P 1100 - Applications

Chapter 3

Application III-1:

A particle moves in a force field given by: $\vec{f} = \frac{25}{6} y\hat{i} + (z - x)\hat{j} + (2z^2 - x)\hat{k}$

following the trajectory defined by parametric equations:

$$x = 3t\ ,\ y = 2t^2\ ,\ z = t - 2.$$

a- Calculate the power received by the particle at time t.

b- What is the position of the particle when this power is minimal?

Application III-2:

A car weighing 2000 kg is moving at a constant speed (v=50m/s) on a horizontal road. The wind applies a resistive force defined by $f = 1.2v^2$.

If the engine efficiency is 0.62, calculate the power that the engine must develop for the car to maintain the same speed.

Application III-3:

Calculate the work done by the weight of a body with mass m, where $\vec{F} = m\vec{g}$. It is assumed that the acceleration due to gravity $\vec{g}$ is constant.

Application III-4:

To explain the motion of the Moon around the Earth and that of the planets around the Sun, Newton postulates the existence of a universal gravitational force acting between all material bodies and formulates the fundamental law of universal gravitation as $\vec{f} = -G\frac{Mm}{r^2}\hat{r}$

Calculate the work done by this force between two points A and B.

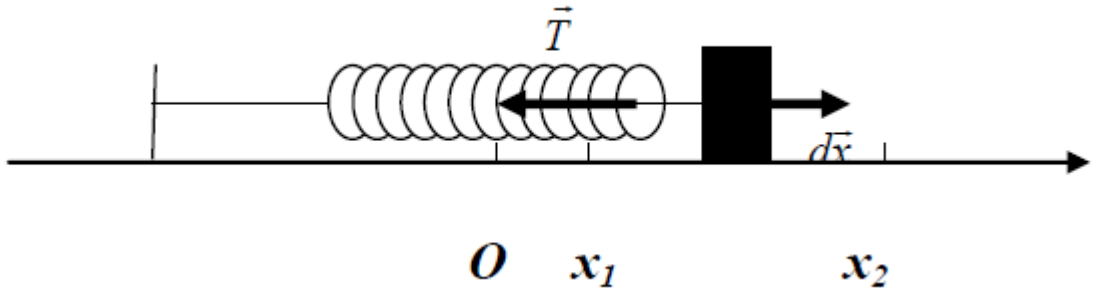
Application III-5:

In electrostatics, Coulomb's law states that two charges Q and q, located at a distance r from each other, attract or repel each other with a force along the line joining them, proportional to each of the charges and inversely proportional to the square of their distance. Calculate the work done by this force between two points A and B.

$$\vec{f} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}\hat{r} \qquad \varepsilon_0 = 8{,}85.10^{-12}\,F/m$$

Application III-6:

- Calculate the work done by tension of a spring when its point of application varies between two positions x_1 and x_2.



Application III-7:

- Calculate the work of the central force $\vec{F} = F(r)\hat{r} = -\frac{k}{r}.\hat{r}$ that depends on r. Deduce the potential function.

Application III-8:

Consider a particle moving in the force field: $\vec{f} = (x - ay)\hat{i} + (3y - 2x)\hat{j}$

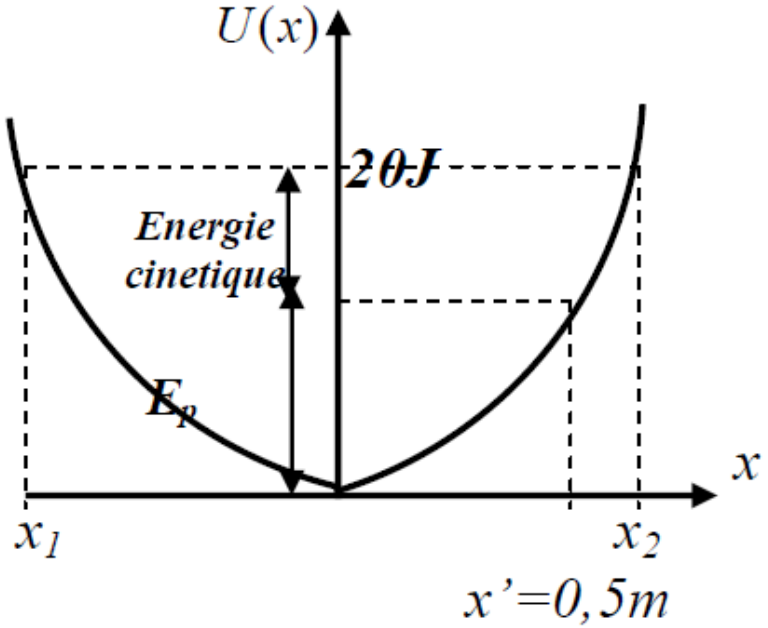
1- Calculate, in terms of a, the work done on the particle as it completes one revolution along the circle centered at the origin with a radius of 2 in the counterclockwise direction.

2- For what value of a does the force field f derive from a potential energy V? Determine V(x, y).

Application III-9:

It is given that the potential of a particle with mass M moving along the x-axis is described by the function $U(x) = \frac{1}{2}cx^2$.

Study the motion of this particle.



Application III-10:

A particle with a mass m = 2g moves along the x-axis and has the potential energy function $U = x^2 - 2x - 1$.

a- Find the force acting on the particle at any point x.

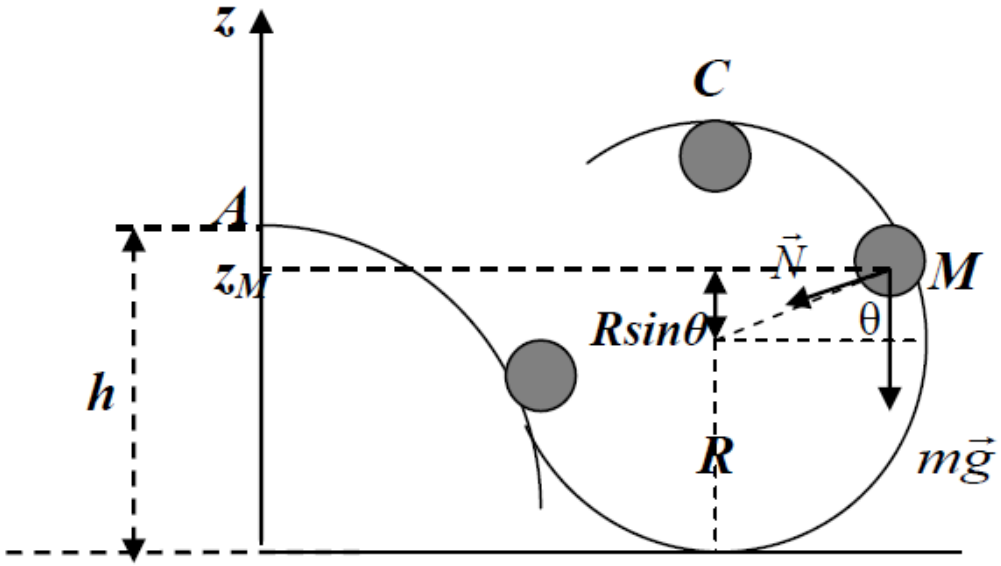
b- Determine the equilibrium position of the particle as well as the corresponding potential energy.

c- At time t=0, the particle is at the equilibrium position, moving with velocity $\vec{v}_o = -2\hat{i}$. Find its total mechanical energy, as well as the values of x for which the velocity becomes zero.

d- Show that the particle undergoes harmonic motion around its equilibrium position; find the period and amplitude of the motion.

Application III-11:

- A particle slides without friction in a trough ending with a circular loop. Calculate the minimum value of the initial altitude h for the particle, released at A without any initial velocity, to remain in contact with the trough throughout the entire trajectory.



Application III-12:

Calculate the work of the dry friction force, for example $\vec{F}_{fr} = -\mu mg\hat{v}$.

Application III-13:

Calculate the work of the viscous friction force $\vec{F}_{fr} = -\lambda v^n \hat{v}$.