

1 Question 1

We have $z_4^{(1)} = z_1^{(1)}$; $z_2^{(1)} = z_6^{(1)}$ and $z_3^{(1)} = z_5^{(1)}$; $\alpha_{1,2}^{(2)} = a^{(2)T} \cdot [W^{(2)} \cdot z_1^{(1)} || W^{(2)} \cdot z_2^{(1)}] = \alpha_{4,2}^{(2)} = \alpha_{4,6}^{(2)}$.

Then $\alpha_{4,5}^{(2)} = a^{(2)T} \cdot [W^{(2)} \cdot z_4^{(1)} || W^{(2)} \cdot z_5^{(1)}] = \alpha_{4,3}^{(2)}$.

$$z_1^{(2)} = \text{softmax}(\text{LeakyRelu}(\alpha_{1,2}^{(2)})) \cdot z_2^{(1)} + \text{softmax}(\text{LeakyRelu}(\alpha_{1,3}^{(2)} \cdot z_3^{(1)})).$$

Thus

$$z_4^{(2)} = \text{softmax}(\text{LeakyRelu}(\alpha_{4,2}^{(2)})) \cdot z_2^{(1)} + \text{softmax}(\text{LeakyRelu}(\alpha_{4,3}^{(2)})) \cdot z_3^{(1)} + \text{softmax}(\text{LeakyRelu}(\alpha_{4,5}^{(2)})) \cdot z_4^{(1)} + \text{softmax}(\text{LeakyRelu}(\alpha_{4,6}^{(2)})) \cdot z_6^{(1)}$$

$$\text{so } z_4^{(2)} = 2 \cdot \text{softmax}(\text{LeakyRelu}(\alpha_{4,2}^{(2)})) \cdot z_2^{(1)} + 2 \cdot \text{softmax}(\text{LeakyRelu}(\alpha_{4,3}^{(2)})) \cdot z_3^{(1)}$$

$$\text{we have then, } \text{softmax}(\text{LeakyRelu}(\alpha_{4,2}^{(2)})) = \frac{\text{softmax}(\text{LeakyRelu}(\alpha_{1,2}^{(2)}))}{2} \text{ and } \text{softmax}(\text{LeakyRelu}(\alpha_{4,3}^{(2)})) = \frac{\text{softmax}(\text{LeakyRelu}(\alpha_{1,2}^{(2)}))}{2}$$

$$\text{So finally } z_4^{(2)} = z_1^{(2)}$$

2 Question 2

The accuracy drops drastically, in fact it went from 1 to only 0.4, this is due to the fact that when using the same features values for all nodes the final output will depend only on the degree of the node. In other words, nodes with same degree will be viewed as the same thing.

3 Question 3

sum :

$$z_{G1} = [2.9 \quad 2.3 \quad 1.9]$$

$$z_{G2} = [3.4 \quad 1.9 \quad 4.3]$$

$$z_{G3} = [1.8 \quad 1.2 \quad 1.6]$$

max :

$$z_{G1} = [2.2 \quad 1.1 \quad 1.5]$$

$$z_{G2} = [2.2 \quad 1.8 \quad 1.5]$$

$$z_{G3} = [2.2 \quad 1.8 \quad 1.5]$$

mean :

$$z_{G1} = [0.96 \quad 0.76 \quad 0.63]$$

$$z_{G2} = [0.85 \quad 0.47 \quad 1.075]$$

$$z_{G3} = [0.9 \quad 0.6 \quad 0.8]$$

Hence the best way to separate the graphs is with using the sum or the mean.

4 Question 4

If we set all the features to 1, in the two regular graphs all the nodes have two edges. So the output vector z for each node will be the same for every node in G_1 or in G_2 . Now as we use the sum as readout function, and the output is Z with 12 identical rows (4+8). 4 for G_1 and 8 for G_2 we will have then,

$Z_{G1} = 4.Z[0]$ and $Z_{G2} = 8.Z[0]$ with x the vector representation for each node. In final we have, $Z_{G2} = 2.Z_{G1}$