Judul Artikel

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- Pengantar -

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

1 Judul 1

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1.1 Judul 1 1

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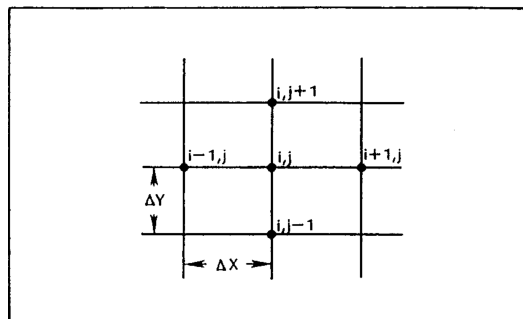

Figure 1: Titik grid formulasi numerik [1]

1.1.1 Judul 111

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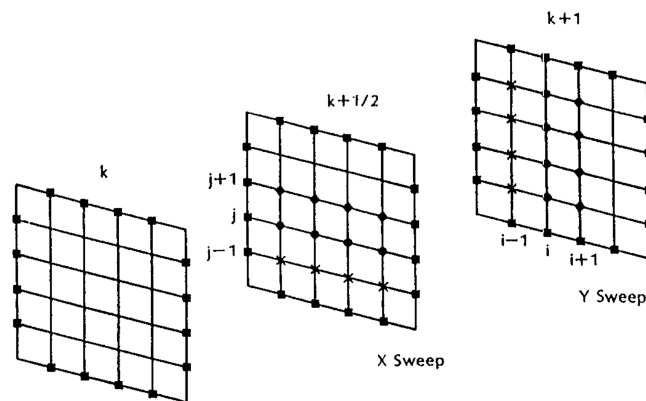

Figure 2: Proses langkah pengerjaan metode ADI [2]

1.2 Judul 1 2

Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetuer.

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Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

1.2.1 Judul 1 2 1

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2 Kesimpulan

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3 Tugas Mandiri

1. Tentukan luas daerah apabila daerah tersebut dibatasi oleh fungsi $f(x) = -(x - 0.5)^2 + 2$, $g(x) = \dfrac{2}{3}x$, serta garis $x = 0$ dan $x = 1$. Kemudian gambarkan grafik dari daerah tersebut.

2. Hitunglah nilai integral berikut

$$\int_0^1 -2x^{4/3} + 2x^{1/3}\,dx = \left[-\frac{6}{7}x^{\frac{7}{3}} + \frac{3}{2}x^{\frac{4}{3}} \right]_0^1$$

3. Hitunglah nilai integral berikut

$$\int_{-2}^{-1} (2x + 2)(x^2 + 2x)\,dx = \int_{x=-2}^{x=-1} (x^2 + 2x)\,d(x^2 + 2x)$$

4 Daftar Pustaka

[1] [5] halaman 153.

[2] [5] halaman 166.

[3] K. Fahim, "Konstruksi transformasi wavelet menggunakan aljabar max plus," Master's thesis, Institut Teknologi Sepuluh Nopember, 2014.

[4] D. d. Goswami, "A discrete wavelet transform based cryptographic algorithm," *International Journal of Computer Science and Network Security*, vol. 11, 2011.

[5] K. A. Hoffmann and S. T. Chiang, *Computational Fluid Duynamics For Engineers Volume I*. New York, NY, USA: *Engineering Education System*, 1995.

[6] S. Keshav, "How to read a paper," *SIGCOMM Comput. Commun. Rev.*, vol. 37, pp. 83–84, July 2007.

[7] *Aljabar Min Max Plus Dan Terapannya*. Institut Teknologi Sepuluh Nopember, 2015.

Lampiran

Persamaan Laplace 2D

Diberikan persamaan Laplace berikut

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \ \ 0 \le x \le 1, \ \ 0 \le y \le 1$$

$$u(0, y) = 10, \ \ u(1, y) = 0, \ \ u(x, 0) = 0, \ \ u(x, 1) = 0$$

Dengan menerapkan metode beda hingga ADI didapatkan persamaan berikut

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y} \right)$$

dengan pendekatan metode beda hingga dengan langkah waktu $\frac{1}{2}$ persamaan di atas akan didekati dengan

$$\frac{\partial}{\partial x}\left.\frac{\partial u}{\partial x}\right|_{i,j}^{n+\frac{1}{2}} + \frac{\partial}{\partial y}\left.\frac{\partial u}{\partial y}\right|_{i,j}^{n+\frac{1}{2}} \approx \frac{\partial}{\partial x}\left(\frac{u_{i+1,j}^n - u_{i,j}^n}{\Delta x} + O(\Delta x, \Delta y)\right) + \frac{\partial}{\partial y}\left(\frac{u_{i,j+1}^n - u_{i,j}^n}{\Delta y} + O(\Delta x, \Delta y)\right)$$

$$\approx \frac{1}{\Delta x}\left(\left(\frac{u_{i+1,j}^n - u_{i,j}^n}{\Delta x}\right) - \left(\frac{u_{i,j}^n - u_{i-1,j}^n}{\Delta x}\right) + O(\Delta x, \Delta y)\right) - O(\Delta x, \Delta y) +$$

$$\frac{1}{\Delta y}\left(\left(\frac{u_{i,j+1}^n - u_{i,j}^n}{\Delta y}\right) - \left(\frac{u_{i,j}^n - u_{i,j-1}^n}{\Delta y} - O(\Delta x, \Delta y)\right) + O(\Delta x, \Delta y)\right)$$

$$\approx \left(\frac{u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n}{(\Delta x)^2}\right) + \left(\frac{u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n}{(\Delta y)^2}\right) = 0$$

Persamaan Difusi 2D

Diberikan persamaan difusi berikut

$$\frac{\partial u}{\partial t} = D_x \frac{\partial^2 u}{\partial x^2} + D_y \frac{\partial^2 u}{\partial y^2}, \quad 0 \le x \le 1, \quad 0 \le y \le 1 \tag{1}$$

$$\frac{\partial u}{\partial n} = 0 \mid n = \{x, y\}, \quad u(0.5, 0.5, t) = 10 \mid 0 \le t \le 5, \quad u(x, y, 0) = 0 \mid (x, y) \neq (0.5, 0.5), \quad t \ge 0$$

Dengan menerapkan metode beda hingga dengan beda mundur pada grid waktu dan beda tengah pada grid ruang, sehingga didapatkan persamaan-persamaan berikut

$$\left.\frac{\partial u}{\partial t}\right|_{i,j}^n \approx \frac{u_{i,j}^n - u_{i,j}^{n-1}}{\Delta t} + O(\Delta t) \tag{2}$$

$$\left.\frac{\partial^2 u}{\partial x^2}\right|_{i,j}^n \approx \frac{u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n}{(\Delta x)^2} + O(\Delta x)^2 \tag{3}$$

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{i,j}^n \approx \frac{u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n}{(\Delta y)^2} + O(\Delta y)^2 \tag{4}$$

kemudian Persamaan (2) - (4) di substitusi ke Persamaan (1) dan menghilangkan notasi kesalahan pemotongan maka didapatkan persamaan berikut

$$\frac{u_{i,j}^n - u_{i,j}^{n-1}}{\Delta t} = D_x \frac{u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n}{(\Delta x)^2} + D_y \frac{u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n}{(\Delta y)^2}$$

$$u_{i,j}^n - u_{i,j}^{n-1} = \left(\frac{\Delta t D_x}{(\Delta x)^2}\right)\left(u_{i+1,j}^n - 2u_{i,j}^n + u_{i-1,j}^n\right) + \left(\frac{\Delta t D_y}{(\Delta y)^2}\right)\left(u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n\right)$$

dengan menyusun kembali persamaan di atas maka didapatkan

$$(1 + 2A + 2B)\, u_{i,j}^n - A\left(u_{i+1,j}^n + u_{i-1,j}^n\right) - B\left(u_{i,j+1}^n + u_{i,j-1}^n\right) = u_{i,j}^{n-1} \tag{5}$$

dengan $A = \dfrac{\Delta t D_x}{(\Delta x)^2}$ dan $B = \dfrac{\Delta t D_y}{(\Delta y)^2}$

Contoh Soal

1. Hitunglah nilai integral berikut
   Jawaban;

$$\int_{-2}^{1} 3x^2 + 2 \, dx = \left[x^3 + 2x\right]_{-2}^{1}$$

$$= [(1)^3 + 2(1)] - [(-2)^3 + 2(-2)]$$

$$= [3] - [-8 - 4]$$

$$= 15$$

2. Hitunglah nilai integral berikut
Jawaban;

$$\int_0^1 -2x^{4/3} + 2x^{1/3}\, dx = \left[-\frac{6}{7}x^{\frac{7}{3}} + \frac{3}{2}x^{\frac{4}{3}} \right]_0^1$$

$$= \left[-\frac{6}{7}(1)^{\frac{7}{3}} + \frac{3}{2}(1)^{\frac{4}{3}} \right] - \left[-\frac{6}{7}(0)^{\frac{7}{3}} + \frac{3}{2}(0)^{\frac{4}{3}} \right]$$

$$= \frac{9}{14}$$

3. Hitunglah nilai integral berikut
Jawaban;

$$\int_{-2}^{-1} (2x + 2)(x^2 + 2x)\, dx = \int_{x=-2}^{x=-1} (x^2 + 2x)\, d(x^2 + 2x)$$

$$= \left[\frac{1}{2}(x^2 + 2x)^2 \right]_{-2}^{-1}$$

$$= \left[\frac{1}{2}((-1)^2 + 2(-1))^2 \right] - \left[\frac{1}{2}((-2)^2 + 2(-2))^2 \right]$$

$$= \frac{1}{2}$$

4. Hitunglah nilai integral berikut
Jawaban;

$$\int_{\pi/6}^{\pi} 2\sin x\, dx = [-2\cos x]_{\pi/6}^{\pi}$$

$$= [-2\cos(\pi)] - [-2\cos(\pi/6)]$$

$$= [2] - [-2(1/2\sqrt{3})]$$

$$= 2 + \sqrt{3}$$

5. Tentukan luas daerah apabila daerah tersebut dibatasi oleh fungsi $f(x) = -(x - 0.5)^2 + 2$, $g(x) = \frac{2}{3}x$, serta garis $x = 0$ dan $x = 1$. Kemudian gambarkan grafik dari daerah tersebut.
Jawaban;

$$\int_0^1 \left[-(x - 0.5)^2 + 2 \right] - \left[\frac{2}{3}x \right] dx = \int_0^1 \left[-x^2 + \frac{1}{3}x + \frac{7}{4} \right] dx$$

$$= \left[-\frac{1}{3}x^3 + \frac{1}{6}x^2 + \frac{7}{4}x \right]_0^1$$

$$= \left[-\frac{1}{3}(1)^3 + \frac{1}{6}(1)^2 + \frac{7}{4}(1) \right] - \left[-\frac{1}{3}(0)^3 + \frac{1}{6}(0)^2 + \frac{7}{4}(0) \right]$$

$$= \frac{19}{12} \text{ satuan luas}$$

6. Hitunglah nilai integral berikut
Jawaban;

$$\int_{-2}^1 3x^2 + 2\, dx = \left[x^3 + 2x \right]_{-2}^1$$

$$= [(1)^3 + 2(1)] - [(-2)^3 + 2(-2)]$$

$$= [3] - [-8 - 4]$$

$$= 15$$

7. Hitunglah nilai integral berikut
   Jawaban;

$$\int_0^1 -2x^{4/3} + 2x^{1/3}\,dx = \left[-\frac{6}{7}x^{\frac{7}{3}} + \frac{3}{2}x^{\frac{4}{3}}\right]_0^1$$
$$= \left[-\frac{6}{7}(1)^{\frac{7}{3}} + \frac{3}{2}(1)^{\frac{4}{3}}\right] - \left[-\frac{6}{7}(0)^{\frac{7}{3}} + \frac{3}{2}(0)^{\frac{4}{3}}\right]$$
$$= \frac{9}{14}$$

8. Hitunglah nilai integral berikut
   Jawaban;

$$\int_{-2}^{-1} (2x+2)(x^2+2x)\,dx = \int_{x=-2}^{x=-1} (x^2+2x)\,d(x^2+2x)$$
$$= \left[\frac{1}{2}(x^2+2x)^2\right]_{-2}^{-1}$$
$$= \left[\frac{1}{2}((-1)^2 + 2(-1))^2\right] - \left[\frac{1}{2}((-2)^2 + 2(-2))^2\right]$$
$$= \frac{1}{2}$$

9. Hitunglah nilai integral berikut
   Jawaban;

$$\int_{\pi/6}^{\pi} 2\sin x\,dx = [-2\cos x]_{\pi/6}^{\pi}$$
$$= [-2\cos(\pi)] - [-2\cos(\pi/6)]$$
$$= [2] - [-2(1/2\sqrt{3})]$$
$$= 2 + \sqrt{3}$$

10. Tentukan luas daerah apabila daerah tersebut dibatasi oleh fungsi $f(x) = -(x-0.5)^2 + 2$, $g(x) = \frac{2}{3}x$, serta garis $x = 0$ dan $x = 1$. Kemudian gambarkan grafik dari daerah tersebut.
    Jawaban;

$$\int_0^1 \left[-(x-0.5)^2 + 2\right] - \left[\frac{2}{3}x\right]\,dx = \int_0^1 \left[-x^2 + \frac{1}{3}x + \frac{7}{4}\right]\,dx$$
$$= \left[-\frac{1}{3}x^3 + \frac{1}{6}x^2 + \frac{7}{4}x\right]_0^1$$
$$= \left[-\frac{1}{3}(1)^3 + \frac{1}{6}(1)^2 + \frac{7}{4}(1)\right] - \left[-\frac{1}{3}(0)^3 + \frac{1}{6}(0)^2 + \frac{7}{4}(0)\right]$$
$$= \frac{19}{12} \text{ satuan luas}$$