Summary:

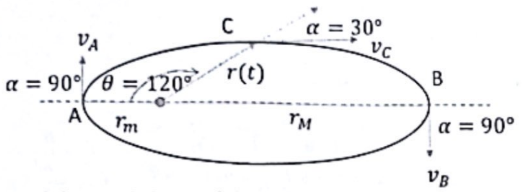
$$\left.\begin{aligned} m(r''-r\theta'^2) &= -\frac{C}{r^2} \\ m(r\theta''+2r'\theta') &= 0 \end{aligned}\right\} \rightarrow E_t = cst \text{ (conservation)}$$

$$\theta' = v/r$$

$$\frac{1}{r}\frac{d}{dt}(m\,r^2\theta') = 0 \rightarrow m\,r^2\theta' = cte \rightarrow mrv = cte$$

طبعي vat يقول $j_a = j_c$ ومعلوم dta

$\vec{j} = m\vec{r}\wedge\vec{v} = mrv\sin\alpha\,\hat{k}$ (conservation of the angular momentum



$$E_t(A) = E_t(B) = E_t(C)$$

$$\frac{1}{2}mv_A^2 - \frac{GMm}{r_m} = \frac{1}{2}mv_B^2 - \frac{GMm}{r_M} = \frac{1}{2}mv_C^2 - \frac{GMm}{r_C}$$

$$= E_t(\text{ellipse}) = -\frac{GMm}{2a} = -\frac{GMm}{r_m + r_M}$$

!

$$J(A) = J(B) = J(C)$$

$$\vec{j} = m\vec{r}\wedge\vec{v} = mrv\sin\alpha\,\hat{k} = \overrightarrow{cst}$$

$$mr_m v_A \sin 90° = mr_M v_B \sin 90° = mr_C v_C \sin 30°$$

$$r = \frac{ep}{1+e\cos\theta} = \frac{\ell_0}{1+e\cos\theta}; \qquad \ell_0 = \frac{h^2}{GM}$$

$$\frac{r_M}{r_m} = \frac{1+e}{1-e} \qquad \boxed{h = v_0 r_0 = v_A r_m}$$

$$\boxed{r_M = \frac{r_m}{\left(\frac{2GM}{r_0 v_0^2}\right) - 1}}$$

✓ !

Escape speed $\quad \boxed{v_\ell = \sqrt{\frac{2GM}{R}}}$

Circular orbital speed $\quad \boxed{v = \sqrt{\frac{GM}{r}}} \qquad \boxed{r = R+h}$

Period $\quad \boxed{T = 2\pi a^{\frac{3}{2}}\sqrt{\frac{1}{GM}}}$

حفظ

! ① $\cdot h = r^2\dot\theta = rv = cst$

$h = r_A v_A = r_m v_m$

② $r = \frac{h^2/GM}{1+e\cos\theta}$

③ $r_M = \frac{r_m}{2GM/r_m v_m^2 - 1}$

④ $\frac{r_M}{r_m} = \frac{1+e}{1-e}$

⑤ $2a = r_m + r_M$

⑥ $E = -\frac{GMm}{2a}$

Escape Velocity + Velocity on a circular orbit

لازم نعرف كيف نبرهنهم

Application II-10-

An artificial satellite, with mass m=1 tonne, orbits the Earth, which has a mass M=6.1024 × 10^{24} kg, on a flat and circular orbit with the center of the Earth as its center and an altitude h=800 km. The Earth's radius is given as R=6400 km, the acceleration due to gravity at the surface is $g_0 = 10\ m/s^2$ and G= 0.66 × 10^{-10} SI. Determine:

a- The potential, kinetic, and total energies of the satellite.

b- Its rotation period around the Earth.

c- The change in speed required to shift its orbit to an ellipse characterized by r_m =7200 km and r_M =8000 km.

Energie ⊖ attractive potential

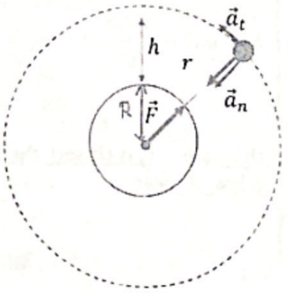
Solution

a- Potential, kinetic, and total energies of the satellite?

Gravitational force oh eyow the normal vector unit

$$U = -\frac{GMm}{r} = -\frac{GMm}{R+h}$$

$$U = -55.10^9 J$$



$$v = \sqrt{\frac{GM}{r_0}} = \sqrt{\frac{GM}{R+h}}$$

Newton 2nd Law (fudemental Relation)

$\vec{F_G} = m\vec{a}$

$\frac{GMm}{(R+h)^2}\hat{n} = m(a_n\hat{n} + a_t\hat{t})$ $\Rightarrow$ Proj on $\hat{n}$: $\frac{GMm}{(R+h)^2} = ma_n = m\frac{v^2}{R+h}$ $\Rightarrow v = \sqrt{\frac{GM}{R+h}} = 7479\ m/s$

$$KE = \frac{1}{2}mv^2 = 27.5 \times 10^9 J$$

$$E_{total} = U + KE = -\frac{GMm}{R+h} + \frac{1}{2}\frac{GMm}{R+h} = -\frac{GMm}{2(R+h)} < 0$$

b- Its period of rotation around the Earth.

$$T = \frac{2\pi(R+h)}{v} = \frac{2\pi(R+h)}{\sqrt{\frac{GM}{R+h}}} = \frac{2\pi(R+h)^{\frac{3}{2}}}{R\sqrt{g_0}} = 1h\,40min$$

c- The change in velocity required to alter its orbit.

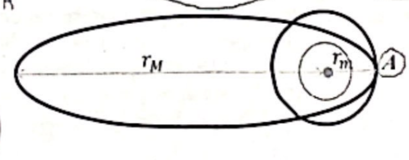
cercle → ellipse change velocity

Calculation of the velocity on the ellipse at point A using total energy.

$$\frac{1}{2}mv_A^2 - \frac{GMm}{r_m} = -\frac{GMm}{r_m + r_M} \rightarrow v_2^2 = 2GM\left(\frac{1}{r_m} - \frac{1}{r_m + r_M}\right)$$

$v_2 = 7603.7\ m/s$

$\Delta v_A = v_{A,ellipse} - v_{A,circle} = 190\ m/s$



R+h Distance ceperating masses

Find Δv in 2 method ①

shift in speed ①

2nd method $r_M = \dfrac{r_m}{\left(\dfrac{2GM}{r_m v_A^2}\right) - 1}$ طلع where v_A is the speed of the ellipse .

3rd meth:

Variation of the total energy between the ellipse and the circle at point A.:

$$E_{T,C} - E_{T,E} =$$

$$\left[\frac{1}{2}mv_c^2 - \frac{GMm}{r_m}\right] - \left[\frac{1}{2}mv_e^2 - \frac{GMm}{r_m}\right] = -\frac{GMm}{2(R+h)} - \left(-\frac{GMm}{r_m + r_M}\right)$$

$$v_e^2 = v_c^2 + \frac{GM}{(R+h)} - \frac{2GM}{r_m + r_M} = 2GM\left(\frac{1}{r_m} - \frac{1}{r_m + r_M}\right)$$

$$\Delta v \sim 190\ m/s$$

only inertial motion $\sum \vec{F}_{ext} = m\vec{a}$

$$r_M = \frac{r_m}{\left(\dfrac{2GM}{r_m v_A^2}\right) - 1}$$ *where* v_A is the speed of the ellipse

Variation of the total energy between the ellipse and the circle at point A.:

$$\left[\frac{1}{2}mv_c^2 - \frac{GMm}{r_m}\right] - \left[\frac{1}{2}mv_e^2 - \frac{GMm}{r_m}\right] = -\frac{GMm}{2(R+h)} - \left(-\frac{GMm}{r_m + r_M}\right)$$

$$v_e^2 = v_c^2 + \frac{GM}{(R+h)} - \frac{2GM}{r_m + r_M} = 2GM\left(\frac{1}{r_m} - \frac{1}{r_m + r_M}\right) \Delta v = 187{,}5\ m/s$$

interaction between a body.

2-7 - Real Forces and Fictitious forces

If a traveler is sitting in a stationary car or in a moving car at a constant speed in a straight line, he does not feel any unusual force and remain at rest in their seat. But when the car decelerates, they feel a force pushing them forward. This sensation is due to the acceleration of the reference frame linked to the car and is manifested by a fictitious force that combines with the real forces to maintain stability and motion.

فمرضي Depalité

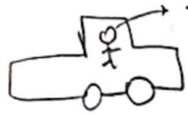
$$\vec{a}_a = \vec{a}_d + \vec{a}_r + \vec{a}_{coriolis}$$ حركة الجر

$$\sum \vec{F}_{real} = m\vec{a}_a = m\vec{a}_d + m\vec{a}_r + m\vec{a}_c = \underbrace{\vec{f}_d + \vec{f}_r + \vec{f}_c}_{\text{fectitous forces}}$$ حركة الدوران Rotation.

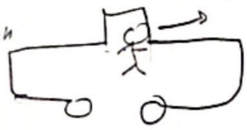
$$\boxed{\vec{f}_r = \sum \vec{F}_{real} - \vec{f}_e - \vec{f}_c}$$

This expression tells us: for an observer linked to the accelerated reference frame to apply the fundamental relation of dynamics, he must add to the real force $\vec{F}_{real}$ the termes $(-m\,\vec{a}_d)$ and $(-m\,\vec{a}_c)$.

Cas 1:



Cas 2: Galileian motion



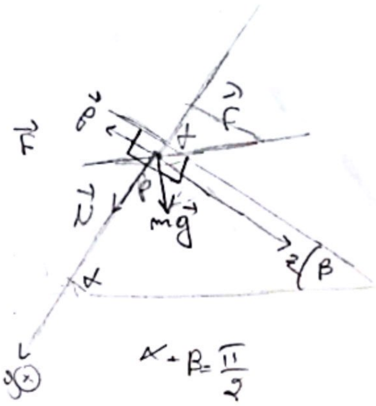
Cas 3: Sudden deceleration

$\vec{V} \neq st$ $a \neq 0$, fictitious forces appear , as a result of the non inertial mot of the car.

Exercise 9:

a) $f_s \leq f_{s\,max} = \mu_s . N$: static friction

let's assume that there is no motion (static case):



FRD on the block : $\sum \vec{F}_{ext} = m\vec{a}$

$m\vec{g} + \vec{N} + \vec{F} + \vec{f} = m\vec{a}$

proj on y : $+mg\cos\beta + N - F\cos\alpha = 0$

$N = F\sin\beta - mg\cos\beta = 8N$

proj on x :

$mg\sin\beta + 0 + F\sin\alpha - f_s = 0$

$f_s = mg\sin\beta + F\cos\beta = 16,16N$

$\underset{16}{f_s} > \underset{4,8}{f_{s\,max}}$ friction is not static

$\Rightarrow$ the Block is sliding downword.

b) $N = 8$ Newton

Kinetic frict

$mg\sin\beta + F\cos\beta - f_K = m.a$

$a = \frac{1}{m}\left[mg\sin\beta + F\cos\beta - \mu_K N\right]$

$= 12,16\ m/s^2$