LIBANESE UNIVERSITY
Faculty of Sciences
Section 3

الجامعة اللبنانية
كلية العلوم
الفرع الثالث

**Course: P1100**
**Duration: 90 min**

**Year: 2024-2025**
**Exam: Session 2**

***Exercise I :*** *(15 min)*

The coordinates of an object moving in the xy plane vary with time according to the equations

$$x(m) = -5\sin(\omega t) \text{ and } y(m) = 4 - 5\cos(\omega t),$$

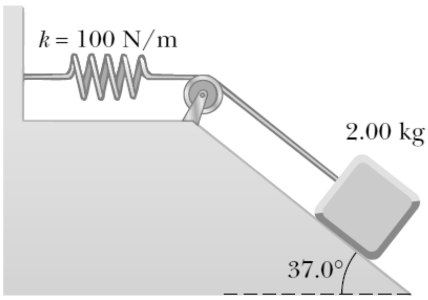
where $\omega$ is a constant, $x$ and $y$ are in meters and $t$ in seconds.

1. Determine the expressions for the position, velocity and acceleration vectors at any time $t > 0$.
2. Determine the normal and tangential components of the acceleration vector as well as the radius of curvature at any time t. Deduce the nature of the motion.
3. Describe the object's trajectory on an $xy$ graph. Verify the nature of the motion and the previously obtained value of the radius of curvature.

***Exercise II :*** *(15 min)*

A 2-kg block situated on a rough incline is connected to a spring of negligible mass having a spring constant of 100 N/m. The pulley is frictionless.

The block is released from rest when the spring is unstretched. The block moves 20 cm down the incline before coming to rest.

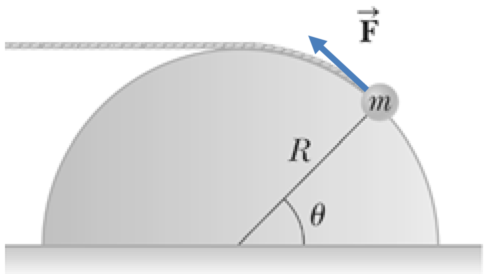



1. Find the normal force exerted by the incline on the block.
2. Calculate the coefficient of kinetic friction between the block and the incline.

***Exercise III :*** *(20 min)*

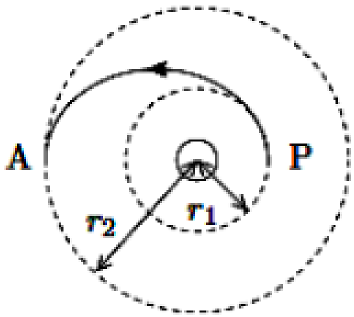
A small particle of mass m is pulled to the top of a frictionless half-cylinder (of radius R) by a light cord that passes over the top of the cylinder as illustrated in the figure below.

1. Assuming the particle moves at a constant speed, find the expression of the force $F$ as a function of $m, g$ and $\theta$.
2. Find the work done by F in moving the particle at constant speed from the bottom to the top of the half-cylinder, as a function of $m, g$ and $R$.
3. Now suppose the particle is pulled by a constant force F and its speed $v$ is no longer constant. Determine an expression for the particle's speed $v$ as a function of $F, \theta, m, R$ and $g$, given that $v = 0$ when $\theta = 0$.



***Exercise IV*** *: (10 min)*

We need to transfer a satellite having a mass m, waiting on a circular orbit with radius $r_1$ = 6700 km to another circular orbit with radius $r_2$ = 42000 km, by passing through an elliptical transfer orbit tangent to both of the circular orbits.



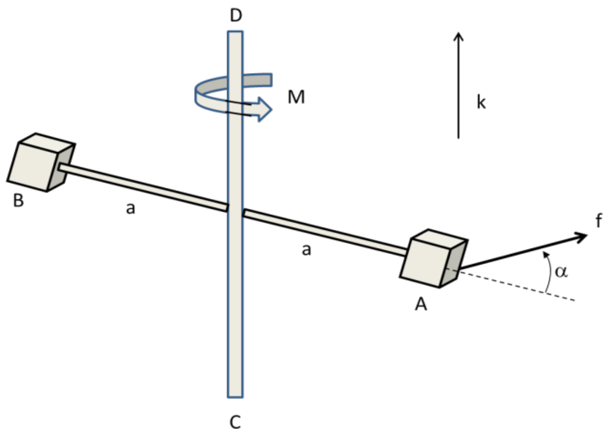

Given: G = $6.67 \times 10^{-11}\ N.m^2.kg^{-2}$, and $M_{Earth} = 6 \times 10^{24}\ kg$.

1. Calculate the speed $v_1$ and $v_2$ of the satellite on the circular orbits with radius respectively $r_1$ and $r_2$.
2. By using the conservation law of the total energy, calculate the satellite speeds $v_P$ and $v_A$ on the ellipse at point P and A respectively.
3. Deduce the variation of the speed at point P and A.

***Exercise V*** *: (15 min)*

Two blocks, A and B, each have a mass of $m_A = m_B = 200\ g$, and rotate in a horizontal plane about the vertical axis CD with an initial speed of $v = 1\ m/s$. A torque with a moment magnitude of $\vec{M} = 0.3\ t\ \vec{k}$ is applied about the axis CD, and a time-dependent force $f = 2t\ (N)$ is applied to block A at an angle α = 30° relative to the line AB. The radius of rotation is a = 0.3 m.

Determine the velocity of both blocks A and B at $t = 2\ s$.




***Exercise VI*** *: (15 min)*

The setup illustrated below is used to measure the speed of a moving object, such as a bullet.

A bullet of mass $m$ is fired with the initial speed $v_i$ into a block of mass $M$ initially at rest at the edge of a frictionless table of height $h$. The bullet remains in the block and after impact, the block lands a distance $d$ from the bottom of the table.

1. Find the expression of the speed $v_f$ of the system (bullet, block) just after collision as a function of $g, d$ and $h$.
2. Deduce the initial speed $v_i$ of the bullet just before collision as a function of $m, M, g, d$ and $h$.