$\ldots = a_n = 42.2 m/s^2$.

$$R = m(g + v^2/\rho) = 150(9.81 + 65^2/100) = 7809N$$

$m\vec{g}$

Proj on $\hat{\tau}$: $0+0 = ma_t \Rightarrow a_t = 0$

$\vec{a} = \vec{a}_m = 42{,}2\,\hat{n}$ (m/s^2)

directed Towards with $\hat{e}_r, \hat{e}_\theta$

$\hat{n} = -\hat{e}_r \rightarrow$ not alway correct

$\hat{n}$ direct Twards the center of curv

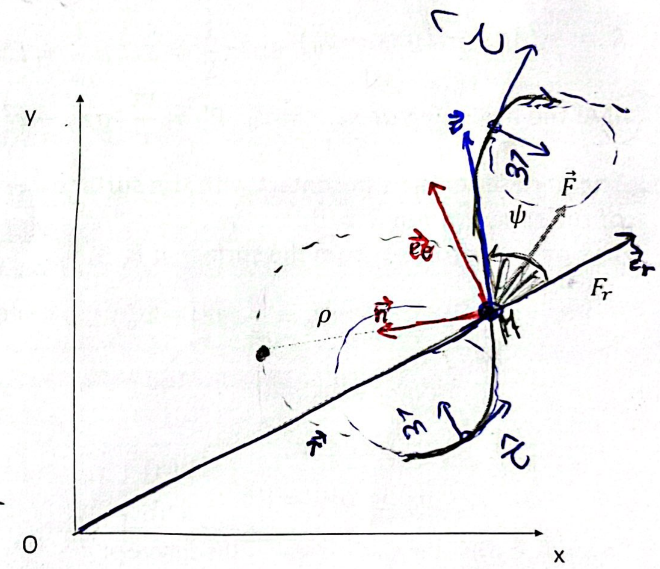
$\hat{\tau} \rightarrow$ Tangent in the direction of motion

– Equation of motion in polar coordinates :

When the particle is required to move in a plane $(\rho;\theta)$, it may be more practical to express the FRD using the two unit vectors $(\vec{e}_\rho, \vec{e}_\theta)$. The equations of motion will then be:

$$\sum F_r\vec{e}_r + \sum F_\theta\vec{e}_\theta = m(a_r\vec{e}_r + a_\theta\vec{e}_\theta)$$

$$\sum F_r = m(r'' - r\theta'^2) \qquad \sum F_\theta = m(r\theta'' + 2r'\theta')$$

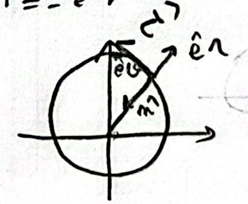


We could determine F_n and F_τ relative to the polar components by the determination of the angle ψ:

$$\psi = (\vec{r}, \vec{\tau})$$

$$\tan\psi = \frac{r}{dr/d\theta}$$

in the case of a circular motion $\hat{n} = -\hat{e}_r$



$$U(r) = -\frac{C}{r} + k$$

We assume that $U(r) \to 0$ $when\ r \to \infty, so\ k = 0$

$C = Gm_1m_2 > 0$, and the potential is attractive

to prove mathematical that tot Energy is conserve

The **fundamental relation of dynamics** on the particule m :

$$m(r'' - r\theta'^2)\vec{e}_r + m(r\theta'' + 2r'\theta')\vec{e}_\theta = -\frac{C}{r^2}\vec{e}_r$$

$$\begin{cases} m(r'' - r\theta'^2) = -\frac{C}{r^2} & \times r' \\ m(r\theta'' + 2r'\theta') = 0 & \times r\theta' \end{cases}$$

2 differential eq. of motion

$$m(r'r'' - rr'\theta'^2 + r\theta' r\theta'' + 2r\theta' r'\theta') = -\frac{C}{r^2}r'$$

$$m(r'r'' + rr'\theta'^2 + r^2\theta'\,\theta'') = -\frac{C}{r^2}r'$$

$$m(r'r'' + rr'\theta'^2 + r^2\theta'\,\theta'') + \frac{C}{r^2}r' = 0$$

The integration of this equation gives:

ش مسؤولين عن صيري

$$\underbrace{\frac{1}{2}m(r'^2 + r^2\theta'^2)}_{\text{kinetic}} \underbrace{- \frac{C}{r}}_{\text{potential}} = cte \to \boxed{E_c + U(r) = E_m = cst}$$

this represents the law of **conservation of total energy.**

The angular momentum is conserved

$$\vec{J} = \vec{r} \wedge m\vec{v} = m\vec{r} \wedge \vec{v} = m\,\vec{h} = \overrightarrow{cst}$$

$$\vec{OP} \wedge V = h$$

2.6.2.

Conic → Cercle $\to (x-a)^2 + (y-b)^2 = r^2$
→ ellipse
→ parabola
→ hyperbola

r = distance between the 2 masses

θ = angle

2-6-2- Trajectories or Orbits

We will start with the 2 diff eq of motion obtained from the FRD

$$m(r'' - r\theta'^2) = -\frac{C}{r^2} \quad (1)$$

$$m(r\theta'' + 2r'\theta') = 0 \to \frac{1}{r}\frac{d}{dt}(r^2\theta') = 0; \quad r^2\theta' = h = cst$$

Eq involving $r'', r', r, \theta'', \theta'$, all changing with time! We need to find another form of this eq → change of variables

$Let\ r = \frac{1}{u}$ where r and u are functions of θ $and\ t$

$$\frac{dr}{dt} = \frac{dr}{du}\frac{du}{dt} = -\frac{1}{u^2}\frac{du}{dt} = -\frac{1}{u^2}\frac{du}{d\theta}\frac{d\theta}{dt} = -\frac{1}{u^2}\theta'\frac{du}{d\theta}$$

$$= -r^2\theta'\frac{du}{d\theta} = -h\frac{du}{d\theta}$$

$$\frac{d^2r}{dt^2} = -h\frac{d}{dt}\left(\frac{du}{d\theta}\right) = -h\frac{d^2u}{dtd\theta} \times \frac{d\theta}{d\theta} = -h\theta'\frac{d^2u}{d\theta^2}$$

$$\boxed{r'' = -h^2u^2\frac{d^2u}{d\theta^2}}$$

$$(1)\ m(r'' - r\theta'^2) = -\frac{C}{r^2} \to m\left(-h^2u^2\frac{d^2u}{d\theta^2} - h^2u^3\right) = -Cu^2$$

$$-h^2u^2\frac{d^2u}{d\theta^2} - h^2u^3 = \frac{-Cu^2}{m} = \frac{-GMmu^2}{m} = -GMu^2$$

$$\boxed{\frac{d^2u}{d\theta^2} + u = \frac{GM}{h^2}}$$

It is a second-order differential equation with a non-homogeneous term, whose **solution** is the sum of a particular solution $(\frac{GM}{h^2})$ with a solution of the equation without a non-homogeneous term ($A\cos\theta$).

$$u(t) = A\cos\theta(t) + \frac{GM}{h^2} \to \boxed{\frac{1}{r} = \frac{GM}{h^2}\left[1 + \frac{Ah^2}{GM}\cos\theta\right]}$$

→ the equation of the conic

General eq of a conic: $\frac{1}{r} = \frac{1}{ep} + \frac{\cos\theta}{p} = \frac{1}{ep}[1 + e\cos\theta]$

Let $e = \frac{Ah^2}{GM}$ is the eccentricity of the conic. Four cases are distinguished based on the value of the eccentricity e:

eccentricity			
$0 < e < 1$	Ellipse	$e = 0$	Circle
$e = 1$	parabola	$e > 1$	Hyperbola

Calculation of h :

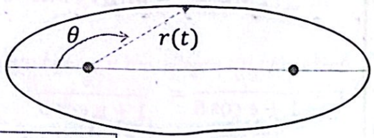
In polar coordinates, the velocity is given by $\vec{v} = r'\vec{e}_r + r\theta'\vec{e}_\theta$. At the initial moment, the radial velocity is zero; only the orthoradial component remains. . $\vec{v}_0 = r_0\theta'\vec{e}_\theta \to v_0 = r_0\theta'$

$$\boxed{h = r_0^2\theta' = \frac{r_0^2 v}{r} = v_0 r_0}$$

$$\vec{v} = \vec{\omega} \wedge \vec{r}$$

To determine the constant A, we use the equation at initial conditions

$$\frac{1}{r} = A\cos\theta(t) + \frac{GM}{h^2}$$



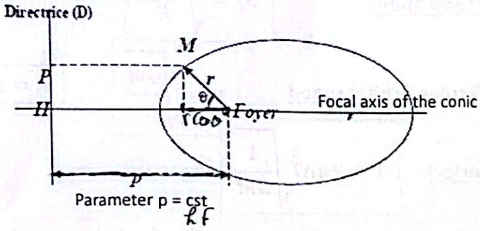
Let $r = r_0$ for $\theta = 0$

$$\frac{1}{r_0} = A + \frac{GM}{h^2} \rightarrow \boxed{A = \frac{1}{r_0} - \frac{GM}{h^2}}$$

The equation of the orbit becomes :

$$\boxed{\frac{1}{r} = \frac{1}{r_0}\left[1 - \frac{GM}{r_0 v_0^2}\right]\cos\theta + \frac{GM}{v_0^2 r_0^2}}$$

Qu'est ce qu'une conique ?



The number e is the excentricity of the conic (mathematical Definition)

$$e = \frac{MF}{MP} = \frac{r}{p - r\cos\theta} \rightarrow r = e(p - r\cos\theta) \rightarrow r(1 + e\cos\theta) = ep$$

$$\frac{1}{r} = \frac{1 + e\cos\theta}{ep} = \frac{1}{ep} + \frac{\cos\theta}{p}$$

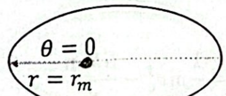
By comparison with $\frac{1}{r} = A\cos\theta(t) + \frac{GM}{h^2}$

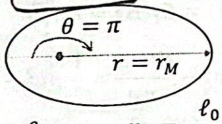
We can deduce that $\boxed{p = \frac{1}{A}}$ and $\boxed{e = \frac{Ah^2}{GM}}$

Case of an elliptic trajectory :

احفظ

$$r = \frac{e\,p}{1 + e\cos\theta} = \frac{\frac{Ah^2}{GM}\cdot\frac{1}{A}}{1 + e\cos\theta} = \boxed{\frac{\ell_0}{1 + e\cos\theta}} \qquad \boxed{\ell_0 = \frac{h^2}{GM} = cst}$$





$\theta = 0$ $\quad r_m = \frac{\ell_0}{1 + e}$ احفظ $\qquad \theta = \pi; \quad r_M = \frac{\ell_0}{1 - e}$

هون بتكون الأرض بالضبط نقطة عالشمس

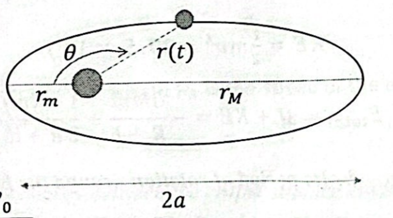
perigee $\qquad$ apogee

منشان ما يكون دائرة

$$\frac{r_M}{r_m} = \frac{1 + e}{1 - e}$$

ابعد نقطة ← r_M ; اقرب نقطة ← r_m

$$2a = r_m + r_M$$ Major axes



$$a = \frac{r_m + r_M}{2} = \frac{\ell_0}{1 - e^2}$$

Relation between excentricity and total energy of a particle :

We admit that the total energy is $E = -\frac{GMm}{2a}$

$$E = \frac{(e^2 - 1)GMm}{2\ell_0}$$

Escape velocity and velocity on a circular orbit:

To determine the escape velocity, we apply the conservation of total energy between two positions:

initial (r_0, v_0) $\qquad$ and $\qquad$ final (∞,v)

$$\frac{1}{2}mv_0^2 - \frac{GMm}{r_0} \qquad \frac{1}{2}mv^2 - \frac{GMm}{\infty}$$

satellite m, Earth M

ME = st

Initial state : r_0, V_0

Final state : $r \rightarrow +\infty$

$$\frac{1}{2}mv_0^2 - \frac{GMm}{r} = \frac{1}{2}mv^2 - \frac{GMm}{\infty} \rightarrow \quad v_0^2 = v^2 + \frac{2GM}{r_0} = \text{Escape velocity}$$

The minimum value of the initial velocity is called the escape velocity and corresponds to v=0 (when it reaches space).

$$v_e = \sqrt{\frac{2GM}{R}} \qquad R: \text{Radius of the planet}$$

(system (m))

To determine the velocity on a circular trajectory, we apply the fundamental relation of dynamics in polar coordinates (the orbital velocity).

$$\vec{F} = m\vec{a} \rightarrow \frac{GMm}{r_0^2} = ma_n = m\frac{v_0^2}{r_0} \rightarrow v = \sqrt{\frac{GM}{r_0}}$$

$(r_0 = R + h)$

If the trajectory is closed, the period can be evaluated using the law of areas:

احفظ

$$T = 2\pi a^{\frac{3}{2}}\sqrt{\frac{1}{GM}}$$

Summary:

$$\left.\begin{array}{l} m(r''-r\theta'^2) = -\dfrac{C}{r^2} \\ m(r\theta''+2r'\theta') = 0 \end{array}\right\} \rightarrow E_t = cst \ (conservation)$$

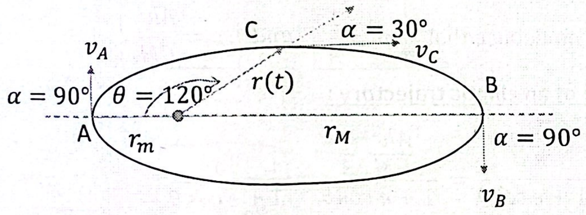
$$\downarrow$$

$$\theta' = v/r$$

$$\frac{1}{r}\frac{d}{dt}(m\,r^2\theta') = 0 \rightarrow m\,r^2\theta' = cte \rightarrow mrv = cte$$

طلعي vat بحول ja = jc وعطيهم α a

$\vec{j} = m\vec{r}\wedge\vec{v} = mrv\sin\alpha\,\hat{k}$ (conservation of the angular momentum



$$\boldsymbol{E_t(A) = E_t(B) = E_t(C)}$$

$$\frac{1}{2}mv_A^2 - \frac{GMm}{r_m} = \frac{1}{2}mv_B^2 - \frac{GMm}{r_M} = \frac{1}{2}mv_C^2 - \frac{GMm}{r_C}$$

$$= E_t(ellipse) = -\frac{GMm}{2a} = -\frac{GMm}{r_m + r_M}$$

$$J(A) = J(B) = J(C)$$

$$\vec{j} = m\vec{r}\wedge\vec{v} = mrv\sin\alpha\,\hat{k} = \overrightarrow{cst}$$

$$mr_m v_A \sin 90° = mr_M v_B \sin 90° = mr_C v_C \sin 30°$$

$$r = \frac{ep}{1+e\cos\theta} = \frac{\ell_0}{1+e\cos\theta}; \qquad \ell_0 = \frac{h^2}{GM}$$

$$\frac{r_M}{r_m} = \frac{1+e}{1-e} \qquad \boxed{h = v_0 r_0 = v_A r_m}$$

$$\boxed{r_M = \frac{r_0}{\left(\dfrac{2GM}{r_0 v_0^2}\right) - 1}}$$

<u>Escape speed</u> $\boxed{v_\ell = \sqrt{\dfrac{2GM}{R}}}$

<u>Circular orbital speed</u> $\boxed{v = \sqrt{\dfrac{GM}{r}}}$ $\boxed{r = R + h}$

<u>Period</u> $\boxed{T = 2\pi a^{\frac{3}{2}}\sqrt{\dfrac{1}{GM}}}$

حفظ

1. $h = r^2\dot{\theta} = rv = cst$

   $h = r_A v_A = r_n v_n$
2. $r = \dfrac{h^2/GM}{1+e\cos\theta}$
3. $r_M = \dfrac{r_m}{2GM/r_m v_m^2 - 1}$
4. $\dfrac{r_M}{r_m} = \dfrac{1+e}{1-e}$
5. $2a = r_m + r_M$
6. $E = -\dfrac{GMm}{2a}$

Escape Velocity + Velocity on a Circular orbit

لازم نعرف كيف نبرهنو

Application II-10-

An artificial satellite, with mass m=1 tonne, orbits the Earth, which has a mass M=6.1024 × 10^{24} kg, on a flat and

$$KE = \frac{1}{2}mv^2 = 27.5 \times 10^9 J$$

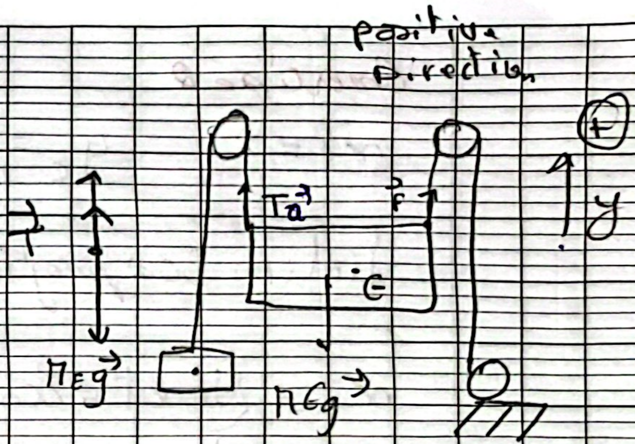
$$GMm \quad 1\,GMm \quad GMm$$

Exercise1:

System: Elevator

$\sum \vec{F}_{ext} = m_E \vec{a}_E$

$\vec{F} + \vec{T} + m_E \vec{g} = m \vec{a}$



Proj on y

$F + T - m_E g = m_E a$ (1)

F: variable, cst; T: cst; a: cst

The force applied at B = The force exerted by the rotor

If a = cst: • $v = at + v_0$

• $v_f^2 - v_i^2 = 2ad$

• $x = \frac{1}{2} a t^2 + v_{0x} t + x_0$

we don't have T then

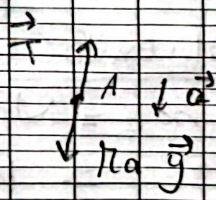
$v_f^2 - \cancel{v_i^2}^{\,0} = 2ad$

$a = \frac{v_f^2}{2d} = \frac{100}{80} = 1,25$ m/s

System A:

$\sum \vec{F}_{ext} = m_A \cdot \vec{a}_A$

$\vec{T} + m_A \vec{g} = m_A \cdot \vec{a}_A$



proj on y:

$T - m_A g = -m_A a$ (2)

$T(1) = T(2)$

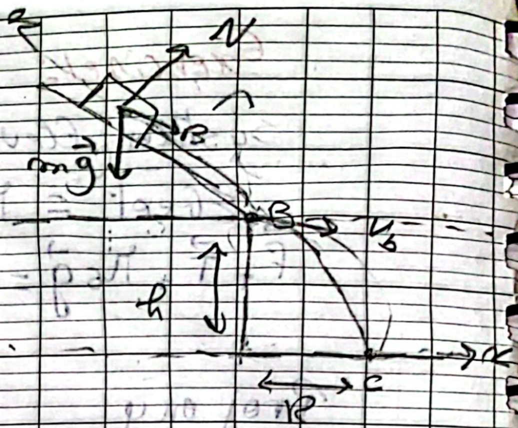
$m_E(a+g) - F = m_A(g-a)$

$F = m_E(a+g) - m_A(g-a) = 4246 N$

$\simeq 4,25 kN$

Exercise 2

$x_C' = ?$ $\quad y_C = 0$



Motion BC = projectile

$x_C = \underbrace{V_{0x}}_{V_B} t + x_0$

$mg_x = mg\cos\theta$

$T = mg\sin\theta$

$y_C = -\frac{1}{2} g t^2 + V_{0y} t + y_0$

Motion AB:

System: mass m

$\sum \vec{F}_{ext} = m\vec{a}$

$m\vec{g} + \vec{N} = m\vec{a}$

proj on x

$mg\sin\theta + 0 = ma$

$a = g\sin\theta = 5 m/s^2$

$V_B^2 - \cancel{V_A^2}^{0} = 2ad \Rightarrow V_B = \sqrt{2\times 5 \times 5} = 7,07$ m/s

Motion BC:

$x_C = V_x t + \cancel{x_0}$

$= V_B\cos\theta . t$

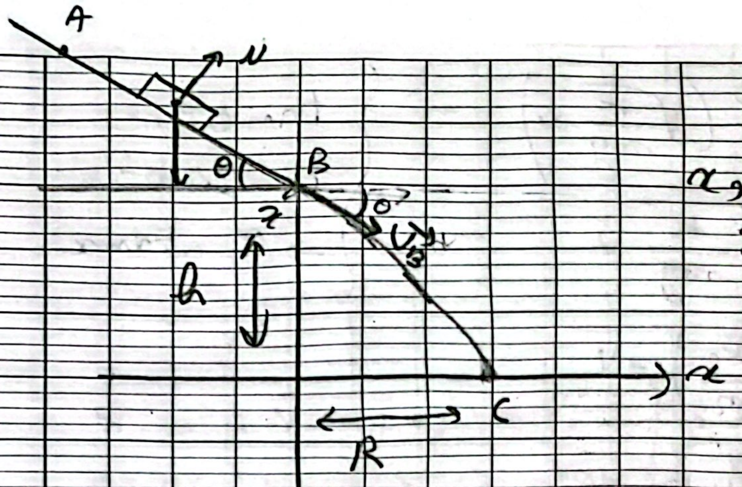
$x_C = 6,12 t$

$y_C = -\frac{1}{2} g t^2 - V_0 \sin\theta t + y_B$

$0 = -5t^2 - 3,5 t + 1$

$t = 0,22$ sec $\Rightarrow x_C = 6,12 \times 0,22$

$= 1,3$



rotion تكل
x, y نأخذ خطة
sin (+) و
جليز

b-) $t_{AC} = t_{AB} + t_{BC}$

$V_B = a t_B + v_0$

$t = \frac{v}{a} = \frac{7,07}{5} = 1,41\,s$

$t_{AC} = 1,63\,s$

~~Exercise 4:~~

Ucte

$\ominus$ Fc $\rightarrow \hat{e}_r$

$\oplus$ $\rightarrow \hat{n}$

Exercise 4:

System: parachuist

$\sum \vec{F}_{ext} = m\vec{a}$

$m\vec{g} + \vec{f} = m\vec{a}$ project y:

$mg - f = ma$

$mg - kv^2 = m\frac{dv}{dt}$

$g - \frac{kv^2}{m} = \frac{dv}{dt}$

$\int_0^t dt = \int_0^v \frac{dv}{g - \frac{k}{m}v^2}$

$t = \frac{1}{g}\int_0^v \frac{dv}{1 - \frac{k}{mg}v^2}$

$v^2 =$

نفرض (+) في الـ motion direction

عرطا ما نتطلع

let $u^2 = \frac{k}{mg}v^2$

$u = \sqrt{\frac{k}{mg}}\,v$; $du = \sqrt{\frac{k}{mg}}\,dv$

$t = \frac{1}{g}\int_0^V \frac{du}{\sqrt{\frac{k}{mg}}\cdot(1-u^2)}$

$t = \frac{1}{g}\sqrt{\frac{mg}{k}}\cdot a\tanh\left(\sqrt{\frac{k}{mg}}\,v\right)\Big|_0^V$

$$t = \sqrt{\frac{m}{kg}}\ \text{atanh}\left(\sqrt{\frac{k}{mg}}\, v\right)$$

$$\left\{ \begin{array}{l} \tan\theta = x \\ \theta = \tan^{-1} x \\ \rightarrow \theta = \text{atan}\, x \end{array} \right.$$

$$\tanh\left(t\sqrt{\frac{kg}{m}}\right) = \sqrt{\frac{k}{mg}}\, v$$

$$\sqrt{\frac{mg}{k}}\ \tanh\left(t\sqrt{\frac{kg}{m}}\right) = v$$

on the arrival on the grnd $t \to +\infty$

$\tanh(t) \Rightarrow 1$ $\quad v \to \sqrt{\frac{mg}{k}}$

Exercice 6:

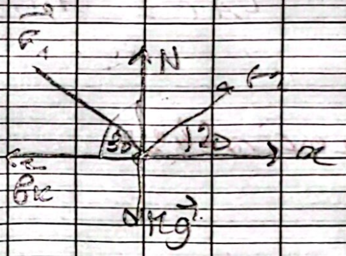
$F_1 = 800\,N$

$F_2 = 1500\,N$

$\mu_k = 0,2$

* system: M

$\sum \vec{F}_{ext} = M\vec{a}$



$\vec{N} + M\vec{g} + \vec{F}_1 + \vec{F}_2 + \vec{f}_k = M\vec{a}$

proj on y

$N - Mg - F_1 \sin 30 - F_2 \sin 20 = 0$

$N = 887\,N$

proj on x:

$0 + 0 + F_1 \cos 30 + F_2 \cos 20 - f_k = M.a$

$$a = \frac{1}{M}\left[F_1 \cos 30 + F_2 \cos 20 - \mu_k N\right] = 19.25\ m/s^2$$

$a = cst$

$v_f^2 - v_i^{2\,(=0)} = 2ad$

$$d = \frac{v_f^2}{2a} = \frac{36}{2 \times 19,25} = 0,94\,m$$

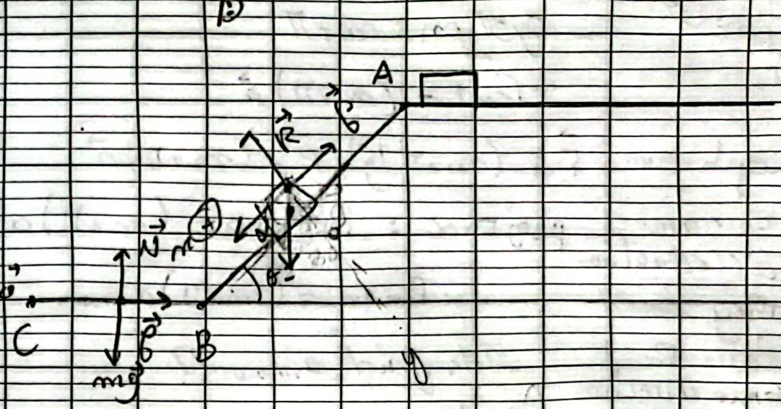
Ex 7:

$V_A = 0,8\ m/s$

$V_C = 0$; $S = ??$

$\mu_K = 0,3$ A → C

Motion BC



$\sum \vec{F}_{ext} = \vec{N} + m\vec{g} + \vec{f} = m\vec{a}$

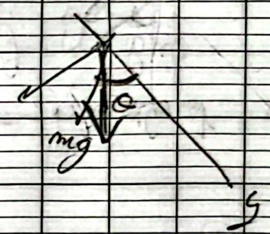
proj on y : $N - mg + 0 = 0$

$N = mg$

proj on x :

$0 + 0 - f = ma$

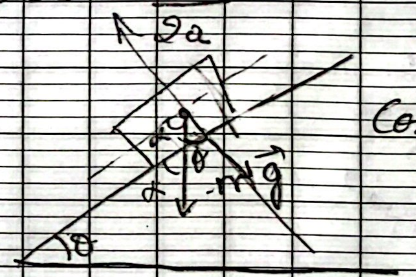
$-\mu_K \cancel{m} g = \cancel{m} a$



$-\mu_K g = a$ $a_{BC} = -3\ m/s^2$

$V_C^2 - V_B^2 = 2a \cdot S \Rightarrow S = -\frac{V_B^2}{2a}$

Let's find V_B



<u>Motion AB:</u>

$\sum \vec{F}_{ext} = m\vec{a}$

$m\vec{g} + \vec{R} + \vec{f} = m\vec{a}$

proj on y : $-mg \cos\theta + R + 0 = 0$

$R = mg \cos\theta$

proj on x : $mg \sin\theta - f_k = ma$

$ma = mg \sin\theta - \mu_K\ mg \cos\theta$

$a_{AB} = 2,4\ m/s^2 = 6t$

$V_B^2 - V_A^2 = 2ad$

$V_B = \sqrt{V_A^2 + 2ad} = 3,2\ m/s \Rightarrow S = -\frac{V_B^2}{2a_{BC}} = -\frac{3,2^2}{2 \times -3}$

$= 1,7\ m$

وإذا في friction يعمل على الـ g إذا فقط

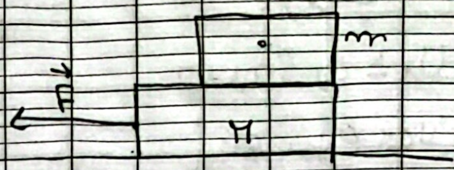
they have the same acceleration they should have the same direction

a → +a, −a

اشرح بو نفهم إذا طلع ⊖ معناه عكس اتجاه الـ direction بالحل

9/11/19 بالحل نفترض انو static ونشتغل على أساس fs وبعدين نقارن fs بالـ fs max ونعرف اذا static او kinetic

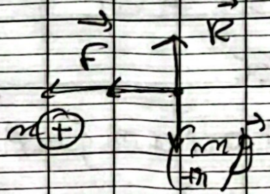
Ex 8:



• System: m+M

$\sum \vec{F}_{ext} = (m+M)\vec{a}$

$\vec{F} + (m+M)\vec{g} + \vec{R} = (m+M)\vec{a}$



proj on x: $F + 0 + 0 = (m+M)a$

$F_{max} = (m+M)a_{max}$

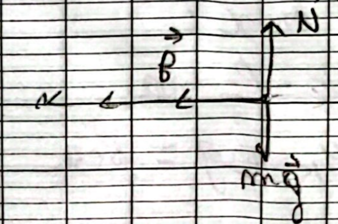
let's find a_{max} ??

$\boxed{f_s \le f_{s\,max} = \mu_s N}$

• System m

$\sum \vec{F}_{ext} = m\vec{a}$

$m\vec{g} + \vec{N} + \vec{f} = m\vec{a}$



proj on y: $-mg + N = 0$

$N = mg$

Proj on x: $f_s = ma$; $f_s \le f_{max}$

$a = \frac{f_s}{m}$; $a_{max} = \frac{f_{s\,max}}{m} = \mu_s g$