$U(x) = \frac{1}{2} kx^2$

$\vec{F} = k\vec{x}.$

III-6 Diagram of the potential energy:

to study the mot° of a particle, we can draw ~~ϕ~~ $U(x)$ as a funct° of x.

① we can find $\vec{F}$. $\vec{F} = -\vec{\nabla}U$, in 1 D: $F_x = -\frac{dU}{dx}$.

② At equilibirium: $F = 0 = -\frac{dU}{dx}$

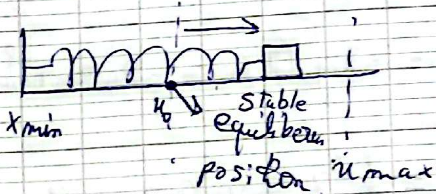
$U(x) = \frac{1}{2} kx^2$

$F = \frac{dU}{dx}$

$= -\frac{1}{2} 2kx$

$= -kx$

we can deduce : x_{min} and x_{max}



③ The stability of equilibrium is given by the single of $U''(x)$

- If $U''(x) > 0$: stable equilibrium: Once the mass is displaced from its eq. position, F resters in to this ⇄ position.

- If $U''(x) < 0$ Unstable equilibirium: once the mass is displaced from eq F tends to move away in from this eq position.

④ The mecanical Energy is cst If there is no friction. ME = cst

Application III-8:

$\vec{F} = (x - ay)\hat{i} + (3y - 2x)\hat{j}.$

1. $W_F = \int \vec{F} \cdot d\vec{x}.$

$x^2 + y^2 = 4$

$x = 2\cos\theta \rightarrow$ ~~$0 \leq \theta \leq 2\pi$~~ $dx = -2\sin(\theta)\, d\theta$

$y = 2\sin(\theta) \rightarrow$ $dy = 2\cos(\theta).\, d\theta$

$0 \leq \theta \leq 2\pi$

$d\vec{r} = dx\,\hat{i} + dy\,\hat{j}$

$w_{\vec{F}} = \int [(x - ay)\hat{i} + (3y - 2x)\hat{j}](dx\hat{i} + dy\hat{j})$

$w_{\vec{F}} = \int [(x - ay)\hat{i} + (3y - 2x)\hat{j}] (dx\,\hat{i} + dy\,\hat{j})$

$\vec{V}_1 \cdot \vec{V}_2 = V_{1x} \cdot V_{2x} + V_{1y} \cdot V_{2y}$

$= \int (x - ay)\,dx + \int (3y - 2x)\,dy.$

$= -2\int (2\cos\theta - 2a\sin\theta)\overset{\sin\theta}{\hat{}}\,d\theta + 2\int (\cancel{8}\sin\theta - 4\cos\theta)\cos\theta\,d\theta.$

$\sin 2a = 2\sin a\cos a$

$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$

$\cos^2\theta = \frac{1 + \cos(2\theta)}{2}$

$= \int (\underbrace{8\sin\theta\cos\theta} + 4a\sin^2\theta - 8\cos^2\theta)\,d\theta$

$= \int [\underbrace{4\sin(2\theta)} + 2a(1 - \cos(2\theta)) - 4(1 + \cos(2\theta))]\,d\theta.$

~~$= -4/2\cos(2\theta) + 2a[\theta - 2\sin(2\theta)] - 4[\theta + 2\sin(2\theta)]$~~

$= \int_0^{2\pi} 4\sin(2\theta) + 2a - 4 + \cos(2\theta)(-2a - 4)\,d\theta.$

$= -2\cos(2\theta) + (2a - 4)\theta + \frac{(-2a - 4)}{2}\sin(2\theta)\Big|_0^{2\pi}.$

$= -2[\cancel{1 - 1}]^{0} + (2a - 4) \cdot 2\pi + 0.$

$= 2\pi(2a - 4).$

2) $a = ??$

$W_{\vec{F}} = 0$ (on a close curve (circle))

$2\pi(2a-4) = 0 \Rightarrow a = 2$

Application 10 : (NoTe: CGS system)

$U_{ergs} = x^2 - 2x - 1$

a) $F = -\frac{dU}{dx}\hat{i}$.

$F\,dx = -dU$

$= -[2x - 2]\hat{i}$.

$= (-2x + 2)\hat{i}$

b) Eq position = ?? $(x_{eq}) = ??$

$F = 0 = -2x + 2 = 0$

$x = \frac{-2}{-2} = 1$.

$U_{eq} = 1^2 - 2 - 1$ ergs.

$= -2$ J.

NoTe.

	mass	length	time	energy
SI	kg	m	1	
CGS	g	Cm	1	ergs

c) At $t=0$; $x = x_{eq}$, $\vec{V_0} = -2\hat{i}$. (Cm/s)

ME = ?? $x = ??$ $V=0$

At $t=0$; $ME = KE + U = \frac{1}{2}mV_0^2 + (x^2 - 2x - 1)$

$= \frac{1}{2}(2)(2)^2 + (1 - 2 - 1)$

$= 2$ ergs.

ME = cst (all the forces are Conservative) $ME = \frac{1}{2}mV^2 + U =$

$ME = (x^2 - 2x - 1)$

$= 2 \Rightarrow x_{min} = -1$ Cm, $x_{max} = 3$ Cm.

d) show that the partical is animated by a harmonic motion about its eq. position; T = ? amplitude ??

$\sum \vec{F_{ext}} = m\vec{a}$; $F = ma$ (one direction)

$-2x + 2 = 2\ddot{x}$

$\ddot{x} + x = 2$ in the form of $\underline{\ddot{x} + w^2 x = 1}$

$w = 1$ rad/s

↓ harmonic motion (equation)

$T = \frac{2\pi}{w} = 2\pi$.

$x_{min} = -1$, 0, $x_{eq} = 1$, $x_{max} = 3$, x (Cm)

amplitude = 2cm .

Work - Kinetic energy theorm :

$$W_{\Sigma \vec{F}} = \int \vec{F} \cdot d\vec{r}$$

$$= \int m\vec{a} \cdot d\vec{r} = \int m \frac{d\vec{v}}{dt} \cdot d\vec{r} = \int m \, d\vec{v} \, \frac{d\vec{r}}{dt}$$

$$= \int m\vec{v} \cdot d\vec{v} = \int_{v_i}^{v_f} m v \, dv$$

$$W_{\Sigma \vec{F}} = \frac{1}{2} m (v_f - v_i)^2 = KE_f - KE_i$$

$$\boxed{W_{\Sigma \vec{F}} = \Delta KE}$$