} netmats 2.npy")
  spatial_maps_2 = load_array_from_outputfolder(f"{input_prefix}_spatial_maps_2.npy")
  return An_1, netmats_1, spatial_maps_1, An_2, netmats_2, spatial_maps_2
# Example usage for loading Group A train and test results
groupA_An_1_train, groupA_netmats_1_train, groupA_spatial_maps_1_train, groupA_An_2_train,
groupA_netmats_2_train, groupA_spatial_maps_2_train =
load_numpy_arrays(os.path.join(fold_outputfolder,"groupA_train"))
groupA_An_1_test, groupA_netmats_1_test, groupA_spatial_maps_1_test, groupA_An_2_test,
groupA_netmats_2_test, groupA_spatial_maps_2_test =
load_numpy_arrays(os.path.join(fold_outputfolder,"groupA_test"))
# Example usage for loading Group B train and test results
groupB_An_1_train, groupB_netmats_1_train, groupB_spatial_maps_1_train, groupB_An_2_train,
groupB_netmats_2_train, groupB_spatial_maps_2_train =
load_numpy_arrays(os.path.join(fold_outputfolder,"groupB_train"))
groupB An 1 test, groupB netmats 1 test, groupB spatial maps 1 test, groupB An 2 test,
groupB_netmats_2_test, groupB_spatial_maps_2_test =
load_numpy_arrays(os.path.join(fold_outputfolder,"groupB_test"))
```

```
# Sanity check for Group A train data
print("Group A Train:")
print(groupA_An_1_train.shape, groupA_netmats_1_train.shape,
groupA_spatial_maps_1_train.shape)
print(groupA_An_2_train.shape, groupA_netmats_2_train.shape,
groupA_spatial_maps_2_train.shape)
# Sanity check for Group A test data
print("Group A Test:")
print(groupA_An_1_test.shape, groupA_netmats_1_test.shape,
groupA_spatial_maps_1_test.shape)
print(groupA_An_2_test.shape, groupA_netmats_2_test.shape,
groupA_spatial_maps_2_test.shape)
# Sanity check for Group B train data
print("Group B Train:")
print(groupB_An_1_train.shape, groupB_netmats_1_train.shape,
groupB_spatial_maps_1_train.shape)
print(groupB_An_2_train.shape, groupB_netmats_2_train.shape,
groupB_spatial_maps_2_train.shape)
# Sanity check for Group B test data
print("Group B Test:")
print(groupB_An_1_test.shape, groupB_netmats_1_test.shape,
groupB_spatial_maps_1_test.shape)
print(groupB_An_2_test.shape, groupB_netmats_2_test.shape,
groupB_spatial_maps_2_test.shape)
# %% [markdown]
#### Netmat Regression
# %%
def extract_phenotype_two(subids,phenotype):
  file path restricted = '../HCP/RESTRICTED zainsou 8 6 2024 2 11 21.csv'
  file_path_unrestricted = '../HCP/unrestricted_zainsou_8_2_2024_6_13_22.csv'
  try:
    data_r = pd.read_csv(file_path_restricted)
    data_ur = pd.read_csv(file_path_unrestricted)
  except FileNotFoundError:
    print(f"File not found: {file_path_restricted} or {file_path_unrestricted}")
    raise
  # Combine restricted and unrestricted data on Subject ID
  data = pd.merge(data_r, data_ur, on='Subject', how='outer')
  # Convert Subject IDs to string for consistency
  data['Subject'] = data['Subject'].astype(str)
  subids = subids.astype(str)
  # Filter data for training subjects
```

```
train_data = data[data['Subject'].isin(subids)]
  # Ensure the order matches the training data
  train_data = train_data.set_index('Subject').loc[subids].reset_index()
  pheno_score = train_data["PicVocab_AgeAdj"] + train_data["PMAT24_A_CR"]
  return pheno score
# Combine train data from Group A and Group B
group_netmats_1_test = np.concatenate((groupA_netmats_1_test, groupB_netmats_1_test),
group_netmats_2_test = np.concatenate((groupA_netmats_2_test, groupB_netmats_2_test),
axis=0)
# Combine train data from Group A and Group B
group_netmats_1_test = np.concatenate((groupA_netmats_1_test, groupB_netmats_1_test),
axis=0)
group_netmats_2_test = np.concatenate((groupA_netmats_2_test, groupB_netmats_2_test),
axis=0)
groupA_train_paths = train_paths[train_labels == 1]
groupA test paths = test paths[test labels == 1]
groupB_train_paths = train_paths[train_labels == 0]
groupB test paths = test paths[test labels == 0]
groupA_train_subject_ids = extract_subject_ids(groupA_train_paths)
groupB_train_subject_ids = extract_subject_ids(groupB_train_paths)
groupA_test_subject_ids = extract_subject_ids(groupA_test_paths)
groupB_test_subject_ids = extract_subject_ids(groupB_test_paths)
train_subid = np.concatenate((groupA_train_subject_ids,groupB_train_subject_ids),axis=0)
test_subid = np.concatenate((groupA_test_subject_ids,groupB_test_subject_ids),axis=0)
print(train subid.shape,test subid.shape)
train_values = np.array(extract_phenotype_two(train_subid,phenotype))
train_age = np.array(extract_phenotype(train_subid, 'Age_in_Yrs'))
test values = np.array(extract phenotype two(test subid,phenotype))
test_age = np.array(extract_phenotype(test_subid, 'Age_in_Yrs'))
# %%
def tan_regression(train_netmats,test_netmats, train_values, test_values, train_age, test_age,
metric=metric):
  # Plot the histogram of training data and calculated weights
  plt.figure(figsize=(10, 6))
  # Plot histogram of training target values
  plt.hist(train_values, bins=10, alpha=0.7, color='blue', edgecolor='black', label='Train Data')
  plt.title(f'Train Data Histogram and Weights')
  plt.xlabel('Target Value')
  plt.ylabel('Frequency / Weight')
  plt.legend()
  plt.show()
```

```
train_values, test_values = preproc(train_values[:,np.newaxis],test_values[:,np.newaxis],
method="center")
  train_age, test_age = preproc(train_age[:,np.newaxis], test_age[:,np.newaxis], method="center")
  train mean = mean covariance(train netmats, metric=metric)
  tan_train = tangent_space(train_netmats, train_mean,metric=metric)
  tan_test = tangent_space(test_netmats, train_mean,metric=metric)
  tan_mean = np.mean(tan_train, axis=0)
  tan_train_centered = tan_train - tan_mean
  tan_test_centered = tan_test - tan_mean
  age_reg = regress_out_age(tan_train_centered, train_age)
  tan_train_centered = tan_train_centered - age_reg.predict(train_age.reshape(-1, 1))
  tan_test_centered = tan_test - age_reg.predict(test_age.reshape(-1, 1))
  reg_model = Ridge(alpha=1)
  # reg_model = LinearSVR(C=1,fit_intercept=True)
  reg model.fit(tan train centered, train values)
  # Evaluate the model on the test set
  test_score = reg_model.score(tan_test_centered, test_values)
  predictions = reg_model.predict(tan_test_centered)
  fold_corr, _ = pearsonr(test_values, predictions)
  print(f"Fold Test R2 Score: {test score}")
  print(f"Fold Test R Score: {fold_corr}")
  # Plot predictions vs true values for test fold
  plot_predictions(test_values, predictions,f'Test Predictions vs True Values - Fold R2:
{test_score:.2f}', 'Test')
  # Now plot for train fold
  predictions_train = reg_model.predict(tan_train_centered)
  plot predictions(train values, predictions train, f'Train Predictions vs True Values - Fold R2:
{test_score:.2f}', 'Train')
   # Plot residuals vs true values for test fold
  plot_residuals(test_values, predictions, f'Test Residuals vs True Values - Fold R2:
{test_score:.2f}', 'Test')
  # Plot residuals vs true values for train fold
  plot residuals(train values, predictions train, f'Train Residuals vs True Values - Fold R2:
{test_score:.2f}', 'Train')
tan regression(group netmats 1 train.group netmats 1 test, train values, test values, train age,
test_age, metric=metric)
tan regression(group netmats 2 train,group netmats 2 test, train values, test values, train age,
test_age, metric=metric)
```

```
# %% [markdown]
#### Analyze Discriminant Information via Netmats
def migp netmat(group data,basis):
     group_data_dm = group_data - group_data.mean(axis=0, keepdims=True)
     basis dm = basis - basis.mean(axis=0, keepdims=True)
    A = ((group_data_dm@np.linalg.pinv(basis_dm.T)))
     # Normalized Time x Components matrix
     An = hcp.normalize(A) # An is Time x Components (T x C)
     del A
     timepoints = An.shape[0]
     group_netmat = (An.T@An)/(timepoints-1)
     return group netmat
def group_dist(group_data1,group_data2,basis,metric="riemann"):
     netmat1 = migp_netmat(group_data1,basis)
     netmat2 = migp_netmat(group_data2,basis)
     return distance(netmat1,netmat2,metric=metric)
ICA mean dist =
group dist(reducedsubsA loaded,reducedsubsB loaded,z scores unthresh major,metric=metric)
ICA_more_mean_dist =
group\_dist(reduced subs A\_loaded, reduced subs B\_loaded, z\_scores\_unthresh\_major\_more, metric = 1000 metric = 10
metric)
IFA_mean_dist =
group_dist(reducedsubsA_loaded,reducedsubsB_loaded,z_scores_unthresh,metric=metric)
save_text_results(f'Distance between Group Average Netmats formed from MIGP on train data
projected on {z_scores_unthresh_major.shape[1]} ICA components:
{ICA_mean_dist}',os.path.join(f"fold_{fold}", "results","results.txt"))
save text results(f'Distance between Group Average Netmats formed from MIGP on train data
projected on {z_scores_unthresh_major_more.shape[1]} ICA components:
{ICA_more_mean_dist}',os.path.join(f"fold_{fold}", "results","results.txt"))
save text results(f'Distance between Group Average Netmats formed from MIGP on train data
projected on {z scores unthresh.shape[1]} IFA components:
{IFA_mean_dist}',os.path.join(f"fold_{fold}", "results", "results.txt"))
# %%
from sklearn.metrics import accuracy_score, classification_report
from sklearn.metrics.pairwise import euclidean_distances
# https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=5662067
def var_diff(group1_train, group1_test, group1_cov_train,group2_train, group2_test,
group2_cov_train, metric):
     # clf = SVC(kernel='linear', class weight='balanced')
     clf = LogisticRegression()
```

# Compute the mean covariances using the training data only

```
group1_mean = mean_covariance(group1_cov_train, metric=metric)
  group2_mean = mean_covariance(group2_cov_train, metric=metric)
  ## Eigen decomposition to get features
  _, feature_all = eigh(group1_mean, group2_mean + group2_mean, eigvals_only=False)
  #_, feature_all, _, _ = tangent_classifier(group1_cov_train, group2_cov_train, TSVM=True,
TLDA=False, tangent_calc=True, metric=metric, visualize=False, n=0)
  # Initialize list to store results (accuracy and distance)
  results = []
  # Loop from n=1 to n=15 for selecting top and bottom eigenvectors
  for n in range(1, feature all.shape[1] // 2 + 1):
     # Perform eigen decomposition based on top and bottom n eigenvectors
     features = np.hstack([feature all[:, :n], feature all[:, -n:]]) # Select top and bottom n
eigenvectors
     group1_train_logvar, group1_test_logvar, group2_train_logvar, group2_test_logvar =
feature_generation(group1_train, group1_test,group2_train,group2_test, features,
metric=metric,method="log-var")
     # Prepare the dataset for classification
     X_train = np.vstack([group1_train_logvar, group2_train_logvar])
     y_train = np.hstack([np.ones(group1_train_logvar.shape[0]),
np.zeros(group2_train_logvar.shape[0])])
     X_test = np.vstack([group1_test_logvar, group2_test_logvar])
     y_test = np.hstack([np.ones(group1_test_logvar.shape[0]),
np.zeros(group2_test_logvar.shape[0])])
     # Train logistic regression classifier on training data
     clf.fit(X_train, y_train)
     # Predict on the test data and calculate accuracy
     y_pred = clf.predict(X_test)
     accuracy = accuracy_score(y_test, y_pred)
     # Calculate class means for distance (using the training data)
     mean_group1_test = np.mean(group1_test_logvar, axis=0)
     mean_group2_test = np.mean(group2_test_logvar, axis=0)
     # Calculate the distance between the two class means
     mean_dist = np.linalg.norm(mean_group1_test - mean_group2_test)
     # Store accuracy and Riemannian distance for this n
     results.append((n, mean_dist, accuracy))
     # Plot when n=1
     if n == 1:
       plt.figure(figsize=(8, 6))
       plt.scatter(group1 test logvar[:, 0], group1 test logvar[:, 1], label='Group A Log Variance
(Test)', color='blue')
       plt.scatter(group2_test_logvar[:, 0], group2_test_logvar[:, 1], label='Group B Log Variance
```

```
(Test)', color='red')
       # Plot the line connecting the two means
       plt.plot([mean_group1_test[0], mean_group2_test[0]], [mean_group1_test[1],
mean_group2_test[1]], 'k--', label=f'Mean Distance: {mean_dist:.2f}')
       # Decision boundary
       x_values = np.array([X_train[:, 0].min(), X_train[:, 0].max()])
       y_values = -(clf.intercept_ + clf.coef_[0][0] * x_values) / clf.coef_[0][1]
       plt.plot(x_values, y_values, 'g-', label='Decision Boundary')
       # Display plot
       plt.xlabel('Log Variance Feature B')
       plt.ylabel('Log Variance Feature A')
       plt.title('Log Variance FKT Feature Comparison and Logistic Regression Decision
Boundary')
       # Display classification accuracy on the plot
       plt.text(0.05, 0.95, f'Accuracy: {accuracy:.2f}', transform=plt.gca().transAxes, fontsize=12,
             verticalalignment='top', bbox=dict(boxstyle='round,pad=0.3', edgecolor='black',
facecolor='lightgrey'))
       plt.legend()
       plt.grid(True)
       plt.show()
       # test_visualize_variance(group1_test, group2_test, features)
  # Return the list of accuracies and distances for each n
  return results
# Updated tangent_class_test function
def tangent_class_test(group1_cov_train, group1_cov_test, group2_cov_train, group2_cov_test,
metric):
  # clf = SVC(kernel='linear', C=.01,class_weight='balanced')
  clf = LogisticRegression()
  tangent_projected_1_train, tangent_projected_2_train, tangent_projected_1_test,
tangent projected 2 test = tangent transform(group1 cov train, group1 cov test,
group2_cov_train, group2_cov_test, metric=metric)
  # Combine the tangent projections for training and testing
  X_train = np.vstack((tangent_projected_1_train, tangent_projected_2_train))
  X_test = np.vstack((tangent_projected_1_test, tangent_projected_2_test))
  y_train = np.hstack((np.ones(tangent_projected_1_train.shape[0]),
np.zeros(tangent_projected_2_train.shape[0])))
  y_test = np.hstack((np.ones(tangent_projected_1_test.shape[0]),
np.zeros(tangent_projected_2_test.shape[0])))
  # Dimensionality reduction
  max_dim = np.min((X_train.shape[0], X_train.shape[1]))
  dims = [2, 3, int((max dim-1)/20), int((max dim-1)/17), int((max dim-1)/15),
       int((max_dim-1)/13), int((max_dim-1)/12), int((max_dim-1)/10),
       int((max_dim-1)/7), int((max_dim-1)/5), int((max_dim-1)/3),
```

```
int((max dim-1)/2), int((max dim-1)/1.7), int((max dim-1)/1.5),
       int((max_dim-1)/1.3), int((max_dim-1)/1.1), max_dim-1]
  results = []
  for i in dims:
    # Reduce dimensionality using PCA
    pca = PCA(n components=i)
    X_train_reduced = pca.fit_transform(X_train)
    X test reduced = pca.transform(X test)
    mean_dist = np.linalg.norm(np.mean(X_test_reduced[y_test == 1],axis=0) -
np.mean(X_test_reduced[y_test == 0],axis=0))
    # Train logistic regression classifier
    clf.fit(X_train_reduced, y_train)
    # Test accuracy
    y_pred = clf.predict(X_test_reduced)
    test_accuracy = accuracy_score(y_test, y_pred)
    results.append((i, mean_dist, test_accuracy))
  return results
def mean diff(group1 covs red,group2 covs red,metric):
  group_1 = mean_covariance(group1_covs_red, metric=metric)
  group_2 = mean_covariance(group2_covs_red, metric=metric)
  return distance(group_1,group_2,metric=metric)
def PSD_diff_all(group1_train, group1_test, group1_cov_train, group1_cov_test, group2_train,
group2_test, group2_cov_train, group2_cov_test, metric):
  psd_mean_distance = mean_diff(group1_cov_test, group2_cov_test, metric)
  tangent_results = tangent_class_test(group1_cov_train, group1_cov_test, group2_cov_train,
group2_cov_test, metric)
  fkt_results = var_diff(group1_train, group1_test, group1_cov_train, group2_train, group2_test,
group2_cov_train, metric)
  result = {
    "psd_mean_distance": psd_mean_distance,
    "tangent_results": tangent_results,
    "fkt_results": fkt_results
  }
  return result
# %%
# For the first comparison (using Group 1 data):
IFA_result = PSD_diff_all(groupA_An_1_train, groupA_An_1_test, groupA_netmats_1_train,
groupA_netmats_1_test, groupB_An_1_train, groupB_An_1_test, groupB_netmats_1_train,
groupB_netmats_1_test, metric=metric)
# For the second comparison (using Group 2 data):
major_result = PSD_diff_all(groupA_An_2_train, groupA_An_2_test, groupA_netmats_2_train,
```

```
groupA_netmats_2_test, groupB_An_2_train, groupB_An_2_test, groupB_netmats_2_train,
groupB netmats 2 test, metric=metric)
def scatter_with_lines(x1, y1, x2, y2, label1='Series 1', label2='Series 2', xlabel='X', ylabel='Y',
title='Scatter Plot with Connecting Lines'):
  Creates a scatter plot with lines connecting corresponding points from two series.
  Parameters:
  - x1, y1: The x and y values for the first series.
  - x2, y2: The x and y values for the second series.
  - label1, label2: Labels for the two series.
  - xlabel, ylabel: Labels for the x and y axes.
  - title: Title for the plot.
  plt.figure(figsize=(12,6)) # Set the size of the figure
  plt.scatter(x1, y1, label=label1, color='blue')
  plt.scatter(x2, y2, label=label2, color='orange')
  # Draw lines connecting corresponding points
  for x 1, y 1, x 2, y 2 in zip(x1, y1, x2, y2):
     plt.plot([x_1, x_2], [y_1, y_2], color='gray', linestyle='--')
  plt.xlabel(xlabel)
  plt.ylabel(ylabel)
  plt.title(title)
  plt.legend()
  plt.show()
save_text_results(f'Distance between Group Average Netmats formed from Subject Dual
Regression using test data and {z_scores_unthresh.shape[1]} IFA Components:
{IFA result["psd mean distance"]}',os.path.join(f"fold {fold}", "results", "results.txt"))
save_text_results(f'IFA Tangent Results: {IFA_result|"tangent_results"]}',os.path.join(f"fold_{fold}",
"results", "results.txt"))
save text results(f'IFA FKT Results: {IFA result["fkt results"]},os.path.join(f"fold {fold}",
"results", "results.txt"))
save_text_results(f'Distance between Group Average Netmats formed from Subject Dual
Regression using test data and {z_scores_unthresh_major_more.shape[1]} ICA Components:
{major_result["psd_mean_distance"]}',os.path.join(f"fold_{fold}", "results","results.txt"))
save_text_results(f'ICA Tangent Results: {major_result["tangent_results"]}',os.path.join(f"fold_{fold}",
"results", "results.txt"))
save_text_results(f'ICA FKT Results: {major_result["fkt_results"]}',os.path.join(f"fold_{fold}",
"results", "results.txt"))
print(IFA result["psd mean distance"], major result["psd mean distance"])
scatter with lines([tup[0] for tup in IFA result["tangent results"]],
            [tup[2] for tup in IFA_result["tangent_results"]],
            [tup[0] for tup in major_result["tangent_results"]],
```

```
[tup[2] for tup in major result["tangent results"]].
            label1='IFA Components', label2='ICA Components',
            xlabel='Dimension', ylabel='Accuracy',
            title='Logistic Regression Accuracy of Tangent Netmats')
scatter_with_lines([tup[0] for tup in IFA_result["tangent_results"]],
            [tup[1] for tup in IFA result["tangent results"]],
            [tup[0] for tup in major_result["tangent_results"]],
            [tup[1] for tup in major result["tangent results"]],
            label1='IFA Components', label2='ICA Components',
            xlabel='Dimension', ylabel='Distance',
            title='Distance Between Group Means of Tangent Netmats')
scatter_with_lines([tup[0]*2 for tup in IFA_result["fkt_results"]],
            [tup[2] for tup in IFA result["fkt results"]],
            [tup[0]*2 for tup in major result["fkt results"]],
            [tup[2] for tup in major_result["fkt_results"]],
            label1='IFA Components', label2='ICA Components',
            xlabel='Dimension (# of Filters)', ylabel='Accuracy',
            title='Logistic Regression Accuracy of LogVar FKT Features')
scatter with lines([tup[0]*2 for tup in IFA result["fkt results"]],
            [tup[1] for tup in IFA_result["fkt_results"]],
            [tup[0]*2 for tup in major_result["fkt_results"]],
            [tup[1] for tup in major_result["fkt_results"]],
            label1='IFA Components', label2='ICA Components',
            xlabel='Dimension (# of Filters)', ylabel='Distance',
            title='Distance Between Group Means of LogVar FKT Features')
# %%
groupA_train_1_tangent, groupB_train_1_tangent, groupA_test_1_tangent, groupB_test_1_tangent
= tangent_transform(groupA_netmats_1_train, groupA_netmats_1_test, groupB_netmats_1_train,
groupB netmats 1 test, metric=metric)
groupA_train_2_tangent, groupB_train_2_tangent, groupA_test_2_tangent, groupB_test_2_tangent
= tangent transform(groupA_netmats_2_train, groupA_netmats_2_test, groupB_netmats_2_train,
groupB_netmats_2_test, metric=metric)
IFA_Class_Result = test_classifiers(groupA_train_1_tangent, groupA_test_1_tangent,
groupB_train_1_tangent, groupB_test_1_tangent)
ICA_Class_Result = test_classifiers(groupA_train_2_tangent, groupA_test_2_tangent,
groupB_train_2_tangent, groupB_test_2_tangent)
save text results(f'Tangent Space Classification Accuracies for Netmats formed from
{z_scores_unthresh.shape[1]} IFA Components: {IFA_Class_Result}',os.path.join(f"fold_{fold}",
"results", "results.txt"))
save_text_results(f'Tangent Space Classification Accuracies for Netmats formed from
{z scores unthresh major more.shape[1]} ICA Components:
{ICA_Class_Result}',os.path.join(f"fold_{fold}", "results","results.txt"))
# %%
```

def scatter\_with\_lines\_dict(result1, result2, label1='Series 1', label2='Series 2',

```
111111
  Creates a scatter plot with lines connecting corresponding points from two dictionaries.
  Parameters:
  - result1: Dictionary containing classifiers and accuracies for the first series.
  - result2: Dictionary containing classifiers and accuracies for the second series.
  - label1, label2: Labels for the two series.
  - xlabel, ylabel: Labels for the x and y axes.
  - title: Title for the plot.
  # Extract classifier names
  classifiers1 = list(result1.keys())
  classifiers2 = list(result2.keys())
  # Ensure both dictionaries have the same classifiers
  assert classifiers1 == classifiers2, "The classifiers (keys) must match between the two result
dictionaries."
  # Extract accuracy values
  accuracies1 = [metrics['accuracy'] for metrics in result1.values()]
  accuracies2 = [metrics['accuracy'] for metrics in result2.values()]
  # Debug: Print extracted accuracies
  print("Accuracies for Series 1:", accuracies1)
  print("Accuracies for Series 2:", accuracies2)
  # Set the figure size
  plt.figure(figsize=(14, 8))
  # Number of classifiers
  num_classifiers = len(classifiers1)
  x_positions = range(num_classifiers)
  # Create scatter plots
  plt.scatter(x_positions, accuracies1, label=label1, color='blue', s=100)
  plt.scatter(x_positions, accuracies2, label=label2, color='orange', s=100)
  # Draw lines connecting corresponding points
  for i in range(num_classifiers):
     plt.plot([x_positions[i], x_positions[i]], [accuracies1[i], accuracies2[i]],
           color='gray', linestyle='--')
  # Set labels and title
  plt.xlabel(xlabel, fontsize=14)
  plt.ylabel(ylabel, fontsize=14)
  plt.title(title, fontsize=16)
  # Set x-ticks to classifier names
  plt.xticks(x positions, classifiers1, rotation=45, ha='right', fontsize=12)
  # Add legend
```

xlabel='Classifier', ylabel='Accuracy', title='Accuracy Comparison'):

```
plt.legend(fontsize=12)
  # Add grid for better readability
  plt.grid(axis='y', linestyle='--', alpha=0.7)
  # Adjust layout to ensure everything fits
  plt.tight_layout()
  # Show the plot
  plt.show()
scatter_with_lines_dict(IFA_Class_Result, ICA_Class_Result, label1='IFA', label2='ICA',
title='Accuracy Comparison Between Tangent Space Classification of Netmats formed via IFA and
ICA')
# %%
train_dist = []
test_dist = []
logistic_dims = []
logistic_dist = []
logistic_acc = []
fkt dims = []
fkt_dist = []
fkt_acc = []
models_acc = []
train_dist_ICA = []
test_dist_ICA = []
logistic_dims_ICA = []
logistic_dist_ICA = []
logistic_acc_ICA = []
fkt_dims_ICA = []
fkt_dist_ICA = []
fkt_acc_ICA = []
models_acc_ICA = []
for i in range(0,5):
  fold = i
  fold_outputfolder = f"fold_{fold}"
  if not os.path.exists(os.path.join(outputfolder, f"fold_{fold}")):
     os.makedirs(os.path.join(outputfolder, f"fold_{fold}"))
  if not os.path.exists(os.path.join(outputfolder, f"fold_{fold}", "results")):
     os.makedirs(os.path.join(outputfolder, f"fold_{fold}", "results"))
  train_parcellated, test_parcellated = data[splits[fold][0]], data[splits[fold][1]]
  train_labels, test_labels = labels[splits[fold][0]], labels[splits[fold][1]]
  train_paths, test_paths = paths[splits[fold][0]], paths[splits[fold][1]]
  groupA train parcellated = train parcellated[train labels == 1]
  groupA_test_parcellated = test_parcellated[test_labels == 1]
  groupA_train_paths = train_paths[train_labels == 1]
```

```
groupA_test_paths = test_paths[test_labels == 1]
  groupB train parcellated = train parcellated[train labels == 0]
  groupB_test_parcellated = test_parcellated[test_labels == 0]
  groupB train paths = train paths[train labels == 0]
  groupB_test_paths = test_paths[test_labels == 0]
  if "reducedsubsA" in locals():
    del reducedsubsA
  if "reducedsubsB" in locals():
    del reducedsubsB
  reducedsubsA_loaded =
load_array_from_outputfolder(os.path.join(fold_outputfolder,'reducedsubsA.npy'))
  reducedsubsB loaded =
load array from outputfolder(os.path.join(fold outputfolder,'reducedsubsB.npy'))
  z scores unthresh =
load array from outputfolder(os.path.join(fold outputfolder,"z scores unthresh.npy"))
  z_scores_thresh =
load_array_from_outputfolder(os.path.join(fold_outputfolder,"z_scores_thresh.npy"))
  z_scores_unthresh_major =
load_array_from_outputfolder(os.path.join(fold_outputfolder,"z_scores_unthresh_major.npy"))
  z_scores_thresh_major =
load array from outputfolder(os.path.join(fold outputfolder,"z scores thresh major.npy"))
  z scores unthresh major more =
load_array_from_outputfolder(os.path.join(fold_outputfolder,"z_scores_unthresh_major_more.npy")
  z_scores_thresh_major_more =
load_array_from_outputfolder(os.path.join(fold_outputfolder,"z_scores_thresh_major_more.npy"))
  # Load function for An, netmats, and spatial maps
  def load_numpy_arrays(input_prefix):
    111111
    Loads the An arrays, netmats, and spatial maps from disk using load_array_from_outputfolder.
    Parameters:
    input_prefix (str): Prefix for the input files.
    tuple: Six numpy arrays (An 1, netmats 1, spatial maps 1, An 2, netmats 2,
spatial_maps_2).
    111111
    An_1 = load_array_from_outputfolder(f"{input_prefix}_An_1.npy")
    netmats_1 = load_array_from_outputfolder(f"{input_prefix}_netmats_1.npy")
    spatial maps 1 = load array from outputfolder(f"{input prefix} spatial maps 1.npy")
    An_2 = load_array_from_outputfolder(f"{input_prefix}_An_2.npy")
    netmats 2 = load array from outputfolder(f"{input prefix} netmats 2.npy")
    spatial_maps_2 = load_array_from_outputfolder(f"{input_prefix}_spatial_maps_2.npy")
    return An_1, netmats_1, spatial_maps_1, An_2, netmats_2, spatial_maps_2
  # Example usage for loading Group A train and test results
  groupA An 1 train, groupA netmats 1 train, groupA spatial maps 1 train,
groupA_An_2_train, groupA_netmats_2_train, groupA_spatial_maps_2_train =
load_numpy_arrays(os.path.join(fold_outputfolder,"groupA_train"))
```

```
groupA_An_1_test, groupA_netmats_1_test, groupA_spatial_maps_1_test, groupA_An_2_test,
groupA_netmats_2_test, groupA_spatial_maps_2_test =
load_numpy_arrays(os.path.join(fold_outputfolder,"groupA test"))
  # Example usage for loading Group B train and test results
  groupB_An_1_train, groupB_netmats_1_train, groupB_spatial_maps_1_train,
groupB_An_2_train, groupB_netmats_2_train, groupB_spatial_maps_2_train =
load_numpy_arrays(os.path.join(fold_outputfolder,"groupB_train"))
  groupB_An_1_test, groupB_netmats_1_test, groupB_spatial_maps_1_test, groupB_An_2_test,
groupB_netmats_2_test, groupB_spatial_maps_2_test =
load_numpy_arrays(os.path.join(fold_outputfolder,"groupB_test"))
  # Sanity check for Group A train data
  print("Group A Train:")
  print(groupA_An_1_train.shape, groupA_netmats_1_train.shape,
groupA spatial maps 1 train.shape)
  print(groupA_An_2_train.shape, groupA_netmats_2_train.shape,
groupA_spatial_maps_2_train.shape)
  # Sanity check for Group A test data
  print("Group A Test:")
  print(groupA_An_1_test.shape, groupA_netmats_1_test.shape,
groupA_spatial_maps_1_test.shape)
  print(groupA_An_2_test.shape, groupA_netmats_2_test.shape,
groupA_spatial_maps_2_test.shape)
  # Sanity check for Group B train data
  print("Group B Train:")
  print(groupB_An_1_train.shape, groupB_netmats_1_train.shape,
groupB_spatial_maps_1_train.shape)
  print(groupB_An_2_train.shape, groupB_netmats_2_train.shape,
groupB_spatial_maps_2_train.shape)
  # Sanity check for Group B test data
  print("Group B Test:")
  print(groupB_An_1_test.shape, groupB_netmats_1_test.shape,
groupB spatial maps 1 test.shape)
  print(groupB_An_2_test.shape, groupB_netmats_2_test.shape,
groupB_spatial_maps_2_test.shape)
  def migp_netmat(group_data,basis):
    group_data_dm = group_data - group_data.mean(axis=0, keepdims=True)
    basis dm = basis - basis.mean(axis=0, keepdims=True)
    A = ((group_data_dm@np.linalg.pinv(basis_dm.T)))
    # Normalized Time x Components matrix
    An = hcp.normalize(A) # An is Time x Components (T x C)
    del A
    timepoints = An.shape[0]
    group\_netmat = (An.T@An)/(timepoints-1)
    return group netmat
  def group_dist(group_data1,group_data2,basis,metric="riemann"):
```

```
netmat1 = migp_netmat(group_data1,basis)
    netmat2 = migp_netmat(group_data2,basis)
    return distance(netmat1,netmat2,metric=metric)
  ICA mean dist =
group_dist(reducedsubsA_loaded,reducedsubsB_loaded,z_scores_unthresh_major,metric=metric)
  ICA more mean dist =
group_dist(reducedsubsA_loaded,reducedsubsB_loaded,z_scores_unthresh_major_more,metric=
metric)
  IFA_mean_dist =
group_dist(reducedsubsA_loaded,reducedsubsB_loaded,z_scores_unthresh,metric=metric)
  train_dist.append(IFA_mean_dist)
  train_dist_ICA.append(ICA_more_mean_dist)
  save text results(f'Distance between Group Average Netmats formed from MIGP on train data
projected on {z_scores_unthresh_major.shape[1]} ICA components:
{ICA_mean_dist}',os.path.join(f"fold_{fold}", "results","results.txt"))
  save_text_results(f'Distance between Group Average Netmats formed from MIGP on train data
projected on {z scores unthresh major more.shape[1]} ICA components:
{ICA_more_mean_dist}',os.path.join(f"fold_{fold}", "results","results.txt"))
  save text results(f'Distance between Group Average Netmats formed from MIGP on train data
projected on {z_scores_unthresh.shape[1]} IFA components:
{IFA_mean_dist}',os.path.join(f"fold_{fold}", "results","results.txt"))
  from sklearn.metrics import accuracy_score, classification_report
  from sklearn.metrics.pairwise import euclidean_distances
  # https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=5662067
  def var_diff(group1_train, group1_test, group1_cov_train,group2_train, group2_test,
group2 cov train, metric):
    # clf = SVC(kernel='linear', class_weight='balanced')
    clf = LogisticRegression()
    # Compute the mean covariances using the training data only
    group1_mean = mean_covariance(group1_cov_train, metric=metric)
    group2 mean = mean covariance(group2 cov train, metric=metric)
    ## Eigen decomposition to get features
     _, feature_all = eigh(group1_mean, group2_mean + group2_mean, eigvals_only=False)
    #_, feature_all, _, _ = tangent_classifier(group1_cov_train, group2_cov_train, TSVM=True,
TLDA=False, tangent_calc=True, metric=metric, visualize=False, n=0)
    # Initialize list to store results (accuracy and distance)
    results = []
    # Loop from n=1 to n=15 for selecting top and bottom eigenvectors
    for n in range(1, feature_all.shape[1] // 2 + 1):
       # Perform eigen decomposition based on top and bottom n eigenvectors
       features = np.hstack([feature_all[:, :n], feature_all[:, -n:]]) # Select top and bottom n
eigenvectors
```

```
group1_train_logvar, group1_test_logvar, group2_train_logvar, group2_test_logvar =
feature_generation(group1_train, group1_test,group2_train,group2_test, features,
metric=metric,method="log-var")
       # Prepare the dataset for classification
       X_train = np.vstack([group1_train_logvar, group2_train_logvar])
       y_train = np.hstack([np.ones(group1_train_logvar.shape[0]),
np.zeros(group2_train_logvar.shape[0])])
       X test = np.vstack([group1 test logvar, group2 test logvar])
       y_test = np.hstack([np.ones(group1_test_logvar.shape[0]),
np.zeros(group2_test_logvar.shape[0])])
       # Train logistic regression classifier on training data
       clf.fit(X_train, y_train)
       # Predict on the test data and calculate accuracy
       y_pred = clf.predict(X_test)
       accuracy = accuracy_score(y_test, y_pred)
       # Calculate class means for distance (using the training data)
       mean_group1_test = np.mean(group1_test_logvar, axis=0)
       mean group2 test = np.mean(group2 test logvar, axis=0)
       # Calculate the distance between the two class means
       mean_dist = np.linalg.norm(mean_group1_test - mean_group2_test)
       # Store accuracy and Riemannian distance for this n
       results.append((n, mean_dist, accuracy))
       # Plot when n=1
       if n == 1:
          plt.figure(figsize=(8, 6))
          plt.scatter(group1_test_logvar[:, 0], group1_test_logvar[:, 1], label='Group A Log Variance
(Test)', color='blue')
          plt.scatter(group2_test_logvar[:, 0], group2_test_logvar[:, 1], label='Group B Log Variance
(Test)', color='red')
          # Plot the line connecting the two means
          plt.plot([mean_group1_test[0], mean_group2_test[0]], [mean_group1_test[1],
mean_group2_test[1]], 'k--', label=f'Mean Distance: {mean_dist:.2f}')
          # Decision boundary
          x_values = np.array([X_train[:, 0].min(), X_train[:, 0].max()])
          y_values = -(clf.intercept_ + clf.coef_[0][0] * x_values) / clf.coef_[0][1]
          plt.plot(x_values, y_values, 'g-', label='Decision Boundary')
          # Display plot
          plt.xlabel('Log Variance Feature B')
          plt.ylabel('Log Variance Feature A')
          plt.title('Log Variance FKT Feature Comparison and Logistic Regression Decision
Boundary')
```

```
# Display classification accuracy on the top-left of the plot
         plt.text(0.05, 0.95, f'Accuracy: {accuracy:.2f}', transform=plt.gca().transAxes, fontsize=12,
               verticalalignment='top', bbox=dict(boxstyle='round,pad=0.3', edgecolor='black',
facecolor='lightgrey'))
          # Set legend in the top-right corner
         plt.legend(loc='upper right')
          # Add grid
          plt.grid(True)
         # Show the plot
         plt.show()
         # test_visualize_variance(group1_test, group2_test, features)
     # Return the list of accuracies and distances for each n
     return results
  # Updated tangent class test function
  def tangent_class_test(group1_cov_train, group1_cov_test, group2_cov_train, group2_cov_test,
metric):
     # clf = SVC(kernel='linear', C=.01,class_weight='balanced')
     clf = LogisticRegression()
     tangent_projected_1_train, tangent_projected_2_train, tangent_projected_1_test,
tangent_projected_2_test = tangent_transform(group1_cov_train, group1_cov_test,
group2_cov_train, group2_cov_test, metric=metric)
     # Combine the tangent projections for training and testing
     X_train = np.vstack((tangent_projected_1_train, tangent_projected_2_train))
     X_test = np.vstack((tangent_projected_1_test, tangent_projected_2_test))
     y_train = np.hstack((np.ones(tangent_projected_1_train.shape[0]),
np.zeros(tangent_projected_2_train.shape[0])))
     y_test = np.hstack((np.ones(tangent_projected_1_test.shape[0]),
np.zeros(tangent_projected_2_test.shape[0])))
     # Dimensionality reduction
     max_dim = np.min((X_train.shape[0], X_train.shape[1]))
     dims = [2, 3, int((max_dim-1)/20), int((max_dim-1)/17), int((max_dim-1)/15),
          int((max_dim-1)/13), int((max_dim-1)/12), int((max_dim-1)/10),
         int((max_dim-1)/7), int((max_dim-1)/5), int((max_dim-1)/3),
          int((max_dim-1)/2), int((max_dim-1)/1.7), int((max_dim-1)/1.5),
          int((max_dim-1)/1.3), int((max_dim-1)/1.1), max_dim-1]
     results = []
     for i in dims:
       # Reduce dimensionality using PCA
       pca = PCA(n components=i)
       X_train_reduced = pca.fit_transform(X_train)
       X test reduced = pca.transform(X test)
       mean_dist = np.linalg.norm(np.mean(X_test_reduced[y_test == 1],axis=0) -
np.mean(X_test_reduced[y_test == 0],axis=0))
```

```
# Train logistic regression classifier
       clf.fit(X_train_reduced, y_train)
       # Test accuracy
       y_pred = clf.predict(X_test_reduced)
       test_accuracy = accuracy_score(y_test, y_pred)
       results.append((i, mean_dist, test_accuracy))
     return results
  def mean_diff(group1_covs_red,group2_covs_red,metric):
     group_1 = mean_covariance(group1_covs_red, metric=metric)
     group_2 = mean_covariance(group2_covs_red, metric=metric)
     return distance(group_1,group_2,metric=metric)
  def PSD_diff_all(group1_train, group1_test, group1_cov_train, group1_cov_test, group2_train,
group2_test, group2_cov_train, group2_cov_test, metric):
     psd_mean_distance = mean_diff(group1_cov_test, group2_cov_test, metric)
     tangent_results = tangent_class_test(group1_cov_train, group1_cov_test, group2_cov_train,
group2_cov_test, metric)
     fkt results = var diff(group1 train, group1 test, group1 cov train, group2 train, group2 test,
group2_cov_train, metric)
     result = {
       "psd_mean_distance": psd_mean_distance,
       "tangent_results": tangent_results,
       "fkt_results": fkt_results
     }
     return result
  # For the first comparison (using Group 1 data):
  IFA_result = PSD_diff_all(groupA_An_1_train, groupA_An_1_test, groupA_netmats_1_train,
groupA_netmats_1_test, groupB_An_1_train, groupB_An_1_test, groupB_netmats_1_train,
groupB_netmats_1_test, metric=metric)
  # For the second comparison (using Group 2 data):
  major_result = PSD_diff_all(groupA_An_2_train, groupA_An_2_test, groupA_netmats_2_train,
groupA_netmats_2_test, groupB_An_2_train, groupB_An_2_test, groupB_netmats_2_train,
groupB_netmats_2_test, metric=metric)
  def scatter_with_lines(x1, y1, x2, y2, label1='Series 1', label2='Series 2', xlabel='X', ylabel='Y',
title='Scatter Plot with Connecting Lines'):
     Creates a scatter plot with lines connecting corresponding points from two series.
     Parameters:
     - x1, y1: The x and y values for the first series.
     - x2, y2: The x and y values for the second series.
     - label1, label2: Labels for the two series.
     - xlabel, ylabel: Labels for the x and y axes.
     - title: Title for the plot.
```

```
plt.figure(figsize=(12,6)) # Set the size of the figure
     plt.scatter(x1, y1, label=label1, color='blue')
     plt.scatter(x2, y2, label=label2, color='orange')
     # Draw lines connecting corresponding points
     for x_1, y_1, x_2, y_2 in zip(x1, y1, x2, y2):
       plt.plot([x_1, x_2], [y_1, y_2], color='gray', linestyle='--')
     plt.xlabel(xlabel)
     plt.ylabel(ylabel)
     plt.title(title)
     plt.legend()
     plt.show()
  save text results(f'Distance between Group Average Netmats formed from Subject Dual
Regression using test data and \{z\_scores\_unthresh.shape[1]\}\ IFA\ Components:
{IFA_result["psd_mean_distance"]}',os.path.join(f"fold_{fold}", "results","results.txt"))
  save_text_results(f'IFA Tangent Results: {IFA_result|"tangent_results"]}',os.path.join(f"fold_{fold}",
"results", "results.txt"))
  save_text_results(f'IFA FKT Results: {IFA_result["fkt_results"]}',os.path.join(f"fold_{fold}",
"results", "results.txt"))
  save_text_results(f'Distance between Group Average Netmats formed from Subject Dual
Regression using test data and {z_scores_unthresh_major_more.shape[1]} ICA Components:
{major_result["psd_mean_distance"]}',os.path.join(f"fold_{fold}", "results", "results.txt"))
  save text results(f'ICA Tangent Results:
{major result["tangent results"]}',os.path.join(f"fold {fold}", "results","results.txt"))
  save_text_results(f'ICA FKT Results: {major_result["fkt_results"]}',os.path.join(f"fold_{fold}",
"results", "results.txt"))
  print(IFA_result["psd_mean_distance"], major_result["psd_mean_distance"])
  test_dist.append(IFA_result["psd_mean_distance"])
  test_dist_ICA.append(major_result["psd_mean_distance"])
  logistic dims.append([tup[0] for tup in IFA result["tangent results"]])
  logistic_acc.append([tup[2] for tup in IFA_result["tangent_results"]])
  logistic_dist.append([tup[1] for tup in IFA_result["tangent_results"]])
  fkt_dims.append([tup[0]*2 for tup in IFA_result["fkt_results"]])
  fkt_acc.append([tup[2] for tup in IFA_result["fkt_results"]])
  fkt_dist.append([tup[1] for tup in IFA_result["fkt_results"]])
  logistic dims ICA.append([tup[0] for tup in major result["tangent results"]])
  logistic_acc_ICA.append([tup[2] for tup in major_result["tangent_results"]])
  logistic_dist_ICA.append([tup[1] for tup in major_result["tangent_results"]])
  fkt_dims_ICA.append([tup[0]*2 for tup in major_result["fkt_results"]])
  fkt_acc_ICA.append([tup[2] for tup in major_result["fkt_results"]])
  fkt_dist_ICA.append([tup[1] for tup in major_result["fkt_results"]])
  scatter_with_lines([tup[0] for tup in IFA_result["tangent_results"]],
             [tup[2] for tup in IFA_result["tangent_results"]],
```

```
[tup[0] for tup in major result["tangent results"]].
            [tup[2] for tup in major_result["tangent_results"]],
            label1='IFA Components', label2='ICA Components',
            xlabel='Dimension', ylabel='Accuracy',
            title='Logistic Regression Accuracy of Tangent Netmats')
  scatter with lines([tup[0] for tup in IFA result["tangent results"]],
            [tup[1] for tup in IFA_result["tangent_results"]],
            [tup[0] for tup in major result["tangent results"]],
            [tup[1] for tup in major_result["tangent_results"]],
            label1='IFA Components', label2='ICA Components',
            xlabel='Dimension', ylabel='Distance',
            title='Distance Between Group Means of Tangent Netmats')
  scatter_with_lines([tup[0]*2 for tup in IFA_result["fkt_results"]],
            [tup[2] for tup in IFA result["fkt results"]],
            [tup[0]*2 for tup in major_result["fkt_results"]],
            [tup[2] for tup in major_result["fkt_results"]],
            label1='IFA Components', label2='ICA Components',
            xlabel='Dimension (# of Filters)', ylabel='Accuracy',
            title='Logistic Regression Accuracy of LogVar FKT Features')
  scatter_with_lines([tup[0]*2 for tup in IFA_result["fkt_results"]],
            [tup[1] for tup in IFA_result["fkt_results"]],
            [tup[0]*2 for tup in major_result["fkt_results"]],
            [tup[1] for tup in major_result["fkt_results"]],
            label1='IFA Components', label2='ICA Components',
            xlabel='Dimension (# of Filters)', ylabel='Distance',
            title='Distance Between Group Means of LogVar FKT Features')
  groupA_train_1_tangent, groupB_train_1_tangent, groupA_test_1_tangent,
groupB_test_1_tangent = tangent_transform(groupA_netmats_1_train, groupA_netmats_1_test,
groupB_netmats_1_train, groupB_netmats_1_test, metric=metric)
  groupA_train_2_tangent, groupB_train_2_tangent, groupA_test_2_tangent,
groupB_test_2_tangent = tangent_transform(groupA_netmats_2_train, groupA_netmats_2_test,
groupB netmats 2 train, groupB netmats 2 test, metric=metric)
  IFA Class Result = test classifiers(groupA train 1 tangent, groupA test 1 tangent,
groupB_train_1_tangent, groupB_test_1_tangent)
  ICA_Class_Result = test_classifiers(groupA_train_2_tangent, groupA_test_2_tangent,
groupB_train_2_tangent, groupB_test_2_tangent)
  models_acc.append(IFA_Class_Result)
  models_acc_ICA.append(ICA_Class_Result)
  save_text_results(f'Tangent Space Classification Accuracies for Netmats formed from
{z_scores_unthresh.shape[1]} IFA Components: {IFA_Class_Result},os.path.join(f"fold_{fold}",
"results", "results.txt"))
  save text results(f'Tangent Space Classification Accuracies for Netmats formed from
{z_scores_unthresh_major_more.shape[1]} ICA Components:
{ICA Class Result}',os.path.join(f"fold {fold}", "results", "results.txt"))
os.mkdir(os.path.join(outputfolder,'results'))
```

```
# save array to outputfolder(os.path.join('results', 'train dist FKT.npy'), train dist)
# save array to outputfolder(os.path.join('results','test dist FKT.npy'), test dist)
# save_array_to_outputfolder(os.path.join('results','logistic_dims_FKT.npy'), logistic_dims)
# save_array_to_outputfolder(os.path.join('results','logistic_dist_FKT.npy'), logistic_dist)
# save_array_to_outputfolder(os.path.join('results','logistic_acc_FKT.npy'), logistic_acc)
# save_array_to_outputfolder(os.path.join('results','fkt_dims_FKT.npy'), fkt_dims)
# save array to outputfolder(os.path.join('results','fkt dist FKT.npy'), fkt dist)
# save_array_to_outputfolder(os.path.join('results','fkt_acc_FKT.npy'), fkt_acc)
# save array to outputfolder(os.path.join('results','models acc FKT.npy'), models acc)
save_array_to_outputfolder(os.path.join('results', 'train_dist_TSSF.npy'), train_dist)
save_array_to_outputfolder(os.path.join('results','test_dist_TSSF.npy'), test_dist)
save_array_to_outputfolder(os.path.join('results','logistic_dims_TSSF.npy'), logistic_dims)
save_array_to_outputfolder(os.path.join('results','logistic_dist_TSSF.npy'), logistic_dist)
save_array_to_outputfolder(os.path.join('results','logistic_acc_TSSF.npy'), logistic_acc)
save array to outputfolder(os.path.join('results','fkt dims TSSF.npy'), fkt dims)
save_array_to_outputfolder(os.path.join('results','fkt_dist_TSSF.npy'), fkt_dist)
save_array_to_outputfolder(os.path.join('results','fkt_acc_TSSF.npy'), fkt_acc)
save_array_to_outputfolder(os.path.join('results', 'models_acc_TSSF.npy'), models_acc)
save_array_to_outputfolder(os.path.join('results', 'train_dist_ICA.npy'), train_dist_ICA)
save array to outputfolder(os.path.join('results','test dist ICA.npy'), test dist ICA)
save_array_to_outputfolder(os.path.join('results','logistic_dims_ICA.npy'), logistic_dims_ICA)
save_array_to_outputfolder(os.path.join('results','logistic_dist_ICA.npy'), logistic_dist_ICA)
save_array_to_outputfolder(os.path.join('results','logistic_acc_ICA.npy'), logistic_acc_ICA)
save_array_to_outputfolder(os.path.join('results','fkt_dims_ICA.npy'), fkt_dims_ICA)
save_array_to_outputfolder(os.path.join('results', 'fkt_dist_ICA.npy'), fkt_dist_ICA)
save_array_to_outputfolder(os.path.join('results', 'fkt_acc_ICA.npy'), fkt_acc_ICA)
save array to outputfolder(os.path.join('results','models acc ICA.npy'), models acc ICA)
# %%
os.mkdir(os.path.join(outputfolder,'results'))
save_array_to_outputfolder(os.path.join('results','train_dist_FKT.npy'), train_dist)
save_array_to_outputfolder(os.path.join('results','test_dist_FKT.npy'), test_dist)
save array to outputfolder(os.path.join('results', 'logistic dims FKT.npy'), logistic dims)
save_array_to_outputfolder(os.path.join('results','logistic_dist_FKT.npy'), logistic_dist)
save array to outputfolder(os.path.join('results','logistic acc FKT.npy'), logistic acc)
save_array_to_outputfolder(os.path.join('results','fkt_dims_FKT.npy'), fkt_dims)
save_array_to_outputfolder(os.path.join('results','fkt_dist_FKT.npy'), fkt_dist)
save_array_to_outputfolder(os.path.join('results','fkt_acc_FKT.npy'), fkt_acc)
save_array_to_outputfolder(os.path.join('results','models_acc_FKT.npy'), models_acc)
# %%
os.mkdir(os.path.join(outputfolder,'results'))
# save_array_to_outputfolder(os.path.join('results','train_dist_FKT.npy'), train_dist)
# save_array_to_outputfolder(os.path.join('results', 'test_dist_FKT.npy'), test_dist)
# save_array_to_outputfolder(os.path.join('results','logistic_dims_FKT.npy'), logistic_dims)
# save array to outputfolder(os.path.join('results', 'logistic dist FKT.npy'), logistic dist)
# save_array_to_outputfolder(os.path.join('results','logistic_acc_FKT.npy'), logistic_acc)
# save array to outputfolder(os.path.join('results','fkt dims FKT.npy'), fkt dims)
# save_array_to_outputfolder(os.path.join('results','fkt_dist_FKT.npy'), fkt_dist)
# save_array_to_outputfolder(os.path.join('results','fkt_acc_FKT.npy'), fkt_acc)
```

```
# save_array_to_outputfolder(os.path.join('results','models_acc_FKT.npy'), models_acc)
save_array_to_outputfolder(os.path.join('results','train_dist_TSSF.npy'), train_dist)
save_array_to_outputfolder(os.path.join('results','test_dist_TSSF.npy'), test_dist)
save_array_to_outputfolder(os.path.join('results','logistic_dims_TSSF.npy'), logistic_dims)
save_array_to_outputfolder(os.path.join('results','logistic_dist_TSSF.npy'), logistic_dist)
save array to outputfolder(os.path.join('results','logistic acc TSSF.npy'), logistic acc)
save_array_to_outputfolder(os.path.join('results','fkt_dims_TSSF.npy'), fkt_dims)
save array to outputfolder(os.path.join('results','fkt dist TSSF.npy'), fkt dist)
save_array_to_outputfolder(os.path.join('results','fkt_acc_TSSF.npy'), fkt_acc)
save_array_to_outputfolder(os.path.join('results', 'models_acc_TSSF.npy'), models_acc)
save_array_to_outputfolder(os.path.join('results', 'train_dist_ICA.npy'), train_dist_ICA)
save_array_to_outputfolder(os.path.join('results','test_dist_ICA.npy'), test_dist_ICA)
save_array_to_outputfolder(os.path.join('results','logistic_dims_ICA.npy'), logistic_dims_ICA)
save array to outputfolder(os.path.join('results', 'logistic dist ICA.npy'), logistic dist ICA)
save_array_to_outputfolder(os.path.join('results', 'logistic_acc_ICA.npy'), logistic_acc_ICA)
save_array_to_outputfolder(os.path.join('results','fkt_dims_ICA.npy'), fkt_dims_ICA)
save_array_to_outputfolder(os.path.join('results','fkt_dist_ICA.npy'), fkt_dist_ICA)
save array to outputfolder(os.path.join('results', 'fkt acc ICA.npy'), fkt acc ICA)
save_array_to_outputfolder(os.path.join('results','models_acc_ICA.npy'), models_acc_ICA)
# %%
# TSSF related files
test dist TSSF =
np.load(os.path.join('Gender_TangentSVMC1_logeuclid', 'results', 'test_dist_TSSF.npy'))
logistic_dims_TSSF =
np.load(os.path.join('Gender_TangentSVMC1_logeuclid', 'results', 'logistic_dims_TSSF.npy'))
logistic dist TSSF =
np.load(os.path.join('Gender_TangentSVMC1_logeuclid', 'results', 'logistic_dist_TSSF.npy'))
logistic_acc_TSSF =
np.load(os.path.join('Gender_TangentSVMC1_logeuclid','results','logistic_acc_TSSF.npy'))
fkt dims TSSF =
np.load(os.path.join('Gender_TangentSVMC1_logeuclid','results','fkt_dims_TSSF.npy'))
fkt dist TSSF =
np.load(os.path.join('Gender_TangentSVMC1_logeuclid','results','fkt_dist_TSSF.npy'))
fkt acc TSSF =
np.load(os.path.join('Gender_TangentSVMC1_logeuclid','results','fkt_acc_TSSF.npy'))
# Loading models_acc_TSSF with allow_pickle=True
models_acc_TSSF =
np.load(os.path.join('Gender_TangentSVMC1_logeuclid','results','models_acc_TSSF.npy'),
allow_pickle=True)
# ICA related files
train dist ICA =
np.load(os.path.join('Gender_TangentSVMC1_logeuclid','results','train_dist_ICA.npy'))
test dist ICA =
np.load(os.path.join('Gender_TangentSVMC1_logeuclid','results','test_dist_ICA.npy'))
logistic dims ICA =
np.load(os.path.join('Gender_TangentSVMC1_logeuclid', 'results', 'logistic_dims_ICA.npy'))
logistic_dist_ICA =
```

```
np.load(os.path.join('Gender TangentSVMC1 logeuclid', 'results', 'logistic dist ICA.npy'))
logistic acc ICA =
np.load(os.path.join('Gender_TangentSVMC1_logeuclid','results','logistic_acc_ICA.npy'))
fkt_dims_ICA =
np.load(os.path.join('Gender TangentSVMC1 logeuclid','results','fkt dims ICA.npy'))
fkt_dist_ICA = np.load(os.path.join('Gender_TangentSVMC1_logeuclid','results','fkt_dist_ICA.npy'))
fkt acc ICA = np.load(os.path.join('Gender TangentSVMC1 logeuclid','results','fkt acc ICA.npy'))
models acc ICA =
np.load(os.path.join('Gender_TangentSVMC1_logeuclid','results','models_acc_ICA.npy'),
allow_pickle=True)
# FKT related files
train dist FKT = np.load(os.path.join('Gender FKT2 logeuclid','results','train dist FKT.npy'))
test_dist_FKT = np.load(os.path.join('Gender_FKT2_logeuclid','results','test_dist_FKT.npy'))
logistic dims FKT =
np.load(os.path.join('Gender_FKT2_logeuclid','results','logistic_dims_FKT.npy'))
logistic_dist_FKT = np.load(os.path.join('Gender_FKT2_logeuclid','results','logistic_dist_FKT.npy'))
logistic_acc_FKT = np.load(os.path.join('Gender_FKT2_logeuclid','results','logistic_acc_FKT.npy'))
fkt dims FKT = np.load(os.path.join('Gender FKT2 logeuclid', 'results', 'fkt dims FKT.npy'))
fkt_dist_FKT = np.load(os.path.join('Gender_FKT2_logeuclid','results','fkt_dist_FKT.npy'))
fkt acc FKT = np.load(os.path.join('Gender FKT2 logeuclid', 'results', 'fkt acc FKT.npy'))
models_acc_FKT = np.load(os.path.join('Gender_FKT2_logeuclid','results','models_acc_FKT.npy'),
allow_pickle=True)
# %%
# Function to compute mean and std across folds
def compute mean std across folds(data):
  n_{folds} = data.shape[0]
                                      # Number of folds (axis 0 gives the number of rows/folds)
  mean_across_folds = np.mean(data, axis=0) # Mean across folds (axis 0)
  std across folds = np.std(data, axis=0) # Std deviation across folds (axis 0)
  sem_across_folds = std_across_folds / np.sqrt(n_folds) # Compute SEM
  print(n folds)
  return mean_across_folds, sem_across_folds
# Modified function to plot means and std dev for a measure across TSSF, ICA, and FKT
def plot_comparison(dims, mean_TSSF, std_TSSF, mean_ICA, std_ICA, mean_FKT, std_FKT,
measure_name, ax):
  # Use seaborn color palette
  palette = sns.color_palette("Set1", 3)
  # Map dims to positions on x-axis
  spacing_factor = 2 # Adjust this factor to control spacing between bins
  x_positions = np.arange(len(dims)) * spacing_factor
  # Width for offsets (should be less than half of spacing factor)
  width = spacing_factor / 5 # Adjust as needed
  # Offsets for each method
  offsets = [-width, 0, width]
```

```
# Adjust x-values for each method
  x_TSSF = x_positions + offsets[0]
  x_ICA = x_positions + offsets[1]
  x_FKT = x_positions + offsets[2]
  # Plotting TSSF, ICA, and FKT with adjusted x-values
  ax.errorbar(x_TSSF, mean_TSSF, yerr=std_TSSF, fmt='o', label='TSSF IFA', color=palette[0],
capsize=0)
  ax.errorbar(x_FKT, mean_FKT, yerr=std_FKT, fmt='o', label='FKT IFA', color=palette[2],
capsize=0)
  ax.errorbar(x_ICA, mean_ICA, yerr=std_ICA, fmt='o', label='ICA', color=palette[1], capsize=0)
  # Set x-ticks to x positions without offsets, labels to dims
  ax.set xticks(x positions)
  ax.set xticklabels(dims)
  # Formatting plot
  ax.set_xlabel('Dimensions')
  ax.set ylabel(f'{measure name} (Mean ± SEM)')
  ax.set_title(f'Comparison of {measure_name} Across Methods')
  ax.legend()
  ax.grid(True)
# Function to plot measures for a single fold without binning the x-axis
def plot_single_fold(dims, measure_TSSF, measure_ICA, measure_FKT, measure_name,
fold_index, ax):
  # Use seaborn color palette
  palette = sns.color_palette("Set1", 3)
  # Plotting TSSF, ICA, and FKT
  ax.plot(dims, measure_TSSF, 'o', label='TSSF IFA', color=palette[0])
  ax.plot(dims, measure FKT, 'o', label='FKT IFA', color=palette[2])
  ax.plot(dims, measure_ICA, 'o', label='ICA', color=palette[1])
  # Formatting plot
  ax.set_xlabel('Dimensions')
  ax.set ylabel(f'{measure name}')
  ax.set_title(f'{measure_name} (Fold {fold_index})')
  ax.legend()
  ax.grid(True)
# Function to plot both averaged data and specific fold data in subplots
def plot_measure_with_fold_and_average(dims, mean_TSSF, std_TSSF, mean_ICA, std_ICA,
mean FKT, std FKT,
                        fold_TSSF, fold_ICA, fold_FKT, measure_name, fold_index):
  fig, axs = plt.subplots(1, 2, figsize=(18, 6))
  # Left subplot: Averaged data with categorical bins
  plot_comparison(dims, mean_TSSF, std_TSSF, mean_ICA, std_ICA, mean_FKT, std_FKT,
measure name, axs[0])
  # Right subplot: Specific fold data without adjusted x-positions
```

```
plot_single_fold(dims, fold_TSSF, fold_ICA, fold_FKT, measure_name, fold_index, axs[1])
  plt.tight_layout()
  plt.show()
# Loading the data for each method (TSSF, ICA, and FKT)
# Replace the paths with your actual data paths
#TSSF related files
# Loading the data for each method (TSSF, ICA, and FKT)
#TSSF related files
logistic_acc_TSSF =
np.load(os.path.join('PicVocab_AgeAdj_PMAT24_A_CR_TangentSVM_3_logeuclid','results','logistic
_acc_TSSF.npy'))
logistic dist TSSF =
np.load(os.path.join('PicVocab_AgeAdj_PMAT24_A_CR_TangentSVM_3_logeuclid','results','logistic
dist TSSF.npy'))
fkt_acc_TSSF =
np.load(os.path.join('PicVocab_AgeAdj_PMAT24_A_CR_TangentSVM_3_logeuclid', 'results', 'fkt_acc
_TSSF.npy'))
fkt dist TSSF =
np.load(os.path.join('PicVocab_AgeAdj_PMAT24_A_CR_TangentSVM_3_logeuclid', 'results', 'fkt_dist
_TSSF.npy'))
logistic_dims_TSSF =
np.load(os.path.join('PicVocab_AgeAdj_PMAT24_A_CR_TangentSVM_3_logeuclid', 'results', 'logistic
_dims_TSSF.npy'))
# ICA related files
logistic acc ICA =
np.load(os.path.join('PicVocab_AgeAdj_PMAT24_A_CR_TangentSVM_2_logeuclid', 'results', 'logistic
_acc_TSSF.npy'))
logistic_dist_ICA =
np.load(os.path.join('PicVocab_AgeAdj_PMAT24_A_CR_TangentSVM_2_logeuclid', 'results', 'logistic
_dist_TSSF.npy'))
fkt acc ICA =
np.load(os.path.join('PicVocab_AgeAdj_PMAT24_A_CR_TangentSVM_2_logeuclid','results','fkt_acc
TSSF.npy'))
fkt_dist_ICA =
np.load(os.path.join('PicVocab_AgeAdj_PMAT24_A_CR_TangentSVM_2_logeuclid', 'results', 'fkt_dist
_TSSF.npy'))
logistic dims ICA =
np.load(os.path.join('PicVocab AgeAdj PMAT24 A CR TangentSVM 2 logeuclid', 'results', 'logistic
_dims_TSSF.npy'))
# FKT related files
logistic_acc_FKT =
np.load(os.path.join('PicVocab_AgeAdj_PMAT24_A_CR_TangentSVM_logeuclid', 'results', 'logistic_a
cc_TSSF.npy'))
logistic dist FKT =
np.load(os.path.join('PicVocab_AgeAdj_PMAT24_A_CR_TangentSVM_logeuclid', 'results', 'logistic_d
ist_TSSF.npy'))
```

```
fkt acc FKT =
np.load(os.path.join('PicVocab_AgeAdj_PMAT24_A_CR_TangentSVM_logeuclid', 'results', 'fkt_acc_
TSSF.npy'))
fkt_dist_FKT =
np.load(os.path.join('PicVocab_AgeAdj_PMAT24_A_CR_TangentSVM_logeuclid', 'results', 'fkt_dist_
TSSF.npy'))
fkt dims FKT =
np.load(os.path.join('PicVocab AgeAdj PMAT24 A CR TangentSVM logeuclid', 'results', 'fkt dims
TSSF.npy'))
# Since logistic_dims and fkt_dims are the same across all methods, we'll just use one of them
dims_logistic = logistic_dims_TSSF[0]
dims_fkt = fkt_dims_FKT[0]
# Compute means and std for all metrics
# Logistic accuracy
mean_logistic_acc_TSSF, std_logistic_acc_TSSF =
compute_mean_std_across_folds(logistic_acc_TSSF)
mean_logistic_acc_ICA, std_logistic_acc_ICA =
compute_mean_std_across_folds(logistic_acc_ICA)
mean logistic acc FKT, std logistic acc FKT =
compute_mean_std_across_folds(logistic_acc_FKT)
# Logistic distance
mean_logistic_dist_TSSF, std_logistic_dist_TSSF =
compute_mean_std_across_folds(logistic_dist_TSSF)
mean_logistic_dist_ICA, std_logistic_dist_ICA =
compute mean std across folds(logistic dist ICA)
mean_logistic_dist_FKT, std_logistic_dist_FKT =
compute_mean_std_across_folds(logistic_dist_FKT)
# FKT accuracy
mean_fkt_acc_TSSF, std_fkt_acc_TSSF = compute_mean_std_across_folds(fkt_acc_TSSF)
mean fkt acc ICA, std fkt acc ICA = compute mean std across folds(fkt acc ICA)
mean_fkt_acc_FKT, std_fkt_acc_FKT = compute_mean_std_across_folds(fkt_acc_FKT)
# FKT distance
mean_fkt_dist_TSSF, std_fkt_dist_TSSF = compute_mean_std_across_folds(fkt_dist_TSSF)
mean_fkt_dist_ICA, std_fkt_dist_ICA = compute_mean_std_across_folds(fkt_dist_ICA)
mean_fkt_dist_FKT, std_fkt_dist_FKT = compute_mean_std_across_folds(fkt_dist_FKT)
# Select the fold you want to plot (e.g., fold 0)
fold index = 0 # Change this index to select a different fold
# Extract data for the selected fold
# Logistic accuracy
logistic_acc_TSSF_fold = logistic_acc_TSSF[fold_index, :]
logistic_acc_ICA_fold = logistic_acc_ICA[fold_index, :]
logistic acc FKT fold = logistic acc FKT[fold index, :]
# Logistic distance
```

```
logistic_dist_TSSF_fold = logistic_dist_TSSF[fold_index, :]
logistic_dist_ICA_fold = logistic_dist_ICA[fold_index, :]
logistic_dist_FKT_fold = logistic_dist_FKT[fold_index, :]
## FKT accuracy
# fkt_acc_TSSF_fold = fkt_acc_TSSF[fold_index, :]
# fkt acc ICA fold = fkt acc ICA[fold index, :]
# fkt_acc_FKT_fold = fkt_acc_FKT[fold_index, :]
## FKT distance
# fkt_dist_TSSF_fold = fkt_dist_TSSF[fold_index, :]
# fkt_dist_ICA_fold = fkt_dist_ICA[fold_index, :]
# fkt_dist_FKT_fold = fkt_dist_FKT[fold_index, :]
# Plotting comparisons for each measure
# Logistic accuracy comparison
plot_measure_with_fold_and_average(dims_logistic,
                     mean_logistic_acc_TSSF, std_logistic_acc_TSSF,
                     mean_logistic_acc_ICA, std_logistic_acc_ICA,
                     mean logistic acc FKT, std logistic acc FKT,
                     logistic_acc_TSSF_fold, logistic_acc_ICA_fold, logistic_acc_FKT_fold,
                     'Logistic Regression TS AIRM Accuracy', fold index)
# Logistic distance comparison
plot_measure_with_fold_and_average(dims_logistic,
                     mean_logistic_dist_TSSF, std_logistic_dist_TSSF,
                     mean_logistic_dist_ICA, std_logistic_dist_ICA,
                     mean_logistic_dist_FKT, std_logistic_dist_FKT,
                     logistic dist TSSF fold, logistic dist ICA fold, logistic dist FKT fold,
                     'Logistic Regression TS_AIRM Distance', fold_index)
## FKT accuracy comparison
# plot_measure_with_fold_and_average(dims_fkt,
                       mean_fkt_acc_TSSF, std_fkt_acc_TSSF,
#
                       mean fkt acc ICA, std fkt acc ICA,
#
                       mean_fkt_acc_FKT, std_fkt_acc_FKT,
#
                       fkt acc TSSF fold, fkt acc ICA fold, fkt acc FKT fold,
#
                       'TSSF_Var_1_Step Accuracy', fold_index)
## FKT distance comparison
# plot_measure_with_fold_and_average(dims_fkt,
                       mean_fkt_dist_TSSF, std_fkt_dist_TSSF,
#
                       mean_fkt_dist_ICA, std_fkt_dist_ICA,
#
                       mean fkt dist FKT, std fkt dist FKT,
#
                       fkt_dist_TSSF_fold, fkt_dist_ICA_fold, fkt_dist_FKT_fold,
#
                       'TSSF_Var_1_Step Distance', fold_index)
# %%
# Function to compute mean and std across folds for model accuracies
def compute mean std model acc(models acc list):
  models_acc_list: List of dictionaries, one for each fold.
```

```
Returns:
     models: List of model names.
     mean acc: List of mean accuracies for each model across folds.
     std_acc: List of std deviations for each model across folds.
  # Get all model names from the first fold
  models = list(models acc list[0].keys())
  # Initialize dictionaries to hold accuracies
  acc_dict = {model: [] for model in models}
  # Collect accuracies from each fold
  for fold_data in models_acc_list:
     for model in models:
       acc = fold_data[model]['accuracy']
       acc dict[model].append(acc)
  # Compute mean and std for each model
  mean_acc = []
  sem acc = []
  n_folds = models_acc_list.shape[0]
                                                 # Number of folds (axis 0 gives the number of
rows/folds)
  print(n_folds)
  for model in models:
     accs = acc_dict[model]
     mean_acc.append(np.mean(accs))
     sem_acc.append(np.std(accs)/np.sqrt(n_folds))
  return models, mean_acc, sem_acc
# Modified function to plot means and std dev for model accuracies across TSSF, ICA, and FKT
def plot_model_comparison(models, mean_TSSF, std_TSSF, mean_ICA, std_ICA, mean_FKT,
std FKT, measure name, ax):
  # Use seaborn color palette
  palette = sns.color_palette("Set1", 3)
  # Increase the spacing between bins
  spacing_factor = 10 # Increased spacing factor from 2 to 3
  x_positions = np.arange(len(models)) * spacing_factor
  # Width for offsets (should be less than half of spacing_factor)
  width = spacing_factor / 5
  # Offsets for each method
  offsets = [-width, 0, width]
  # Adjust x-values for each method
  x_TSSF = x_positions + offsets[0]
  x_ICA = x_positions + offsets[1]
  x_FKT = x_positions + offsets[2]
  # Plotting TSSF, ICA, and FKT with adjusted x-values
```

```
ax.errorbar(x_TSSF, mean_TSSF, yerr=std_TSSF, fmt='o', label='TSSF IFA 3', color=palette[0],
capsize=0)
  ax.errorbar(x_ICA, mean_ICA, yerr=std_ICA, fmt='o', label='TSSF IFA 2', color=palette[1],
capsize=0)
  ax.errorbar(x_FKT, mean_FKT, yerr=std_FKT, fmt='o', label='TSSF IFA 1', color=palette[2],
capsize=0)
  # Set x-ticks to x_positions without offsets, labels to models
  ax.set xticks(x positions)
  ax.set_xticklabels(models, rotation=45, ha='right')
  # Formatting plot
  ax.set_xlabel('Models')
  ax.set_ylabel(f'{measure_name} (Mean ± SEM)')
  ax.set title(f'Comparison of {measure name} Across Methods')
  ax.legend()
  ax.grid(True)
# Modified function to plot measures for a single fold with categorical bins
def plot_single_fold_models(models, acc_TSSF, acc_ICA, acc_FKT, measure_name, fold_index,
ax):
  # Use seaborn color palette
  palette = sns.color_palette("Set1", 3)
  # Increase the spacing between bins (same as in the averaged plot)
  spacing_factor = 10 # Increased spacing factor from 2 to 3
  x_positions = np.arange(len(models)) * spacing_factor
  # Width for offsets (should be less than half of spacing factor)
  width = spacing factor / 10
  # Offsets for each method
  offsets = [-width, 0, width]
  # Adjust x-values for each method
  x_TSSF = x_positions + offsets[0]
  x ICA = x positions + offsets[1]
  x_FKT = x_positions + offsets[2]
  # Plotting TSSF, ICA, and FKT with adjusted x-values
  # Changed markers to squares ('s')
  ax.plot(x_TSSF, acc_TSSF, 's', label='TSSF IFA 3', color=palette[0])
  ax.plot(x_ICA, acc_ICA, 's', label='TSSF IFA 2', color=palette[1])
  ax.plot(x_FKT, acc_FKT, 's', label='TSSF IFA 1', color=palette[2])
  # Set x-ticks to x positions without offsets, labels to models
  ax.set_xticks(x_positions)
  ax.set_xticklabels(models, rotation=45, ha='right')
  # Formatting plot
  ax.set_xlabel('Models')
  ax.set_ylabel(f'{measure_name}')
```

```
ax.set_title(f'{measure_name} (Fold {fold_index})')
  ax.legend()
  ax.grid(True)
# Function to plot both averaged data and specific fold data in subplots
def plot_model_measure_with_fold_and_average(models, mean_TSSF, std_TSSF, mean_ICA,
std ICA, mean FKT, std FKT,
                          fold_TSSF, fold_ICA, fold_FKT, measure_name, fold_index):
  fig, axs = plt.subplots(1, 1, figsize=(20, 6))
  # Left subplot: Averaged data with categorical bins
  plot_model_comparison(models, mean_TSSF, std_TSSF, mean_ICA, std_ICA, mean_FKT,
std_FKT, measure_name, axs)
  # Right subplot: Specific fold data with categorical bins and square markers
  # plot single fold models(models, fold TSSF, fold ICA, fold FKT, measure name, fold index,
axs[1])
  plt.tight_layout()
  plt.show()
# Loading the data for each method (TSSF, ICA, and FKT)
models acc TSSF 3 =
np.load(os.path.join('PicVocab_AgeAdj_PMAT24_A_CR_TangentSVM_3_logeuclid', 'results',
'models_acc_TSSF.npy'), allow_pickle=True)
models_acc_TSSF_2 =
np.load(os.path.join('PicVocab_AgeAdj_PMAT24_A_CR_TangentSVM_2_logeuclid', 'results',
'models_acc_TSSF.npy'), allow_pickle=True)
models acc TSSF 1 =
np.load(os.path.join('PicVocab_AgeAdj_PMAT24_A_CR_TangentSVM_logeuclid', 'results',
'models_acc_TSSF.npy'), allow_pickle=True)
# Since models are the same across methods, we'll extract them from one
models = list(models_acc_TSSF_1[0].keys())
print(models)
# Compute means and std for model accuracies
models_TSSF_3, mean_acc_TSSF_3, std_acc_TSSF_3 =
compute_mean_std_model_acc(models_acc_TSSF_3)
models_TSSF_2, mean_acc_TSSF_2, std_acc_TSSF_2 =
compute_mean_std_model_acc(models_acc_TSSF_2)
#FKT
models_TSSF_1, mean_acc_TSSF_1, std_acc_TSSF_1 =
compute mean std model acc(models acc TSSF 1)
# Ensure the models are in the same order
assert models_TSSF_1 == models_TSSF_2 == models_TSSF_3 == models, "Models are not the
same across methods"
# Select the fold you want to plot (e.g., fold 0)
```

```
fold_index = 0 # Change this index to select a different fold
## Extract data for the selected fold
##TSSF
# fold_acc_TSSF_3 = [models_TSSF_3[fold_index][model]['accuracy'] for model in models]
# # ICA
# fold_acc_TSSF_2 = [models_TSSF_2[fold_index][model]['accuracy'] for model in models]
# # FKT
# fold_acc_TSSF_1 = [models_TSSF_1[fold_index][model]['accuracy'] for model in models]
# Plotting comparisons for model accuracies
plot_model_measure_with_fold_and_average(models,
                         mean_acc_TSSF_3, std_acc_TSSF_3,
                         mean_acc_TSSF_2, std_acc_TSSF_2,
                         mean_acc_TSSF_1, std_acc_TSSF_1,
                         None. None. None.
                         'Model Accuracy', fold_index)
# %%
# Function to compute mean and std across folds for model accuracies
def compute mean std model acc(models acc list):
  models_acc_list: List of dictionaries, one for each fold.
  Returns:
    models: List of model names.
    mean_acc: List of mean accuracies for each model across folds.
    std_acc: List of std deviations for each model across folds.
  # Get all model names from the first fold
  models = list(models_acc_list[0].keys())
  # Initialize dictionaries to hold accuracies
  acc_dict = {model: [] for model in models}
  # Collect accuracies from each fold
  for fold data in models acc list:
    for model in models:
       acc = fold_data[model]['accuracy']
       acc_dict[model].append(acc)
  # Compute mean and std for each model
  mean_acc = []
  sem_acc = []
  n_folds = models_acc_list.shape[0]
                                                # Number of folds (axis 0 gives the number of
rows/folds)
  print(n_folds)
  for model in models:
    accs = acc_dict[model]
    mean acc.append(np.mean(accs))
```

sem\_acc.append(np.std(accs)/np.sqrt(n\_folds))

```
return models, mean_acc, sem_acc
# Modified function to plot means and std dev for model accuracies across TSSF, ICA, and FKT
def plot_model_comparison(models, mean_TSSF, std_TSSF, mean_ICA, std_ICA, mean_FKT,
std FKT, measure name, ax):
  # Use seaborn color palette
  palette = sns.color_palette("Set1", 3)
  # Increase the spacing between bins
  spacing_factor = 10 # Increased spacing factor from 2 to 3
  x_positions = np.arange(len(models)) * spacing_factor
  # Width for offsets (should be less than half of spacing_factor)
  width = spacing factor / 5
  # Offsets for each method
  offsets = [-width, 0, width]
  # Adjust x-values for each method
  x TSSF = x positions + offsets[0]
  x_ICA = x_positions + offsets[1]
  x FKT = x positions + offsets[2]
  # Plotting TSSF, ICA, and FKT with adjusted x-values
  ax.errorbar(x_TSSF, mean_TSSF, yerr=std_TSSF, fmt='o', label='TSSF IFA', color=palette[0],
capsize=0)
  ax.errorbar(x_ICA, mean_ICA, yerr=std_ICA, fmt='o', label='ICA', color=palette[1], capsize=0)
  ax.errorbar(x_FKT, mean_FKT, yerr=std_FKT, fmt='o', label='FKT IFA', color=palette[2],
capsize=0)
  # Set x-ticks to x_positions without offsets, labels to models
  ax.set_xticks(x_positions)
  ax.set_xticklabels(models, rotation=45, ha='right')
  # Formatting plot
  ax.set xlabel('Models')
  ax.set ylabel(f'{measure name} (Mean ± SEM)')
  ax.set_title(f'Comparison of {measure_name} Across Methods')
  ax.legend()
  ax.grid(True)
# Modified function to plot measures for a single fold with categorical bins
def plot_single_fold_models(models, acc_TSSF, acc_ICA, acc_FKT, measure_name, fold_index,
ax):
  # Use seaborn color palette
  palette = sns.color_palette("Set1", 3)
  # Increase the spacing between bins (same as in the averaged plot)
  spacing_factor = 10 # Increased spacing factor from 2 to 3
```

x positions = np.arange(len(models)) \* spacing factor

# Width for offsets (should be less than half of spacing\_factor)

```
width = spacing_factor / 10
  # Offsets for each method
  offsets = [-width, 0, width]
  # Adjust x-values for each method
  x TSSF = x positions + offsets[0]
  x_ICA = x_positions + offsets[1]
  x FKT = x positions + offsets[2]
  # Plotting TSSF, ICA, and FKT with adjusted x-values
  # Changed markers to squares ('s')
  ax.plot(x_TSSF, acc_TSSF, 's', label='TSSF IFA', color=palette[0])
  ax.plot(x_ICA, acc_ICA, 's', label='ICA', color=palette[1])
  ax.plot(x_FKT, acc_FKT, 's', label='FKT IFA', color=palette[2])
  # Set x-ticks to x_positions without offsets, labels to models
  ax.set_xticks(x_positions)
  ax.set_xticklabels(models, rotation=45, ha='right')
  # Formatting plot
  ax.set xlabel('Models')
  ax.set ylabel(f'{measure name}')
  ax.set_title(f'{measure_name} (Fold {fold_index})')
  ax.legend()
  ax.grid(True)
# Function to plot both averaged data and specific fold data in subplots
def plot model measure with fold and average(models, mean TSSF, std TSSF, mean ICA,
std_ICA, mean_FKT, std_FKT,
                           fold_TSSF, fold_ICA, fold_FKT, measure_name, fold_index):
  fig, axs = plt.subplots(1, 2, figsize=(20, 6))
  # Left subplot: Averaged data with categorical bins
  plot model comparison(models, mean TSSF, std TSSF, mean ICA, std ICA, mean FKT,
std FKT, measure name, axs[0])
  # Right subplot: Specific fold data with categorical bins and square markers
  plot_single_fold_models(models, fold_TSSF, fold_ICA, fold_FKT, measure_name, fold_index,
axs[1])
  plt.tight_layout()
  plt.show()
# Loading the data for each method (TSSF, ICA, and FKT)
models acc TSSF =
np.load(os.path.join('PicVocab_AgeAdj_PMAT24_A_CR_TangentSVM_3_logeuclid', 'results',
'models acc TSSF.npy'), allow pickle=True)
models_acc_FKT =
np.load(os.path.join('PicVocab AgeAdj PMAT24 A CR TangentSVM 2 logeuclid', 'results',
'models_acc_TSSF.npy'), allow_pickle=True)
models_acc_ICA =
```

```
np.load(os.path.join('PicVocab_AgeAdj_PMAT24_A_CR_TangentSVM_logeuclid', 'results',
'models_acc_TSSF.npy'), allow_pickle=True)
# Since models are the same across methods, we'll extract them from one
models = list(models_acc_TSSF[0].keys())
# Compute means and std for model accuracies
#TSSF
models TSSF, mean acc TSSF, std acc TSSF =
compute_mean_std_model_acc(models_acc_TSSF)
# ICA
models_ICA, mean_acc_ICA, std_acc_ICA = compute_mean_std_model_acc(models_acc_ICA)
# FKT
models FKT, mean acc FKT, std acc FKT = compute mean std model acc(models acc FKT)
# Ensure the models are in the same order
assert models_TSSF == models_ICA == models_FKT == models, "Models are not the same across
methods"
# Select the fold you want to plot (e.g., fold 0)
fold_index = 0 # Change this index to select a different fold
# Extract data for the selected fold
#TSSF
fold_acc_TSSF = [models_acc_TSSF[fold_index][model]['accuracy'] for model in models]
# ICA
fold_acc_ICA = [models_acc_ICA[fold_index][model]['accuracy'] for model in models]
#FKT
fold acc FKT = [models acc FKT[fold index][model]['accuracy'] for model in models]
# Plotting comparisons for model accuracies
plot_model_measure_with_fold_and_average(models,
                        mean acc TSSF, std acc TSSF,
                        mean_acc_ICA, std_acc_ICA,
                        mean acc FKT, std acc FKT,
                        fold_acc_TSSF, fold_acc_ICA, fold_acc_FKT,
                        'Model Accuracy', fold index)
# %% [markdown]
#### Z Scores on Dual Regressed
fkt_riem_eigs_netmat, filters_netmatIFA, _, _ = tangent_classifier(groupA_netmats_1_train,
groupB_netmats_1_train, TSVM=True, TLDA=False, tangent_calc=True,
metric=metric, visualize=True, n=0)
# fkt_riem_eigs_netmat, filters_netmat, _, _ = tangent_classifier(groupA_netmats_1_test,
groupB netmats 1 test, TSVM=True, TLDA=False, tangent calc=True.
metric=metric, visualize=True, n=0)
fkt riem eigs netmat, filters netmatICA, , = tangent classifier(groupA netmats 2 train,
groupB_netmats_2_train, TSVM=True, TLDA=False, tangent_calc=True,
metric=metric, visualize=True, n=0)
```

```
# fkt_riem_eigs_netmat, filters_netmat, _, _ = tangent_classifier(groupA_netmats_2_test,
groupB netmats 2 test, TSVM=True, TLDA=False, tangent calc=True,
metric=metric, visualize=True, n=0)
# %%
print(((groupA spatial maps 1 test.transpose((0,2,1))@filters netmatIFA[:,
[0,1,-2,-1]).transpose((0,1,2))).shape)
# %%
print(filters_netmatIFA[:,[0,1,-2,-1]].shape)
np.sum(groupA_spatial_maps_1_train,axis=1).shape
transformedA1 = ((groupA_spatial_maps_1_train.transpose((0,2,1))@filters_netmatIFA[:,
[0,1,-2,-1])
transformedB1 = ((groupB_spatial_maps_1_train.transpose((0,2,1))@filters_netmatlFA[:,
[0,1,-2,-1]
transformedA2 = ((groupA spatial maps 2 train.transpose((0,2,1))@filters netmatICA[:,
[0,1,-2,-1]]))
transformedB2 = ((groupB_spatial_maps_2_train.transpose((0,2,1))@filters_netmatICA[:,
[0,1,-2,-1])
# %%
from scipy.stats import ttest_ind
for i in range(0,transformedA1.shape[2]):
  t_stat, p_val = ttest_ind(transformedA1[:,:,i], transformedB1[:,:,i], axis=0, equal_var=False,
nan_policy='raise',permutations=None, random_state=None, alternative='two-sided')
  reject, _, _, _ = multipletests(p_val, alpha=0.05,method='fdr_bh')
  _, p_val2 = ttest_ind(transformedA2[:,:,i], transformedB2[:,:,i], axis=0, equal_var=False,
nan_policy='raise',permutations=None, random_state=None, alternative='two-sided')
  reject2, _, _, _ = multipletests(p_val2, alpha=0.05,method='fdr_bh')
  print(np.sum(reject) - np.sum(reject2))
# %%
for i in range(0,groupA_spatial_maps_1_test.shape[1]):
  t_stat, p_val = ttest_ind(groupA_spatial_maps_1_train[:,i,:], groupB_spatial_maps_1_train[:,i,:],
axis=0, equal_var=True, nan_policy='raise',permutations=None, random_state=None,
alternative='two-sided')
  reject, _, _, _ = multipletests(p_val, alpha=0.05,method='fdr_bh')
  _, p_val2 = ttest_ind(groupA_spatial_maps_2_train[:,i,:], groupB_spatial_maps_2_train[:,i,:],
axis=0, equal_var=True, nan_policy='raise',permutations=None, random_state=None,
alternative='two-sided')
  reject2, _, _, _ = multipletests(p_val2, alpha=0.05,method='fdr_bh')
  print(np.sum(reject) - np.sum(reject2))
```

```
# %%
import numpy as np
from scipy.stats import ttest_ind
from statsmodels.stats.multitest import multipletests
from scipy.ndimage import label
# Step 1: Calculate t-statistics and uncorrected p-values
t_stat, p_val = ttest_ind(
  np.sum(groupA spatial maps 2 train, axis=1),
  np.sum(groupB_spatial_maps_2_train, axis=1),
  axis=0.
  equal_var=True,
  nan_policy='raise'
)
# Step 2: Perform FDR correction on p-values
reject, p_values_corrected, _, _ = multipletests(p_val, alpha=0.05, method='fdr_bh')
print(f"Number of significant voxels (FDR corrected): {np.sum(reject)}")
# Step 3: Threshold based on voxel-wise uncorrected p-values for initial cluster formation
voxel_threshold = 0.01 # Set an uncorrected threshold for defining clusters
significant voxels = p val < voxel threshold
# Step 4: Identify clusters of contiguous significant voxels
labeled_array, num_clusters = label(significant_voxels)
# Step 5: Calculate cluster sizes
cluster_sizes = np.array([(labeled_array == i).sum() for i in range(1, num_clusters + 1)])
# Step 6: Apply a cluster size threshold (based on empirical threshold or permutation testing)
# Here, we choose an arbitrary cluster size threshold (e.g., 20)
# In practice, you should determine this threshold using permutation testing
cluster_size_threshold = 9 # Example threshold for demonstration
significant_clusters = [i for i, size in enumerate(cluster_sizes, 1) if size >= cluster_size_threshold]
# Step 7: Create a final thresholded map of significant clusters
final significant map = np.isin(labeled array, significant clusters)
# Output results
print(f"Number of clusters found: {num_clusters}")
print(f"Number of significant clusters (size > {cluster_size_threshold}): {len(significant_clusters)}")
#`final_significant_map` is a binary map of significant clusters
# %%
print(final_significant_map.shape)
# %%
plotting.view surf(hcp.mesh.inflated, hcp.cortex data(transformedA1[0,:,-2]),
threshold=np.percentile(np.abs(transformedA1[0,:,-2]),90), bg_map=hcp.mesh.sulc)
```

```
# %%
plotting.view_surf(hcp.mesh.inflated, hcp.cortex_data(transformedB1[0,:,0]),
threshold=np.percentile(np.abs(transformedB1[0,:,0]),90), bg_map=hcp.mesh.sulc)
# %%
plotting.view_surf(hcp.mesh.inflated, hcp.cortex_data(z_scores_thresh[:,0]), threshold=0,
bg map=hcp.mesh.sulc)
# %%
plotting.view_surf(hcp.mesh.inflated, hcp.cortex_data(groupA_spatial_maps_2_test[0,0,:]),
threshold=np.percentile(np.abs(groupA_spatial_maps_1_test[0,0,:]),95), bg_map=hcp.mesh.sulc)
# %%
plotting.view_surf(hcp.mesh.inflated, hcp.cortex_data(groupB_spatial_maps_2_test[0,0,:]),
threshold=np.percentile(np.abs(groupB_spatial_maps_1_test[0,0,:]),95), bg_map=hcp.mesh.sulc)
# %%
# plotting.view_surf(hcp.mesh.inflated, hcp.cortex_data(groupA_spatial_maps_1_train[10,20,:]),
threshold=np.percentile(np.abs(groupA_spatial_maps_1_train[10,20,:]), 95),
bg_map=hcp.mesh.sulc)
# %%
# plotting.view_surf(hcp.mesh.inflated, hcp.cortex_data(groupA_spatial_maps_1_train[20,20,:]),
threshold=np.percentile(np.abs(groupA_spatial_maps_1_train[20,20,:]), 95),
bg_map=hcp.mesh.sulc)
# %%
# plotting.view_surf(hcp.mesh.inflated, hcp.cortex_data(groupB_spatial_maps_1_train[11,20,:]),
threshold=np.percentile(np.abs(groupB_spatial_maps_1_train[11,20,:]), 95),
bg_map=hcp.mesh.sulc)
# %%
# plotting.view_surf(hcp.mesh.inflated, hcp.cortex_data(groupB_spatial_maps_1_train[1,20,:]),
threshold=np.percentile(np.abs(groupB_spatial_maps_1_train[1,20,:]), 95),
bg map=hcp.mesh.sulc)
# %%
# plotting.view_surf(hcp.mesh.inflated, hcp.cortex_data(z_scores_unthresh[:,20]),
threshold=np.percentile(np.abs(z_scores_unthresh[:,20]), 95), bg_map=hcp.mesh.sulc)
```