Université Libanaise

Faculté des Sciences, B3

Tripoli - Liban

الجامعة اللبنانية

كلية العلوم، الفرع 3

طرابلس – لبنان

Matière : Mécanique Classique (P1100)

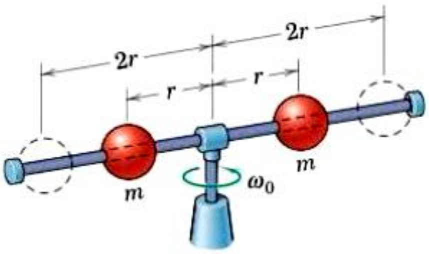
TD E

DYNAMICS FOR ROTATIONAL MOTION

1) The two particles of equal mass m are able to slide along the horizontal rotating rod. Neglect the small mass of the rod. Each particle is initially located in position a distance r from the rotating axis with the assembly rotating freely with an angular velocity ω_0 .

a) Determine the new angular velocity ω after the particles are released and finally assume positions at the ends of the rod at a radial distance of $2r$.

b) Find the fraction n of the initial kinetic energy of the system which is lost.



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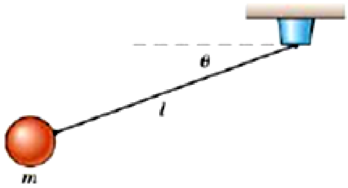
2) The simple pendulum of mass m and length l is released from rest at $\theta = 0$.

a) Determine the expression for $\ddot{\theta}$ in terms of θ.

b) Find the angular velocity $\dot{\theta}$ of the pendulum at $\theta = 90°$ and deduce the linear velocity v.



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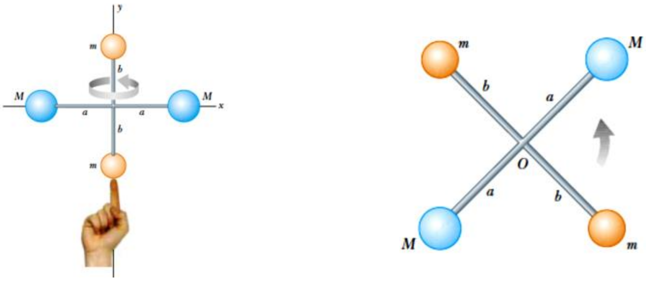
3) Four tiny spheres are fastened to the ends of two rods of negligible mass lying in the vertical (xy) plane. We shall assume the radii of the spheres are small compared with the dimensions of the rods.

a) If the system rotates about the y axis (Fig. 1) with an angular speed ω, find the moment of inertia and the rotational kinetic energy about this axis.

b) Suppose the system rotates in the (xy) plane about the z axis through O (Fig. 2). Calculate the moment of inertia and rotational kinetic energy about this axis.



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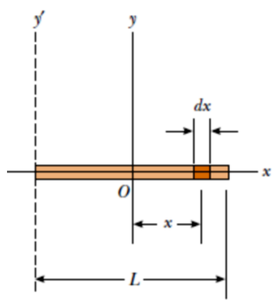
4) a) Calculate the moment of inertia of a uniform rigid rod of length L and mass M about an axis perpendicular to the rod (the y axis) and passing through its center of mass.

b) Find the moment of inertia of the rod about an axis perpendicular to the rod through one end (the y' axis).



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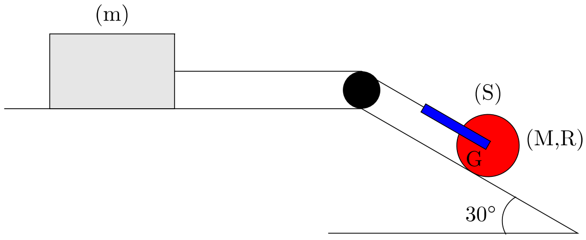
5) A sphere (S) of mass M = 4 kg, radius R = 20 cm and moment of inertia $I = 0,4MR^2$ can roll without sliding on an inclined plane by an angle α = 30 ° with respect to the horizontal. A block of mass m = 2 kg, is attached to the sphere through an inextensible wire and negligible mass. The wire passes around a massless pulley and the block can slide without friction on the horizontal plane.



a) Show that the tension T of the wire is given by $T = Mg\sin\theta - \frac{7}{5}Ma$, with a is the linear acceleration of the sphere. Deduce the value of « a » and « T ».

b) Calculate the minimum coefficient of static friction $\mu_{S_{min}}$ between the sphere and the incline to preserve the condition for rolling without sliding.

c) Find the angular speed of the sphere when it moves of 2 m.

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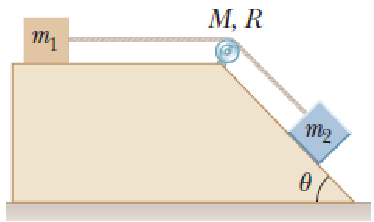
6) A block of mass $m_1 = 2\ kg$ and a block of mass $m_2 = 6\ kg$ are connected by a massless string over a pulley in the shape of a solid disk having radius R = 0.25 m, mass M = 10 kg and moment of inertia $I = \frac{1}{2}MR^2$. These blocks are allowed to move as shown in the figure where $\theta = 30°$. The coefficient of kinetic friction is 0.36 for both blocks. Assume the string is inextensible and does not slide on the pulley.



a) Draw free-body diagrams of both blocks and of the pulley.

b) Determine the acceleration of the two blocks.

c) Determine the tensions in the string on both sides of the pulley.

d) Assume that the string is cut and block 2 continues to move down the inclined plane. Use the work energy theorem to find the acceleration of this block.

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7) A particle of mass $m = 10$ g travelling horizontally with an initial speed $V = 100$ m/s strikes a homogeneous rod ($M = 1$ kg; $L = 1.2$ m) embedding itself at $t = 0$ at point C ($OC = 1$ m). The new rigid system (S) (particle, rod) can rotate without friction around an axis (Δ) perpendicular to the rod at the fixed pivot O as shown in the figure (Fig. 1). Take: $I_{rod/G} = \frac{1}{12}M.L^2$; G is the center of mass of the rod.

Fig. 1

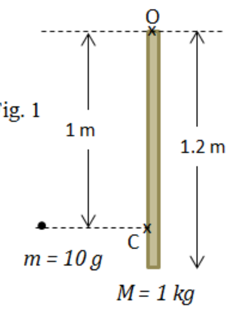
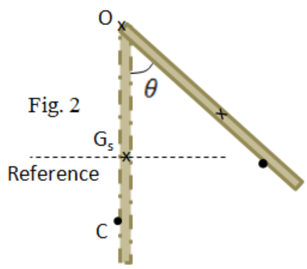


Fig. 2



1) a. Determine the moment of inertia of the system (S).

b. Determine the angular speed of the system (S) immediately after collision.

2) Show that the collision is not elastic and give a physical interpretation to this result.

3) Find the center of mass G_S of the system (S) relative to O.

4) After collision, find the maximum deviation angle, θ_{max}, of (S) with respect to the vertical. The gravitational potential energy is taken to be zero at the horizontal level passing by G_S at $t = 0$ (Fig. 2).

5) After reaching the maximum deviation, the system (S) heads back to its equilibrium position.

a. Using Newton's second law for a rigid object in rotation, find the angular acceleration of (S) when it makes an angle $\theta = 15°$ with the vertical. Deduce the linear acceleration of point C at this position.

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b. Determine the angular speed of the system at $\theta = 15°$ and deduce the linear velocity, V_C, of point C at this position.

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8) A solid homogeneous cylinder of mass m and radius R, rolls without slipping down an inclined plane of angle β with the horizontal (see Figure 1). The moment of inertia of a cylinder is $I_{CM} = \frac{1}{2}mR^2$.

a) Write the conditions for a rolling motion of a cylinder without slipping (see Figure 2).

b) Using Newton's second law find the acceleration of the center of mass of the cylinder (Figure 1).

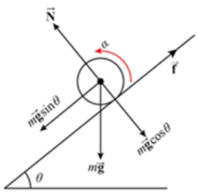

Figure 1

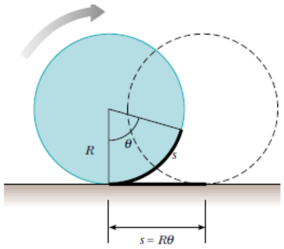

Figure 2

c) Find the minimum coefficient of friction so that slipping will not occur.

d) All points on a rolling object move in a direction perpendicular to an axis through the instantaneous point of contact P as shown in Figure 3. In other words, all points rotate about P. Using this property find the total kinetic energy of the rolling cylinder.

e) Although there is a force of friction, yet one can still use the principle of conservation of energy. Explain.

f) Let h the vertical distance covered by the center of mass. Using the energy methods find the corresponding velocity of the center of mass of the cylinder.

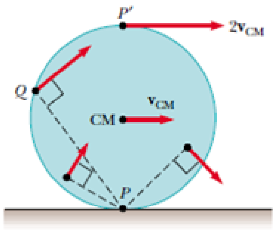

Figure 3

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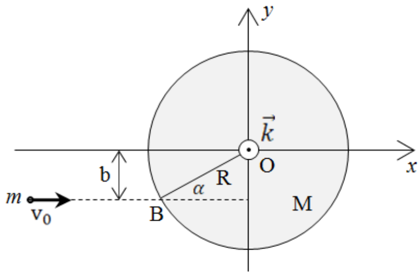
9) Consider a particle of mass $m = 200\ g$ moving horizontally at a distance $b = 15\ cm$ below the x-axis with a velocity $v_0 = 15\ m/s$. The particle strikes a disk (mass $M = 1kg$ and radius $R = 20\ cm$) that can rotate freely about the z-axis perpendicular to the plane of the figure below, embedding itself at B. The moment of inertia of the disk with respect to its center of mass is: $I_{C.M} = \frac{1}{2}MR^2$.



a) Show that the angular momentum of the system (particle, disk) before collision is given by: $\vec{L_i} = mv_0 b\,\vec{k}$ where $\vec{k}$ is the unit vector of the z-axis.

b) Calculate the angular velocity ω of the system (particle, disk) just after the collision.

c) Calculate the percentage loss of the initial kinetic energy due to the collision.

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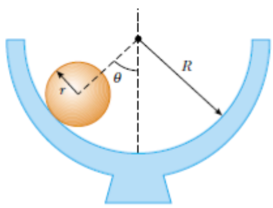
10) A uniform solid sphere of radius r is placed on the inside surface of a hemispherical bowl with much larger radius R. The sphere is released from rest at an angle θ to the vertical and rolls without slipping as shown in the figure. Use the conservation law of energy to determine the angular speed of the sphere when it reaches the bottom of the bowl.



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11) Two particles of equal mass $m_A = m_B = 1\ kg$ are positioned at the ends of two rods of negligible mass. Each rod of length $L = 1\ m$ makes an angle of 30^0 with the horizontal as shown in the figure. The system (rods, particles) rotates around an axis (Δ) with an initial angular velocity $\omega_0 = 8\ rad/sec$.

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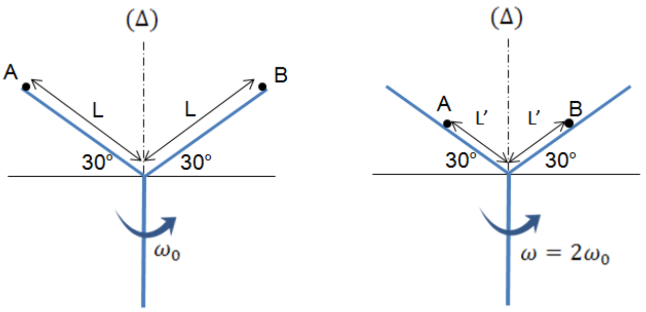

a) Show that the angular momentum of the system is conserved.
b) Calculate the initial angular momentum of the system.
c) Calculate the new position *L'* of each particle for an angular velocity $\omega = 2\omega_0$ at an instant *t*.
d) Find the variation of the kinetic energy ΔE_k between $t = 0$ and *t*.

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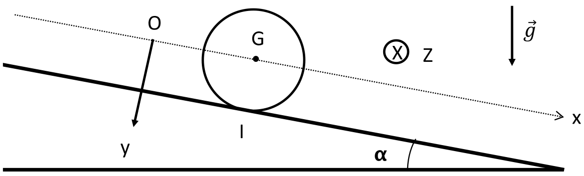
12) We consider a homogeneous sphere of mass m, radius R, and moment of inertia $I_G = \frac{2}{5}mR^2$.

Suppose that $\overrightarrow{OG} = x\vec{u}_x$ and $\vec{\omega} = \omega\vec{u}_z$.
The coefficient of friction between the sphere and the ground is μ.



a) What is the angle α to have a rolling without sliding? Determine ω in terms of the time t (two methods).
b) In the case of sliding, determine ω in terms of t.
