P1100
Chapter 0

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Mathematical notions

1. Scalar Quantities and Vector Quantities
2. Vector addition
3. Vector projection
4. Scalar product and cross product
5. Derivative – differential – integral notions
6. Trigonometric relationships

1. Scalar Quantities and Vector Quantities

SCALAR QUANTITY

- **A quantity defined by magnitude only**

Ex: mass, length, time, power, energy

VECTOR QUANTITY

- A quantity that requires both <u>magnitude and direction</u>.

Ex: Position, Velocity, Acceleration, Momentum, Force.

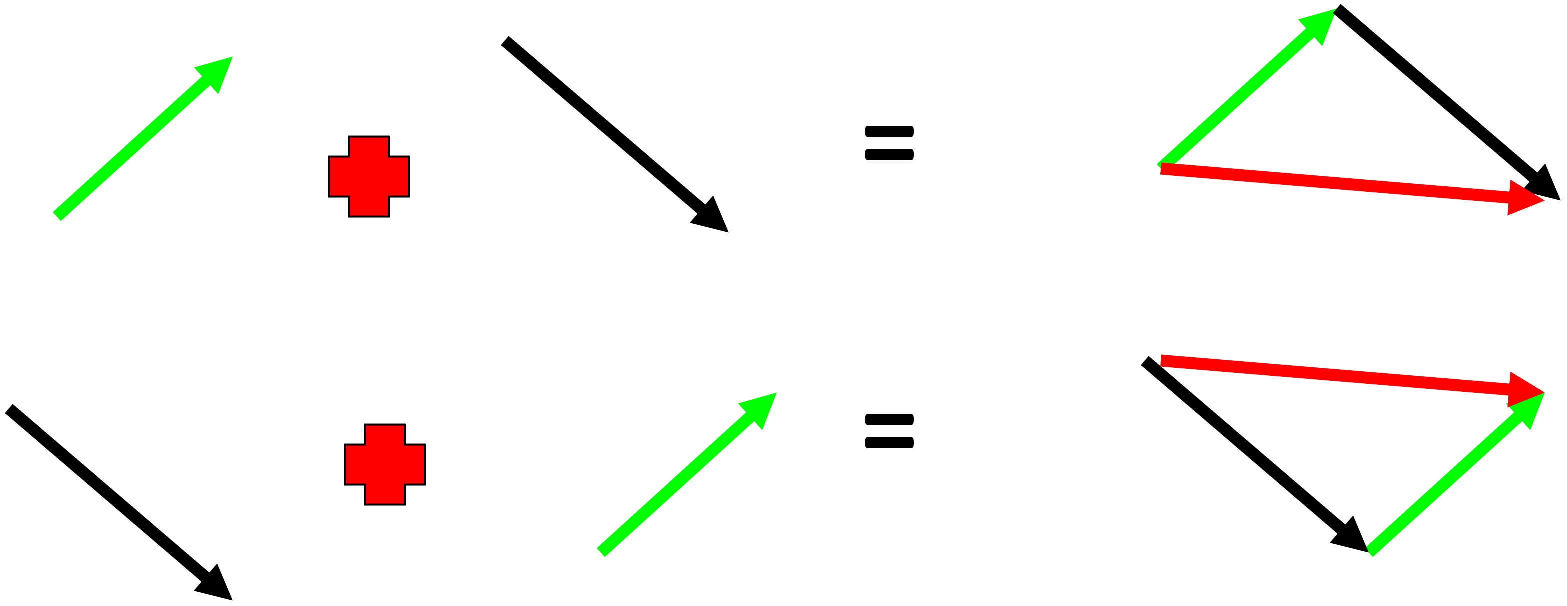
- **Vector quantity is represented by an arrow; length of the arrow indicates the magnitude and direction of the arrow indicates its direction.**

- Vectors have four characteristics:

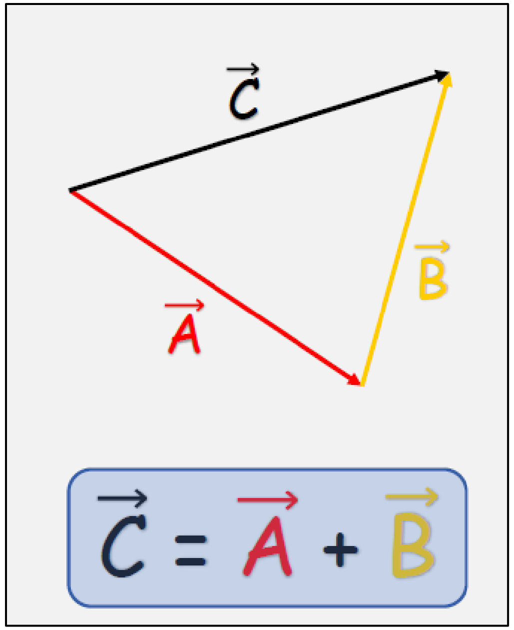
Point of application
Line of application
Direction
Magnitude

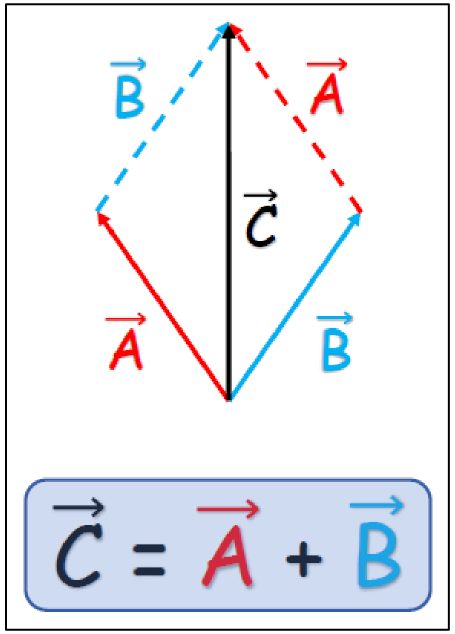
2. Vector Addition

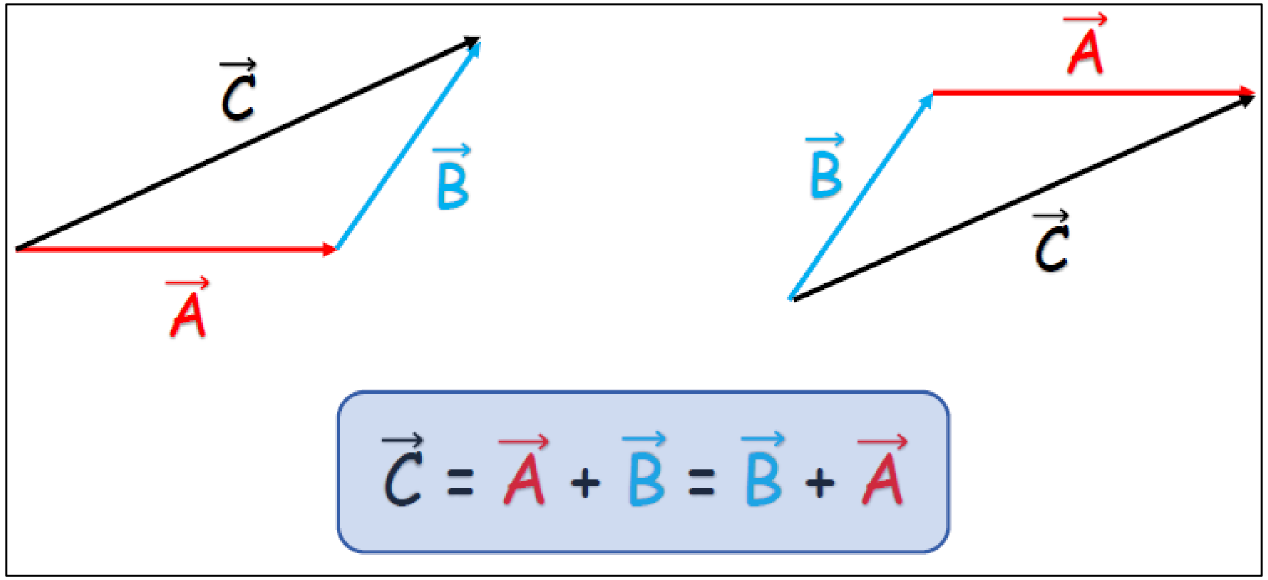
- The vectors are added using the head-to-tail method.

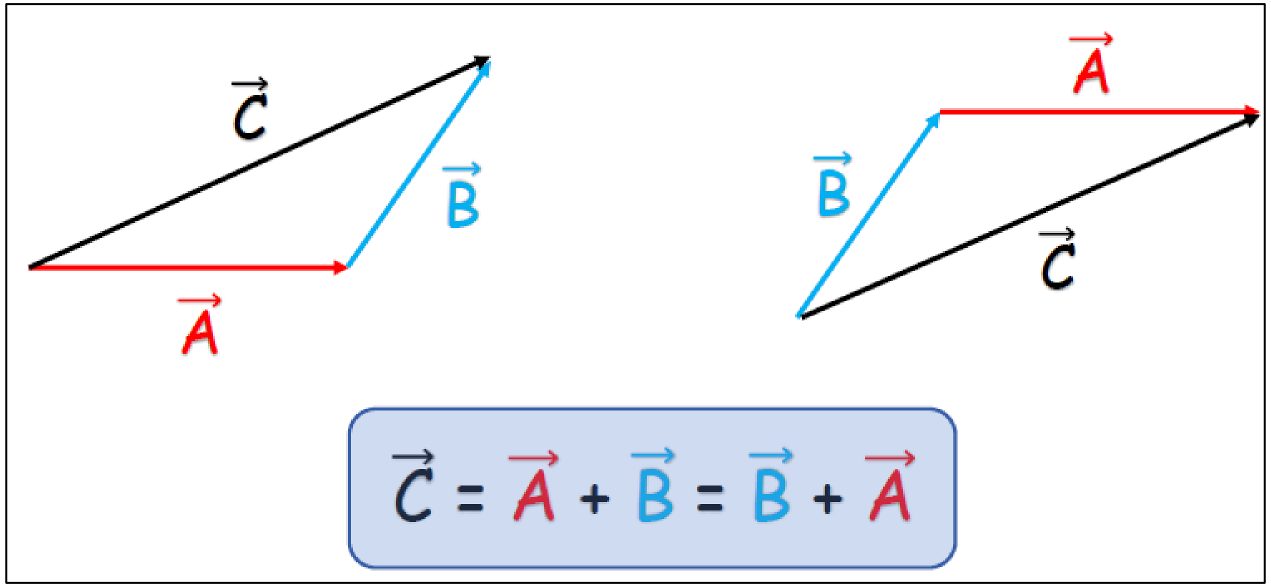


Vector addition



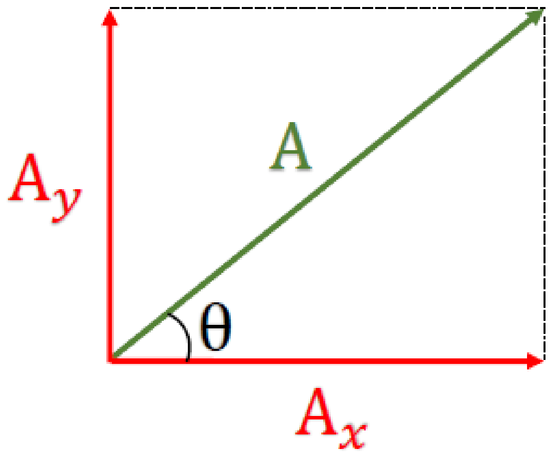






3. Vector projection

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$



Magnitude:

$$A = \sqrt{A_x^2 + A_y^2}$$

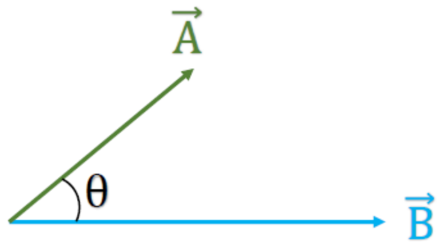
Direction:

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

4. Scalar product and cross product

Scalar product

- **Scalar product**: definition



$$\boxed{\vec{A}\cdot\vec{B} = AB\cos\theta}$$

In an orthonormal coordinate system:

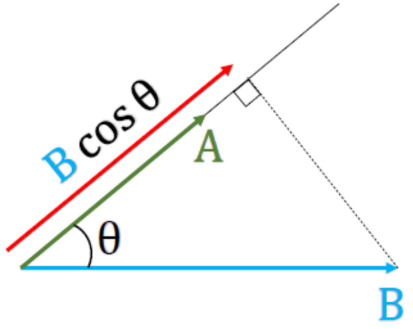
$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

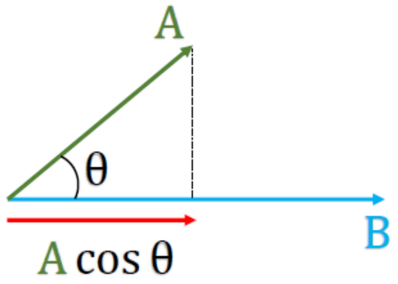
$$\boxed{\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z}$$

Scalar product: geometrical meaning

$$\vec{A}\cdot\vec{B} = A(B\cos\theta)$$



$$\vec{A}\cdot\vec{B} = (A\cos\theta)B$$



Examples of scalar product

$$W = \vec{F} \cdot \vec{s}$$

$$W = Fs\cos\theta$$

W = work done

F = force

s = displacement

$$P = \vec{F} \cdot \vec{v}$$

$$P = Fv\cos\theta$$

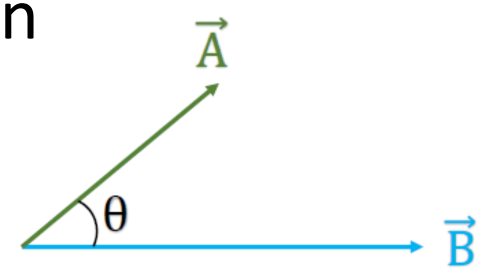
P = power

F = force

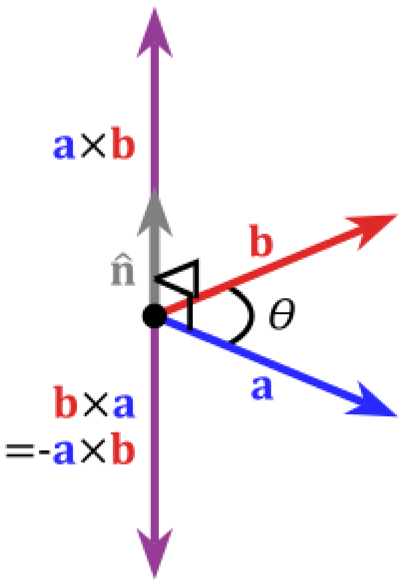
v = velocity

Vector Product

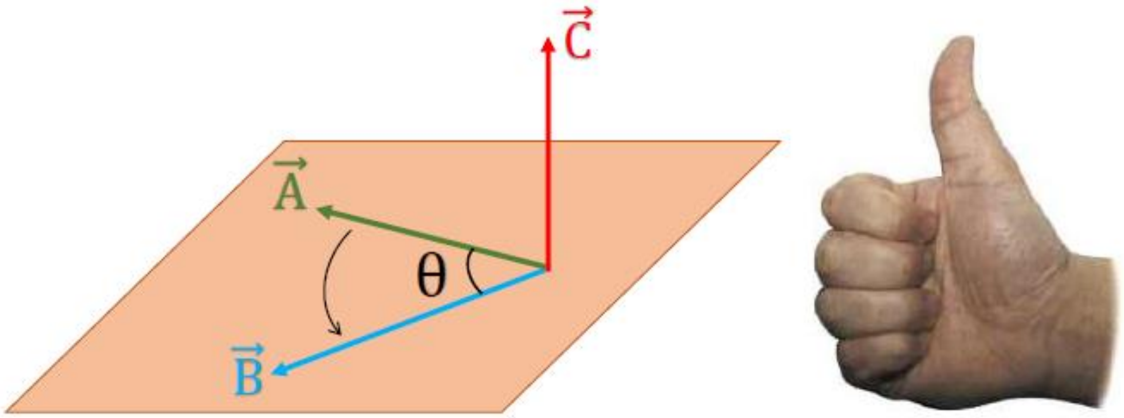
Definition



$$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n} = \vec{C}$$



Right hand rule



Examples of vector product

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = rF \sin\theta\, \hat{n}$$

τ = torque

r = position

F = force

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{L} = rp \sin\theta\, \hat{n}$$

L = angular momentum

r = position

p = linear momentum

$\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$	$(\lambda\vec{a}) \times \vec{b} = \lambda(\vec{a} \times \vec{b})$
$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$	$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

The vector product of two different unit vectors is a third unit vector.

$\hat{\imath} \times \hat{\jmath} = \hat{k}$	$\hat{\jmath} \times \hat{\imath} = -\hat{k}$
$\hat{\jmath} \times \hat{k} = \hat{\imath}$	$\hat{k} \times \hat{\jmath} = -\hat{\imath}$
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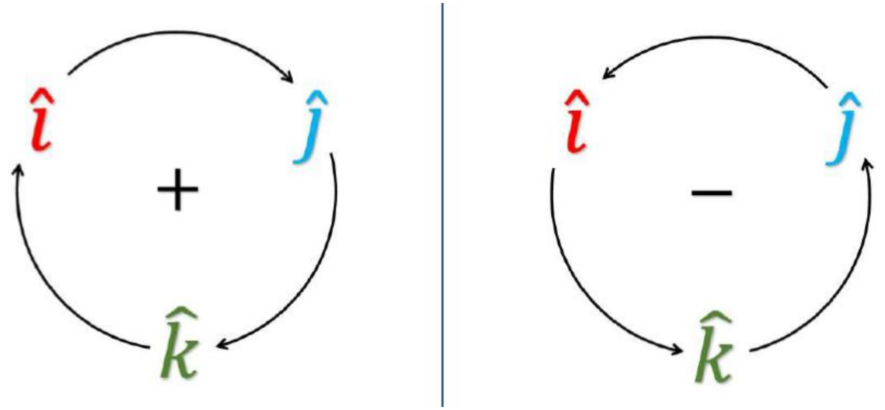


5. Derivative

Derivative is very important in physics, it allow us to:

- Know the variation of a function:
  - If $f' \geq 0$, f is increasing
  - If $f' < 0$, f is decreasing

- Find the equation of the tangent
  - $f'(\mathrm{x})$represents the slope of the tangent line to $f(x)$ at any point x.
  - Example: find f'(1). We should first evaluate f'(x) at any point x, then evaluate it for x = 1.
  - Application:

    Evaluate the slope of the function $f(x) = x^3 e^x$ at point x = 0.

    1st step: $f'(x) = 3x^2 e^x + x^3 e^x$

    2nd step: $f'(0) = \ 3.0^2 . e^0 + 0^3 . e^0 = 0$

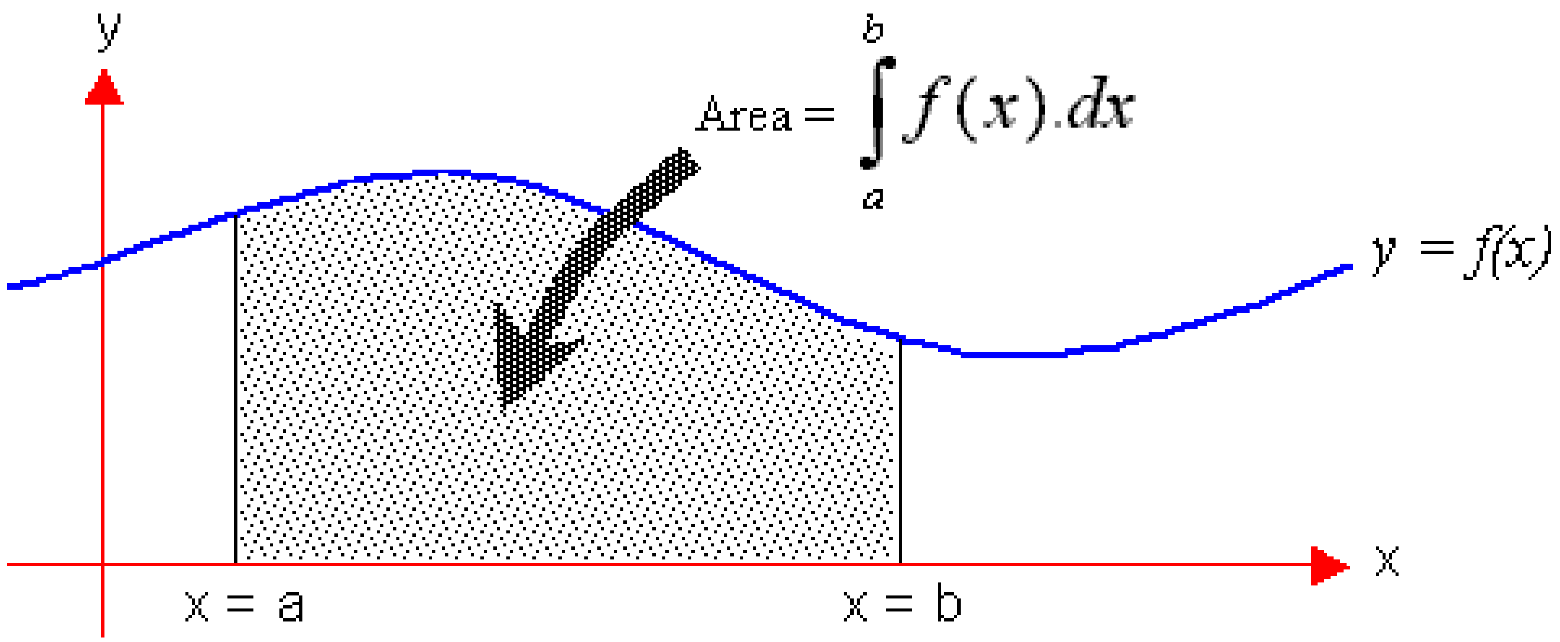
Differentiation and Integration are branches of calculus where we determine the derivative and integral of a function. Differentiation is the process of finding the ratio of a small change in one quantity with a small change in another which is dependent on the first quantity. On the other hand, the process of finding the area under a curve of a function is called integration. We can find the differentiation and integration of a function at particular values and within a particular range of finite limits. Integration of a function that is done within a defined and finite set of limits, then it is called definite integration.

Differentiation and Integration

$$\boxed{\frac{dy}{dx} \rightleftharpoons \int y \, dx}$$

Integration

Many ideas in physics require the prior knowledge of integration. The big idea of integral calculus is the **calculation of the area under a curve using integrals**.



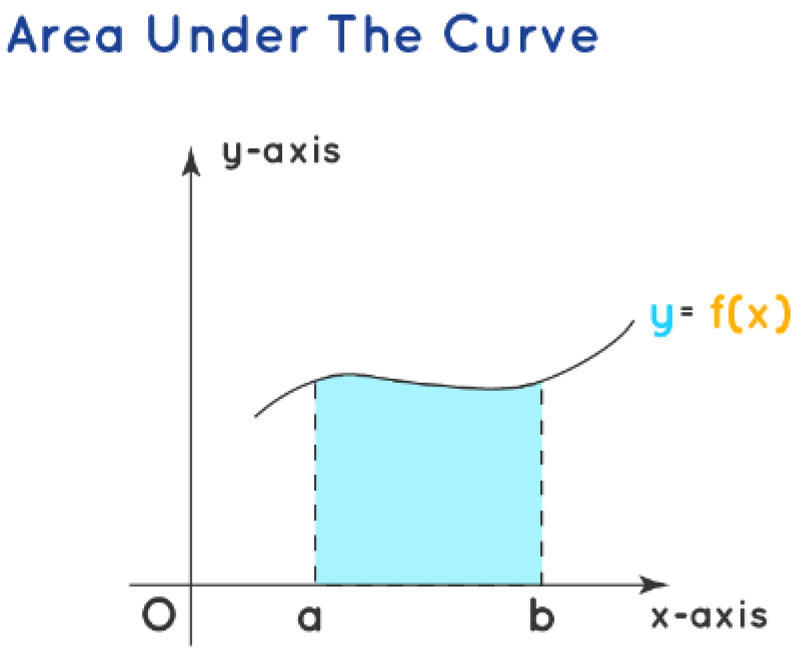
The area under the curve can be calculated through three simple steps. First, we need to know the equation of the curve($y = f(x)$), the limits across which the area is to be calculated, and the axis enclosing the area. Secondly, we have to find the integration (antiderivative) of the curve. Finally, we need to apply the upper limit and lower limit to the integral answer and take the difference to obtain the area under the curve.

$$\text{Area} = \int_a^b y.\,dx$$

$$= \int_a^b f(x).\,dx$$

$$= [g(x)]_a^b$$

$$= g(b) - g(a)$$



Trigonometric relationships

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$ $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$ $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$ $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$	$\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$ $\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$
$\sin\alpha+\sin\beta=2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$ $\sin\alpha-\sin\beta=2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$	$\cos\alpha+\cos\beta=2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$ $\cos\alpha-\cos\beta=-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$
$\sin\alpha\sin\beta=\frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$ $\cos\alpha\cos\beta=\frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$	$\sin\alpha\cos\beta=\frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$ $\cos\alpha\sin\beta=\frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$

1. Chap 1:
    - Units and dimensions
    - Kinematics of a point
2. Chap 2:
    - Classical dynamics
    - Central force and space dynamics
    - Real forces and fictive forces
3. Chap 3: Conservation of energy
4. Chap 4: Momentum – translation and rotation
5. Chap 5: Kinematics and Dynamics of Solid Bodies