LIBANESE UNIVERSITY
Faculty of Sciences
Section 3

الجامعة اللبنانية
كلية العلوم
الفرع الثالث

Course: P1100 | **Year: 2024-2025**
Duration: 90 min | **Exam: Final**

$\frac{dv}{dt} = kv^2$ $\frac{dv}{V^2} = kd$ $-\frac{1}{V} = k$

Problem I *: (5 min)*

A particle moves along a straight line with an acceleration $\vec{a} = kv^2\hat{\imath}$.

1. Find the **dimension** of the constant k.
2. Determine the **velocity** of the particle as a function of x, given that for $x = 0$; $v = 0$.

Problem II: *(8 min)*

The position vector of a particle M at any time t in the cartesian coordinate system xy, is given by:
$\vec{r} = (t^2 + 1)\vec{\imath} - 6t\vec{\jmath}$

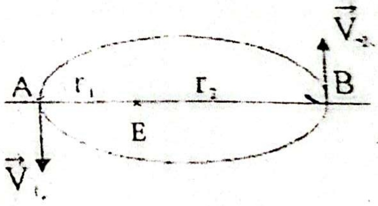
$adx = vdV$
$v^2kdx = Vd$

a) Find the equation of the **trajectory** of M.
b) Determine the **velocity vector** of M and its **magnitude** at any time t.
c) Find the **acceleration vector** of M and its **magnitude** at any time t.
d) Calculate the magnitudes of the **tangential and normal accelerations**, as well as the **radius of curvature** at any time t.

$V\,kdx$
kdx

Problem III *: (5 min)*

An artificial satellite orbits around the Earth in an elliptical trajectory with radius $r_A = 5R$ and $r_B = 10R$ where R is the radius of Earth. The satellite's speeds at points A and B are V_1 et V_2, respectively (see figure).
Using conservation laws, determine V_1 and V_2.
Given :$G = 6.67 \times 10^{-11} SI,$ $\quad M_T = 5.9 \times 10^{24} Kg$
and $\quad R = 6400Km.$

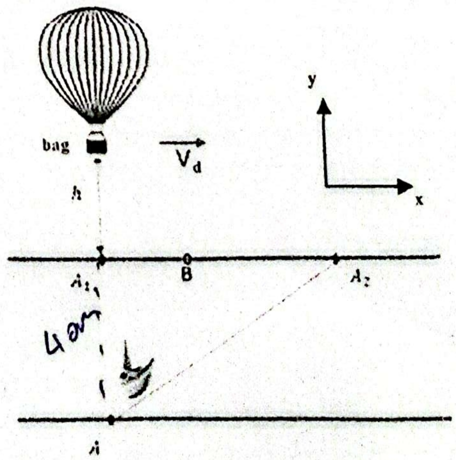


kdt
$kx =$

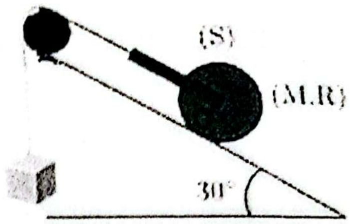
Problem IV *: (20 min)*

A dirigible flies at an altitude ***h*** with a horizontal velocity $\overrightarrow{v_d}$. At time t_0, it is located above A_1, a point on one shore of a river. On the opposite shore, at point A, a boat is ready to cross the river.
The river has a width of AA_1=40 m, and its current flows at a speed of v_{water} = *10 m/s*, parallel to the shores. The boat moves at a speed of v_b = 20 m/s relative to the water. To reach the other shore at point A_2, the captain directs the boat along a path (AB) that forms an angle of 15° with (AA_1).

1. Determine the **relative velocity vector** $\vec{v}_r$ of the boat with respect to the water in the river.
2. Find the **absolute velocity vector** of the boat as well as the **angle** that defines the direction followed by the boat with respect to the shore.
3. Calculate the **time** required to reach A_2.
4. At time t_0, the dirigible drops a bag.
   Determine the **altitude** ***h*** and the **speed** v_d required for the bag to meet the boat at A_2.

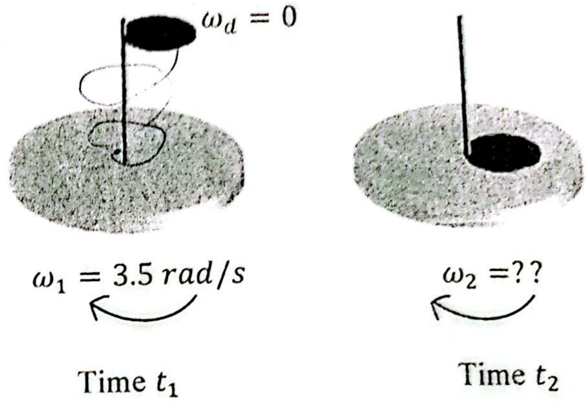


Problem V *: (15 min)*
A sphere (S) with masse M = 4 kg, radius R = 20 cm and moment of inertia I, $I = \frac{2}{5}MR^2$ **can roll without slipping** on an inclined plane at an angle $\alpha = 30°$ to the horizontal. A block with mass m = 2 kg is attached to the sphere by an inextensible and massless wire. The wire passes over a pulley with negligible mass.
Calculate the tension T in the wire and the linear acceleration of the sphere, specifying the direction of motion.

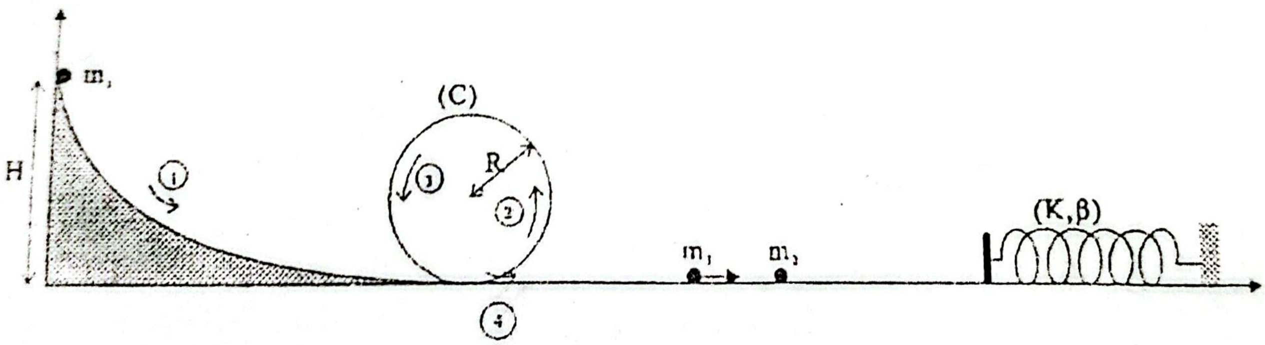


Problem VI *: (7 min)*
A disk (d) with a radius of $r = 5\ cm$ and a mass of 100 g, initially at rest at time t_1, falls onto a rotating platform (P) at time t_2 as illustrated in the figure. **What is the angular speed ω_2 of the platform-disk system at time t_2?**
Given :

- Moment of inertia of the platform: $I_p = 0.5\ Kg.m^2$.
- Moment of inertia of the disk about its center of mass: $I_{d\%G} = \frac{1}{2}mr^2$.



Problem VII *: (30 min) **(Parts A and B are independent)***



A. A mass $m_1 = 2\ kg$ is released from rest at the top of a curved inclined track connected to the end of a vertical circular loop (C) with a radius $R = 40\ cm$. The other end of the loop leads to a horizontal track. $= 0{,}4 m$
   1. Determine the **minimum height H** from which mass m_1 must be released so that it can complete the loop without losing contact with the track?
   2. The mass m_1 continues moving horizontally and undergoes a **perfectly elastic collision** with another mass $m_2 = 3m_1$ initially at rest. Calculate the **velocity V_2** of mass m_2 after the collision.

B. After collision, we assume that mass m_2 continues its motion with a speed $V_0 = 2\sqrt{5}\ m/s$ and strikes a horizontal spring with a stiffness constant $K = 500\ N/m$, fixed to a vertical support. The friction forces in the mass-spring system are modeled as $f = \beta x$, with $\beta = 250$ (SI units).
   1. Calculate the **work** done by the spring force T and by the friction force f as functions of the spring compression x and speed V.
   2. Using the work-kinetic energy theorem, determine the **maximum compression** of the spring.

Good job !

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Course: P1100
Duration: 90 min

Year: 2024-2025
Exam: Session 2

Exercise I ***: (15 min)***

The coordinates of an object moving in the xy plane vary with time according to the equations

$$x(m) = -5\sin(\omega t) \text{ and } y(m) = 4 - 5\cos(\omega t),$$

where ω is a constant, x and y are in meters and t in seconds.

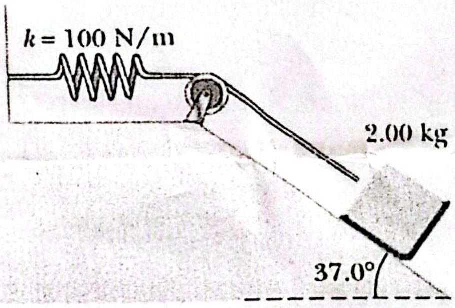
1. Determine the expressions for the position, velocity and acceleration vectors at any time $t > 0$.
2. Determine the normal and tangential components of the acceleration vector as well as the radius of curvature at any time t. Deduce the nature of the motion.
3. Describe the object's trajectory on an xy graph. Verify the nature of the motion and the previously obtained value of the radius of curvature.

Exercise II ***: (15 min)***

A 2-kg block situated on a rough incline is connected to a spring of negligible mass having a spring constant of 100 N/m. The pulley is frictionless.

The block is released from rest when the spring is unstretched. The block moves 20 cm down the incline before coming to rest.

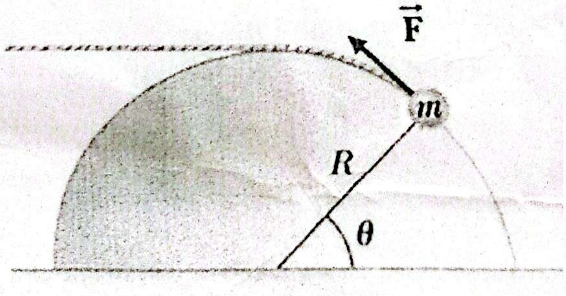
1. Find the normal force exerted by the incline on the block.
2. Calculate the coefficient of kinetic friction between the block and the incline.



Exercise III ***: (20 min)***

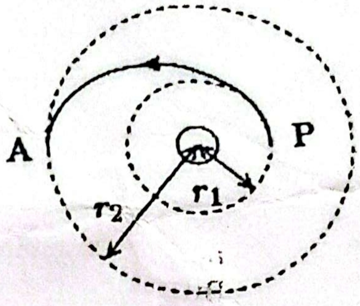
A small particle of mass m is pulled to the top of a frictionless half-cylinder (of radius R) by a light cord that passes over the top of the cylinder as illustrated in the figure below.

1. Assuming the particle moves at a constant speed, find the expression of the force F as a function of m, g and θ.
2. Find the work done by F in moving the particle at constant speed from the bottom to the top of the half-cylinder, as a function of m, g and R.
3. Now suppose the particle is pulled by a constant force F and its speed v is no longer constant. Determine an expression for the particle's speed v as a function of F, θ, m, R and g, given that $v = 0$ when $\theta = 0$.



Exercise IV : (10 min)

We need to transfer a satellite having a mass m, waiting on a circular orbit with radius $r_1 = 6700$ km to another circular orbit with radius $r_2 = 42000$ km, by passing through an elliptical transfer orbit tangent to both of the circular orbits.



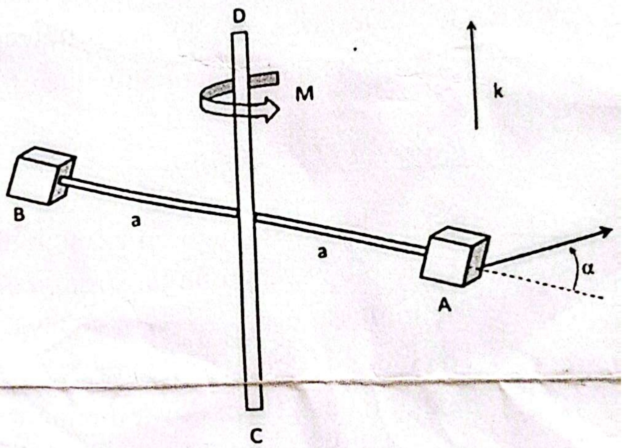
Given: $G = 6.67 \times 10^{-11}\ N.m^2.kg^{-2}$, and $M_{Earth} = 6 \times 10^{24}\ kg$.

1. Calculate the speed v_1 and v_2 of the satellite on the circular orbits with radius respectively r_1 and r_2.
2. By using the conservation law of the total energy, calculate the satellite speeds v_P and v_A on the ellipse at point P and A respectively.
3. Deduce the variation of the speed at point P and A.

Exercise V : (15 min)

Two blocks, A and B, each have a mass of $m_A = m_B = 200\ g$, and rotate in a horizontal plane about the vertical axis CD with an initial speed of $v = 1\ m/s$. A torque with a moment magnitude of $\vec{M} = 0.3\ t\ \vec{k}$ is applied about the axis CD, and a time-dependent force $f = 2t\ (N)$ is applied to block A at an angle $\alpha = 30°$ relative to the line AB. The radius of rotation is $a = 0.3$ m.

Determine the velocity of both blocks A and B at $t = 2\ s$.



Exercise VI : (15 min)

The setup illustrated below is used to measure the speed of a moving object, such as a bullet.
A bullet of mass m is fired with the initial speed v_i into a block of mass M initially at rest at the edge of a frictionless table of height h. The bullet remains in the block and after impact, the block lands a distance d from the bottom of the table.

1. Find the expression of the speed v_f of the system (bullet, block) just after collision as a function of g, d and h.
2. Deduce the initial speed v_i of the bullet just before collision as a function of m, M, g, d and h.