$f_k = \mu_k N$

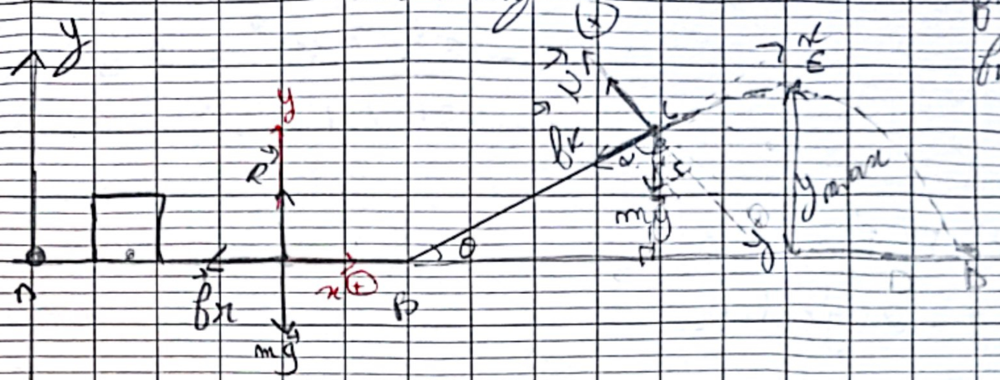
$f_{s\,max} = \mu_s N$

نطلع f_s الكتلة توقف ? يبدا يتحرك f_k

Exercise 15 :

$AB = 2m$; AB $\vec{f_r} = -b\vec{v}$; b) o BCD: kinetic friction $f_k = \mu_k N$

air drag



a) <u>Motion AB:</u> $\sum \vec{F}_{ext} = m\vec{g} + \vec{R} + \vec{f_r} = m\vec{a}$

$f_r = \mu_k R$ proj on x : $0 + 0 - |f_r| = m.a$

$-bv = m.a$

$a(x)$ $\int a\,dx = v\,dv$ $(v = v(x))$

we will have an eq. $v = v(x)$

$-\frac{b}{m} v\,dx = v\,dv$

$-\frac{b}{m} \int_{x_A}^{x_B} dx = \int_{v_A}^{v_B} dv$

$-\frac{b}{m}(x_B - x_A) = v_B - v_A$

$v_A - \frac{b}{m} AB = v_B$

$v_B = 8 - \frac{0,65}{2} \times 2 = 7,35\ m/s$

b) $BC = ?$; $v_C = 5\ m/s$

Motion BC :

FBD: $\sum \vec{F}_{ext} = m\vec{g} + \vec{N} + \vec{f_k} = m.\vec{a}$

proj on y : $-mg\cos\theta + N + 0 = 0$

$N = mg\cos\theta$

proj x : $-mg\sin\theta + 0 - f_k = m.a$

$-mg\sin\theta - \mu_k mg\cos\theta = m a$

$a = -g(\sin\theta + \mu_k\cos\theta)$

$= -7,17\ m/s^2$

$= cst$

$a = 5t \Rightarrow V_C^2 - V_B^2 = 2 \cdot a \cdot d$

$$d = BC = \frac{V_C^2 - V_B^2}{2a_{BC}}$$

$$BC = \frac{5^2 - 7{,}35^2}{2(-7{,}17)} = 2{,}02m$$

Derive
تعني دورنا
كيف
طلع

c) Motion CD:
projectile

$$\vec{a}\begin{cases} a_x = 0 \\ a_y = -g \end{cases} \rightarrow \vec{V}\begin{cases} V_x = V_{0x} = V_0\cos\theta \\ V_y = \int a_y\,dt = -gt + V_0\sin\theta \end{cases}$$

$$\rightarrow \vec{r}\begin{cases} x = V_C\cos\theta t + x_C \,? \\ y = -\frac{1}{2}gt^2 + V_C\sin\theta t + y_C \,? \end{cases}$$

$x_C = AB + BC\cos\theta = 3{,}75m$

$y_C = BC\sin\theta = 1{,}01m$

$x = 4{,}33t + 3{,}75$

$y = -5t^2 + 2{,}5t + 1{,}01$

d) 1st method

$V_{y_E} = 0 = -gt_E + V_C\sin\theta$

$$t_E = \frac{V_C\sin\theta}{g} = \frac{2{,}5}{10} = 0{,}25s$$

$y_E = -5t^2 + 2{,}5t + 1{,}01 = 1{,}32m$

2nd method:

Cst accelleration on y: $V_{fy}^2 - V_{iy}^2 = 2a(y_f - y_i)$

$-V_{Cy}^2 = 2g(y_E - y_C)$

$$y_E = \frac{V_{Cy}^2}{2g} + y_C = 1,32\ m$$

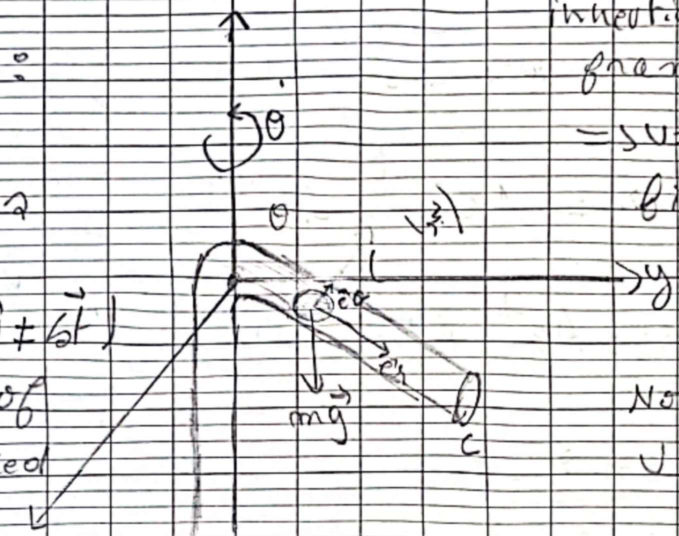
e) $x_D = ?$; $y_D = 0 = -5t_D^2 + 2,5\,t_D + 1,01$

$\Rightarrow t_D = 0,76\ s$

$x_D = 4,33\,t_D + 3,75$

$= 7,06\ m$

Exercise 18:



inertial frame $= \vec v = cst$

fictitious force $\Rightarrow$ non inertial

Non inertial $\vec v \neq cst$

$\dot\theta = w = 4\ rad/s$

a- the tube is rotating ($\vec v \neq cst$) so the frame of reference linked to the tube is non-inertial (non-Galilean) frame

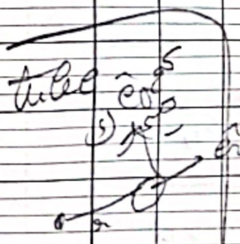
[Note: fictitous forces appear in non inertial frames]

b- Real forces: $m\vec g$ and $\vec N$

Fictitous forces: $\vec F_d = m\vec a_d$; $\vec F_c = m\vec a_c$

2nd law

c- FBD: $\sum_{real} \vec F_{ext} = m\vec a$ absolute



$m\vec g + \vec N = m[\vec a_d + \vec a_c + \vec a_{relative}]$

$\vec N = N_z \vec k + N_\theta \vec e_\theta = m[-w^2 r\,\vec e_r + 2w\dot r\,\vec e_\theta + \ddot r\,\vec e_r]$

proj on $\vec k$: $-mg + N_z = 0 \Rightarrow N_z = mg$

proj on $\vec e_\theta$: $N_\theta = 2mw\dot r$

proj on $\vec e_r$: $0 = m[-w^2 r + \ddot r]$,

$\vec{a}_d = \vec{a}_o + \vec{\alpha} \wedge \vec{r} + \vec{w} \wedge \vec{w} \wedge \vec{r}$

$\vec{a}_d = 0 + 0 + w\hat{k} \wedge (w\hat{k} \wedge r\hat{e}_r) = w\hat{k} \wedge w r \hat{e}_\theta$

$\vec{a}_d = -w^2 r \hat{e}_r$

$\vec{a}_c = 2\vec{w} \wedge \vec{V}_{relative} = 2w\hat{k} \wedge \dot{r}\hat{e}_r = 2w\dot{r}\hat{e}_\theta$

$\vec{a}_{relative} = \ddot{r}\hat{e}_r$

$\ddot{r} - w^2 r = 0$

$r = Ae^{-wt} + Be^{wt}$ (we know that the solution is in this form)

$r = Ae^{-4t} + Be^{4t}$

Ex 20:

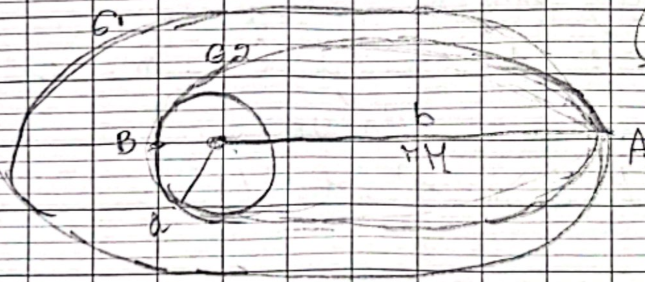
Catelyte : circle of radius $a = 10\ Mm = 10 \times 10^6 m = 10^7 m$ (Megameter)

Rocket : ellipse 1 (big) $e = 0{,}5$; $r_M = b = 120\ Mm$

$r_m = ?$

ellipse 2 : $e' = ?$

$r'_M = b$; $r'_m = a$



ΔV_B

circle

a) $\Delta V = V_A(E_1) - V_A(E_2)$

$\vec{h} = r V_\perp$

$r = \dfrac{h^2/GM}{1 + e\cos\theta}$

for $r = r_A$: $\theta_A = \pi$

$r_A = \dfrac{r_A^2 \cdot V_A^2}{GM(1 + e\cos\pi)}$

$V_A = \sqrt{\dfrac{GM(1-e)}{r_A}} = 1173{,}6\ m/s$

W

On ellipse (1) : $V_A = \sqrt{\frac{Gr(1-e)}{b}} = 1173{,}6 \ m/s$

On ellipse (2) : $V_{A\,ellipse(2)} = \sqrt{\frac{Gr(1-e')}{b}}$; $e' = ??$

$\frac{r_{M}'}{r_m'} = \frac{1+e'}{1-e'} \Rightarrow r_M'(1-e') = r_m'(1+e')$

$e' = \frac{r_M' - r_m'}{r_M' + r_m} = \frac{b-a}{b+a} = 0{,}84$

$V_A = 710{,}2 \ m/s$

b) $\Delta V_B = \underbrace{V_{ellipse(2)}}_{?} - \underbrace{V_{circle\,(B)}}_{?}$

$V_{circle} = \sqrt{\frac{Gr}{r}} = \sqrt{\frac{Gr}{a}} = 6273{,}2 \ m/s$

$V_{B\,ellipse} = ??$

1st method : $r_B = \frac{k^2}{Gr(1+e'\cos 0)}$

$= \frac{r_B^2 V_B^2}{Gr(1+e')}$

2nd method :

$k = c t = r_A V_A = r_B V_B$

$V_B = \frac{r_A}{r_B} \cdot V_A = \frac{b}{a} V_{A\,ellipse(2)} \Rightarrow V_{B\,ellipse(2)} = \sqrt{\frac{Gr(1+e')}{a}}$

$= 8523{,}6 \ m/s$

$\Delta V_B = 8523{,}6 - 6273{,}2$

$= 2250 \ m/s$

Ex 21:

a- $r = \dfrac{h^2/Gr}{1+e\cos\theta}$ $\quad r_p = \dfrac{r_p^2 V_p^2}{Gr(1+e\cos\theta)}$ $\quad$ où P, $\theta = 0$, $\cos\theta = 1$

$V_p = \sqrt{\dfrac{Gr(1+e)}{r_p}} = 9538,6 \, m/s$

b) $r = \dfrac{h^2/Gr}{1+e\cos\theta}$; $r_B = ??$

$r_B = \dfrac{r_p^2 V_p^2}{Gr(1+e\cos 135°)}$ $\quad h = r_p V_p = r_B V_B$

$= 6,5 \times 10^6 \, m = 6,5 \, Mm$