

P 1100 - Applications

Chapter 4 - Momentum

Application IV-1:

Determine the center of mass of a homogeneous semicircular wire with radius 'a'."

Application IV-2:

In the laboratory reference frame, the velocities of two particles with masses m_1 and m_2 are v_1 and v_2 .

a- Prove that the velocities of these two particles in the center of mass frame are proportional to their relative velocity.

b- Deduce an expression for their linear momenta and total kinetic energy in the center of mass frame.

Application IV-3:

A box with a mass of 25 kg is pulled downward on an inclined plane with a force $F_N=100t$.

Determine its velocity after two seconds. Given: its initial velocity $v_i=1$ m/s, the angle of the inclined plane $\alpha=30^\circ$, and the coefficient of friction $\mu=0.3$.

Application IV-4:

A car traveling at a speed of 10 m/s collides with a tree.

a- A passenger without a seatbelt strikes the windshield with his head and comes to a stop in 0.002 s. The contact area between the head and the windshield is $6 \cdot 10^{-4} \text{ m}^2$. The mass of the head is 5 kg. Find the average force and the force per unit area exerted on the head.

b- A passenger with a mass of 70 kg, wearing a seatbelt, comes to a stop in 0.2 s. The contact area between the seatbelt and the passenger is 0.1 m^2 . Find the average force and the force per unit area.

Application IV-5:

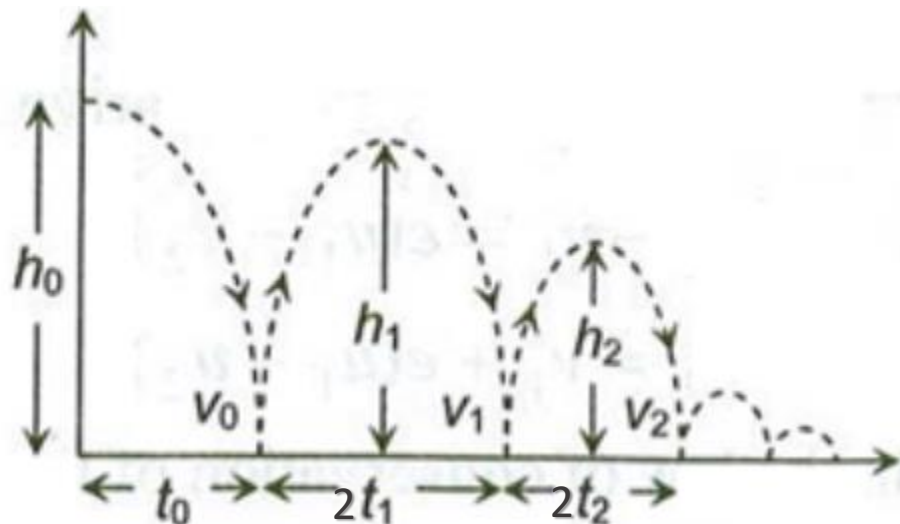
A fisherman weighing 70kg is at the back of a rectangular boat weighing 140kg and measuring 3m in length, with the center of gravity G equidistant from the ends. The fisherman brings the front of his boat into contact with the shore, then moves forward with a velocity $v = 2\text{m/s}$.

- 1- Considering the fisherman-boat system as isolated, at what speed has the boat moved backward?
- 2- When the fisherman reaches the front of the boat, by what distance has the boat moved backward?

Application IV-8:

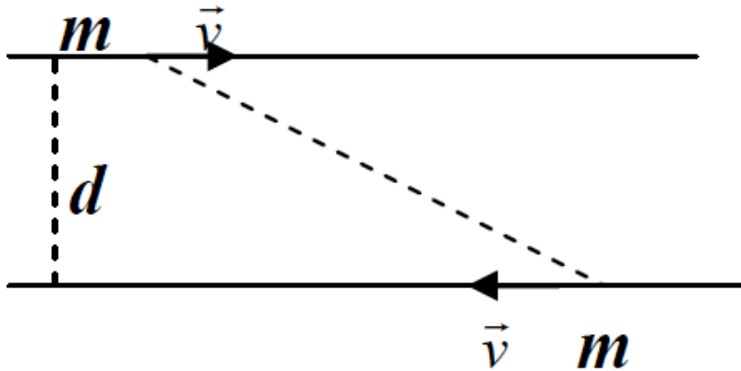
A steel ball, considered as a point mass m , is dropped without initial velocity from a height h_0 above a horizontal table. It bounces vertically up to a height h_1 , then falls, bounces again, and so on. Determine:

- 1- The coefficient e of restitution and the height h_n to which the ball rebounds after the n^{th} bounce on the table.
- 2- The energy absorbed during the collision after the n^{th} bounce. This is the difference between the kinetic energy just before the first collision and the kinetic energy just after the n^{th} collision.
- 3- The time t at which the ball comes to a stop.



Application IV-9:

Two particles of mass m and speed v move in opposite directions along two parallel lines separated by a distance d . Show that the total angular momentum of the system is the same regardless of the chosen origin.



Application IV-10:

A 5 kg block rests on a horizontal surface and is connected to the axis of rotation by a rod of length 0.4m and negligible mass. The system rotates under the influence of a torque defined as $\tau = 3t$ (N.m) and a force $F = 10\text{N}$ applied to the block perpendicular to the rod. Calculate the speed of the block after 4 seconds if it started from rest.

Application IV-11:

On the edge of a homogeneous disk with a radius R and mass M , a thin rope is wound, and a constant tension T is exerted downward. The disk rotates freely around its axis.

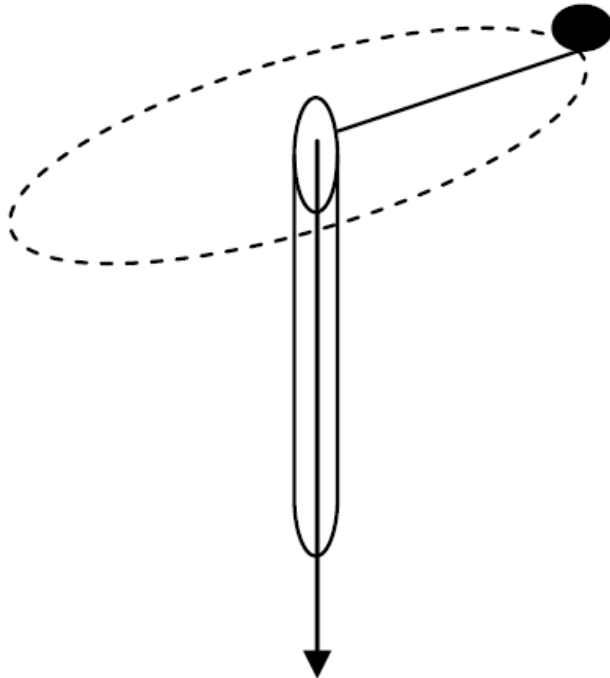
a- Find the angular acceleration of the wheel and the tangential acceleration of a point on the edge.

b- Imagine that an object of mass m is suspended from the rope. Find the angular acceleration of the disk and the tangential acceleration of a point on the edge.

c- Suppose the disk is initially at rest. Calculate the work done by the torque on the disk in 2 seconds. Also, calculate the increase in the rotational kinetic energy of the disk. Deduce the variation in mechanical energy. Note: $M = 2.5 \text{ kg}$, $R = 0.2 \text{ m}$, $mg = 5 \text{ N}$.

Application IV-12:

We attach an object of mass m to a light rope passing through a tube. Holding the tube with one hand and the rope with the other, the object is set into circular motion with radius r at a speed of v_1 . Finally, we pull the rope at the bottom end of the tube to decrease the circle's radius from r_1 to r_2 . Find the new linear velocity v_2 and the new angular velocity ω_2 of the object in terms of v_1 , r_1 , and r_2 . Gravity effects are neglected.



Additional Exercise 1

We displace a bloc of mass $m = 0.5 \text{ kg}$ under the action of a force $F = 100x^2$ with a distance $x = 1 \text{ m}$. We give $k = 50 \text{ N/m}$.

1. Find the work done by F .
2. Find the work done by the tension of the spring
3. If $W_{\vec{F}} = 30 \text{ J}$ and $|W_{\vec{T}}| = 26 \text{ J}$, what is the final speed?



Additional Exercise 2

A 50-kg boy jumps on a 5-kg skateboard with a horizontal velocity of 5 m/s. Determine the distance s the boy reaches up the inclined plane before momentarily coming to rest. Neglect the skateboard's rolling resistance.

