TP5 : Descente de gradient

Ce script présente la DG sur la minimisation de la fonction Unexpected text node: ' E(x) ' quelconque. Leproblème est de trouver la valeur dexqui minimise Unexpected text node: 'E(x)'. Pour trouver analytiquementle minimum de la fonction E, il faut trouver les racines de l'équation Unexpected text node: 'E′(x) = 0', donctrouver ici les racines d'un polynôme de degré 3, ce qui est des fois "difficile". Donc onva utiliser la DG. La DG consiste à construire une suite de valeurs xi(avec x0 fixé auhasard) de manière itérative :

xi+1=xi−ηE′(xi)

On peut donner un critère de fin à la DG par exemple si xi+1 − xi < eps ou si i > nombre_max.
Pour ce problème, nous utilisons eps = 0.01 et nombre_max= 1000.

Descente de gradient

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1. Calculez l'expression analytique de la fonction Unexpected text node: ' E(x) = (x−1)(x−2)(x−3)(x−5) ' et sa dérivée.

In [1]:

```
from sympy import Symbol, expand, diff

x = Symbol('x')
exp = expand((x-1)*(x-2)*(x-3)*(x-5))
#La dérivée
div_exp = diff(exp, x)

print('E(x) = ', exp, '\nE\'(x) = ',div_exp)
```

```
E(x) =  x**4 - 11*x**3 + 41*x**2 - 61*x + 30
E'(x) =  4*x**3 - 33*x**2 + 82*x - 61
```

2. Implémentez l'algorithme DG sous Python pour la fonction E(x).

Fonction Descente de gradient

- **div_e :** la dérivée de la fonction E(x)
- **x0 :** la valeur intiale de x
- **n :** le pas de Descente de gradient
- **eps :** pour le condition d'arrête
- **nb_max_iter :** nombre maximum des itérations

In [5]:

```
def DG(div_e, x0, n, eps = 0.01, nb_max_iter = 1000):
    cpp = 0 # compteur pour calculer le nombre d'itération
    while True:
        # calculer le nouveau x (xi+1)
        x1 = x0 - n*div_e.subs(x, x0).evalf()
        cpp += 1
        # condition d'arrêt
        if ( abs(x1 - x0) < eps ) | ( cpp > nb_max_iter ):
            break
        x0 = x1
    # résultat return le nouveau x et le nombre d'itération
    return [x1, cpp]
```

3. Pour comprendre ce que fait effectivement la DG, testez l'algorithme implémenté en utilisant des exemples d'exécutions avec des valeurs initiales de x0 et η suivantes :

- x0 = 5 et η = 0.001
- x0 = 5 et η = 0.01
- x0 = 5 et η = 0.1
- x0 = 5 et η = 0.17
- x0 = 5 et η = 1
- x0 = 0 et η = 0.001

In [6]:

```
print("x0 = 5 et η = 0.001 \t: ", DG(div_exp, x0=5, n=0.001))
print("x0 = 5 et η = 0.01 \t: ", DG(div_exp, x0=5, n=0.01))
print("x0 = 5 et η = 0.1 \t: ", DG(div_exp, x0=5, n=0.1))
print("x0 = 5 et η = 0.17 \t: ", DG(div_exp, x0=5, n=0.17))
print("x0 = 5 et η = 1 \t: ", DG(div_exp, x0=5, n=1))
print("x0 = 0 et η = 0.001 \t: ", DG(div_exp, x0=0, n=0.001))
```

```
x0 = 5 et η = 0.001     :  [4.66148331568632, 22]
x0 = 5 et η = 0.01      :  [4.36100829491372, 10]
x0 = 5 et η = 0.1       :  [4.48300247305520, 1001]
x0 = 5 et η = 0.17      :  [1.23789802112164, 1001]
x0 = 5 et η = 1         :  [-5.59079531307448e+2165218897387849575449569542280595721495861785890200169487984
640726689692623705307933951769342049549605036164832782173103034129608036149295080551194764987554074946681668
978958079023089253294582547639268975015256397044116579963969577226087629587121543654371017833958596520034967
414619393227754352672765506194522912424105740720275128204523067986059345070180384910080092271110636563888155
462862848820544652709611781766626003237259515385703156691028186064669180469086499802903989431, 1001]
x0 = 0 et η = 0.001     :  [0.949407118070548, 39]
```

4. Affichez le minimum trouvé, ainsi que E(xmin) et le nombre d'itérations. Que constatez-vous?

In [7]:

```
args = [[5, 0.001], [5, 0.01], [5, 0.1], [5, 0.17], [5, 1], [0, 0.001]]
all_xmin = []
for arg in args:
    xmin, nb_ite = DG(div_exp, arg[0], arg[1])
    print('-'*60)
    print('| \tPour x0 = ', arg[0],' et η = ', arg[1], '\t:\t\t   |')
    print('-'*60)
    print('| xmin \t\t\t| ', xmin, "\t\t   |")
    xmin_f = exp.subs(x, xmin).evalf()
    print('| E(xmin) \t\t| ', xmin_f, "\t\t   |")
    print('| nombre d’itérations \t| ', nb_ite, "\t\t\t\t   |")
    all_xmin.append([xmin, xmin_f, nb_ite])

all_xmin.pop(4)
```

```
------------------------------------------------------------
|       Pour x0 =  5  et η =  0.001     :                  |
------------------------------------------------------------
| xmin                  |  4.66148331568632                |
| E(xmin)               |  -5.48096300209431               |
| nombre d’itérations   |  22                              |
------------------------------------------------------------
|       Pour x0 =  5  et η =  0.01      :                  |
------------------------------------------------------------
| xmin                  |  4.36100829491372                |
| E(xmin)               |  -6.90117578513082               |
| nombre d’itérations   |  10                              |
------------------------------------------------------------
|       Pour x0 =  5  et η =  0.1       :                  |
------------------------------------------------------------
| xmin                  |  4.48300247305520                |
| E(xmin)               |  -6.63072895912501               |
| nombre d’itérations   |  1001                                    |
------------------------------------------------------------
|       Pour x0 =  5  et η =  0.17      :                  |
------------------------------------------------------------
| xmin                  |  1.23789802112164                |
| E(xmin)               |  -1.20189221322028               |
| nombre d’itérations   |  1001                                    |
------------------------------------------------------------
|       Pour x0 =  5  et η =  1         :                  |
------------------------------------------------------------
| xmin                  |  -5.59079531307448e+21652188973878495754495695422805957214958617858902001694879846
407266896926237053079339517693420495496050361648327821731030341296080361492950805511947649875540749466816689
789580790230892532945825476392689750152563970441165799639695772260876295871215436543710178339585965200349674
146193932277543526727655061945229124241057407202751282045230679860593450701803849100800922711106365638881554
6286284882054465270961178176662600323725951538570315669102818606466918046908649980290398 9431           |
| E(xmin)               |  9.76999563434790e+866087558955139830179827816912238288598344714356080067795193856
290675877049482123173580707736819819842014465933112869241213651843214459718032220477905995021629978672667591
583231609235701317833019055707590006102558817646631985587830890435051834848617461748407133583438608013986965
847757291101741069106202477809164969642296288110051281809227194423738028072153964032036908444254625555262185
145139528217861083844712706650401294903806154281262676411274425867672187634599921161595772 6           |
| nombre d’itérations   |  1001                                    |
------------------------------------------------------------
|       Pour x0 =  0  et η =  0.001     :                  |
------------------------------------------------------------
| xmin                  |  0.949407118070548               |
| E(xmin)               |  0.441491059862557               |
| nombre d’itérations   |  39                              |
```

Out[7]:

```
[-5.59079531307448e+2165218897387849575449569542280595721495861785890200169487984640726689692623705307933951
769342049549605036164832782173103034129608036149295080551194764987554074946681668978958079023089253294582547
639268975015256397044116579963969577226087629587121543654371017833958596520034967414619393227754352672765506
194522912424105740720275128204523067986059345070180384910080092271110636563888155462862848820544652709611781
766626003237259515385703156691028186064669180469086499802903989431,
 9.76999563434790e+8660875589551398301798278169122382885983447143560800677951938562906758770494821231735807 0
773681981984201446593311286924121365184321445971803222047790599502162997867266759158323160923570131783301905
570759000610255881764663198558783089043505183484861746174840713358343860801398696584775729110174106910620247
780916496964229628811005128180922719442373802807215396403203690844425462555526218514513952821786108384471270
66504012949038061542812626764112744258676721876345999211615957726,
 1001]
```

5. Visualisez l'évolution des minimums de la fonction E(x) trouvés au cours des itérations.

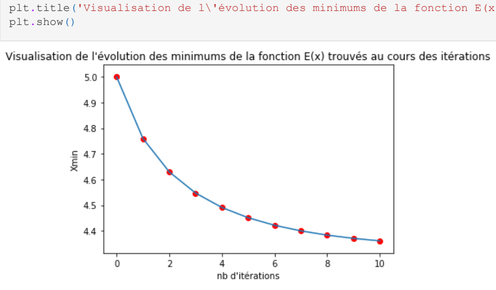
In [9]:

```
def DG(div_e, x0, n, eps = 0.01, nb_max_iter = 1000):
    cpp = 0 # compteur pour calculer le nombre d'itération
    Xmins = []
    Xmins.append(x0)
    while True:
        # calculer le nouveau x (xi+1)
        x1 = x0 - n*div_e.subs(x, x0).evalf()
        cpp += 1
        # condition d'arrêt
        Xmins.append(x1)
        if ( abs(x1 - x0) < eps ) | ( cpp > nb_max_iter ):
            break
        x0 = x1
    # resultat return le nouveau x et le nombre d'itération
    return [Xmins, cpp]

import numpy as np
import matplotlib.pyplot as plt

xmins, nb_ite = np.array(DG(div_exp, 5, 0.01))

plt.scatter(range(nb_ite+1), xmins, c='r')
plt.plot(range(nb_ite+1), xmins)
plt.xlabel('nb d\'itérations')
plt.ylabel('Xmin')
plt.title('Visualisation de l\'évolution des minimums de la fonction E(x) trouvés au cours des itérations')
plt.show()
```



6. Testez votre algorithme avec d'autres valeurs de eps et nombre max.

In [10]:

```
def sprint(div_e, data):
    xmin, ite = DG(div_e, data[0], data[1])
    print('-'*60)
    print('| \tPour x0 = ', data[0],' et η = ', data[1], '\t:\t\t   |')
    print('-'*60)
    print('| xmin \t\t\t| ', xmin, "\t\t   |")
    xmin_f = exp.subs(x, xmin).evalf()
    print('| E(xmin) \t\t| ', xmin_f, "\t\t   |")
    print('| nomber des iteration \t| ', ite, "\t\t\t\t   |")
    return xmin, xmin_f

dict1 = [[1, 0.001], [6, 0.001], [6, 0.01], [4, 0.1], [4, 0.17]]

for d in dict1:
    sprint(div_exp, d)
```

```
------------------------------------------------------------
|       Pour x0 =  1  et η =  0.001     :                  |
------------------------------------------------------------
```

```
| xmin                  |  [1, 1.00800000000000]                   |
| E(xmin)               |  x**4 - 11.0*x**3 + 41.0*x**2 - 61.0*x + 30.0                    |
| nomber des iteration  |  1                               |
------------------------------------------------------------
|       Pour x0 =  6  et η =  0.001     :                  |
------------------------------------------------------------
| xmin                  |  [6, 5.89300000000000, 5.79818438917200, 5.71344305858812, 5.63714841968474, 5.568
02121065637, 5.50503952526897, 5.44737544559007, 5.39434985365062, 5.34539953823634, 5.30005281371908, 5.257
91115469927, 5.21863516112886, 5.18193369253159, 5.14755535614586, 5.11528176723680, 5.08492216009623, 5.056
30904013533, 5.02929464677417, 5.00374805382113, 4.97955277551832, 4.95660477698370, 4.93481081053332, 4.914
08701648401, 4.89435774003588, 4.87555452579438, 4.85761525918669, 4.84048343001917, 4.82410749812207, 4.808
44034473993, 4.79343879627631, 4.77906320936224, 4.76527710811615, 4.75204686599971, 4.73934142592307, 4.727
13205327424, 4.71539211738605, 4.70409689764592, 4.69322341102674, 4.68275025829436, 4.67265748654537, 4.662
92646606287]               |
| E(xmin)               |  x**4 - 11.0*x**3 + 41.0*x**2 - 61.0*x + 30.0                    |
| nomber des iteration  |  41                              |
------------------------------------------------------------
|       Pour x0 =  6  et η =  0.01      :                  |
------------------------------------------------------------
| xmin                  |  [6, 4.93000000000000, 4.72509072000000, 4.60846931728543, 4.53307608572641, 4.481
08358851332, 4.44380531005350, 4.41638377433183, 4.39585145084092, 4.38028037898460, 4.36836062409739, 4.359
17182179604]               |
| E(xmin)               |  x**4 - 11.0*x**3 + 41.0*x**2 - 61.0*x + 30.0                    |
| nomber des iteration  |  11                              |
------------------------------------------------------------
|       Pour x0 =  4  et η =  0.1       :                  |
------------------------------------------------------------
| xmin                  |  [4, 4.50000000000000, 4.07500000000000, 4.49139375000000, 4.09033596314547, 4.487
46424287683, 4.09723514586139, 4.48545025898770, 4.10074620644149, 4.48436628136908, 4.10262895433799, 4.483
76856250918, 4.10366503293212, 4.48343472943744, 4.10424304807444, 4.48324697284468, 4.10456793543805, 4.483
14096304211, 4.10475130627544, 4.48308097818159, 4.10485504459495, 4.48304699454819, 4.10491380953348, 4.483
02772821786, 4.10494712298755, 4.48301680127560, 4.10496601610128, 4.48301060265721, 4.10497673353596, 4.483
00708587130, 4.10498281400109, 4.48300509048017, 4.10498626397670, 4.48300395826814, 4.10498822153239, 4.483
00331582084, 4.10498933229930, 4.48300295127436, 4.10498996258552, 4.48300274441674, 4.10499032023380, 4.483
00262703730, 4.10499052317791, 4.48300256043128, 4.10499063833691, 4.48300252263617, 4.10499070368302, 4.483
00250118960, 4.10499074076323, 4.48300248901989, 4.10499076180413, 4.48300248211426, 4.10499077374368, 4.483
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| E(xmin)               |  x**4 - 11.0*x**3 + 41.0*x**2 - 61.0*x + 30.0                    |
| nomber des iteration  |  1001                            |
------------------------------------------------------------
|       Pour x0 =  4  et η =  0.17      :                  |
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| xmin                  |  [4, 4.85000000000000, 1.99502000000001, 1.48336482859609, 1.29985340908275, 1.535
20416176282, 1.26598771774773, 1.59966520234823, 1.24239070441334, 1.64867602179201, 1.23755864714305, 1.659
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39377699626]             |
| E(xmin)              |  x**4 - 11.0*x**3 + 41.0*x**2 - 61.0*x + 30.0                    |
| nomber des iteration  |  1001                                        |
```

Descente de gradient pour la régression linéaire

---

1. Calculation des dérivées partielles de la fonction G(a; b) selon a et b

In [11]:

```
# In our example, G(x,y) = SUM(ax+y - y)
def G(x, y, a, b):
    return np.sum((a*x+b-y)**2)
# define derive of fonction
def derive_G(x, y, a, b):
    return {'dx': 2*(a*x**2+b*x-y*x), 'dy': 2*(a*x+b-y)}

def y(x, a, b):
    return (a*x+b)
```

2. Implémentez l'algorithme DGR

In [12]:

```
#Gradient Descent implied for a regression linear case with data
def gradient_reg(max_iter, step, x, y, df,eps = 0.01):
    #Const
    nb_sample_data = x.shape[0]
    rg_sample = range(1, nb_sample_data)

    #initialisation
    a = 5 #np.random.random(x.shape[1])
    b = 5 #np.random.random(x.shape[1])
    E = sum([a*x[j]+b-y[j] for j in rg_sample])

#     print(E)
    for i in range(max_iter):
        #Compute gradient
        grad_a = sum([2*(a*x[j]**2+b*x[j]-y[j]*x[j]) for j in rg_sample])
        grad_b = sum([2*(a*x[j]+b-y[j]) for j in rg_sample])

        #Descente de gradient
        a = a - step * grad_a
        b = b - step * grad_b


        #Compute error
        e = sum([a*x[j]+b-y[j] for j in rg_sample])

        if abs(E-e) <= eps:
            return ({'a': a, 'b': b}, i)

        E = e

    return ({'a': a, 'b': b}, max_iter)
```

3. test d'algorithme implémenté

In [13]:

```
from sklearn import datasets
x_reg, y_reg = datasets.make_regression(n_samples=100, n_features=1, noise=7)
all_final_reg = []
print(x_reg.reshape(100).shape)
print(y_reg.shape)
```

```
(100,)
(100,)
```

In [14]:

```
def show_res_reg(vect,derive_G):
    print("***************************************************************")
    res,nb_ite = gradient_reg(vect[0],vect[1],vect[2],vect[3],derive_G)
    print("for initial max iteration = ",vect[0]," , step = ",vect[1])
    print(' ==> final_a = ',res['a'],' final_b = ',res['b'],'nb_iteration = ',nb_ite)
    pry = [y(x_d[0],res['a'],res['b']) for x_d in x_reg]
#     print("final result =  ",G(x_reg,y_reg,res['a'],res['b']))
    print('E(', x_reg[0][0], ') = ', pry[0])
    all_final_reg.append(pry)
    print("***************************************************************")
```

4. Affichage des resultas trouvés

In [15]:

```
all_final_reg = []
show_res_reg([100,0.001,x_reg,y_reg],derive_G)
show_res_reg([500,0.001,x_reg,y_reg],derive_G)
show_res_reg([1000,0.001,x_reg,y_reg],derive_G)
show_res_reg([100,0.01,x_reg,y_reg],derive_G)
show_res_reg([100,1,x_reg,y_reg],derive_G)
```

```
***************************************************************
for initial max iteration =  100  , step =  0.001
 ==> final_a =  [93.66725942]  final_b =  [0.01392354] nb_iteration =  49
E( -0.7493494090740423 ) =  [-70.17558196]
***************************************************************
***************************************************************
for initial max iteration =  500  , step =  0.001
 ==> final_a =  [93.66725942]  final_b =  [0.01392354] nb_iteration =  49
E( -0.7493494090740423 ) =  [-70.17558196]
***************************************************************
***************************************************************
for initial max iteration =  1000  , step =  0.001
 ==> final_a =  [93.66725942]  final_b =  [0.01392354] nb_iteration =  49
E( -0.7493494090740423 ) =  [-70.17558196]
***************************************************************
***************************************************************
for initial max iteration =  100  , step =  0.01
 ==> final_a =  [-2.57176602e+08]  final_b =  [-76344971.72586623] nb_iteration =  100
E( -0.7493494090740423 ) =  [1.16370163e+08]
***************************************************************
***************************************************************
for initial max iteration =  100  , step =  1
 ==> final_a =  [-1.51482288e+235]  final_b =  [-4.49648641e+234] nb_iteration =  100
E( -0.7493494090740423 ) =  [6.85482986e+234]
***************************************************************
```

5. utilisation des stats.linregress de scipy

In [16]:

```
from scipy import stats
slope, intercept, r_value, p_value, std_err = stats.linregress(x_reg.reshape(100), y_reg)
np.array(all_final_reg[0]).shape
```

Out[16]:

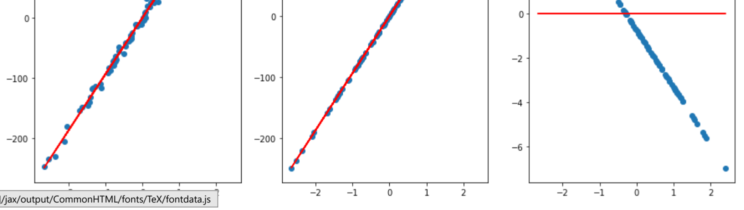
```
(100, 1)
```

6. Visualisation

In [17]:

```
print("r-squared:", r_value**2)

plt.rcParams["figure.figsize"] = (15,7)
plt.subplot(1, 3, 1)
plt.plot(x_reg, y_reg, 'o', label='original data')
plt.plot(x_reg, intercept + slope*x_reg, 'r', label='fitted line')
plt.title('make_regression data')
plt.legend()


print("r-squared:", r_value**2)
plt.subplot(1, 3, 2)
plt.plot(x_reg, all_final_reg[0], 'o', label='original data')
plt.plot(x_reg, intercept + slope*x_reg, 'r', label='fitted line')
plt.title('a) with (a =  82.04, b =  5.54e-5)')
plt.legend()

print("r-squared:", r_value**2)
plt.subplot(1, 3, 3)
plt.plot(x_reg, all_final_reg[3], 'o', label='original data')
plt.plot(x_reg, intercept + slope*x_reg, 'r', label='fitted line')
plt.title('d) with (a =  39.30, b =  8.28)')
plt.legend()

plt.show()
```

```
r-squared: 0.9953015220740956
r-squared: 0.9953015220740956
r-squared: 0.9953015220740956
```



Links

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