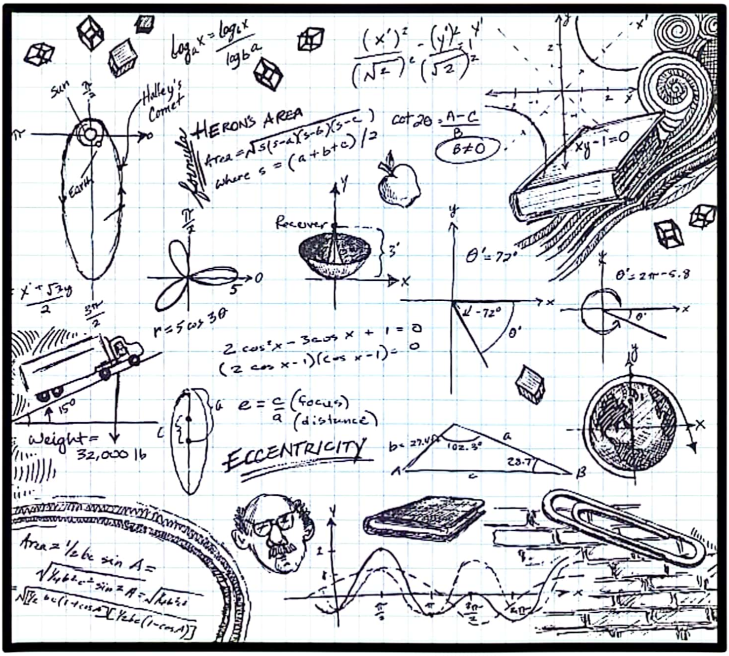
Summary - PHYSICS
P1100 : " MECHANICS "



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2023 - 2024

①

Pl100

Ch 2: Motion along a st line (ملخص)

- Displacement: $\Delta x = x_2 - x_1$ (m)

$\Delta x > 0 \Rightarrow$ along +ve x-direction

$\Delta x < 0 \Rightarrow$ along -ve x-direction

- Average Velocity:

$$V_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \quad (m/s)$$

- Graphical determination of V_{avg}:

draw line $(t_1, x_1, - t_2, x_2 ---)$

determine the slope of the line

- Average Speed S_{avg}:

$$S_{avg} = \frac{\text{Total distance}}{\Delta t} \quad (m/s)$$

- Instantaneous Velocity:

$$V = \frac{dx}{dt} \quad (m/s)$$

- Average Acceleration:

$$a_{avg} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{\Delta V}{\Delta t} \quad (m/s^2)$$

- Instantaneous Acceleration:

$$a = \frac{dv}{dt} = \frac{d^2x}{d^2t} \quad (m/s^2)$$

- Motion with Constant Acceleration:

$V = at + C$, $C = V_0$

$a = 0 \Rightarrow V = V_0 \Rightarrow x = vt + x_0$

a = Const $\begin{cases} V = V_0 + at & (m/s) \\ x = x_0 + V_0 t + \frac{at^2}{2} & (m) \end{cases}$

$$V^2 - V_0^2 = 2a(x - x_0)$$

Ch 4: Motion in two & Three dimentions (ملخص)

- Displacement: $\Delta r = r_2 - r_1$ (m)
$= (x_2 - x_1, y_2 - y_1, z_2 - z_1)$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\|\vec{r}\| = \sqrt{x^2 + y^2 + z^2}$

- Average velocity: $\vec{V}_{av} = \frac{\Delta \vec{r}}{\Delta t}$
- Instantenous ,, : $\vec{v} = \frac{d\vec{r}}{dt}$ $\|\vec{v}\| = \sqrt{v_x^2 + v_y^2}$
- Average acceleration: $\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$

- Instantenous ,, : $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$ $\|\vec{a}\| = \sqrt{a_x^2 + a_y^2}$
- Uniform Circular Motion:

$\vec{v}$ = constant $\Rightarrow$ velocity vector is constant $\Rightarrow \vec{a} = \frac{d\vec{v}}{dt} = \vec{0}$

$\Rightarrow$ Uniform motion is St line

- Speed = constant $\Rightarrow$ Velocity is not constant $\Rightarrow$ changing direction

$\Rightarrow$ motion in a circle $\Rightarrow \vec{a} \neq \vec{0}$

$\|\vec{v}\| \neq cte$

$\vec{v}$ tangent to the circle

The acceleration is radial & towards center

$a_r = \frac{v^2}{r}$ $\quad a_t = \frac{d\|\vec{v}\|}{dt} = 0 \Rightarrow \vec{a} = \vec{a}_n + \vec{a}_t$

tangential acceleration along $\vec{v}$

The time T to make a full revoluation is:

$$\boxed{T = \frac{2\pi r}{v}}$$

$$\vec{a}_t = C\,\vec{v}$$
$$\Downarrow$$
$$\frac{d\|\vec{v}\|}{dt}$$

Ch5: Force & Motion I (ملغى)

- Newton's Second Law:
$$\vec{F}_{net} = m\vec{a}$$

- The Gravitational Force:
$$|\vec{F_g}| = mg$$

- Weight:
$$W = mg \qquad (N)$$

- Newton's Third Law:

2 bodies interact → Forces are equal in mag → opp in direction

$$\vec{F}_{CB} = -\vec{F}_{BC}$$

Ch6: Force & Motion II

- Friction:
- Static:

$$f_{s.max} = \mu_s F_N \qquad 0 < f_s \leq \mu_s F_N$$

- Kinetic:

$$f_K = \mu_K F_N \qquad f_K < f_{s\,max}$$

- Uniform Circular Motion, Centripetal Force:

$$a_n = \frac{v^2}{r} \Rightarrow F = \frac{mv^2}{r} \qquad a_t = \frac{dv}{dt} = 0 \quad (v = \text{const})$$

Ch7: Kinetic Energy & Work.

- Kinetic Energy:

$$K = \frac{mv^2}{2} \qquad (J)$$

- Work done by Force: $W = \vec{F}\cdot\vec{d} = F.d.\cos\phi$

- Work-Kinetic Energy Theorem:

$$\Delta K = K_f - K_i = W_{net}$$

$$W_{net} > 0 \longrightarrow K_f - K_i > 0 \longrightarrow K_f > K_i$$

$$W_{net} < 0 \longrightarrow K_f - K_i < 0 \longrightarrow K_f < K_i$$

- Work Done by the Gravitational Force:

$$W_{g(A \to B)} = mgd \cos(180°) = -mgd$$

$$W_{g(B \to A)} = mgd \cos(0°) = mgd$$

- Work done by a variable Force $F(x)$:

$$W = \int_{x_i}^{x_f} F(x)\,dx$$

- Spring Force: $F = -Kx$

- Work done by spring Force: $W_s = -K\frac{x^2}{2} = \frac{1}{2}Kx_i^2 - \frac{1}{2}Kx_f^2$

- Three dimensional Analysis:

$$W = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz$$

- Power:

$$P_{avg} = \frac{W}{\Delta t}$$ average power (W: watt).

$$P = \frac{dW}{dt}$$ Instantaneous power.

$$P = Fv\cos\phi = \vec{F}\cdot\vec{v}$$

ch8: Conservation of Energy

- potential energy:
  - gravitational: $U_{mg} = -W_{mg} = mgh$.
- Elastic energy:
  - Spring: $U_s = -W_s = -\frac{1}{2}K(x_i^2 - x_f^2) = \frac{1}{2}K(x_f^2 - x_i^2)$

- In general: $\boxed{\Delta U = -\int \vec{F}\cdot d\vec{r}}$

- Conservation of Energy:

$\Delta K + \Delta U + \Delta E_{th} = W_{ext\ Force}$.

$\Delta K = K_f - K_i$

$\Delta U = (U_f - U_i)_{grav} + (U_f - U_i)_{Elastic}$

$\Delta E_{th} = -$ Work due to friction $= -W_f = -\vec{F}\cdot\vec{d} = F.d$

ch9: Center of Mass & Linear Momentum

- Center of Mass:

1. Discrete System:

$x_{c.m} = \frac{\sum m_i x_i}{M}$ & $y_{c.m} = \frac{\sum m_i y_i}{M}$

2. Continous System:

$x_{c.m} = \frac{1}{M}\int x.dm$, $y_{cm} = \frac{1}{M}\int y.dm$ & $z_{cm} = \frac{1}{M}\int z\,dm$

where $M = \int dm$

1-D: $dm = \lambda dx$

2-D: $dm = \sigma dA = \sigma dx dy$

3-D: $dm = \rho dV = \rho dx dy dz$

Uniform System: $\rho = \lambda = \sigma =$ consta

- Newton's second law for c.m:

$$\vec{F}_{net} = M\vec{a}_{c.m} = \frac{d}{dt}\vec{P}_{cm}$$

- Collisions & Impulse:

The impulse $\vec{J} = \Delta\vec{p} = \vec{P}_f - \vec{P}_i$

$\vec{J} = \Delta\vec{p} = \vec{F}_{net}\,\Delta t$ (F const)

$\vec{J} = \int \vec{F}\,dt$ (F variable)

P (Kgm/s) - Conservation of linear momentum: Inelastic collision 1) KE loss

Isolated System: $F_r = 0$ 2) Heat created

$\vec{F}_{net} = \vec{0} \Rightarrow \frac{d\vec{p}}{dt} = 0$ $\vec{P}_{sys} = $ cont $\Rightarrow \vec{P}_i = \vec{P}_f$ 3) Damage Don

- Elastic collision:

1. Conservation of linear momentum: 1) No KE loss

$m_1\vec{V}_{1i} + m_2\vec{V}_{2i} = m_1\vec{V}_{1f} + m_2\vec{V}_{2f}$ 2) No Damage

2. Conservation of Kinetic energy: 3) No Heat

$$\frac{1}{2}m_1V_{1i}^2 + \frac{1}{2}m_2V_{2i}^2 = \frac{1}{2}m_1V_{1f}^2 + \frac{1}{2}m_2V_{2f}^2$$

$\Downarrow$

$$V_{1f} = \frac{(m_1 - m_2)V_{1i} + 2m_2V_{2i}}{m_1 + m_2}$$

(+ve or −ve)

$$V_{2f} = \frac{2m_1V_{1i} + (m_2 - m_1)V_{2i}}{m_1 + m_2}$$

Ch 10: Rotation

- Rotational variables:

$S = R\theta$ (θ: radian: rd) If only rotation

. Angular velocity average:

$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t} \quad rd/s$$

- Instanteneous angular velocity:

$$\omega = \frac{d\theta}{dt} \quad rd/s$$

- Direction:

1) $\omega > 0 \Rightarrow$ Counterclockwise
2) $\omega < 0 \Rightarrow$ clockwise
by Right-Hand Rule

- Angular Acceleration:

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t} \quad (rd/s^2) \quad \text{(Average)}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \quad (rd/s^2) \quad \text{(instan)}$$

- Rotation with constant α:

$x \longleftrightarrow \theta$, $v \longleftrightarrow \omega$, $\alpha \longleftrightarrow a$

$v = v_0 + a(t - t_0) \longrightarrow \omega = \omega_0 + \alpha(t - t_0)$

$x = x_0 + v_0 t + \frac{at^2}{2} \longrightarrow \theta = \theta_0 + \omega_0 t + \frac{\alpha t^2}{2}$

$v^2 - v_0^2 = 2a(x - x_0) \longrightarrow \omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)$

- Moment of Inertia:

- Discrete system:

$I = \sum_{i=1}^{N} m_i r_i^2$ $(Kg\,m^2)$ (about axis of rotation)

- Continous System: $I = \int r^2\,dm$

1-D $\rightarrow dm = \lambda\,dx$

2-D $\rightarrow dm = \sigma\,dA$

3-D $\rightarrow dm = \rho\,dV$

- Parallel-Axis theorem:

If we Know I_{cm} + I_D where $(D) \parallel$ axis passing through c.m

$\Rightarrow I_{(D)} = I_{cm} + Mh^2$ h: distance btw 2 Axis

- Torque: $\vec{\tau} = \vec{r} \wedge \vec{F}$ (N.m)
- Magnitude: $|\vec{\tau}| = rF\sin\theta$ $\qquad r_\perp = r\sin\theta$

$$= F(r\sin\theta) = F(r_\perp)$$

- Direction:

$\tau > 0 \rightarrow \vec{F}$ rotate the object in counterclockwise

$\tau < 0 \rightarrow$ " " " " " clockwise

- Newton's Second law for Rotation:

$$\tau = r_\perp F = r_\perp ma = r_\perp m r_\perp \alpha = m r_\perp^2 \alpha = I\alpha$$

$$\boxed{\vec{\tau}_{net} = I\vec{\alpha}}$$

- Rotational Energy:

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(r\omega)^2 = \frac{1}{2}(mr^2)\omega^2 = \frac{1}{2}I\omega^2$$

$$\boxed{K = \frac{1}{2}I\omega^2}$$

2π rad $\rightarrow 360°$

1 rev $\rightarrow 2\pi$

- Work - Kinetic energy theorem:

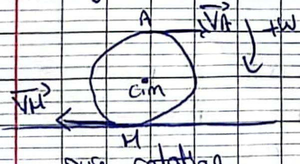
$$\Delta K = W_{ex} = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$$

to know max & min 2nd drive $>$ or $<$ 0

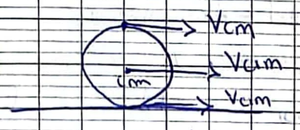
$$W_{ex} = \int_{\theta_i}^{\theta_f} \tau_{net}\, d\theta$$

Ch 11: Rolling, Torque, Angular Momentum

Rolling: Combination of pure rotation + translation



pure rotation



pure translation

$V_A = R\omega$, $V_M = -R\omega$

- Smooth rolling or rolling without slipping:

The velocity of the contact pt with ground $= 0$

$V_M = V_{c.m} + V_M = V_{c.m} - R\omega = 0$

$\Rightarrow \boxed{\begin{array}{l} V_{c.m} = R\omega \\ a_{c.m} = R\alpha \end{array}}$

Note That:

$\underset{\text{Rolling}}{V_{c.m}} = \underset{\text{trans}}{V_{c.m}} + \cancel{\underset{\text{Rotati}}{V_{c.m}}}^{\,0} \Rightarrow \boxed{\underset{\text{Rolling}}{V_{c.m}} = R\omega}$

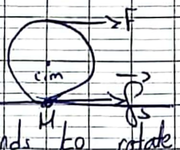
$V_A = V_{c.m} + R\omega = 2R\omega = 2V_{c.m}$

- Kinetic energy for Rolling

$K = K_{trans} + K_{Rot} = \frac{1}{2} m V_{c.m}^2 + \frac{1}{2} I_{c.m}\omega^2$

- Friction + Rolling:

Friction tends to oppose the tendency of motion



$t=0$ $\vec{F}$ tends to rotate the object firstly before any translation

$\Rightarrow$ M rotate to left $\Rightarrow V_M = 0 \Rightarrow$ force should act to the right

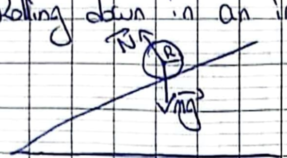
$\Rightarrow$ static to the right.

- Smooth Rolling $\Rightarrow$ Static friction
- Non-smooth Rolling $\Rightarrow$ Kinetic friction.

If F tends to do translation before rotation

$\Rightarrow \vec{f_s}$ to the left (opp to translation).

- Rolling down in an inclined plane:



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rotation before translation
⇓
Counterclockwise $\Rightarrow f_s$ to the right

- Newton's second law for rotation: $\vec{\tau}_{net} = I\vec{\alpha}$

$\tau_{mg} + \cancel{\tau_N} + \tau_{f_s} = I\alpha$

$\boxed{f_s R = I\alpha}$

- Newton's second law for translation

$\vec N + m\vec g + \vec{f_s} = m\vec a$

proj x-axis : $\boxed{mg\sin\theta - f_s = ma}$

" y-axis : $\boxed{N = mg\cos\theta}$

Smooth rolling : $\boxed{a = R\alpha}$

$$\begin{cases} f_s R = I\alpha \quad \text{\_\_\_ ①} \\ mg\sin\theta - f_s = ma \quad \text{\_\_\_ ②} \\ a = R\alpha \end{cases}$$

$$\Rightarrow f_s = \frac{Ia}{R^2} \quad \& \quad a = \frac{g\sin\theta}{1 + I/mR^2} \Rightarrow f_s = I/R^2 \, \frac{g\sin\theta}{1 + I/mR^2}$$

Hoop: $I = MR^2 \Rightarrow a = \frac{g\sin\theta}{2}$

cylinder $\Rightarrow I = \frac{1}{2}MR^2 \Rightarrow a = \frac{2g\sin\theta}{3}$

$a^{cyl} > a^{hoop}$

Angular momentum:

For point-like particles:

Translation: $\vec{p} = m\vec{v}$ (Kgm/s)

Rotation (about a fixed axis or a pt): $\vec{L} = \vec{r} \wedge \vec{p}$ (Kgm^2/s) or Js

Magnitude: $|\vec{L}| = |\vec{r} \wedge \vec{p}| = rp\sin\theta$ θ = angle $(\vec{r}, \vec{p})$

$\Rightarrow \boxed{|\vec{L}| = r_{\perp} p}$

$\perp$ drawn from O to p

Newton's second law for rotation:

Analogy: $\frac{d\vec{P}}{dt} = \vec{F}_{net} \xrightarrow{?} \left(\frac{d\vec{L}}{dt} \overset{?}{=} \vec{\tau}_{net}\right)$

$$\boxed{\frac{d\vec{L}}{dt} = \vec{\tau}_{net}}$$

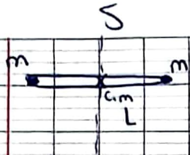
Angular Momentum of a rigid body: (about a fixed axis)

$$\boxed{L_z = I\omega}$$

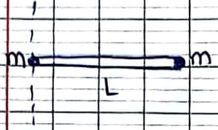
If the system Isolated

$(\vec{\tau}_{net} = \vec{0})$

$\Rightarrow \vec{L}_i = \vec{L}_f$ Conserved



$I = \frac{1}{2} m L^2$



$I = m L^2$

$I_{c.m} = \frac{M L^2}{12}$

$I = \frac{M L^2}{3}$

Rolling with slipping (no smooth)

$S \neq R\theta$ $\quad V \neq R\omega$ $\quad a \neq R\alpha$

Eth في

without slipping (smooth)

stat fr يكون في عنا

Eth بس ما بتحسب

Energy conserved لما ما يتغير الشكل

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Lebanese University
Faculty of Sciences I
Department of Physics

Course: Phys 1100 (Mechanics)
Final 2021-2022
27-04-2022 (90 minutes)

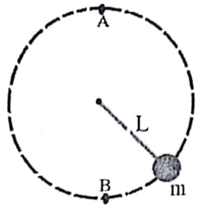
(Take $g = 10\ m/s^2$)

Problem I (16 Points)

A ball of mass $m = 275$ g swings in a vertical circular path on a string $L = 85$ cm long as in the adjacent figure.

a) Draw force diagrams for the ball when it is at the top of the circle (point A) and when it is at the bottom (point B).

b) If its speed is 5.2 m/s at point A, what is the tension in the string there?

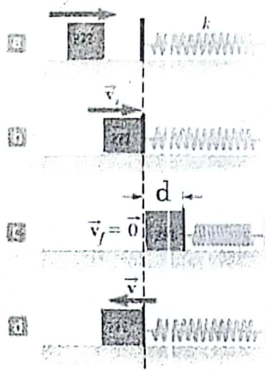


Problem II (20 Points)

An object of mass $m = 1kg$ slides to the right on a rough surface having a coefficient of kinetic friction $\mu_k = 0.25$ (Figure a). The object has a speed of $v_i = 3m/s$ when it makes contact with a massless spring (Figure b) that has a force constant of $k = 50\ N/m$. The object comes to rest $(v_f = 0)$ after the spring has been compressed a distance d (Figure c). The object is then forced toward the left by the spring (Figure d) and continues to move in that direction beyond the spring's unstretched position where it has a speed v. Find:

a) The distance of compression d.

b) The speed v at the unstretched position when the object is moving to the left (Figure d).

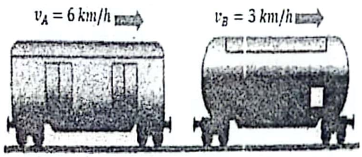


Problem III (20 Points)

In a train station, a wagon A (50 tons) was moving with a speed $v_A = 6\ km/h$ before hitting another wagon B (80 tons) that was moving with a speed $v_B = 3\ km/h$. Both wagons were moving in the same directions before collision, as shown in the figure.

a) Determine the speed of the system of two wagons if they moved together after the collision.

b) What is the relative variation of the kinetic energy due to the collision $\left(\frac{\Delta KE}{KE_i}\right)$?



$\frac{46}{3}$

$(2L)^2 + L^2$

$= 5L^2$ $\quad \frac{\sqrt{5L^2}}{2}$

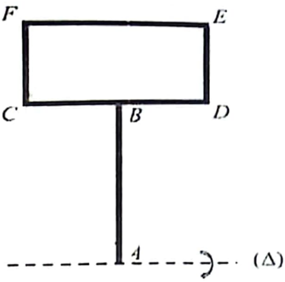
Problem IV (22 Points)

The structure shown in the adjacent figure is formed of a rigid rectangle CDEF (formed of four thin rods) that is connected to a thin rod AB. The rods AB, CD and EF are identical in mass ($2m$) and length ($2l$); the rods CF and ED are identical in mass (m) and length (l).

The system is placed in a vertical plane and able to rotate about a horizontal axis (Δ) that passes by the point A (see figure).

a) Show that the moment of inertia of the structure about the axis of rotation (Δ) is: $I_{structure/\Delta} = \frac{124}{3} ml^2$.

b) The structure starts rotating from rest ($\omega_0 = 0$). Find the expression of its angular speed ω (in terms of g and l) when it rotates about (Δ) an angle of 180°.

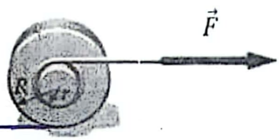


Hint:

The moment of inertia of a rod, about an axis that passes by its center of mass and perpendicular to it, is: $I_{com} = \frac{ml^2}{12}$.

Problem V (22 Points)

A heavy uniform cylinder of mass m, a radius R and $I_{com} = \frac{1}{2} mR^2$ is rolled smoothly by a force $\vec{F}$ on a horizontal table. This force is applied through a rope wound around a light drum of radius $r = R/4$ that is attached to the cylinder, as shown in the adjacent figure.



$\vec{f}_s \leftarrow$

a) Is the frictional force $\vec{f}$ (toward the left) between the cylinder and the table static or kinetic? Justify briefly

b) Find the acceleration $\boldsymbol{a}$ of the center of mass of the cylinder in terms of F and m.

c) Find the expression of the frictional force f in terms of F.