Aerodynamics Homework 05

Zakary Steenhoek

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I. Question 1

The equations for the camber line of a NACA 4-digit series airfoil are seen below:

$$Z_c(x) = \frac{m}{p^2}(2px - x^2) \quad \text{for } x < p \tag{1.1}$$

$$Z_c(x) = \frac{m}{(1-p)^2}[(1-2p) + 2px - x^2] \quad \text{for } x > p \tag{1.2}$$

For these equations, m is the maximum ordinate of the camber line expressed as a fraction of the chord, and p is the chordwise position of the maximum ordinate expressed as a fraction of the chord. For example, for the NACA 4412, $m = 0.04$ and $p = 0.4$. Note that, in the above equations, both Z_c and x are made dimensionless by the airfoil chord.

A. Question 1 Problem Statement:

1. *Compare the zero-lift angle you found for the NACA 4412 in the laboratory and in homework assignment #3, to that predicted by thin-airfoil theory. Discuss the validity of thin-airfoil theory for determining the lift-curve slope and the zero-lift angle of attack.*

Assume: Small disturbances, thin airfoil, comparison of homework and laboratory α_{zl} values as they were submitted

Given: Eq. [1.1,1.2], α_{zl} values for the NACA 4412 from the lab and homework

Solution: To determine the validity of the thin-airfoil theory, we must first derive α_{zl} according to the thin-airfoil theory and calculate the theoretical α_{zl} for NACA 4412. For the derivation, first, consider the fundamental equation of thin-airfoil theory, derived from the governing boundary condition. This BC simply states that there will be no flow crossing the mean camber line, and flow over the surface of the airfoil is in the streamwise direction. Additionally, to satiate the Kutta condition, the vortex strength at the trailing edge must be zero:

$$\frac{\partial \phi_L}{\partial z} = V_\infty \left[\frac{dz_c}{dx} - \alpha\right] = -\frac{1}{2\pi}\int_0^c \frac{\gamma(x)}{x - x_1}\, dx_1 \tag{1.3}$$

$$\frac{dz_c}{dx} = \alpha - \frac{1}{V_\infty}\frac{1}{2\pi}\int_0^c \frac{\gamma(x)}{x - x_1}\, dx_1 \tag{1.4}$$

The derivative $\frac{dz_c}{dx}$ is computed for the fore and aft equations for the NACA 4412 mean camber line, Eqs. [1.1,1.2], and for the case where $x = p$, $\frac{dz_c}{dx} = 0$, according to the definition of a derivative:

$$\frac{d}{dx} Z_c(x)_{fore} = \frac{d}{dx}\left[\frac{m}{p^2}(2px - x^2)\right] = \frac{1}{5} - \frac{x}{2} \quad for\ x < p \tag{1.5}$$

$$\frac{d}{dx}Z_c(x)_{aft} = \frac{d}{dx}\left[\frac{m}{(1-p)^2}[(1-2p)+2px-x^2]\right] = \frac{4}{45} - \frac{2x}{9} \quad for\ x > p \tag{1.6}$$

$$\frac{d}{dx}Z_c(x) = 0 \quad for\ x = p \tag{1.7}$$

Next, for ease of later integration, define a variable substitution for x in terms of θ., as well as a substitution for the differential element dx in terms of θ:

$$x = \frac{c}{2}(1-\cos\theta); \quad dx = \frac{c}{2}\sin\theta\, d\theta, \quad \text{where } 0 < \theta < \pi \tag{1.8}$$

Using Eq. [1.8] in Eqs. [1.5,1.6] yields the following:

$$\frac{d}{dx}Z_c(x)_{fore} = \frac{1}{5} - \frac{x}{2} = \frac{\cos\theta}{4} - \frac{1}{20} \quad for\ x < p \tag{1.9}$$

$$\frac{d}{dx}Z_c(x)_{aft} = \frac{4}{45} - \frac{2x}{9} = \frac{\cos\theta}{9} - \frac{1}{45} \quad for\ x > p \tag{1.10}$$

Next, assume a solution $\gamma(\theta)$ to find $\gamma(x)$ in terms of the substituted variable of integration θ that will satisfy the Kutta condition, as each term evaluates to 0 when $\theta = \pi$:

$$\gamma(\theta) = 2V_\infty\left[A_o\frac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta\right] \tag{1.12}$$

Where A_o and A_n can be found as the following:

$$A_o = \alpha - \frac{1}{\pi}\int_0^\pi \frac{dz_c}{dx}\, d\theta \tag{1.13}$$

$$A_n = \frac{2}{\pi}\int_0^\pi \frac{dz_c}{dx}\cos n\theta\ d\theta \tag{1.14}$$

To determine these coefficients for the NACA 4412 with separate fore and aft equations of mean camber line, the x location where $\frac{d}{dx}Z_c(x) = 0$ must be converted into radians, and the above integrals can then be computed as a sum of the integrals of the fore and aft equations of mean camber line evaluated with respective integral limits. To do this, convert the x-location of maximum camber, where the equations switch, to radians:

$$x = \frac{c}{2}(1-\cos\theta) = 0.4c$$

$$(1-\cos\theta) = 0.8$$

$$\theta = 1.3694\, rad$$

Next, finding A_o for $\frac{dz_c}{dx}$:

$$A_o = \alpha - \frac{1}{\pi}\left[\int_0^{1.3694} \frac{\cos\theta}{4} - \frac{1}{20}\, d\theta + \int_{1.3694}^{\pi} \frac{\cos\theta}{9} - \frac{1}{45}\, d\theta\right] = \alpha - 0.00898577 \tag{1.15}$$

Next, finding A_n for $\frac{dz_c}{dx}$:

$$A_n = \frac{2}{\pi}\left[\int_0^{1.3694} \frac{\cos\theta}{4} - \frac{1}{20}\cos n\theta\ d\theta + \int_{1.3694}^{\pi} \frac{\cos\theta}{9} - \frac{1}{45}\cos n\theta\ d\theta\right] \tag{1.16}$$

To determine the α_{zl} for this airfoil, we need to find the equation for C_l as a function of α, as we can use the slope and y-intercept to determine a value for the zero-lift angle. The equation for C_l is found next, starting from the definition of the lift coefficient:

$$C_l = \frac{l}{\frac{1}{2}\rho_\infty V_\infty^2 c} = \frac{\int_0^c \rho_\infty V_\infty \gamma(x)\ dx}{\frac{1}{2}\rho_\infty V_\infty^2 c}$$

Using our coordinate transform in conjunction with Eq. [1.12], we can obtain the following expression for C_l in terms of coefficients A_o and A_n:

$$C_l = 2\left[\int_0^\pi A_o(1+\cos\theta)\ d\theta + \int_0^\pi \sum_{n=1}^{\infty} A_n \sin n\theta \sin\theta\ d\theta\right] \tag{1.17}$$

Note the following for the second integral:

$$\int_0^\pi A_n \sin n\theta \sin\theta\, d\theta = \begin{cases} A_1, & n = 1 \\ 0, & n \neq 1 \end{cases}$$

Thus, after evaluation:

$$C_l = 2\pi\left(A_o + \frac{A_1}{2}\right)$$

Or, more intuitively:

$$C_l = 2\pi\left[\alpha - \frac{1}{\pi}\int_0^\pi \frac{dz_c}{dx}\, d\theta + \frac{1}{\pi}\int_0^\pi \frac{dz_c}{dx}\cos\theta\, d\theta\right] = 2\pi\left[\alpha + \frac{1}{\pi}\int_0^\pi \frac{dz_c}{dx}(\cos\theta - 1)d\theta\right] = 2\pi(\alpha - \alpha_{zl}) \tag{1.18}$$

Solving for α_{zl}:

$$\alpha_{zl} = -\frac{1}{\pi}\int_0^\pi \frac{dz_c}{dx}(\cos\theta - 1)\ d\theta \tag{1.19}$$

Alternatively, after solving for A_1 in MATLAB:

$$C_l = 2\pi\left(A_o + \frac{A_1}{2}\right) = 2\pi\alpha + 0.4556$$

Where α is in radians. Converting to degrees, we can finally determine α_{zl} for the NACA 4412:

$$\alpha_{zl} = -4.1548° \tag{1.20}$$

Results: Comparing the theoretical value from Eq. [1.20] to the values obtained in lab and in previous homework, we should expect similar results. The theoretical value for α_{zl} is $-4.1548°$, the value for α_{zl} from lab was found to be $-7.9493°$, and the value for α_{zl} from homework was found to be $-3.7894°$.

Discussion: The α_{zl} value for the NACA 4412 from the theoretical calculation and the α_{zl} value for the NACA 4412 from the homework seem to be the closest together. The α_{zl} value for the NACA 4412 from the lab seems to be an outlier here, as it is almost two times the theoretical value. Assumptions for this problem state that the values will be compared as they were submitted, regardless of 'correctness'. However, discussion on why the lab value is so much different will not be neglected.

First, it seems as though the thin-airfoil theory predicted the α_{zl} value for the NACA 4412 with relative accuracy compared to the α_{zl} value derived using RANS equations. The difference between the two is roughly $0.3654°$, which is almost imperceptible in all actuality. The difference that is present can likely be attributed to the fact that RANS allows for compressible and turbulent flow, while the thin-airfoil theory depends on fundamental solutions that must satisfy continuity, and do not allow for turbulent flow. Both methods factor in viscosity, so that can be factored out of consideration for the difference. Another potential source of error results from the fact that the α_{zl} value derived using RANS equations was personally computed using MATLAB code, and I cannot say for certain that my code, solution technique, or final answer matches the expected value from the RANS data. These sources of error are likely to describe the difference between theoretical and calculated values.

For the α_{zl} value for the NACA 4412 found in lab, there are many more sources of potential error that can contribute to the discrepancy. We have focused so much on the thin-airfoil theory recently, and between lectures and textbook reading, there are many instances where it is explicitly stated that the thin-airfoil theory is a good predictor of real-life conditions of steady flow around an airfoil. Thus, the deduction is made that the lab α_{zl} value is incorrect, not the theoretical value. Sources of error that could contribute to the inconsistency in lab data include, but certainly are not limited to: error propagation through instrument calibration, human error in lab setup/procedures, and incorrect code and/or calculations.

In theory, all the calculated α_{zl} values should be within the same general range. It should be expected that the lab data is the furthest from the theoretical value, as data is collected from real-life simulation, but not by a whole factor of two. Regardless, the theoretical calculations prove useful for modeling flow in ideal conditions.

Regarding the lift slope of the curves, the thin-airfoil theory predicts a slope of 2π per radian, or roughly 0.11 per degree. The values found in lab and homework align within a reasonable range: the slope from the homework was found to be 0.1002, and the slope from the lab was found to be 0.0646. Again, the value from the homework is much closer to the theoretical result, but this time, the value from the lab is half of the expected value. The sources of error mentioned above apply here as well.

Overall, the thin-airfoil theory seems to be a good predictor for theoretical flow and potentially real-life ideal flow, or at least it is much closer than a novice aerodynamics student writing MATLAB code.

II. Question 2

A. Question 2 Problem Statement:

1. *Use the thin-airfoil theory to find the center of pressure (as a function of lift coefficient) and the moment coefficient about the aerodynamic center for the NACA 4412. Plot the center of pressure vs. C_l (be sure to include negative values for lift coefficient).*

Assume: Small disturbances, thin airfoil

Given: Eq. [1.1,1.2]

Solution: To determine the center of pressure, consider the definition of the pitching moment about the leading edge, which is the lift multiplied by a moment arm, and the respective coefficient, of the typical form. Note that, for the reader's and the writer's sake, some integration detail is skipped henceforth due to the similarity to integration detail seen in question 1:

$$M_o = -\int_0^c \rho_\infty V_\infty \gamma(x) x \ dx \tag{2.1}$$

$$C_{M_o} = \frac{M_o}{\frac{1}{2}\rho_\infty V_\infty^2 c^2} \tag{2.2}$$

Using the coordinate transform, vorticity distribution, and aerodynamic coefficients from Eqs. [1.8,1.12-1.14], the moment coefficient can be represented in terms of A_o, and A_n when n=1,2:

$$C_{M_o} = -\frac{\pi}{2}\left(A_o + A_1 - \frac{A_2}{2}\right) \tag{2.3}$$

Let the center of pressure be defined as the pitching moment per span divided by the lift per span, as this results in a moment arm relative to the leading edge. Note that this operation can be done with the coefficients, as most of the terms will be cancelled, resulting in the following:

$$x_{cp} = -\frac{M_o}{l} = \frac{c}{4}\left(\frac{2A_o + 2A_1 - A_2}{2A_o + A_1}\right) = \frac{c}{4}\left[1 + \frac{\pi}{C_l}(A_1 - A_2)\right] \tag{2.4}$$

To determine the moment coefficient about the aerodynamic center, let the aerodynamic center be defined as a quarter-chord from the leading edge:

$$x_{ac} = \frac{c}{4}$$

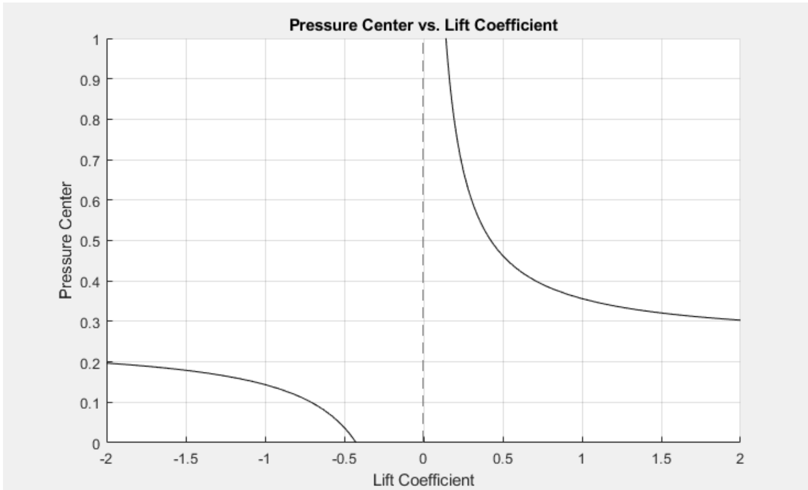
Free-body analysis of the moments on a wing section about this aerodynamic center yield the following after integration and simplification, like Eq. [2.2]:

$$\mathrm{M}_{ac} = \int_0^{c/4} \rho_\infty V_\infty \gamma(x)\left(\frac{c}{4} - x\right) dx - \int_{c/4}^{c} \rho_\infty V_\infty \gamma(x)\left(x - \frac{c}{4}\right) dx = \frac{c}{4} l + M_o$$

Eq. [2.6] can also be represented in terms of the respective dimensionless coefficients, as well as the aerodynamic coefficients seen above, which were computed using MATLAB:

$$\mathrm{C}_{\mathrm{M}_{ac}} = \frac{C_l}{4} + C_{M_o} = \frac{\pi}{4}(A_2 - A_1) = -0.1062$$

Results: The dimensionless x-location of the center of pressure was plotted against the lift coefficient, seen below:



Note that in the above graph, since pressure center is made dimensionless w.r.t. a quantity that is defined as positive for this case, it cannot have a negative value, so only the upper plane of the graph is shown.

Discussion: From the graph of the dimensionless pressure center vs. lift coefficient, which is dimensionless by nature, there are a couple things to note. We can see that the center of pressure w.r.t. the lift coefficient, and therefore the angle of attack, is a complex one. When the lift coefficient is negative, i.e. the airfoil is producing lift in the downward direction, the pressure center is at about 0.2 on the dimensionless chordwise station. As the airfoil approaches a zer0-lift angle, the pressure center reaches a discontinuity but *returns to a similar value* as the airfoil starts producing positive lift. Thus, in all reality, it can be found that the pressure center is typically between $\bar{x} = 0.2$ to $\bar{x} = 0.3$ for angles of attack that do not result in a stall condition.

III. Question 3

In some situations, it is necessary for an airfoil to have a positive moment about the aerodynamic center, but to also generate positive lift at $\alpha = 0$. This type of airfoil could have a camberline given by a cubic equation:

$$Z_c = kx(x-1)(x-b) \tag{3.1}$$

$$cos^2\theta = \frac{1}{2}(1 + cos2\theta) \tag{3.2}$$

where Z_c and x have been made dimensionless by the airfoil chord, and k and b are dimensionless constants.

A. Question 3 Problem Statements:

1. *Find the lower limit on b; i.e., what is the minimum value for b that would make C_l positive at $\alpha = 0$?*
2. *Find the upper limit on b; i.e., what is the maximum value for b that would make $C_{m_{ac}}$ positive?*
3. *For a value of b that gives $C_l = 0$ at $\alpha = 0$, plot the camber line. Discuss the similarities and differences between this camber line and a 'normal' camber line and explain why this airfoil has positive moment coefficient about the aerodynamic center at $\alpha = 0$. (Recall that an airfoil with 'normal' camber, such as the NACA 4412, has negative moment about the aerodynamic center.)*

Assume: Small disturbances, thin airfoil.

Given: Eqs. [3.1,3.2]

Solution: To determine a symbolic C_l, steps like the ones taken in question 1 were followed using Eq. [3.1] for Z_c, seen below:

$$Z_c = kx(x-1)(x-b) \tag{3.3}$$

$$\frac{d}{dx}Z_c = -\frac{k}{4}(1 + 2\cos\theta - 3\cos^2\theta - 4b\cos\theta)$$

$$A_o = \alpha - \frac{1}{\pi}\int_0^{\pi}\frac{dz_c}{dx}\,d\theta = \alpha - \frac{k}{8} \tag{3.4}$$

$$A_n = \frac{2}{\pi}\int_0^{\pi}\frac{dz_c}{dx}\cos n\theta\ d\theta = \begin{cases} \mathrm{A}_1 = \frac{k}{2}(2b-1), & n = 1 \\ \mathrm{A}_2 = \frac{3k}{8}, & n = 2 \end{cases} \tag{3.5}$$

After computing the necessary coefficients in MATLAB, we can plug the values into the known equations for C_l and $C_{m_{ac}}$ derived in question 1. First, to find C_l, since we know that $\alpha = 0$:

$$C_l = 2\pi\left(A_o + \frac{A_1}{2}\right) = 2\pi\left(-\frac{k}{8} + \frac{k}{4}(2b-1)\right) = 2\pi k\left(-\frac{1}{8} + \frac{1}{4}(2b-1)\right) \tag{3.6}$$

Solving Eq. [3.6] for $C_l = 0$ will yield the value for the lower limit on b:

$$2\pi k\left(-\frac{1}{8} + \frac{1}{4}(2b-1)\right) = 0$$

$$\frac{1}{8} = \frac{1}{4}(2b-1) \xrightarrow{yields} \frac{1}{2} = 2b - 1 \xrightarrow{yields} b_{lower} = \frac{3}{4}$$

Next, to find the upper limit of b:

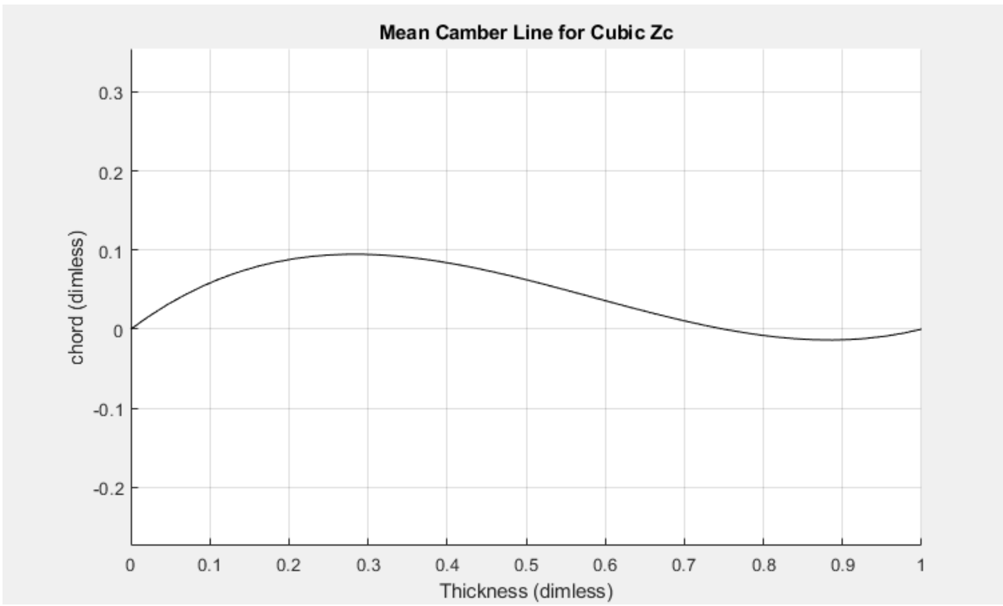
$$C_{m_{ac}} = \frac{\pi}{4}(A_2 - A_1) = \frac{\pi}{4}\left(\frac{3k}{8} - \frac{k}{2}(2b-1)\right) = \frac{\pi k}{4}\left(\frac{3}{8} - \frac{1}{2}(2b-1)\right) \tag{3.7}$$

Solving Eq. [3.7] for $C_{m_{ac}} = 0$ will yield the value for the lower limit on b:

$$\frac{\pi k}{4}\left(\frac{3}{8} - \frac{1}{2}(2b-1)\right) = 0$$

$$\frac{3}{8} - \frac{1}{2}(2b-1) = 0 \xrightarrow{yields} \frac{3}{8} = \frac{1}{2}(2b-1) \xrightarrow{yields} b_{\text{upper}} = \frac{7}{8}$$

Results: The camber line of this airfoil was plotted using the determined value of b_{lower}, as seen below:



Discussion: An analysis of the cubic camber line seen in the figure above shows that it is atypical when compared to a ‘normal’ mean camber line, e.g. the one seen on the NACA 4412. This is immediately obvious upon visual inspection, as the MCL has two critical points along the dimensionless chord. This means that there will be a more complicated free-body diagram analysis required for the moments produced by the surface of this airfoil. This airfoil will likely generate a positive moment about the aerodynamic center at $\alpha = 0$ since, regardless of the positive lift produced, the asymmetrical curves will end up changing the sign of the moment that this airfoil produces. This asymmetry in question refers to the fact that the magnitude of the camber of the forward portion is greater than the magnitude of the camber aft of the aerodynamic center. When compared to the camber of a ‘traditional’ airfoil, such as the NACA 4412, there is an entirely second contribution to the moment produced due to the cubic nature. This can be thought of as almost a ‘reverse’ flap in the context of a typical aircraft wing. If the camber line was modified such that the magnitude of the camber was greater in magnitude, but in the same ‘direction’ as the forward portion of the airfoil, it would likely generate greater lift at the same $\alpha = 0$.

IV. Question 4

Alter the MATLAB code you have been using throughout the semester for lift and drag calculations to also calculate the moment coefficient about the quarter-chord. Note that, for a "thick" airfoil, pressure forces in both x and z directions produce a moment:

$$m_{c/4} = \int_{airfoil} (x - x_{c/4})p \; dx + \int_{airfoil} zp \; dz \tag{4.1}$$

B. Question 1 Problem Statement:

4. *Make the above equation dimensionless to find $C_{m_{c/4}}$. Use the TACAA data for the NACA 4412 at $\alpha = -6°$ through $\alpha = 15°$, and run your code to calculate and plot the $C_{m_{c/4}}$ vs. angle of attack. Compare to the theoretical value of $C_{m_{ac}}$ found in problem 2. Comment on the validity of thin-airfoil theory relative to the more 'exact' TACAA data. Comment on whether the theoretical value of $\overline{x_{ac}} = 0.25$ is valid.*

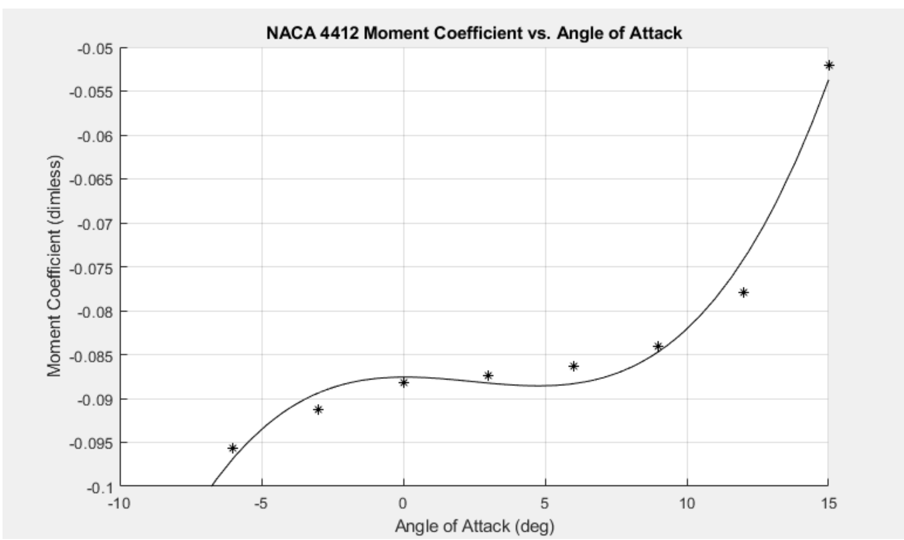
Assume: Small disturbances, thick airfoil.

Given: Eq. [4.1]

Solution: The code from homework 3 was modified to also compute the moment coefficient, which can be derived from Eq. [4.1] by substituting dimensionless coefficients for all variables present:

$$C_{m_{c/4}} = \int_{airfoil} (\overline{x} - 0.25)C_p \; d\overline{x} + \int_{airfoil} \overline{z}C_p \; d\overline{z}$$

Results: Using the code included in appendix A, the following graph of moment coefficient vs. angle of attack was determined. Note that the order of the polyfit was set to 3 so that it fits the computed data:



Discussion: From the graph above, the moment coefficient at most angles of attack between $-6°$ and $9°$ fall between -0.095 and -0.085. This is like the value obtained in question 2, there the moment coefficient was computed to be -0.1062. It is expected that the data computed from the 'exact' solution does not match the theoretical data, as RANS equations allow for compressibility and turbulent flow. However, the computed and theoretical values seem to be within reasonable range. Thus, the thin-airfoil theory seems to be a valid and relatively close approximation to the exact data *for non-stall angles of attack*. The theoretical $\overline{x_{ac}} = 0.25$ seems to be valid in this case, as $\overline{x_{ac}}$ is defined as the chordwise station where the moment and its coefficient do not change as a function of angle of attack. Since the moment coefficient is seen to be relatively stable throughout the range of typical, usable, non-stall angles of attack, it can be determined that the theoretical $\overline{x_{ac}} = 0.25$ is a good approximation, even with the 'exact' RANS data. Note that, even at negative angles of attack, the moment coefficient is still stable, further proving the validity of the theoretical $\overline{x_{ac}}$ for this case.

I. Appendix A

```
%% Header
% Author: Zakary Steenhoek
% Date: 24 October 2024
% AEE 360 HW05

clc; clear; clf; %close all;

%% Definitions

m = 0.04;                                                         % dimless max
ordinate (x-value)
p = 0.4;                                                          % dimless
chord position of max ordinate

% Symbolic variables
syms a T X n Vi
assume(n > 0);
%assume(T > 0);

% Anon functions for Z_c(x)
Zc_front = @(X) (m/p^2).*(2*p.*X-X.^2);                           % Z_c for x <
p, i.e. front of the airfoil
Zc_back = @(X) (m/((1-p)^2)).*((1-2*p)+2*p.*X-X.^2);              % Z_c for x >
p, i.e. back of the airfoil

% Derivitives
dZc_fore = diff(Zc_front, X);
dZc_aft = diff(Zc_back, X);

% Variable substitution
X_TERMS_THETA = 0.5*(1-cos(T));
dZc_fore = subs(dZc_fore, X, X_TERMS_THETA);
dZc_aft = subs(dZc_aft, X, X_TERMS_THETA);
theta_limit = abs(solve(X_TERMS_THETA == p, T));

if (theta_limit(1,1) == theta_limit(2, 1))
    theta_limit = theta_limit(1,1);
end

% Define Ao
A_o = a-(1/pi)*(int(dZc_fore, T, 0, theta_limit)+int(dZc_aft, T, theta_limit, pi));

% Define An
A_n = (2/pi)*(int(dZc_fore.*cos(n.*T), T, 0, theta_limit)+int(dZc_aft.*cos(n.*T), T,
theta_limit, pi));

% Assume solution
gamma = 2*Vi*(A_o.*(1+cos(T))./(sin(T))+A_n.*sin(T));

% Extract peicewise solution where n = 1
A_1 = subs(A_n, n, 1);
A_2 = subs(A_n, n, 2);
```

```
% Compute C_l
C_l = 2*pi*(A_o+(A_1/2));
a_zl = rad2deg(solve(C_l == 0,a));

% Pressure center
x_cp = @(CL) 0.25.*(1+((pi./CL).*(A_1-A_2)));
c_l = linspace(-1.5, 1.5, 100);

% Moment about quarter chord
Cm_ac = (pi/4)*(A_2-A_1);

figure(1); hold on; grid on;
xlim([-2,2]); ylim([0,1]);
title('Pressure Center vs. Lift Coefficient');
xlabel('Lift Coefficient'); ylabel('Pressure Center');
%fplot(x_cp, 'k-');
plot(c_l,x_cp(c_l), 'k-')

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% AEE 360 HW05

clc; clear; clf; %close all;

%% Definitions

% Symbolic variables
syms a T X n Vi k b
assume(n > 0);
%assume(T > 0);

% Function for Z_c(x)
%Zc = X.*(X-1).*(X-b);
Zc = k.*X.*(X-1).*(X-b);

% Derivitive
dZc = diff(Zc, X);
%dZc = k.*((3.*X.^2)-(2.*b.*X)-(2.*X)+b);

% Variable substitution
X_TERMS_THETA = 0.5*(1-cos(T));
COS_IDENT = 0.5*(1+cos(2*T));
dZc = subs(dZc, X, X_TERMS_THETA);
dZc = simplify(dZc);
dZc = subs(dZc, cos(T)^2, COS_IDENT);

% Define Ao
A_o = a-(1/pi)*(int(dZc, T, 0, pi));
A_o = subs(A_o, a, 0);

% Define An
A_n = (2/pi)*(int(dZc.*cos(n.*T), T, 0, pi));
```

```
% Extract peicewise solution where n = 1
A_1 = subs(A_n, n, 1);
A_2 = subs(A_n, n, 2);

% Compute lift coefficient
C_l = 2*pi*(A_o+(0.5*A_1));
b_min = solve(C_l == 0, b);
k_comp = solve(subs(C_l, b, b_min) == 0, k);

% Compute moment coefficient
Cm_ac = (pi/4)*(A_2-A_1);
b_max = solve(Cm_ac == 0, b);

a_zl = rad2deg(solve(C_l == 0,a));

% Pressure center
x_cp = @(CL) 0.25.*(1+((pi./CL).*(A_1-A_2)));
c_l = linspace(-1.5, 1.5, 100);

% Plot
figure(1); hold on; grid on; axis equal;
xlim([0,1]);
title('Mean Camber Line for Cubic Zc');
xlabel('Thickness (dimless)'); ylabel('chord (dimless)');
Zc = subs(Zc, b, b_min); Zc = subs(Zc, k, 1);
fplot(Zc, 'k-');

%% Header
% Author: Zakary Steenhoek
% Date: 24 October 2024
% AEE 360 HW05

clc; clear; clf; %close all;

%% Find data files for NACA 4412

% Load data from the text files
dataPath_4412 = 'C:\Users\zaste\OneDrive\Documents\<path>';
dataFiles_4412 = dir(fullfile(dataPath_4412));

% Get rid of standard listed dirs
dataFiles_4412(1:2)=[];

% To record the data
allC_4412 = zeros(length(dataFiles_4412), 4);

% Define known p
p = 0.4;

%% Generate Cl & Cd for NACA 4412

% For the number data files
for itr = 1:length(dataFiles_4412)
    % Build path to the file
    filePath = fullfile(dataPath_4412, dataFiles_4412(itr).name);
```

```
    dataFile = importdata(filePath);

    % Parse the filename to get the angle of attack
    subs = split(dataFiles_4412(itr).name, '_'); lastSub = subs{end};
    alphadeg = erase(lastSub, '.txt');

    % Determine if alpha is negative and convert to int
    if contains(lastSub, 'm')
        alphadeg = erase(alphadeg, 'm');
        alphadeg = -1*str2double(alphadeg);
    else
        alphadeg = str2double(alphadeg);
    end

    % Convert to rad
    alpha = alphadeg*pi/180;

    % Extract x and z columns and ipc column
    dimlessX = flipud(dataFile.data(:, 1));
    dimlessZ = flipud(dataFile.data(:, 3));
    ipc = flipud(dataFile.data(:, 4));

    % Find force coefficients for x and z
    cfx = trapz(dimlessZ, ipc); cfz = trapz(dimlessX, -ipc);

    % Lift and drag coefficient calculations
    cl = cfz*cos(alpha)-cfx*sin(alpha);
    cd = cfz*sin(alpha)+cfx*cos(alpha);

    % Moment calculation
    Cm_ac = trapz(dimlessX, (dimlessX-0.25).*ipc) + trapz(dimlessZ, dimlessZ.*ipc);

    allC_4412(itr,1) = alphadeg;
    allC_4412(itr,2) = cl;
    allC_4412(itr,3) = cd;
    allC_4412(itr,4) = Cm_ac;
end

% Sort the data
allC_4412 = sort(allC_4412);

%% Cl-Alpha Plot

% Plot Cl data for airfoil 4412
figure(1); hold on;
scatter(allC_4412(:,1), allC_4412(:,2), 'k*');
ClCv_4412 = polyfit(allC_4412(:,1), allC_4412(:,2), 1);
fplot(poly2sym(ClCv_4412), 'k-');
xlabel('Angle of Attack (deg)'); ylabel('Lift Coefficient (dimless)');
title('NACA 4412 Lift Coefficient vs. Angle of Attack');
grid on; hold off; zeroLiftAoA_4412 = roots(ClCv_4412);

%% Cd-Alpha Plot

% Plot Cd data for airfoil 4412
```

```
figure(2); hold on;
scatter(allC_4412(:,1), allC_4412(:,3), 'k*');
CdCv_4412 = polyfit(allC_4412(:,1), allC_4412(:,3), 3);
fplot(poly2sym(CdCv_4412), 'k-');
xlabel('Angle of Attack (deg)'); ylabel('Drag Coefficient (dimless)');
title('NACA 4412 Drag Coefficient vs. Angle of Attack');
grid on; hold off;

%% Cmac-Alpha Plot

% Plot Cmac data for airfoil 4412
figure(3); hold on;
scatter(allC_4412(:,1), allC_4412(:,4), 'k*');
CmacCv_4412 = polyfit(allC_4412(:,1), allC_4412(:,4), 3);
fplot(poly2sym(CmacCv_4412), 'k-');
xlabel('Angle of Attack (deg)'); ylabel('Moment Coefficient (dimless)');
title('NACA 4412 Moment Coefficient vs. Angle of Attack');
grid on; hold off;
```