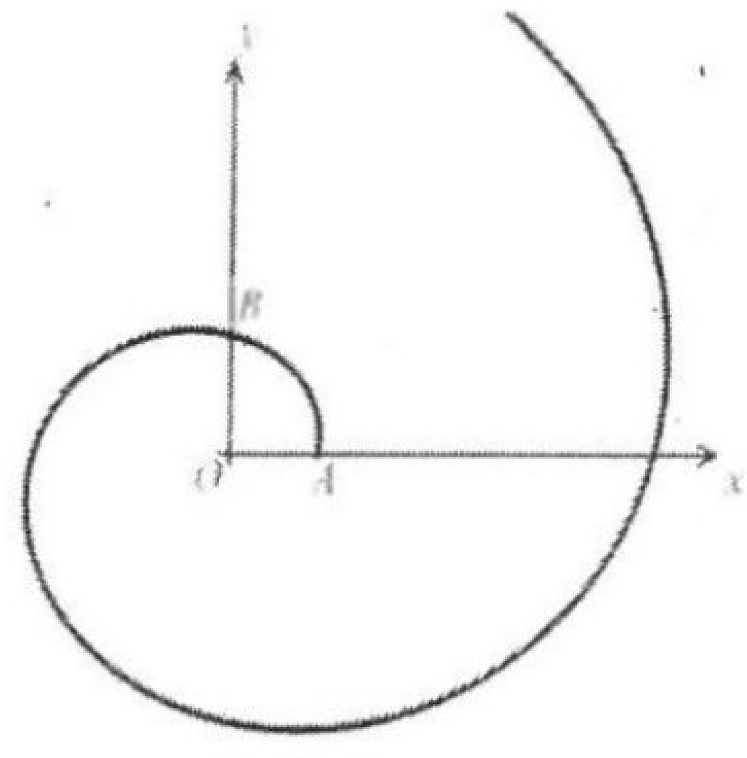
Lebanese University Faculty of Sciences V	Classical Mechanics P1100 - Partial	Date: Nov. 16th 2018 Duration: 60 min

Problem 1. *Spiral Motion.* (36 points)
A point M moves on a spiral polar equation:

$$r = r_0 e^{\theta}$$



with $\theta = \omega t$ is the direction with the x-axis; r_0 and ω are constants.

a) What are the cartesian coordinates of A and B?
b) Knowing that the polar unit vectors are $\vec{e_r}$ and $\vec{e_\theta}$;
$(\vec{e_r}, \vec{e_\theta}) = \frac{\pi}{2}$ and $\overrightarrow{OM} = r\vec{e_r}$.
Calculate, in terms of ω and r, the radial component V_r and transverse component V_θ , and the magnitude of the velocity vector $\vec{V}$.
c) Determine the angle between $\vec{V}$ and $\vec{e_r}$.
d) Find the radial component a_r and transverse component a_θ, and the magnitude of the acceleration vector $\vec{a}$.

Problem 2. *Lennard-Jones Potential.* (36 points)
A commonly used potential energy function to describe the interaction between two atoms is the Lennard-Jones 6, 12 potential:

$$E_p(r) = E_0\left[\left(\frac{r_0}{r}\right)^{12} - 2\left(\frac{r_0}{r}\right)^{6}\right] \ ; r > 0$$

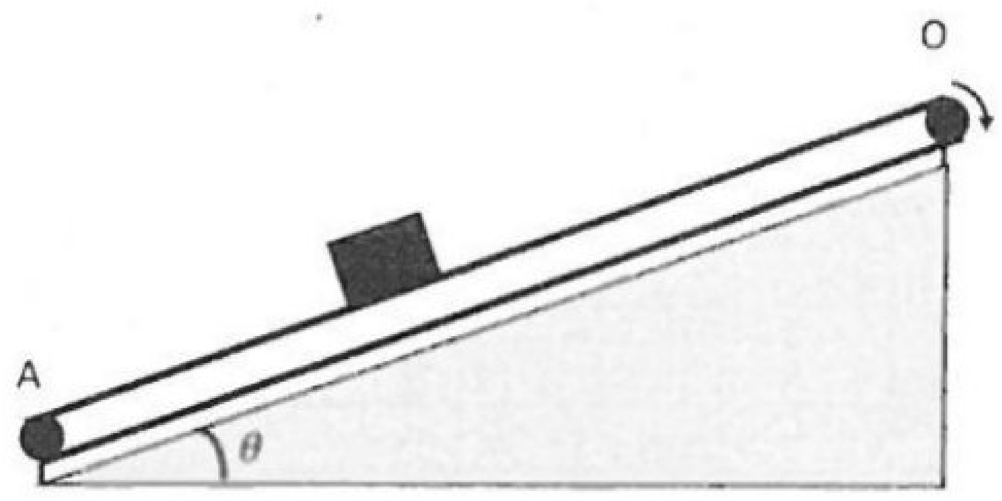
Where r is the distance between the atoms; E_0 and r_0 are positive constant.

a) Find the force associated to this potential.
b) What is the equilibrium position, and explain whether the equilibrium is stable or not?
c) Determine the corresponding potential energy.

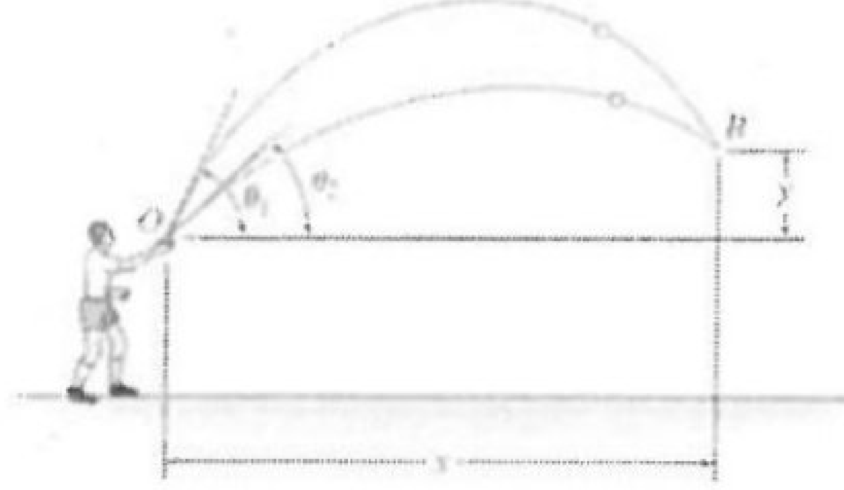
Problem 3. *Rolling carpet - Projectile.* (28 points)
The two parts A and B are independent.

<u>Part A.</u>

An object of mass m = 3 kg is placed on a conveyor that makes an angle $\theta = 30°$ with the horizontal. The coefficient of static friction between the object and the conveyor is μ_s. The system (object + conveyor) are driven upward by a traction force $F = 30\ N$. Find the minimum value of μ_s that permits the system to move with a constant speed $v = 2\ m/s$. *Given*: g = 10 m/s²



<u>Part B.</u> (In this part all types of friction forces are neglected)
When the object arrives at point O, it is ejected with a speed v at an angle θ_2 with respect to the horizontal. If another object is launched with the same speed v but at an angle $\theta_1 > \theta_2$ then **show** that the time interval separating the launch of the two objects is equal to:



$$\Delta t = |t_1 - t_2| = \frac{2v}{g}\frac{\sin(\theta_1 - \theta_2)}{(\cos\theta_1 + \cos\theta_2)}$$

Such that the two objects reach B at the same instant.
Use the identity: $\sin(a - b) = \sin(a)\cos(b) - \cos(a)\sin(b)$

Good reflection

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PI) 26 pts $r = r_0 e^{\theta} = r_0 e^{\omega t}$. r_0 and ω are constant.

a) $\begin{cases} x = r\cos\theta = r_0 e^{\theta}\cdot\cos\theta \; (1) \\ y = r\sin\theta = r_0 e^{\theta}\sin\theta \; (1) \end{cases}$ $A(\theta = 0)$ $(1\tfrac{1}{2})$ $B(\theta = \tfrac{\pi}{2})$ $(1\tfrac{1}{2})$

$\begin{cases} x_A = r_0 \; (1) \\ y_A = 0 \; (1) \end{cases}$; $\begin{cases} x_B = r_0 e^{\pi/2}\cos\tfrac{\pi}{2} = 0 \; (1) \\ y_B = r_0 e^{\pi/2}\sin\tfrac{\pi}{2} = r_0 e^{\pi/2} \; (1) \end{cases}$

$\boxed{A(r_0, 0)}$ $\boxed{B(0; r_0 e^{\pi/2})}$.

b) $\overrightarrow{OM} \overset{(1)}{=} r\vec{e_r} \Rightarrow \vec{V} \overset{(1)}{=} \dfrac{d\overrightarrow{OM}}{dt} = \overset{(1)}{\dot{r}\vec{e_r}} + \overset{(1)}{r\dot{\theta}\vec{e_\theta}}$

$\begin{cases} v_r = \dot{r} = r_0\omega e^{\omega t} \; (1) \\ v_\theta = r\dot{\theta} = r_0\omega e^{\omega t} \; (1) \end{cases}$ $V \overset{(1)}{=} \sqrt{V_r^2 + V_\theta^2} = \sqrt{2}\cdot r_0\omega e^{\omega t}$ (2)

c) $\vec{V}\cdot\vec{e_r} \; (2) = (\dot{r}\vec{e_r} + r\dot{\theta}\vec{e_\theta})\cdot\vec{e_r} \; (1) = \dot{r} \; (1) = |V|\cdot\cos\alpha$ (1).

$\Rightarrow \cos\alpha = \dfrac{\dot{r}}{|V|} = \dfrac{r_0\omega e^{\omega t}}{\sqrt{2}\, r_0\omega e^{\omega t}} \; (1) = \dfrac{\sqrt{2}}{2} \; (1) \Rightarrow \boxed{\alpha = \dfrac{\pi}{4}}$ (2).

d) $\begin{cases} a_r = \ddot{r} - r\dot{\theta}^2 \; (1\tfrac{1}{2}) = r_0\omega^2 e^{\omega t} - r_0 e^{\omega t}\cdot\omega^2 = 0 \; (2) \\ a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \; (1\tfrac{1}{2}) = 0 + 2r_0\omega^2 e^{\omega t} = 2r_0\omega^2 e^{\omega t} \; (2) \end{cases}$

$a = \sqrt{a_r^2 + a_\theta^2} \; (1) = \boxed{2r_0\omega^2 e^{\omega t} = a}$ (1)

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PII) 40 $E_p(r) = E_0\left[\left(\frac{r_0}{r}\right)^{12} - 2\left(\frac{r_0}{r}\right)^{6}\right]$ $r > 0$.
$E_0, r_0 > 0$.

a) $\vec{F} = -\overrightarrow{\text{grad}}\, E_p(r) = -\frac{dE_p}{dr}\vec{u}_r$. 6

$\vec{F} = 12 E_0 r_0^6\left[r_0^6 r^{-13} - r^{-7}\right]\vec{u}_r$ 6

b) * $\vec{F} = \vec{0}$ 3 Position d'équilibre (Equilibrium position)

$\Rightarrow r_0^6 r^{-13} - r^{-7} = 0$ 3 $\Rightarrow r_0^6 r^{-6} - 1 = 0 \Rightarrow \boxed{r_0 = r} > 0$ 3

* stability: $\frac{d^2E}{dr^2} > 0$ or < 0 ?

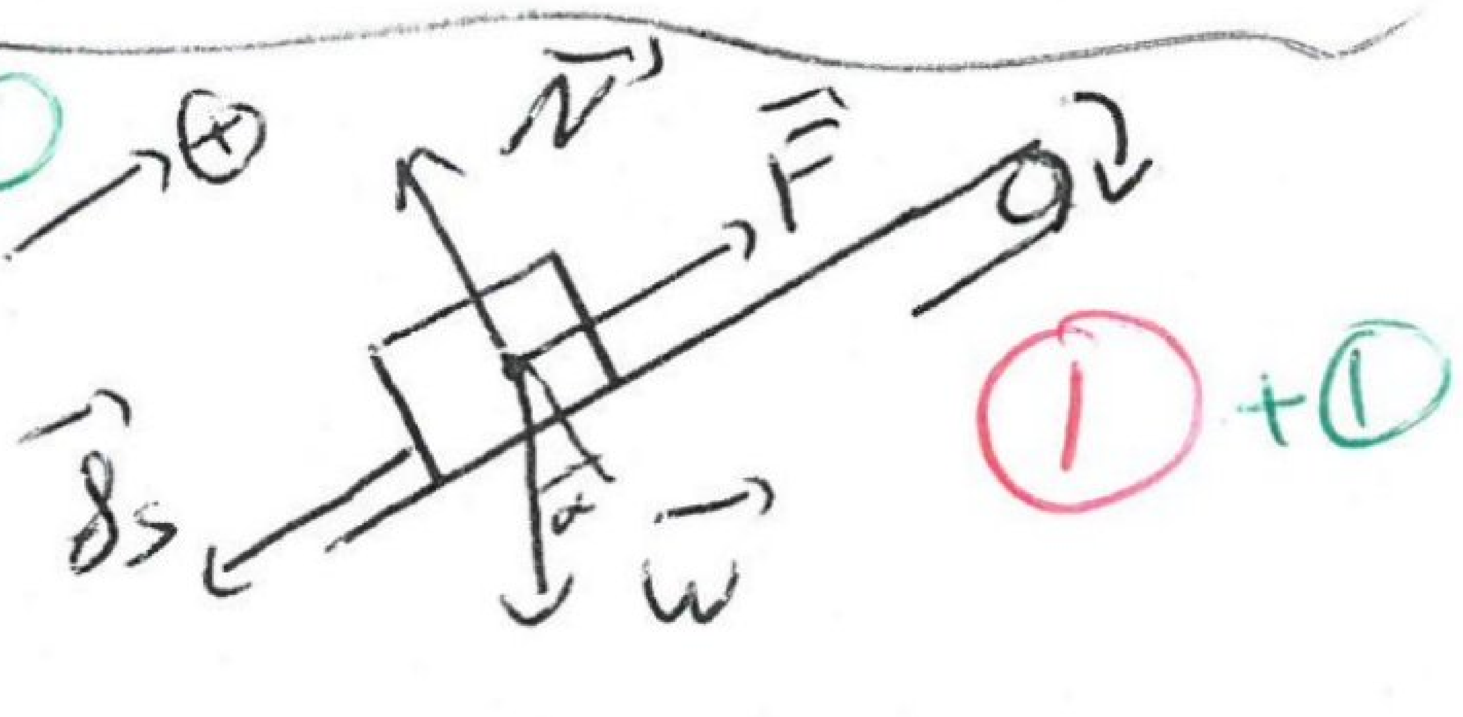
$\frac{dE}{dr} = E_0\left[-12 r_0^{12} r^{-13} + 12 r_0^6 r^{-7}\right]$ 3

$\frac{d^2E}{dr^2} = E_0\left[12\times 13\, r_0^{12} r^{-14} - 12\times 7\times r_0^6 r^{-8}\right]$ 3

For $r = r_0 \Rightarrow \left.\frac{d^2E}{dr^2}\right)_{r_0} = 72 E_0 r_0^{-2} > 0 \Rightarrow$ Stable. 3

c) $E_p(r_0) = E_0\left[\left(\frac{r_0}{r_0}\right)^{12} - 2\left(\frac{r_0}{r_0}\right)^{6}\right] = \boxed{-E_0 = E_p(r_0)}$ 3 3

PIII) 28 Part A



NSL: $\vec{F} + \vec{w} + \vec{N} + \vec{f}_s = m\vec{a}$ 2

$v = 2 = cte \Rightarrow a = 0$. 2 +1

Projection $\vec{i}$: $-f_s - w\sin\alpha + F = 0 \Rightarrow f_s = F - w\sin\alpha = 15N$ 2 +1

" $\vec{j}$: $N - w\cos\alpha = 0$ 2 +1 $\Rightarrow f_{s\,max} = \mu_s N = \mu_s m g \cos 30$ 2 +1

$f_{s\,max} = 15\sqrt{3}\,\mu_s (N)$. 2 +1

Object at rest % carpet $\Rightarrow f_s \le f_{s\,max} \Rightarrow \mu_s > \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \boxed{= 0{,}58 = \mu_{s\,min}}$ +1 1

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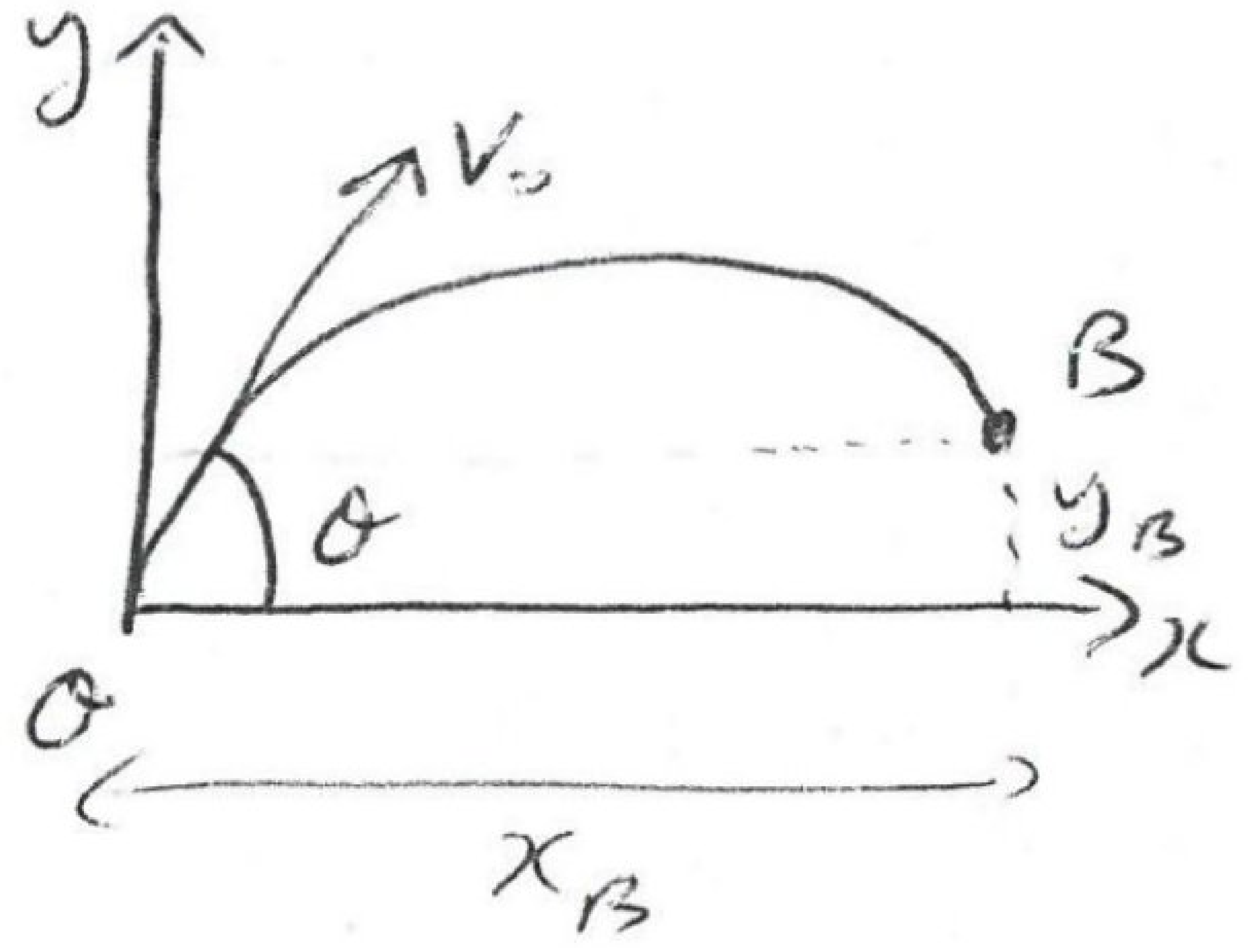
Part B



NSL $\quad \cancel{m}\vec{g} = \cancel{m}\vec{a} \Rightarrow \vec{a} = -g\vec{j}$ 1

$a_x = 0$ 1 $\quad \Rightarrow \vec{V} = \begin{cases} V_0\cos\theta \\ -gt + (V_0\sin\theta) \end{cases}$ 1

$a_y = -g$

$\vec{OM}(t) \begin{cases} x(t) = (V_0\cos\theta)t + x_0 \\ y(t) = -\frac{1}{2}gt^2 + (V_0\sin\theta)t + \cancel{y_0} \end{cases}$ 1

Particle 1: $x_1 = (V_0\cos\theta_1)\,t_1$ $\quad$ **Particle 2**: $x_2 = (V_0\cos\theta_2)\,t_2$

$y_1 = -\frac{1}{2}gt_1^2 + (V_0\sin\theta_1)t_1$ $\quad$ $y_2 = -\frac{1}{2}gt_2^2 + (V_0\sin\theta_2)t_2$

at B : $x_1 = x_2$ $\frac{1}{2}$ and $y_1 = y_2$ $\frac{1}{2}$

$\Rightarrow \boxed{(\cos\theta_1)t_1 = (\cos\theta_2)t_2}$ 1 $\Rightarrow t_1 = \frac{\cos\theta_2}{\cos\theta_1}t_2$

and $-\frac{1}{2}gt_2^2 + (V_0\sin\theta_2)t_2 = -\frac{1}{2}gt_1^2 + (V_0\sin\theta_1)t_1$.

$\Rightarrow -\frac{g}{2}\left[t_2^2 - t_1^2\right] = V_0\left[\sin\theta_1\, t_1 - \sin\theta_2 t_2\right]$

$\Rightarrow -\frac{g}{2}\left[t_2^2 - \left(\frac{\cos\theta_2}{\cos\theta_1}\right)^2 t_2^2\right] = V_0\left[\sin\theta_1 \cdot \frac{\cos\theta_2}{\cos\theta_1}t_2 - \sin\theta_2 t_2\right]$

$\Rightarrow t_2 = \frac{2V_0}{g}\left[\frac{\overset{(a)}{\sin\theta_1\cos\theta_2\cos\theta_1} - \overset{(b)}{\cos^2\theta_1\sin\theta_2}}{+\cos^2\theta_2 - \cos^2\theta_1}\right]$ $\cancel{3}$

and $\Rightarrow t_1 = \frac{2V_0}{g}\left[\frac{\overset{(c)}{\sin\theta_1\cos^2\theta_2} - \overset{(d)}{\cos\theta_1\cos\theta_2\sin\theta_2}}{+\cos^2\theta_2 - \cos^2\theta_1}\right]$

$\Delta t = |t_1 - t_2| = \frac{2V_0}{g}\;\frac{(c) - (d) - (a) + (b)}{(\cos\theta_2 - \cos\theta_1)(\cos\theta_2 + \cos\theta_1)}$

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$$\Delta t = |t_1 - t_2| = \frac{2V_0}{g}\,\frac{\sin\theta_1\cos^2\theta_2 - \cos\theta_1\cos\theta_2\sin\theta_2 - \sin\theta_1\cos\theta_2\cos\theta_1 + \cos^2\theta_1\sin\theta_2}{(\cos\theta_2 - \cos\theta_1)(\cos\theta_2 + \cos\theta_1)}$$

$\cancel{4}$ 1

$$\Delta t = \frac{2V_0}{g}\,\frac{\sin\theta_1\cos\theta_2\,\cancel{(\cos\theta_2 - \cos\theta_1)} + \cos\theta_1\sin\theta_2(\cos\theta_1 - \cos\theta_2)}{\cancel{(\cos\theta_2 - \cos\theta_1)}(\cos\theta_2 + \cos\theta_1)}$$

$$\boxed{\Delta t = \frac{2V_0}{g}\,\frac{\sin(\theta_1 - \theta_2)}{\cos\theta_2 + \cos\theta_1}}$$

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