Lebanese University Faculty of Sciences V	Classical Mechanics P1100 - Partial	Date: Dec. 2nd 2019 Duration: 60 min

Exercise 1. *Spiral motion.* (40 points)

The equation of motion of a particle M is given in the frame $\Re(Oxy)$ by:

$$\overrightarrow{OM}(t) = (sin\omega t)\vec{\imath} + (1 - cos\omega t)\vec{\jmath} \qquad \omega \text{ is a constant}$$

a) Determine the trajectory of M and plot it.
b) Calculate the velocity vector $\vec{V}$ and its magnitude. Deduce the tangential unit vector $\vec{e_t}$.
c) Calculate the acceleration vector $\vec{\gamma}$ and its magnitude.
d) Calculate the tangential and normal accelerations γ_t and γ_n.
e) Determine the polar coordinates (ρ *and* θ) of M. Deduce the radial γ_ρ and transverse $\gamma_\perp$ accelerations in term of ω and θ.

At time t_1, the point M is located on the first bisector ($x = y$). This position is called A.

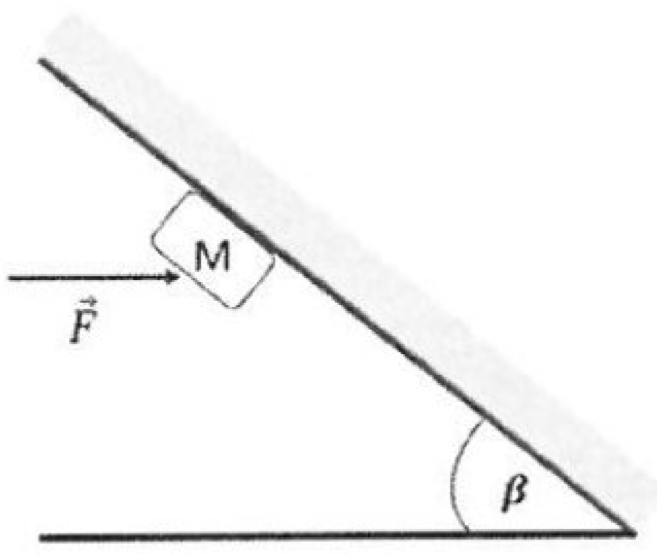
f) Calculate t_1 and place the point A on the trajectory.

Let $\overrightarrow{OA}$ be the position vector with $\vec{e}_\rho$ its radial unit vector making an angle $\theta = (\vec{\imath}, \vec{e}_\rho)$ with $\overrightarrow{Ox}$.

g) Plot at A the unit vectors $\vec{e}_\rho$, $\vec{e}_\theta$, $\vec{e}_t$, and $\vec{e}_n$.

Exercise 2. *Newton Second Law.* (35 points)

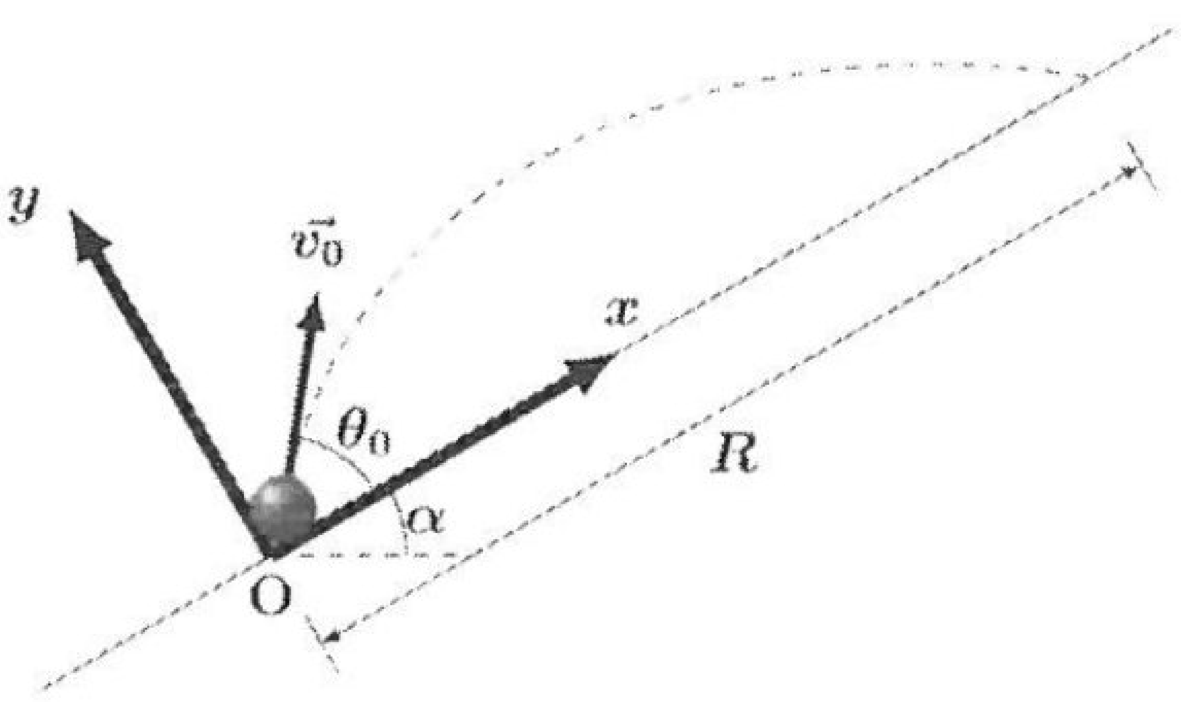
A block of mass $M = 1kg$ is held at rest against a wall, that makes an angle $\beta = 60°$ with the ground, as shown in the adjacent figure. The block is subjected to a force $F = 15\ N$ parallel to the horizontal and to a friction force f. The coefficients of static and kinetic friction are respectively $\mu_S = 0{,}6$ and $\mu_K = 0{,}5$. Given g = 10 N / Kg.



a) Does the block slides downward?
b) If yes, calculate its acceleration.

Exercise 3. *Projectile motion on an incline.* (25 points)

A projectile is fired up an incline plan, with an initial velocity $\vec{v}_0$ at an angle θ_0 with respect to the oriented x-axis. Denote by α the angle between the inclined plane and the horizontal.



a) Express the two components of the acceleration (a_x *and* a_y) as a function of α and g.
b) Show that the range on inclined plane $\boldsymbol{R}$ traveled by the projectile is given by:

$$\boldsymbol{R} = \frac{2v_0^2 cos(\theta_0 + \alpha) sin\theta_0}{g.cos^2\alpha}$$

Given: cos (a + b) = cos(a)cos(b) - sin(a)sin(b)

Good reflection

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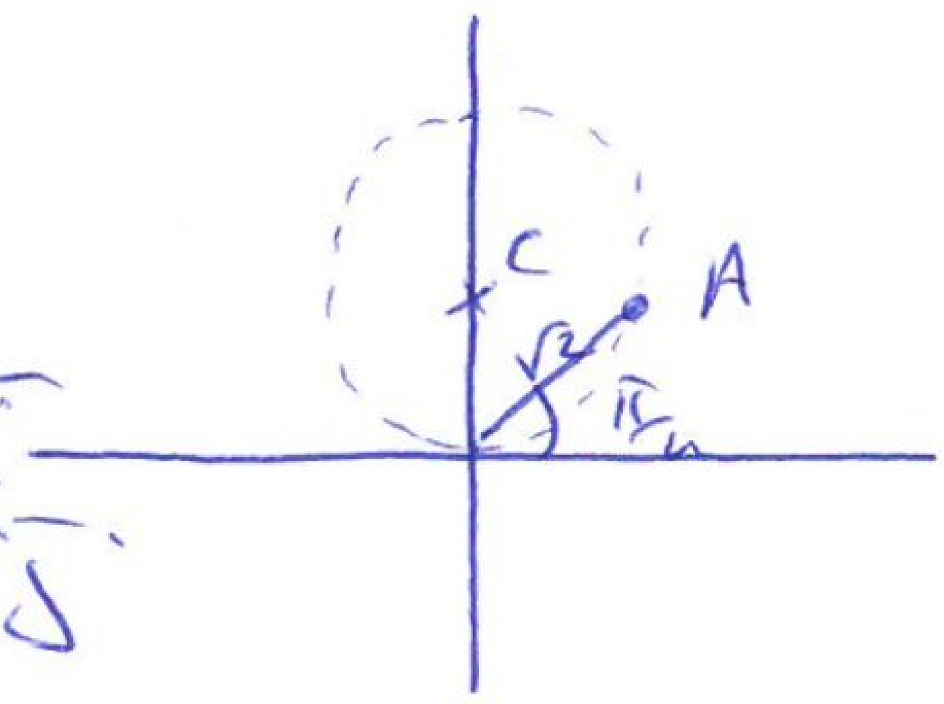
P1) $\overrightarrow{OM}(t) = (\sin\omega t)\vec{i} + (1-\cos\omega t)\vec{j}$

a) $\begin{cases} x = \sin\omega t \\ y = 1-\cos\omega t \end{cases} \Rightarrow \sin^2\omega t + \cos^2\omega t = \boxed{1 = x^2 + (1-y)^2}$.

Trajectory is a circle, $R=1$; $C(0;1)$

b) $\vec{V} = \dfrac{d\overrightarrow{OM}}{dt} = (\omega\cos\omega t)\vec{i} + (\omega\sin\omega t)\vec{j}$



$\boxed{\|V\| = \omega} = \sqrt{V_x^2 + V_y^2}$, $\vec{e}_t = \dfrac{\vec{V}}{\|V\|} = \cos\omega t\,\vec{i} + \sin\omega t\,\vec{j}$

$\vec{V} \parallel \vec{e}_t \Rightarrow \vec{e}_t = \dfrac{\vec{V}}{\|V\|} = \cos\omega t\,\vec{i} + \sin\omega t\,\vec{j}$.

c) $\vec{\gamma} = \dfrac{d\vec{V}}{dt} = \omega^2[-\sin\omega t\,\vec{i} + \cos\omega t\,\vec{j}] \Rightarrow \|\gamma\| = \omega^2$.

d) $\gamma_t = \dfrac{d\|V\|}{dt} = 0$ since $\|V\| = \omega = $ cte ; $\gamma_n = \gamma = \omega^2$.

e) $\begin{cases} \gamma_r = \ddot{r} - r\theta'^2 \\ \gamma_\perp = \gamma_\theta = 2r'\theta' + r\theta'' \end{cases}$ and $\begin{cases} x = r\cos\theta = \sin\omega t \\ y = r\sin\theta = 1-\cos\omega t \end{cases}$

$x^2+y^2 = r^2 = (\sin\omega t)^2 + (1-\cos\omega t)^2 = 2 - 2\cos\omega t = 2(1-\cos\omega t)$

$\Rightarrow r^2 = 4\sin^2\dfrac{\omega t}{2} \Rightarrow \boxed{r = 2\sin\dfrac{\omega t}{2}}$ since $2\sin^2\alpha = 1-\cos 2\alpha$.

$x = r\cos\theta = \sin\omega t = 2\sin\left(\dfrac{\omega t}{2}\right)\cdot\cos\theta \Rightarrow \cos\theta = \cos\dfrac{\omega t}{2}$

$\Rightarrow \boxed{\theta = \dfrac{\omega t}{2}} \Rightarrow \begin{cases} \gamma_r = -\dfrac{\omega^2}{2}\sin\dfrac{\omega t}{2} - 2\left(\sin\dfrac{\omega t}{2}\right)\left(\dfrac{\omega}{2}\right)^2 \\ \gamma_\perp = 2\cdot\dfrac{\omega}{2}\cdot\omega\cdot\cos\dfrac{\omega t}{2} = \omega^2\cos\dfrac{\omega t}{2}. \end{cases}$

or $\begin{cases} \gamma_r = -\dfrac{\omega^2}{2}\sin\theta - 2\dfrac{\omega^2}{4}\sin\theta = -\omega^2\sin\theta \\ \gamma_\perp = \omega^2\cos\theta. \end{cases}$

8) 1^{st} bisector $\Rightarrow x = y \Rightarrow \sin\omega t_1 = 1 - \cos\omega t_1$.

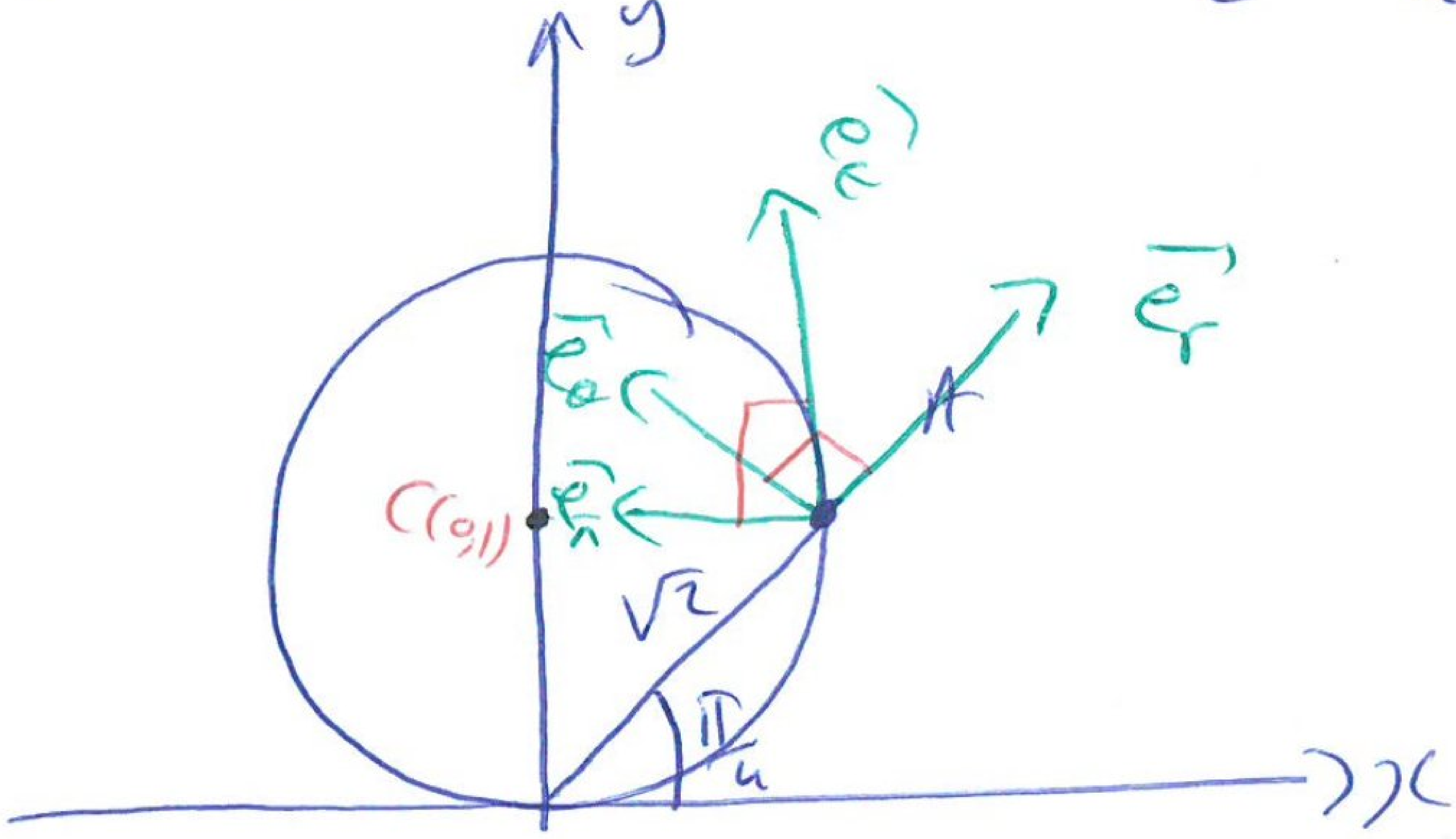
$\Rightarrow (\sin\omega t_1 + \cos\omega t_1)^2 = 1^2 \Rightarrow \sin^2\omega t_1 + \cos^2\omega_1 t + 2\sin\omega_1 t\cdot\cos\omega_1 t = 1$

$\Rightarrow \sin(2\omega_1 t) = 0 \Rightarrow 2\omega t_1 = 0 \Rightarrow t_1 = 0$ (origin).

or $2\omega t_1 = \pi \Rightarrow t_1 = \frac{\pi}{2\omega}$ ✓

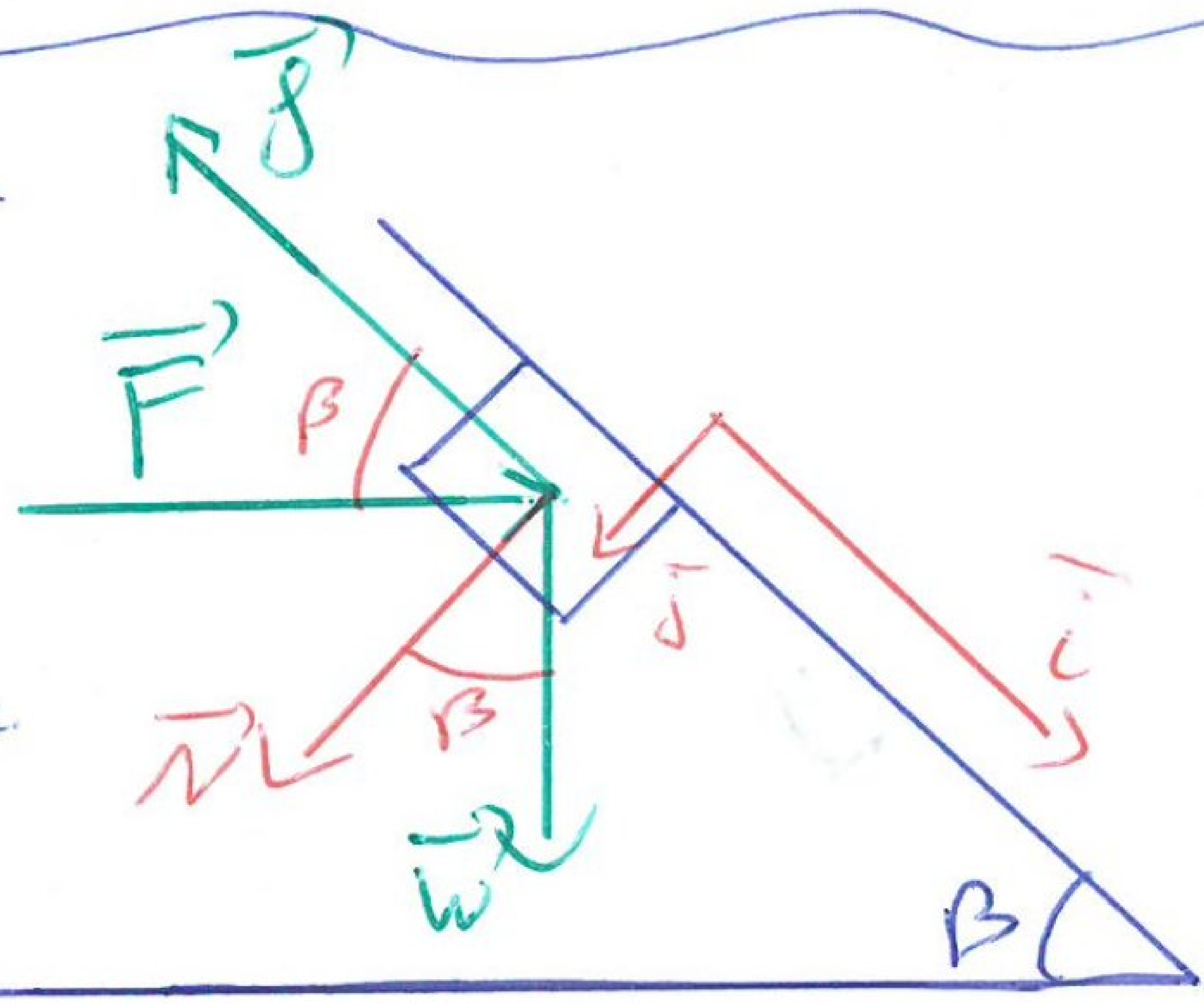
$\Rightarrow r = 2\sin\frac{\omega t_1}{2} = \boxed{\sqrt{2} = r}$ at t_1: $\theta = \frac{\omega t_1}{2} = \frac{\pi}{4}$. (See the Figure)

9)



P2) NSL: $\vec{N} + \vec{f_s} + \vec{F} + m\vec{g} = \vec{0}$

We suppose that non-motion and we compare: f_s to f_{smax}



$\vec{i}$: $-f_s + F\cos\beta + W\sin\beta = 0$

$\Rightarrow f_s = mg\sin\beta + F\cos\beta = 1\times10\times\sin 60 + 15\cos 60 = \underline{16{,}16 = f_s}$

$\vec{j}$: $N + W\cos\beta - F\sin\beta = 0 \Rightarrow N = F\sin\beta - mg\cos\beta$

$N = 15\sin 60 - 10\cos 60 = \underline{8\,N = N}$

$f_{smax} = \mu_s . N = 0.6\times 8 = 4{,}8N$

$\Rightarrow f_s = 16N > 4{,}8N = f_{smax} \Rightarrow$ motion.

b) $\vec{N} + \vec{f_k} + \vec{w} + \vec{F} = m\vec{a}$

$\vec{i}$: $-f_k + mg\sin\beta + F\cos\beta = ma$.

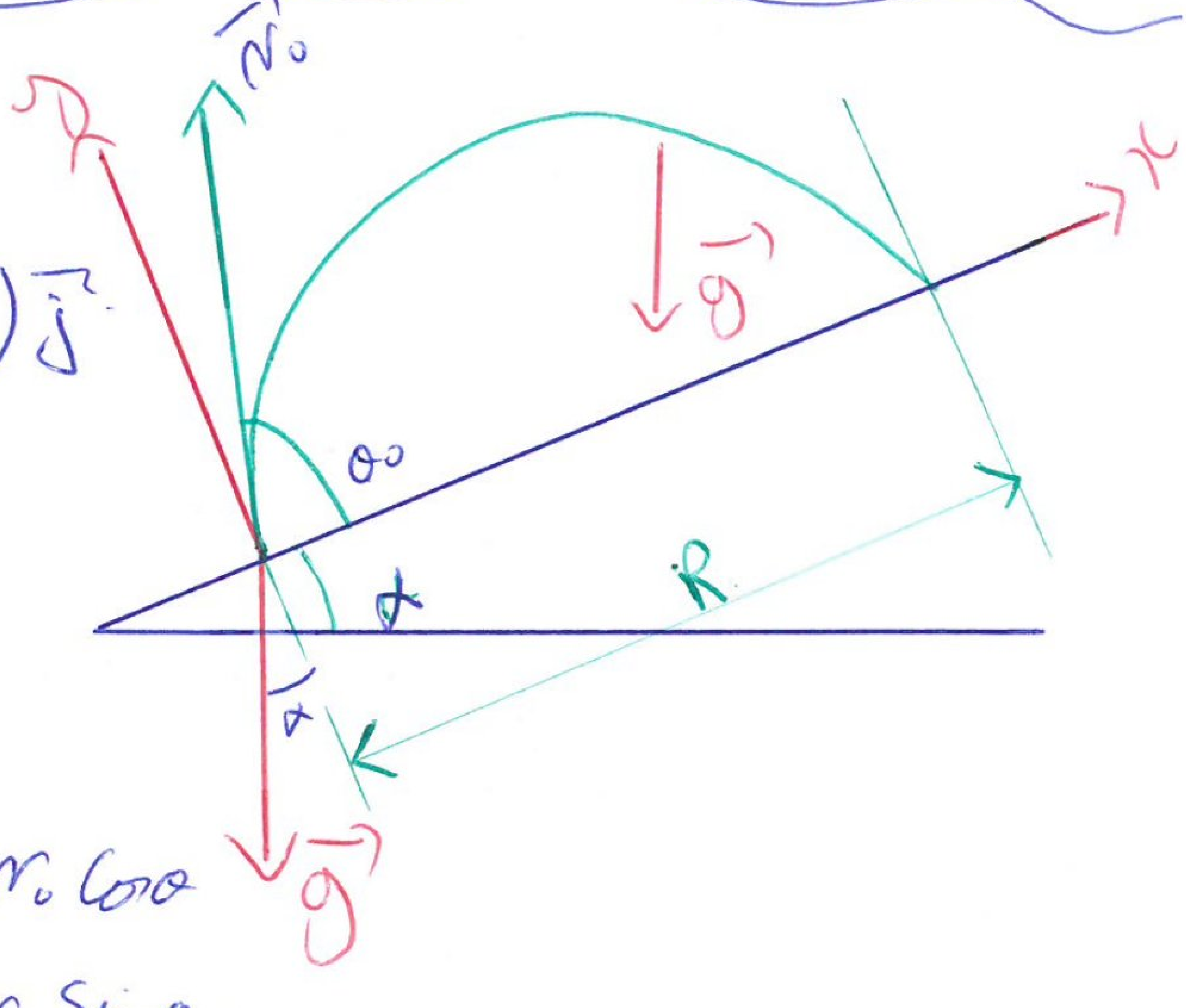
$\vec{j}$: $N + w\cos\beta - F\sin\beta = 0$ and $f_k = \mu_k \cdot N$

$\Rightarrow N = F\sin\beta - w\cos\beta \Rightarrow f_k = 0.5[8] = 4$ N.

$N = 8$ N.

$\Rightarrow a = \frac{1}{m}[-f_k + mg\sin\beta + F\cos\beta] = 12{,}16\ m/s^2$

P3) NSL: $m\vec{g} = m\vec{a}$



a) $\vec{g} = (-g\sin\alpha)\vec{i} - (g\cos\alpha)\vec{j}$

$\vec{a} = \begin{cases} a_x = -g\sin\alpha \\ a_y = -g\cos\alpha \end{cases}$

b) $\vec{v}(t) = \begin{cases} v_x = (-g\sin\alpha)t + v_0\cos\theta \\ v_y = (-g\cos\alpha)t + v_0\sin\theta \end{cases}$

$\vec{r}(t) = \begin{cases} x(t) = -\frac{g\sin\alpha}{2}t^2 + (v_0\cos\theta)t + x_0 \\ y(t) = -\frac{g\cos\alpha}{2}t^2 + (v_0\sin\theta)t + y_0 \end{cases}$

$y = 0 \Rightarrow t\left[v_0\sin\theta - \frac{g\cos\alpha}{2}t\right] = 0$

$t_1 = 0$ initial position or $v_0\sin\theta = \frac{g\cos\alpha}{2}t_2$

$t_2 = \frac{2v_0}{g}\frac{\sin\theta}{\cos\alpha} \xrightarrow{x} R = x(t_2)$

$$R = x(t_2) = -\frac{g\sin\alpha}{2}\left(\frac{2v_0}{g}\frac{\sin\theta}{\cos\alpha}\right)^2 + v_0\cdot\cos\theta\cdot\frac{2v_0}{g}\frac{\sin\theta}{\cos\alpha}$$

$$R = -\frac{2v_0^2}{g}\left[\frac{\sin\alpha\cdot\sin^2\theta}{\cos^2\alpha} - \frac{\cos\theta\cdot\sin\theta}{\cos\alpha}\right].$$

$$R = \frac{2v_0^2}{\cos^2\alpha\cdot g}\left[\cos\theta\cdot\sin\theta\cdot\cos\alpha - \sin\alpha\cdot\sin^2\theta\right]$$

$$R = 2v_0^2\frac{\sin\theta}{g\cos^2\alpha}\left[\cos\theta\cdot\cos\alpha - \sin\alpha\sin\theta\right]$$

$$R = 2v_0^2\frac{\sin\theta}{g\cos^2\alpha}\left[\cos(\alpha+\theta)\right].$$

$$\boxed{R = \frac{2v_0^2\sin\theta\cdot\cos(\alpha+\theta)}{g\cos^2\alpha}}$$