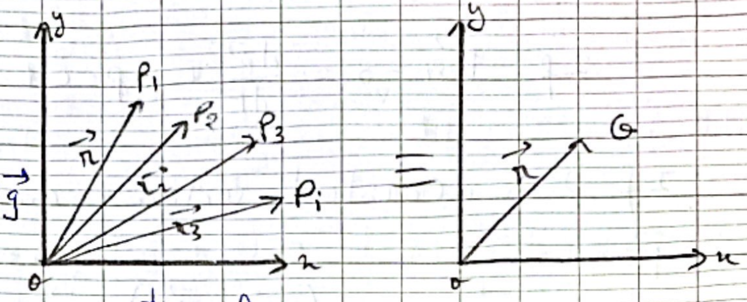
chap4 : Momentum

1. Center of mass:

$\vec{F} = \vec{F_1} + \vec{F_2} + \dots \vec{F_n}$

$= (m_1 + m_2 + \dots + m_n)\vec{g}$

$= M_{CM}\, \vec{g}$



system of n particles $1 \leq i \leq n$

$M_{CM} = \sum_{i=1}^{n} m_i = M_G$

CM : Center of mass or Center of gravity or Center of inertia.

$$\vec{OG} = \vec{r}_{CM} = \frac{\sum_{i=1}^{n} m_i \vec{r_i}}{\sum_{i=1}^{n} m_i}$$

2. Linear momentum $\vec{p}$.

For 1 particle : linear momentum $\vec{p} = m\vec{v}$

For a system of n of n particles: $\vec{P} = \vec{P_1} + \vec{P_2} + \dots + \vec{P_n} = m_1\vec{v_1} + m_2\vec{v_2} + \dots + m_n\vec{v_n}$

$= M\,\vec{V}_{CM}$

$M = \sum_{i=1}^{n} m_i$

3. Theorem of linear momentum :

fondamental Relation of dinamic

FRD for 1 particle : $\vec{F}_{ext} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$.

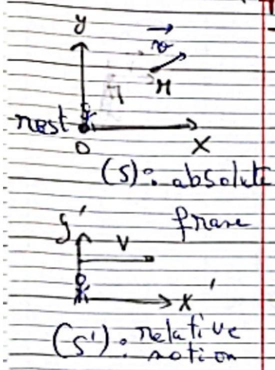
For a system of n particles: $\vec{F}_{ext} = M\vec{a}_{CM} = \frac{d\vec{P}}{dt}$.

$\vec{F}_{ext} = \frac{d\vec{p}}{dt} \Rightarrow \int \vec{F}_{ext}\, dt = \int_{\vec{p_i}}^{\vec{p_f}} d\vec{p}$.

$\int \vec{F}_{ext}\, dt = \vec{p_f} - \vec{p_i}$.

$$\boxed{\vec{p_f} = \vec{p_i} + \int \vec{F}_{ext}\, .dt}$$

4- Conservation of the linear momentum:



If $\vec{F}_{ext} = 0 = \frac{d\vec{p}}{dt}$; $\vec{p} = cst$; $\vec{p}_f = \vec{p}_i$.

If $\vec{p}$ is a constant vector in an absolute frame S; $\frac{d\vec{p}}{dt} = 0$,

$\vec{p}'$ is also cst vector in a relative initeal fram S'; $\frac{d\vec{p}'}{dt} = 0$

$\vec{p}_{(ab)} = m\vec{v}_{ad} = m(\vec{v}' + \vec{v}_d) = \vec{p}' + m\vec{v}_d$. Graliléan $\vec{v}_d = \vec{v}_{s'} = cst$

absolute

relative velocity

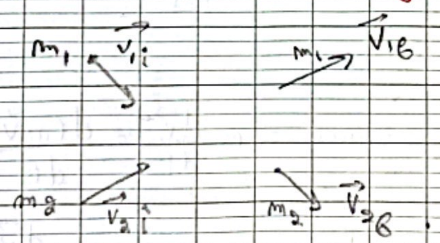
$$\frac{d\vec{p}}{dt} = \frac{d\vec{p}'}{dt} + \frac{d\, m\vec{v}_d}{dt}$$

$$\text{If } \frac{d\vec{p}}{dt} = 0 = \frac{d\vec{p}'}{dt} + \cancel{\frac{d}{d}(m\vec{v}_d)}$$

$$\frac{d\vec{p}'}{dt} = 0$$

The linear momentum is invariant under a Galiléan transformation

5- Collision (and disintegration):



isolated system

In a Collesion m_1 and m_2 move toward each other and interact, then can go away from each other. We Consider two states:

Initial state: well before Collesion

Final state: very after collesion

where the system is an isolated System

System : $m_1 + m_2$

$$\vec{P_i} = \vec{P_f}$$

$$\vec{P_{1i}} + \vec{P_{2i}} = \vec{P_{1f}} + \vec{P_{2f}}$$

$$m_1 \vec{V_{1i}} + m_2 \vec{V_{2i}} = m_1 \vec{V_{1f}} + m_2 \vec{V_{2f}} .$$

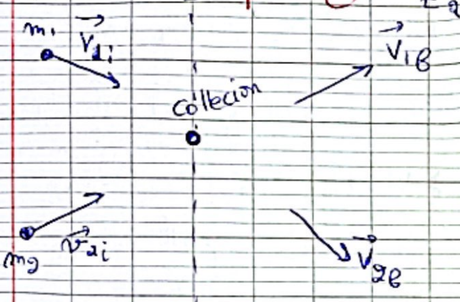
Restitution Coefficient:

نحنا منا نبرهن

$$e = -\frac{\vec{V_{f2}} - \vec{V_{f1}}}{\vec{V_{i2}} - \vec{V_{i1}}}$$; if $e = 1$: elastic collotion.

$0 < e < 1$: inelastic

t_1 phase ① t pase ② t_2



phase ① $(t_1 \to t)$

phase ② $(t \to t_2)$

In a Collesion, we have two phases:

phase 1: phase of deformation under the act° of $\vec{f}$ (~~deformation force~~). May cause deformation

phase 2: phase of restitution under the effect of $\vec{R}$ (restitution force).

~~Theorem of lin~~

Theorem of linear momentum:

$$\vec{P_f} = \vec{P_i} + \int \vec{F_{ext}} \cdot dt .$$

phase ① $(t_1 \to t)$

on m_1 : $m_1 \vec{V} = m_1 \vec{V_{1i}} + \int \vec{f} \cdot dt$

on m_2 : $m_2 \vec{V} = m_2 \vec{V_{2i}} + \int \vec{f} \cdot dt$

phase ② $(t \to t_2)$

$m_1 \vec{V_{1f}} = m_1 \vec{V} + \int \vec{R} \cdot dt .$

$m_2 \vec{V_{2f}} = m_2 \vec{V} + \int \vec{R} \cdot dt .$ restitution

the Coefficion of restitution is defined as:

$$e = \frac{\int \vec{R} \cdot dt}{\int \vec{f} \cdot dt} = \frac{m_1 (\vec{V_{1f}} - \vec{V})}{m_1 (\vec{V} - \vec{V_{1i}})} = \frac{\vec{V_{1f}} - \vec{V}}{\vec{V} - \vec{V_{2i}}}$$

By eliminating $\vec{V}$: $$\boxed{e = -\frac{\vec{V_{2f}} - \vec{V_{1f}}}{\vec{V_2} - \vec{V_1}} \geq 0}$$ → حفظ

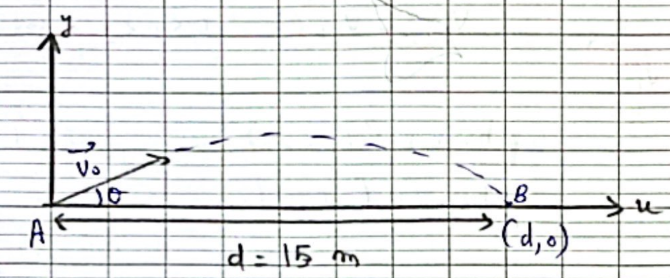
3 Cases:

* $e = 1$: elastic Collision : $\vec{P}_i = \vec{P}_f$. Collision
  $KE_i = KE_f$. (Elastic Collision)

* $e = 0$: soft Collision : $\vec{P}_i = \vec{P}_f$; $KE_i \neq KE_f$

* $0 < e < 1$: inelastic collision: $\vec{P}_i = \vec{P}_f$; $KE_i \neq KE_f$.

Exercice 1:



$\vec{P}_0 = m\vec{v}_0$??

projectiles $x = v_{0x}t + x_0 = v_0 \cos\theta\, t + 0$

$x_B = \boxed{v_0 \cos\theta\, t_B = d}$ ① $\Rightarrow t_B = \dfrac{d}{v_0 \cos\theta}$

$y = -gt^2 + v_{0y}t + y_0 = -\frac{1}{2}gt^2 + v_0 \sin(\theta) t.$

$y_B = \boxed{-\frac{1}{2}gt_B^2 + v_0 \sin\theta\, t_B = 0}$ ②

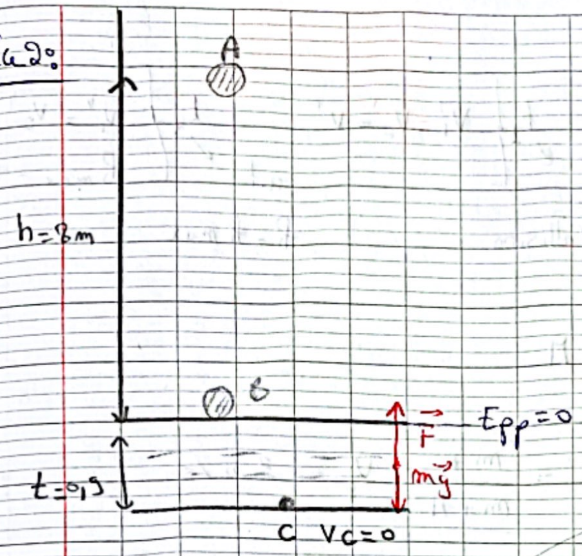
$-\frac{1}{2}g t_B + v_0 \sin\theta$

$-\frac{1}{2}g\left(\dfrac{d}{v_0\cos\theta}\right)^2 + v_0 \sin\theta \left(\dfrac{d}{v_0\cos\theta}\right) = 0.$

$v_0 = \sqrt{\dfrac{gd}{2\sin\theta\cos\theta}} = 13{,}16 \text{ m/s}.$

$P_0 = \underbrace{m}_{Kg}\,\underbrace{v_0}_{m/s} = 2{,}63 \text{ Kg m/s or N/s}.$

Exercice 2:



Theorem of linear momentum:

$$\vec{P}_B = \vec{P}_i + \int \Sigma \vec{F}_{ext}\, dt$$

$$\vec{P}_C = \vec{P}_B + \int (m\vec{g} + \vec{F})\, dt.$$

$$0 = m\vec{V}_B + \int (m\vec{g} + \vec{F})\, dt.$$

proj ony:

$$0 = mV_B + \int_0^t (mg - F)\, dt.$$

$$0 = mV_B + mgt - Ft.$$

$$F = \frac{m(V_B + gt)}{t} = 121\ N$$

let's find V_B:

$$ME_A = ME_B$$

$$\frac{1}{2} m V_A^2 + mgh_A = \frac{1}{2} m V_B^2 + mg\overset{0}{h_B}$$

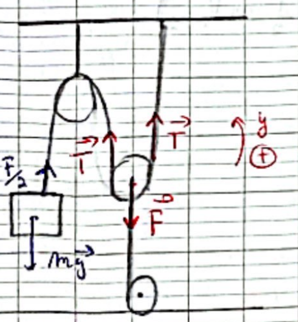
$$V_B = \sqrt{V_A^2 + 2gh}$$

Exercice 3:

$2\vec{T} = \vec{F}$

$\downarrow$

$T = \frac{F}{2}$



$\Rightarrow$ (1) $\vec{F} + \vec{T} + \vec{T} = 0$

$-F + 2T = 0 \Rightarrow T = \frac{F}{2}$.

Since the box is inially at rest, It will Start moving upward when the Force $\frac{F}{2}$ exceeds the weight of the box.

$$\frac{F}{2} + m\vec{g} = m\vec{a}$$

proj ony: $\frac{F}{2} - mg = ma \geq 0$

$$\frac{5}{2}(30 + t^2) - mg \geq 0$$

$$\Rightarrow t = \sqrt{\frac{2mg}{5} - 30} = \sqrt{2}\ s.$$

يباشي يطلع في الوقت $t = \sqrt{2}s$

$\Rightarrow$ (2)

$$\vec{P}_B = \vec{P}_i + \int \Sigma \vec{F}_{ext}\, dt$$

$$\vec{P}_B = \vec{P}_i + \int \left(\frac{\vec{F}}{2} + m\vec{g}\right) dt$$

$$m\vec{V}_B = m\vec{V}_i + \int_{t_i=\sqrt{2}}^{t_B=4} \left(\frac{\vec{F}}{2} + m\vec{g}\right) dt$$

projection ony: $mV_B = m\overset{0}{V_i} + \int_{\sqrt{2}}^{4} \left(\frac{F}{2} - mg\right) dt$

$$mV_B = \int \left[\frac{5}{2}(30 + t^2) - mg\right] dt$$

$$= 75t + \frac{5}{6}t^3 - mgt \Big|_{\sqrt{2}}^{4}$$

$$V_B = 4,76\ m/s.$$

Exercice 4. (a)

before Collision: $t_0 \begin{cases} m_i \quad V_1 = V_0 = 30\ m/s \\ M_i \quad V_2 = 0 \end{cases}$

Just after Collision: $t_1 \begin{cases} V_1' = V_2' = V' \end{cases}$

at $h = h_{max}$: $t_2 \begin{cases} V_1'' = V_2'' = 0 \\ h_{max} = ?? \end{cases}$

$\vec{P}_{t_0} = \vec{P}_{t_1}$ System : $m + M$

$m\vec{V}_1 + \cancel{M\vec{V}_2} = (m+M)\vec{V}'$

proj on y: $m V_1 = (m+M)V' \Rightarrow V' = \frac{m}{m+M} \cdot V_0 = 3\ m/s$.

let's find h_{max}: $ME_{t_1} = ME_{t_2}$

$\frac{1}{2}(m+M)V'^2 + \cancel{(m+M)gh} = \frac{1}{2}\cancel{(m+M)V''^2} + (m+M)g\,h_{max}$

$h_{max} = \frac{V'^2}{2g} = 0{,}45\ m.$

Kinetic Energy

(b) $\Delta_{KE} = KE_{t_1} - KE_{t_0}$

$= \frac{1}{2}(m+M)V'^2 - \left[\frac{1}{2} m V_0^2 + \cancel{\frac{1}{2} M V_2^2}\right]$

$= -81\ J.$