Lebanese University
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Tripoli - Lebanon

P1100 - Mechanics

Chapter 1 - Kinematics

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1- Introduction and mathematical recap

2- Unit systems and dimensions

3- kinematics : Motion in cartesian coordinates

I-3-1- Three dimensional motion

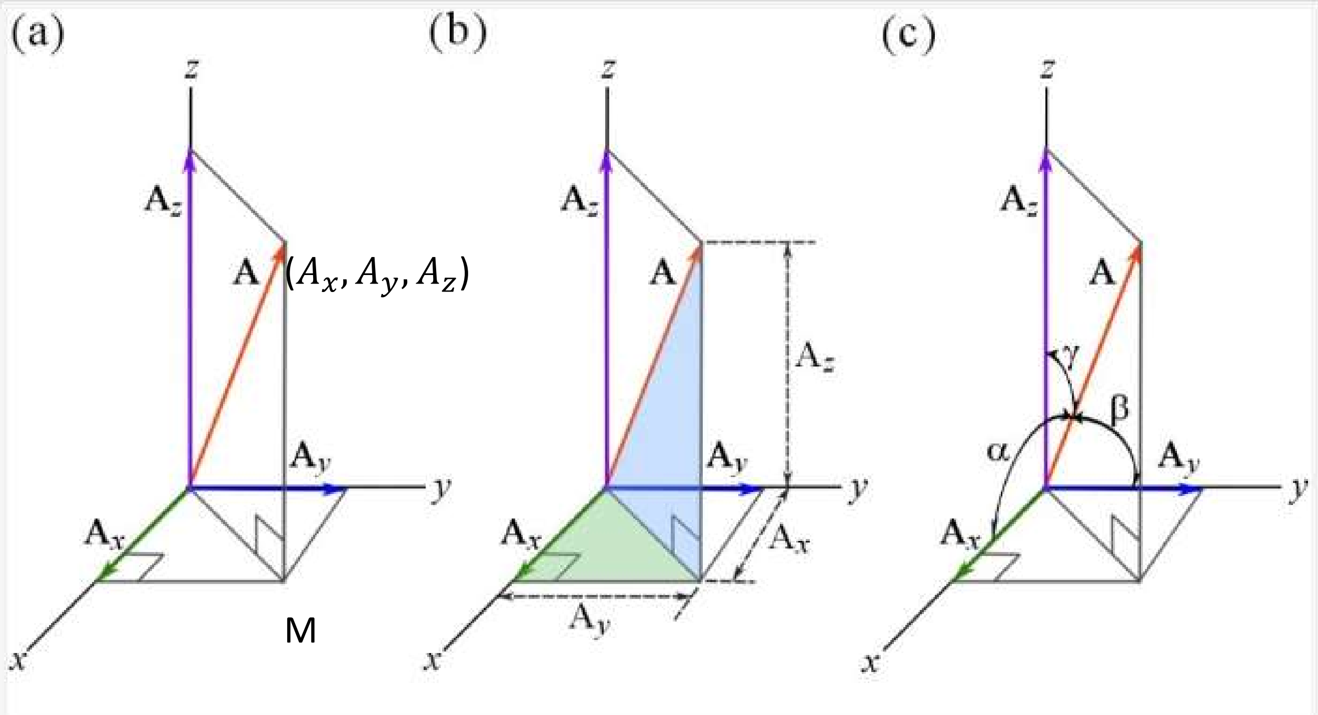
I-3-2- Rectilinear motion

I-3-3- Planar motion: projectile

I-4- kinematics : Motion in cylindrical coordinates

I-5- kinematics : Motion in polar coordinates

Recap
I- 3- Motion in cartesian coordinates



Components of a vector in a three-dimensional Cartesian coordinate system.

[Engineering at Alberta Courses » Cartesian vector notation]

I-4- Motion in cylindrical coordinates:

[A merry-go-round rotating ride]

Cylindrical coordinates:

Sometimes we deal with the motion of a system that has **rotational symmetry around an axis**, such as the *oz* axis. It is convenient to use cylindrical coordinates, which are:

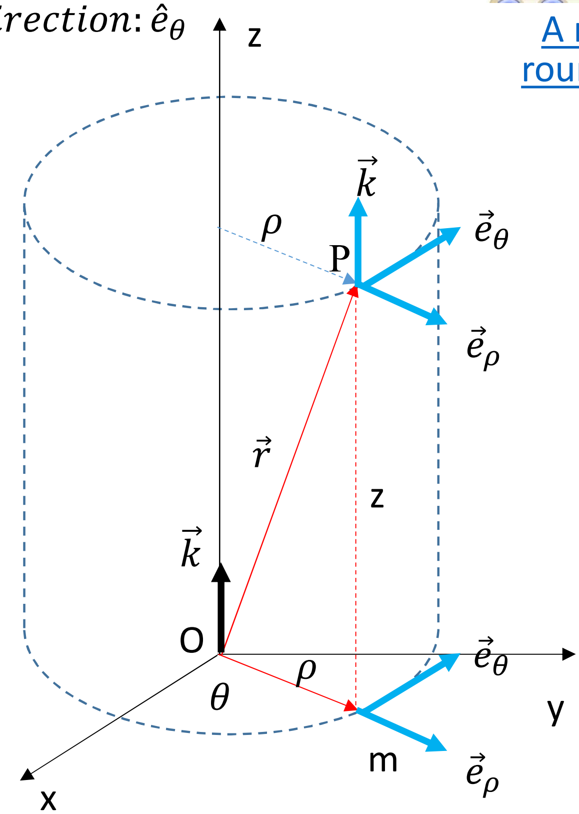
$z =$ *distance from P to XOY* $\rightarrow$ *direction:* $\hat{k}$

$\rho =$ *distance from P to OZ* $\rightarrow$ *direction:* $\hat{e}_\rho$

$\theta =$ *angle between* $\overrightarrow{OX}$ *and* $\overrightarrow{OM}$ $\rightarrow$ *direction:* $\hat{e}_\theta$

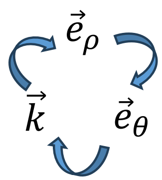
a- Position Vector:

$$\overrightarrow{OP} = \overrightarrow{Om} + \overrightarrow{mP} \rightarrow \boxed{\vec{r} = \rho\hat{e}_\rho + z\,\hat{k}}$$

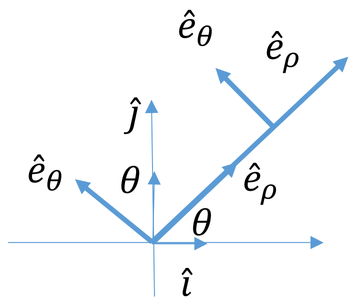


b- Velocity Vector:

Orthonormal base:



$$\vec{v} = \frac{d\vec{r}}{dt} = \rho'\hat{e}_\rho + \rho\hat{e}'_\rho + z'\,\hat{k}$$



Let's calculate $\hat{e}'_\rho$=?

$$\hat{e}_\rho = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$$

$\hat{e}'_\rho = -\theta'\sin\theta\,\hat{\imath} + \theta'\cos\theta\,\hat{\jmath},$ but $\hat{e}_\theta = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$

So $\hat{e}'_\rho = \theta'\hat{e}_\theta,$ Similarily, we can prove $\hat{e}'_\theta = -\theta'\hat{e}_\rho$

Therefore $\boxed{\vec{v} = \rho'\hat{e}_\rho + \rho\theta'\hat{e}_\theta + z'\,\hat{k}}$

c- Acceleration Vector :

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[\rho'\hat{e}_\rho + \rho\theta'\hat{e}_\theta + z'\,\hat{k}\right] = \rho''\hat{e}_\rho + \rho'\hat{e}'_\rho + \rho'\theta'\hat{e}_\theta + \rho\theta''\hat{e}_\theta + \rho\theta'\widehat{e'}_\theta + z''\,\hat{k}$$

But $\hat{e}'_\rho = \theta'\hat{e}_\theta$, et $\hat{e}'_\theta = -\,\theta'\hat{e}_\rho$

$$\vec{a} = \rho''\hat{e}_\rho + \rho'(\theta'\hat{e}_\theta) + \rho'\theta'\hat{e}_\theta + \rho\theta''\hat{e}_\theta + \rho\theta'\left(-\,\theta'\hat{e}_\rho\right) + z''\hat{k} = \left(\rho'' - \rho\theta'^2\right)\hat{e}_\rho + (\rho\theta'' + 2\rho'\theta')\hat{e}_\theta + z''\,\hat{k}$$

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I-5- Motion in cylindrical coordinates:

In the plan *z = 0* these coordinates are called polar coordinates (2 dimensions)

Position Vector: $\vec{r} = \rho\hat{e}_\rho$

Velocity Vector:

$\vec{v} = \rho'\hat{e}_\rho + \rho\theta'\hat{e}_\theta = v_\rho\hat{e}_\rho + v_\theta\hat{e}_\theta = \overrightarrow{v_\rho} + \overrightarrow{v_\theta}$

$\overrightarrow{\boldsymbol{v}_\rho}$ **is the radial velocity, and**
$\overrightarrow{v_\theta}$ **is the orthoradial velocity**

Magnitude of $\vec{v}$ is $\boldsymbol{v} = \sqrt{\rho'^2 + (\rho\theta')^2}$ **direction : $\tan\alpha = \frac{\rho\theta'}{\rho'}$**

Acceleration vector: $\vec{a} = (\rho'' - \rho\theta'^2)\hat{e}_\rho + (\rho\theta'' + 2\rho'\theta')\hat{e}_\theta = a_\rho\hat{e}_\rho + a_\theta\hat{e}_\theta = \overrightarrow{a_\rho} + \overrightarrow{a_\theta}$

$\overrightarrow{\boldsymbol{a}_\rho}$ **is the radial acceleration, and $\overrightarrow{\boldsymbol{a}_\theta}$ is the orthoradial acceleration**

Magnitude of $\vec{a}$ is $\boldsymbol{a} = \sqrt{(\rho'' - \rho\theta'^2)^2 + (\rho\theta'' + 2\rho'\theta')^2}$ **direction : $\tan\alpha = \frac{(\rho\theta'' + 2\rho'\theta')}{(\rho'' - \rho\theta'^2)}$**

$\overrightarrow{v_\theta} = \overrightarrow{v_\tau}$ in circular motion!
Where $\overrightarrow{v_r}$ *is perp to the tang*

Application : I-13-

The horse in a merry-go-round moves according to the following equations: r=2.4 (m), θ=0.6t (rad), and z = 0.45 sinθ (m) with t in seconds.

1) Calculate the velocity and acceleration vectors of the horse at t=4 s.

2) Determine the maximum and minimum values of speed and acceleration during the motion.

Solution:

We prepare the derivatives of the coordinates:

$$\rho = 2,4m \qquad \dot{\rho} = 0 \qquad \ddot{\rho} = 0$$

$$\theta = 0,6t \qquad \dot{\theta} = 0,6 \qquad \ddot{\theta} = 0$$

$$z = 0,45\sin\theta \qquad \dot{z} = 0,45\dot{\theta}\cos\theta \qquad \ddot{z} = 0,45\ddot{\theta}\cos\theta - 0,45\dot{\theta}^2\sin\theta$$

$$\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{MP} = 2,4\hat{e}_\rho + 0,45\sin(0,6\times 4)\hat{k} = 2,4\hat{e}_\rho + 0,304\hat{k}\,.$$

1) The velocity vector: $\vec{v} = \rho\dot{\theta}\,\hat{e}_\theta + 0,45\dot{\theta}\cos\theta\,\hat{k} = 1,44\hat{e}_\theta - 0,199\hat{k}$

The acceleration vector: $\vec{a} = \left(\rho'' - \rho\theta'^2\right)\hat{e}_\rho + (\rho\theta'' + 2\rho'\theta')\hat{e}_\theta + z''\,\hat{k} = -\rho\dot{\theta}^2\hat{e}_\rho - 0,45\dot{\theta}^2\sin\theta\,\hat{k} = -0,86\hat{e}_\rho - 0,11\hat{k}$

2) We must start by calculating the magnitude of the velocity: $v = \sqrt{(1,44)^2 + (0,27\cos\theta)^2} \Rightarrow \begin{array}{l} v_{\max,\theta=0} = 1,464m/s \\ v_{\min,\theta=\pi/2} = 1,44m/s \end{array}$

Magnitude of $\vec{a}$ is $\boldsymbol{a}$ $= \sqrt{\left(\rho'' - \rho\theta'^2\right)^2 + (\rho\theta'' + 2\rho'\theta')^2} \rightarrow \; a_{\max,\theta=\pi/2} = 0,875m/s^2 \quad a_{\min,\theta=0} = 0,86m/s^2\,.$

Application I-14:

A child is sitting on a merry-go-round 3 meters away from the center of rotation. The merry-go-round is initially at rest, and it is given an angular acceleration of $\ddot{\Theta} = 2\ rd/s^2$.
Determine the child's velocity when the acceleration becomes a= $8m/s^2$.
Solution:

$$\vec{v} = \dot{\rho}\hat{e}_r + \rho\dot{\theta}\hat{e}_\theta + \dot{z}\hat{k} \Rightarrow v = \sqrt{\dot{\rho}^2 + \rho^2\dot{\theta}^2 + \dot{z}^2} \quad ,$$

$\rho = 3 = cst$; $\rho' = 0$; $\rho'' = 0$

$z = cst$; $z' = 0$; $z'' = 0$

$\theta'' = 2\ rd/s$ Let's find θ'=? $\ddot{\theta} = \frac{d\dot{\theta}}{dt} \rightarrow \int_0^{\dot{\theta}} d\dot{\theta} = \int_0^t \ddot{\theta}\, dt \rightarrow \dot{\theta} = \int_0^t 2\, dt = 2t$; t=??

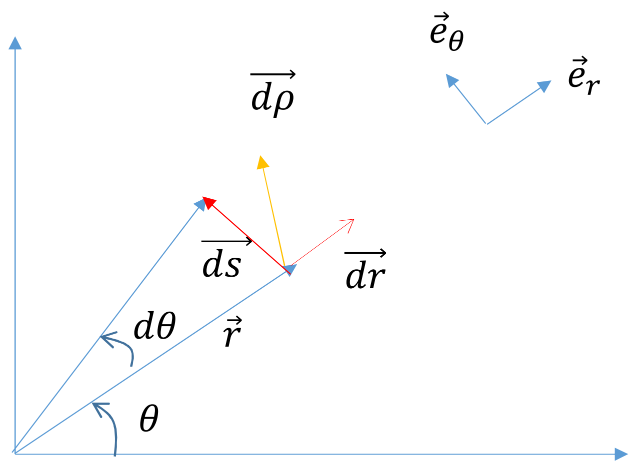
$$\vec{a} = (\rho'' - \rho\theta'^2)\hat{e}_\rho + (\rho\theta'' + 2\rho'\theta')\hat{e}_\theta + z''\,\hat{k}$$

$$a = \sqrt{\rho^2\dot{\theta}^4 + \rho^2\ddot{\theta}^2} = \sqrt{144t^4 + 36} = 8$$

$$\Rightarrow t^4 = 0{,}2s \Rightarrow t = 0{,}66s$$

$$v = \rho\dot{\theta} = 2\rho t = 3{,}96\ m/s$$

The length of the path traveled between the two instants t_1 and t_2 (curvilinear abscissa) in polar coordinates:



Considering θ constant and increasing r by dr in the radial direction, then: $\overrightarrow{dr} = dr\vec{e}_r$

Now, considering r constant and increasing θ by dθ, the particle then moves an elementary distance: $\overrightarrow{ds} = rd\theta\vec{e}_\theta$

In general, if r and θ vary simultaneously, the elementary displacement is the vector sum of the two terms:

$$\overrightarrow{d\rho} = dr\vec{e}_r + rd\theta\vec{e}_\theta,$$

$$dr = r'dt \quad and\ d\theta = \theta' dt$$

$$\overrightarrow{d\rho} = r'dt\ \vec{e}_r + r\theta' dt\vec{e}_\theta$$

The length of the path traveled between the two t_1 and t_2 (curvilinear abscissa):

$$s = \int_{t_1}^{t_2} vdt = \int \frac{d\rho}{dt} dt = \int_{t_1}^{t_2} \sqrt{(r')^2 + (r\theta')^2} dt$$