Magnitudes from spectra and the pivot wavelength

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Suppose we have a spectrum f_λ and a response function for a filter R (transmission curve; contribution to the detector signal per photon entering the atmosphere of Earth). We can determine the mean f_λ as follows:

$$\langle f_\lambda \rangle = \frac{\int \mathrm{d}\lambda\, \lambda\, R\, f_\lambda}{\int \mathrm{d}\lambda\, \lambda\, R}$$

Note the weighting here is λR for a photon-counting device, e.g., CCD. Then we can apply the same to a reference spectrum to obtain the mean flux of the reference, and then the magnitude is

$$m = -2.5 \log_{10} \left(\frac{\langle f_\lambda \rangle}{\langle f_{\lambda,ref} \rangle} \right)$$

Alternatively, we can do the same using f_ν:

$$\langle f_\nu \rangle = \frac{\int \frac{\mathrm{d}\nu}{\nu}\, R\, f_\nu}{\int \frac{\mathrm{d}\nu}{\nu}\, R}$$

also integrating photons. The AB system is in principle defined this way with $f_{\nu,ref} = 3631\ \mathrm{Jy}$, i.e., $m_{\mathrm{ab}} = -2.5 \log_{10} (\langle f_\nu \rangle / 3631\ \mathrm{Jy})$.

Using:

$$f_\nu = \frac{\lambda^2}{c} f_\lambda \quad \text{and} \quad \frac{\mathrm{d}\nu}{\nu} = -\frac{\mathrm{d}\lambda}{\lambda}$$

we can write

$$\langle f_\nu \rangle = \frac{\int \mathrm{d}\lambda\, \lambda\, R\, f_\lambda}{c \int \frac{\mathrm{d}\lambda}{\lambda} R}$$

For consistency, we need

$$\langle f_\nu \rangle = \frac{\lambda_{\mathrm{eff}}^2}{c} \langle f_\lambda \rangle$$

and from this, we get

$$\lambda_{\mathrm{eff}}^2 = \frac{\int \mathrm{d}\lambda\, \lambda\, R}{\int \frac{\mathrm{d}\lambda}{\lambda} R}$$

where R is the combined atmospheric-instrument transmission and detector-conversion efficiency as a function of wavelength, with a photon counting device.

With a bolometer, energy measuring device, where T is the transmission curve; or if you are using a redefined version of a transmission curve such that it is the contribution per incident energy (i.e. λR renormalized), then λR and $\underline{R/\nu}$ are replaced by T in the above equations, e.g.

$$\lambda_{\mathrm{eff}}^2 = \frac{\int \mathrm{d}\lambda\, T}{\int \frac{\mathrm{d}\lambda}{\lambda^2} T}$$

This λ_{eff} is known as the **pivot wavelength**. This is also given in equation A11 of Tokunaga & Vacca (2005PASP..117..421T), who refer back to Koorneef et al. (1986HiA.....7..833K), and is used for HST filters. It seems to me that this is the most useful definition of `effective wavelength of a filter'.

Using the filter data distributed with the kcorrect v4.2 code, I determined the pivot wavelengths for the following filters.

filter	pivot wavelength (Å)		filter	pivot wavelength (Å)
SDSS *u*	3557		Bessell U	3585
SDSS *g*	4702		Bessell B	4371
SDSS *r*	6175		Bessell V	5478
SDSS *i*	7491		Bessell R	6504

SDSS z	8946		Bessell I	8020
UKIDSS Y	10310			
UKIDSS J	12500		2MASS J	12350
UKIDSS H	16360		2MASS H	16460
UKIDSS K	22060		2MASS K	21600
GALEX FUV	1535		UK Schmidt b_j	4602
GALEX NUV	2301			

Note that in earlier versions of kcorrect, the 2MASS response functions were given as λR renormalized as stated in Cohen et al. (2003). The correction back to the transmission curve R was implemented for kcorrect v4.2 for consistency with the other filter curve data.

In my opinion, authors of filter curve data should provide the natural transmission curve, and then note whether the detector is photon counting or a bolometer, i.e., the weighting factor. Providing λR renormalized can create confusion, and cause some codes to be in error if they have assumed that the transmission curve does not include the weighting factor for a photon-counting device.

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