Session 2 2024_2025

Ex 1:

$x(t) = -5\sin(\omega t)$; $y = 4 - 5\cos(\omega t)$; $\omega = cst$

1. $\vec{r} = -5\sin(\omega t)\hat{i} + (4 - 5\cos(\omega t))\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = -5\omega\cos(\omega t)\hat{i} + 5\omega\sin(\omega t)\hat{j}$

$\vec{a} = \frac{d\vec{v}}{dt} = 5\omega^2\sin(\omega t)\hat{i} + 5\omega^2\cos(\omega t)\hat{j}$

2. $a_t = \frac{dv}{dt}$

$v = \sqrt{(5\omega)^2\cos^2(\omega t) + (5\omega)^2\sin^2(\omega t)}$

$= 5\omega\sqrt{\sin^2(\omega t) + \cos^2(\omega t)} = 5\omega = cst$ (m/s)

$a_t = 0$

$a_n = \sqrt{a^2 - \underbrace{a_t^2}_{=0}} = a = \sqrt{(5\omega^2)^2\sin^2(\omega t) + (5\omega^2)^2\cos^2(\omega t)}$

$= 5\omega^2$ (m/s²)

$a_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{v^2}{a_n} = \frac{5^2\omega^2}{5\omega^2} = 5$ m.

$\rho = cst \longrightarrow$ circular motion with cst speed.

$\Longrightarrow$ uniform circular motion.

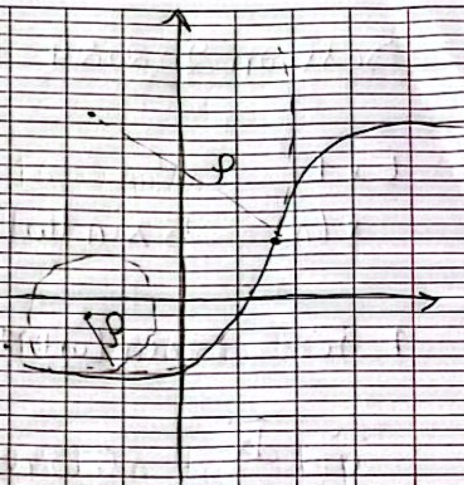
3. $x^2 + (y-4)^2 = 5^2\sin^2(\omega t) + 5^2\cos^2(\omega t) = 25$

$x^2 + (y-4)^2 = 5^2 = R^2$ (cercle with center (0;4)

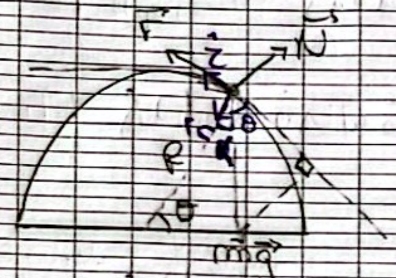
radius 5 m)

$(x-a)^2+(y-b)^2=R^2$

$a = x_{center}$, $b = y_{center}$.



Ex3:



1- cst speed ; $F = F(m, g, \theta)$?

FBD on m : $\sum \vec{F}_{ext} = m\vec{a}$

$m\vec{g} + \vec{F} + \vec{N} = m\vec{a}$

proj on $\hat{t}$: $-mg\cos\theta + F = m a_t = m\dfrac{dv}{dt} = 0$

$F = mg\cos\theta$

2- $W_{\vec{F}} = \int \vec{F}\cdot d\vec{r} = \int mg\cos\theta\,\hat{t}\cdot r\,d\theta\,\hat{t} = mgR\sin\theta\Big]_0^{\pi/2} = mgR$

3- $F = cst$; $v \neq cst$

$v = v(F, \theta, m, R, g)$

For $\theta = 0 \longrightarrow v = 0$

proj on $\hat{t}$: $-mg\cos\theta + F = m a_t = m\dfrac{dv}{dt}$

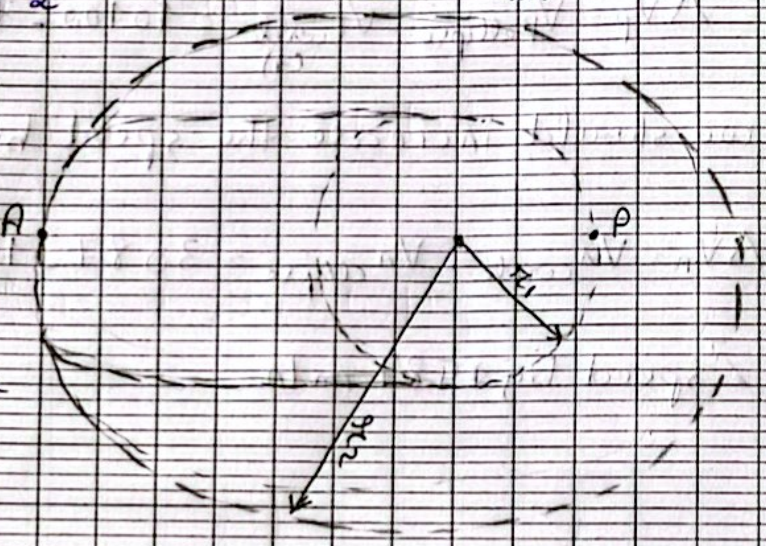
$a_t = a_t(\theta) \longrightarrow \boxed{a_t\,ds = v\,dv}$

$$\int_0^\theta \left(-g\cos\theta + \frac{F}{m}\right) R\,d\theta = \int_0^v v\,dv$$

$$-Rg\sin\theta\Big]_0^\theta + \frac{F}{m}R\theta\Big]_0^\theta = \frac{v^2}{2}$$

$$-Rg\sin\theta + \frac{F}{m}R\theta = \frac{v^2}{2} \Rightarrow v = \sqrt{-2Rg\sin\theta + \frac{F}{m}R\theta}$$

Ex4:



circle (1) : r_1

$\downarrow \Delta v_P$

ellipse : $r_m = r_1$; $r_M = r_2$

$\downarrow \Delta v_A$.

circle (2) : r_2.

1 – $v_1 = \sqrt{\frac{GM}{r_1}} = 7711$ m/s

$v_2 = \sqrt{\frac{GM}{r_2}} = 3087$ m/s.

2 – $KE + PE = E_T$

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{r_1+r_2}.$$

at P : $\frac{1}{2}\cancel{m}v_{p\,ellipse}^2 - \frac{GM\cancel{m}}{r_1} = -\frac{GM\cancel{m}}{r_1+r_2}$

$$v_p = \sqrt{2GM\left(\frac{1}{r_1} - \frac{1}{r_1+r_2}\right)} = \sqrt{\frac{2GM}{r_1+r_2}\cdot\frac{r_2}{r_1}} = 10100 \text{ m/s}$$

$$\frac{\cancel{r_1}+r_2-\cancel{r_1}}{r_1}$$

$h = cst = r_A v_A = r_p v_p$

$$V_{A_{ellipse}} = \frac{r_p}{r_A} \cdot V_{P_{ellipse}} = \frac{r_1}{r_2} \cdot V_{P_{ellipse}} = 1600\ m/s.$$

3 - $\Delta V_p = V_{P_{ellipse}} - V_{P_{cercle\,(1)}} = 10100 - 7711 = 2390\ m/s.$

we should increase the speed by 2390 m/s at P.

$\Delta V_A = V_{A_{cercle}} - V_{A_{ellipse}} = 3087 - 1600 = 1487\ m/s.$

$\nearrow$ speed by 1487 m/s.

Chap 3: Conservation of energy

1) Introduction:

2) power and efficiency:

$$P = \vec{F}(w) \cdot v(m/s)$$

power developped by $\vec{F}$ — force — velocity

$$\text{efficiency} = \varepsilon = \frac{\text{output power}}{\text{input power}}.$$

$$P = \frac{\Delta E}{\Delta t}$$

3- work done by a force.

$$W_{\vec{F}} = \int_{t_1}^{t_2} p\,dt = \int \vec{F}\,\underbrace{\vec{v}}_{\frac{\vec{dr}}{dt}}\,dt = \int \vec{F}\,\vec{dr}$$

In cartesian coordinates:

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \longrightarrow \vec{dr} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

In cylendrical coordinates:

$$\vec{V} = \frac{\vec{dr}}{dt} = \frac{dr}{dt}\hat{e}_r + r\cdot\frac{d\theta}{dt}\hat{e}_\theta + \frac{dz}{dt}\hat{k}$$

$$\vec{dr} = dr\,\hat{e}_r + r\,d\theta\,\hat{e}_\theta + dz\,\hat{k}$$

$$\rightarrow W_{\vec{F}} = \int (F_r\hat{e}_r + F_\theta\hat{e}_\theta + F_z\hat{k})(dr\,\hat{e}_r + r\,d\theta\,\hat{e}_\theta + dz\,\hat{k})$$

$$= \int (F_r\,dr + F_\theta \cdot r\,d\theta + F_z\,dz)$$

in cartesian coordinates:

$$\vec{W}_F = \int \vec{F}\,\vec{dr} = \int (F_x\,\hat{i} + F_y\,\hat{j} + F_z\,\hat{k})(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$$

$$= \int F_x\,dx + F_y\,dy + F_z\,dz \,.$$