Lebanese University
Faculty of Sciences, B3
Tripoli - Lebanon

الجامعة اللبنانية
كلية العلوم، الفرع 3
طرابلس – لبنان

Course: Classic Mechanics (P1100)

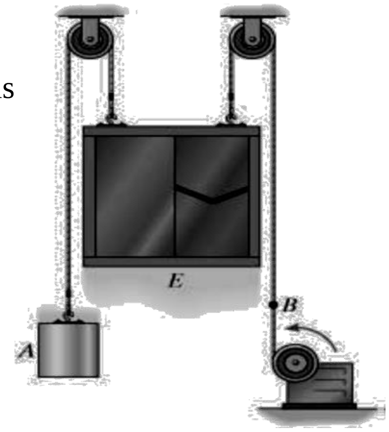
Part B: Dynamics

Exercise 1

It is assumed that the mass of the elevator is 500 kg and that of its weight is 150 kg.

If the elevator reaches the speed 10 m/s after ascending 40 m.

Calculate the force applied at point B.

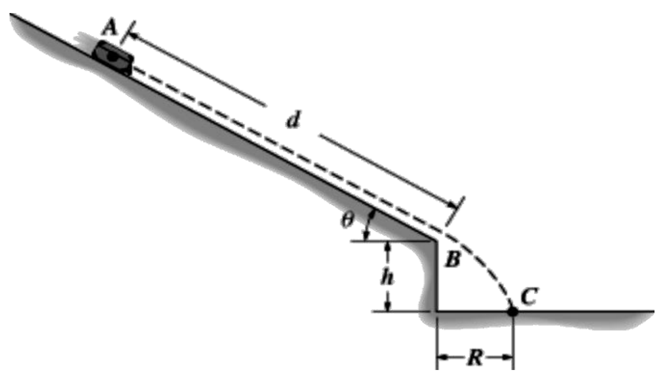


Exercise 2

A 20 kg mass box starts at rest and slides without friction on an inclined plane.

a. Determine the position of point C where it touches the ground.
b. Calculate the total time to go from A to C.

We give: $d=5$ m, $h=1$ m, and $\theta=30^o$.



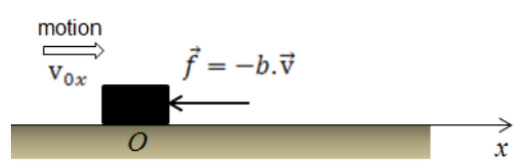
Exercise 3

Consider a block launched with an initial velocity v_{0x} on a horizontal smooth surface, and subjected to an air resistance: $\vec{f} = -b.\vec{v}$ (linear with v) where b is a positive constant.

A 1) a. Write the equation of motion to show that: $v_x(t) = v_{0x}.e^{-\frac{t}{\tau}}$ where $\tau = \frac{m}{b}$.

b. Give the units of τ and b.

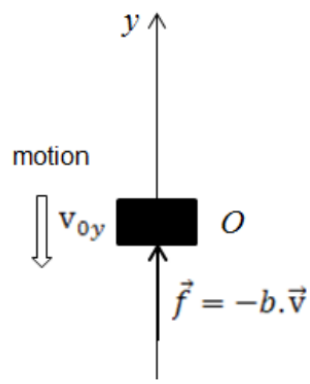
c. The block practically stops after a time t_s. Give the expression of t_s.



2) a. Show that: $x(t) = v_{0x}.\tau.\left(1 - e^{-\frac{t}{\tau}}\right)$

b. Deduce that the block covers a total distance $d = \frac{m.v_{0x}}{b}$ before coming to rest.

B. The same block is launched vertically downward with an initial velocity v_{0y} and is subjected to an air resistance: $\vec{f} = -b.\vec{v}$ (linear with v) where b is a positive constant.



1) a. Write the equation of motion and show that:

$v_y(t) = v_{0y}.e^{-\frac{t}{\tau}} + v_{ter}\left(1 - e^{-\frac{t}{\tau}}\right)$ where: $\tau = \frac{m}{b}$ and $v_{ter} = -m.\frac{g}{b}$.

b. Give the physical meaning of v_{ter} and find its expression directly.

2) Show that: $y(t) = v_{ter}.t + (v_{0y} - v_{ter}).\tau.\left(1 - e^{-\frac{t}{\tau}}\right)$.

C. Using the results of **A** and **B**; show that the path equation (trajectory) of a particle launched with the initial conditions of **A** and **B** and subjected to an air resistance $\vec{f} = -b.\vec{v}$ is given by:

$$y = \left(\frac{v_{0y} - v_{ter}}{v_{0x}}\right)x - v_{ter}.\tau.\ln\left(1 - \frac{x}{v_{0x}.\tau}\right)$$

Exercise 4

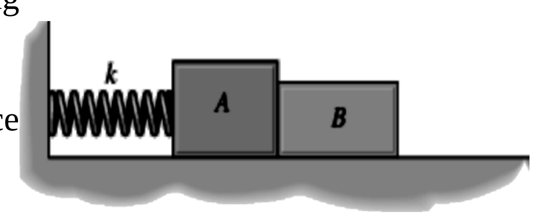
Calculate the speed of a parachutist falling in a straight fall in air which resistance is given by $f = kv^2$.

Deduce the speed on the arrival on ground.

We give $\int du/(1-u^2) = \text{atanh}(u) + cst$

Exercise 5

Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. Another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d,



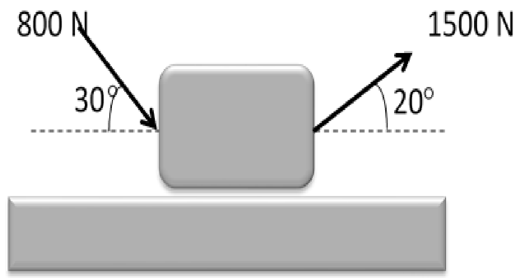
a) What is the distance the blocks slide on the surface before they separate?

b) Show that for separation to occur it is necessary that $d > 2\mu_k g(m_A + m_B)/k$, where μ_k is the coefficient of kinetic friction between the blocks and the ground.

Hint: find the speed for 0 < x < d.

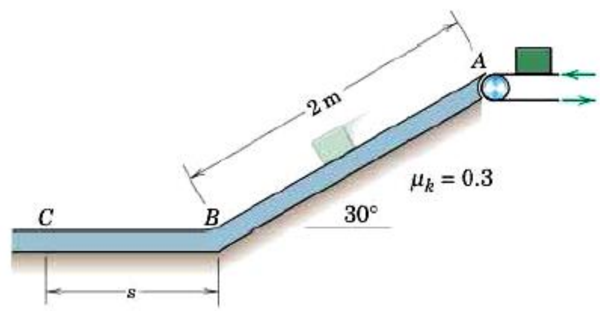
Exercise 6

Two forces $F_1 = 800$ N and $F_2 = 1500$ N are applied to a body of mass M = 100 Kg, see figure. If the system is initially at rest. Determine its distance traveled when it reaches a speed of v = 6 m/s. The kinetic coefficient of friction between the body and the surface is $\mu_k = 0.2$.



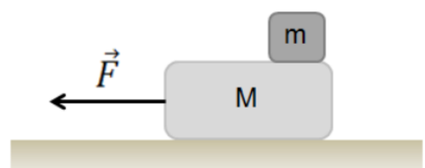
Exercise 7

A block is deposited at *A* using a conveyor belt at a speed v = 0.8 m/s. The incline has an angle $\theta = 30°$ with the horizontal. Calculate the distance *s* at the surface *BC* where the block stops. The coefficient of kinetic friction between *A* and *C* is $\mu_k = 0.3$.



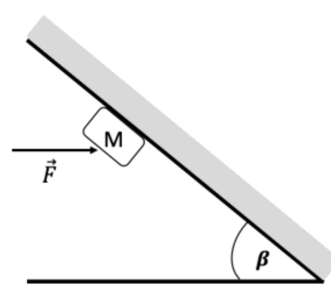
Exercise 8

A block of mass $m = 5\ kg$ rides on top of a second block of mass $M = 10\ kg$. A person attaches a string to the bottom block and pulls the system horizontally across a frictionless surface as shown. Friction between the two blocks keeps the $5\ kg$ block from slipping off. If the coefficient of static friction is $\mu_s = 0.35$ determine the maximum force that can be exerted by the string on the $10\ kg$ block without causing the $5\ kg$ block to slip.



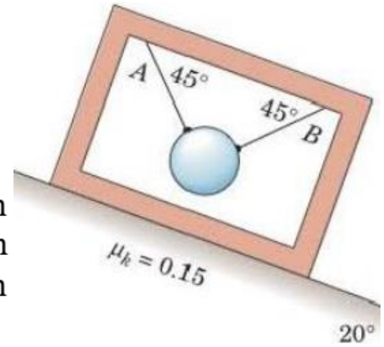
Exercise 9

A block of mass $M = 1kg$ is held at rest against a wall, that makes an angle $\beta = 60°$ with the ground, as shown in the adjacent figure. The block is subjected to a force $F = 15\ N$ parallel to the horizontal and to a friction force f. The coefficients of static and kinetic friction are respectively $\mu_S = 0{,}6$ and $\mu_K = 0{,}5$. Given g = 10 N / Kg.



a) Does the block slides downward?
b) If yes, calculate its acceleration.

Exercise 10

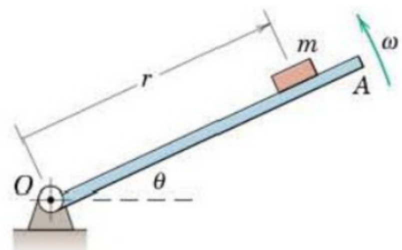


The 10-kg steel sphere is suspended from the 15-kg frame which slides down the 20° incline. If the coefficient of kinetic friction between the frame and incline is 0.15, compute the tension in each of the supporting wires A and B.

Exercise 11

The member OA rotates about a horizontal axis through O with a constant counterclockwise velocity $\omega = 3\, rad/sec$. As it passes the position $\theta = 0$; a small block of mass m is placed on it at a radial distance $r = 50\ cm$.

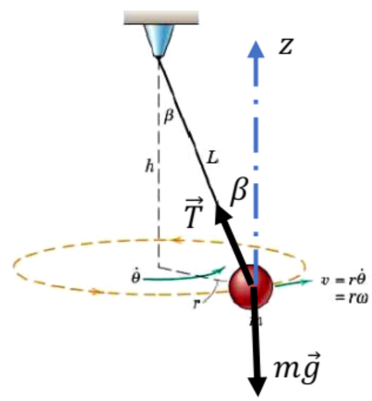
If the block is observed to slip at $\theta = 50°$, determine the coefficient of static friction μ_s between the block and the member.



Exercise 12

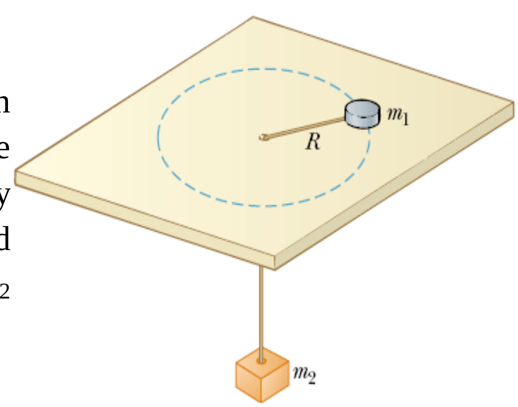
The small ball of mass m is attached to a light cord of length L and moves as a conical pendulum in a horizontal circle with a tangential velocity v.

Show that $h = \frac{g}{\omega^2}$ and the tension in the cord $T = mL\omega^2$ where g is the acceleration of gravity and ω is the angular velocity about the vertical axis.



Exercise 13

A small mass m_1 is moving along a circle of radius R with an angular speed ω. The mass m_1 moves on a horizontal table without friction. The mass m_1 is connected to a mass m_2 by an inextensible massless wire passing through a hole. Find the value that the angular speed ω must have so that m_2 remains stationary.

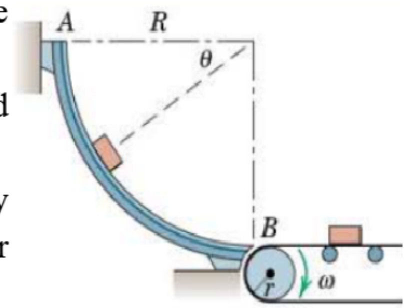


Exercise 14

Small objects are released from rest at A and slide down the smooth circular surface of radius R to a conveyor B.

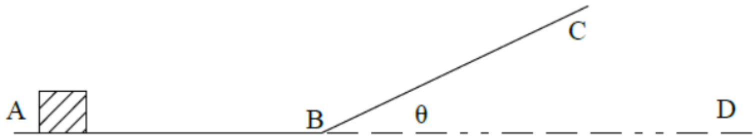
a) Determine the normal contact force N between the guide and each object in terms of θ.

b) Specify the correct angular velocity ω of the conveyor pulley of radius r to prevent any sliding on the belt as the objects transfer to the conveyor.



Exercise 15

A block of mass $m = 2kg$ is launched at an initial speed $v_A = 8\ m/s$ along the path AB of length $2m$. It is subjected to a single resistive force, the air drag, which is linear with the velocity and is given by: $\vec{f_r} = -b\vec{v} = -0.65\,\vec{v}$. From point B, the block slides up a rough incline BC of angle $\theta = 30°$ with the horizontal where it is subjected to a kinetic friction of coefficient $\mu_k = 0.25$. At point C, the block starts moving as a projectile before landing at point D. The air drag is negligible along the path B-C-D.



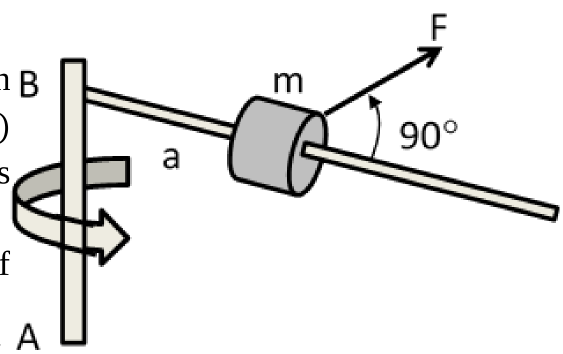
a) Show that the speed of the block at B is $v_B = 7.35\ m/s$.
b) Determine the length BC so that the block can reach point C with a speed $v_C = 5\ m/s$.
c) Derive the parametric equations $x(t)$ and $y(t)$ of the motion of the block between C and D.
d) Determine with respect to the ground the maximum height reached by the block.
e) Determine with respect to A the coordinates of the point of impact D.

Exercise 16

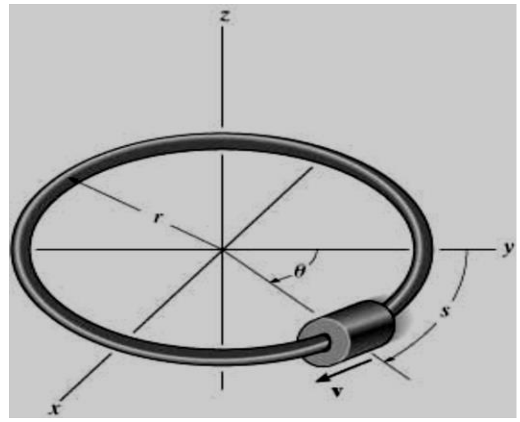
A particle of mass $m = 0.1$ Kg can slide without friction on a rod of negligible mass. We apply a force $F = 2t$ (N) to the particle located at a distance $a = 1$ m from the axis AB.

a) We fix the particle. Calculate the angular speed of the system after 2 seconds.

b) We release the particle and we cancel the force F. What becomes the new position of the particle m when its angular speed becomes 10 rad/s.



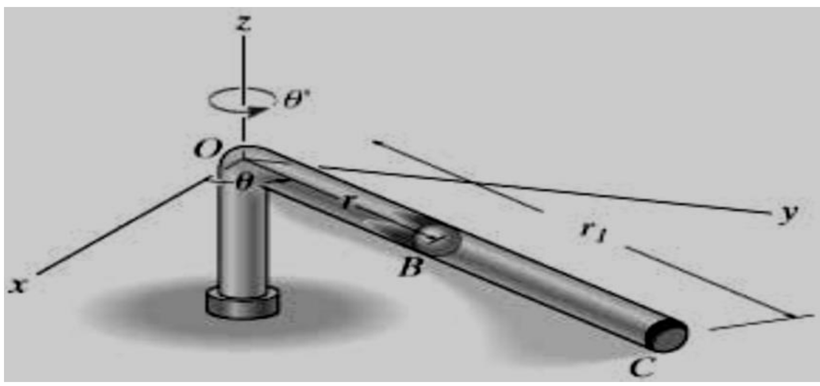
Exercise 17



A 0.75 kg-mass washer slides with friction along a circular ring with a radius of 10 cm. It is launched with an initial speed 4 m/s at the point $\theta = 0$. Determine the slipped distance before stopping.
We give $\mu_k = 0.3$.

Exercise 18

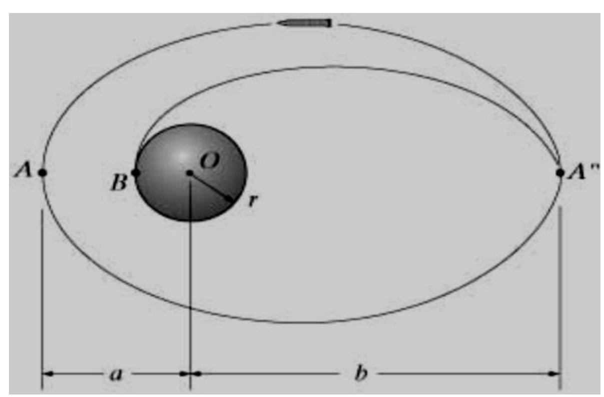
The tube rotates in a horizontal plane at constant angular speed $\dot{\theta} = 4\ \mathrm{rad/s}$. We suppose that the particle B starts from the origin O with an initial radial speed $\dot{r} = 1.5\ \mathrm{m/s}$.



a- What is the nature of the reference linked to the tube?
b- Determine the two types of forces applied to B (magnitude and direction).
c- Show that the instantaneous position of B is given by $r = Ae^{-4t} + Be^{4t}$.
d- Determine the components of the velocity vector just at point C ($r_C = 0.5$ m).

Exercise 19

A rocket is in free flight along an elliptical trajectory around a planet. The rocket has to land on the surface of the planet at point B. Determine:

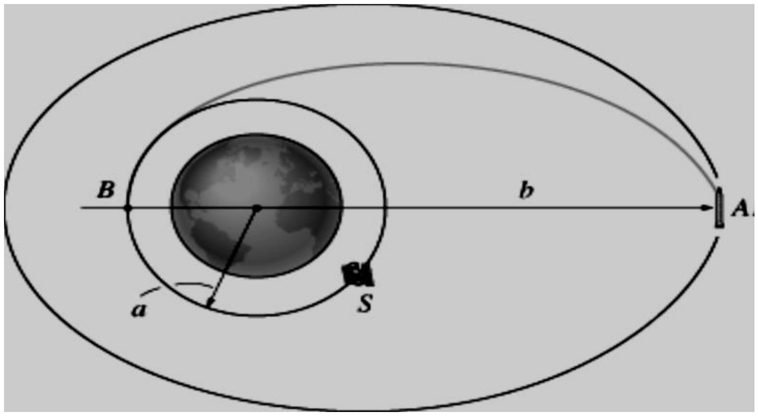


a- The variation in speed required at point A'.
b- The time required to travel the path A'B.

We give: $v_A = 7170\ \mathrm{m/s}$, mass of the planet $m = 3.6 \times 10^{24}$ kg, $r = 3200$ km, $a = 6400$ km and $b = 16000$ km.

Exercise 20

We consider a satellite which revolves around the earth on a circular trajectory, and a rocket which revolves around the earth on an ellipse of eccentricity $e = 0.58$. We want to transport the rocket from its ellipse to the circle of the satellite through an ellipse that joins the two orbits. Determine:



a- The variation of the speed necessary at point A to pass from one ellipse to another.

b- The variation of speed at point B to keep the rocket in a circular orbit. We give $a = 10$ Mm and $b = 120$ Mm.

Exercise 21

A satellite revolves around the earth on an ellipse of eccentricity $e = 0.156$. Determine:

a. Its speed at point P.
b. The length of the radius vector OB.

We give: $r_P = 5$ Mm and $\theta = 135°$.