Objectives

- **Newton's Second Law (NSL) of Motion.**
- Analysis of the accelerated motion of a particle using the equation of motion with **different coordinate systems.**
- Investigation of **central-force motion** and application to **space mechanics.**

Classical Dynamics

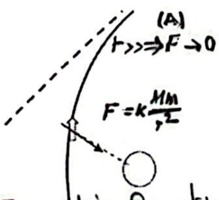
II-1- Newton's First Law, or the Principle of Inertia:

Suppose we send a spacecraft outside the solar system. After passing beyond the orbits of the farthest planets, the spacecraft follows a hyperbolic trajectory relative to the Sun. At a very great distance from the solar system, the spacecraft will have an almost straight-line trajectory, which is none other than the asymptote of the hyperbola.

Its velocity will be constant, and its acceleration will be zero.

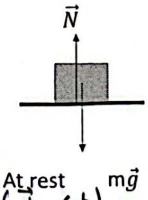
Such motion is called uniform rectilinear motion or inertial motion.

Note: A **straight line** and **constant velocity** are two essential concepts in the definition of **inertial motion.** Motion at constant speed along a circle is not inertial.

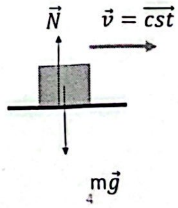


Thus, a body placed on a smooth horizontal table (no friction) is subjected to its weight $\vec{W}$ and the vertical reaction $\vec{N}$ that exactly counteracts the weight. It is experimentally verified that in this setup, when the body is launched from one edge of the table to the other, it exhibits inertial motion between its launch point A and its arrival point B. Everything happens as if the body were not subjected to any external influence.

An object that is not influenced by any other objects (isolated object):
- **Possesses uniform rectilinear motion (inertial motion) if it's moving OR**
- **Remains at rest if it were initially at rest**



$$\boxed{\sum \vec{F}_{est} = \vec{0}}$$



Isolated object: → $V=0$ or . $\vec{V} = cst$ } Inertial motion ($\vec{V} = cst$) or Galilean motion

$\Sigma \vec{F}_{ext} = 0$

Newton's second law:

This principle establishes the relationship between the cause of motion and the variation of its state. Let a material point be subjected to a resultant force $\vec{F}$. Under the action of this force, the point experiences an acceleration $\vec{a}$. Newton's second law postulates that the relationship between $\vec{F}$ and $\vec{a}$ is given by $\vec{F} = m\vec{a}$, where m is a characteristic property of the body called mass. $\vec{F}$ and $\vec{a}$ are both vector quantities. This relationship is known as the fundamental law of dynamics.

$$\boxed{\sum \vec{F}_{ext} = m\vec{a}} \qquad \vec{a} = \frac{\Sigma \vec{F}}{m}$$

If we have $\vec{F}$, $\vec{v}$ will change

The mass is a measure of an object's resistance to acceleration when a force is applied. كتلة ال weight هو الوزن

Newton's Third Law or the Principle of Action and Reaction:

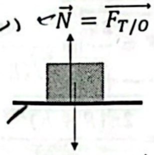
الفعل ورد الفعل

رمز Direction عال الموجودين

ما فينا نحطها دون مكان

$\vec{N} + m\vec{g} = 0 \rightarrow \quad \vec{N} = -m\vec{g}$

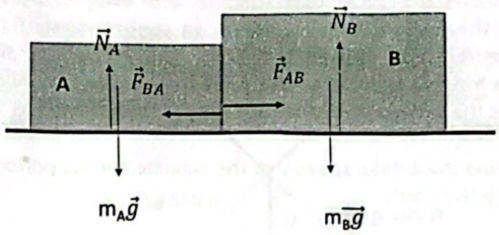
ردّ الفعل ← $\vec{N} = \vec{F_{T/O}}$



$N - mg = 0$

$N = mg$

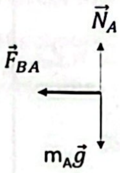
الفعل → $m\vec{g} = \vec{F_{O/T}}$ object on Table



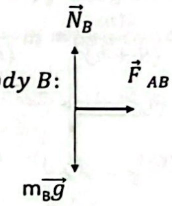
$m_A\vec{g} = -\vec{N}_A \qquad \vec{F}_{AB} = -\vec{F}_{BA} \qquad m_B\vec{g} = -\vec{N}_B$

Free − body diagram for body A:

منرسم ال object على انو نقطة



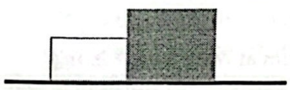
Free − body diagram for body B:



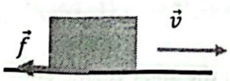
2.2: Types of forces

1- Contact forces : ال forces يلي بيتلامس يكون في Contact

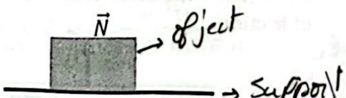
- Forces caused by other objects touching the object of interest.



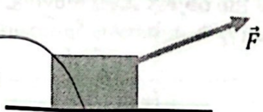
- The forces of friction (frottement)



- The Normal contact force



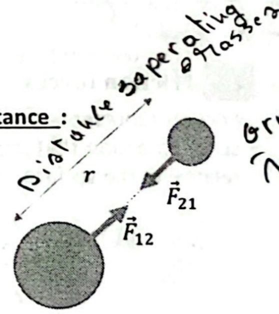
- The forces exerted by ropes (Tension)



- The force exerted by spring (ressort)

2- Forces acting at distance :

- The Gravitational force



Gravity جاذبية

$$\vec{F}_{12} = -\vec{F}_{21}$$

$$F_G = F_{12} = F_{21} = G\frac{M_1 M_2}{r^2}$$

- The weak nuclear force (hold the neutrons together)

- The strong nuclear force (hold neutrons and protons)

- The electromagnetic force → magnetic force, Electrical force

Applications: II-1- An artificial satellite, with mass m, orbits around the Earth with mass M, on an approximately flat orbit centered on that of the Earth, and located at an altitude h = 800 km. The Earth's radius is given as R = 6400 km, and the acceleration due to gravity at the surface is g = 10 m/s².
Determine the orbital speed v of the satellite and its period T around the Earth.

Solution

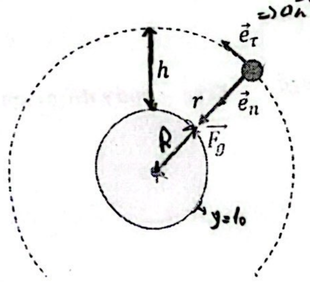
$$\boxed{F_g = G\frac{Mm}{r^2 = (R+h)^2}}$$

Newton's second law $\quad \vec{F_g} = m\vec{a} = m(\vec{a}_n + \vec{a}_t)$

Projection $\vec{n}$: $\quad F = ma_n \rightarrow$

$$G\frac{Mm}{(R+h)^2} = m\frac{v^2}{(R+h)}$$

Circular motion $\rightarrow a_n \hat{n} + a_t \hat{t}$



h = altitude → الارتفاع عن سطح الارض

Vsatellite = ??

T = ??

$$v = \sqrt{\frac{GM}{R+h}} \qquad GM = ?$$

The only force acting on the satellite is its weight, mg.

At the height h=R, on the surface of the Earth: $F = mg$

(2) → $$G\frac{Mm}{(R+h)^2} = mg$$

$$GM = gR^2$$

Note: g changes with altitude!
$g = 10 m/s^2$ at h=0 only

$$v = \sqrt{\frac{g_0 R^2}{R+h}}$$

$$v = 6400.10^3\sqrt{\frac{10}{7200.10^3}} = 7{,}55\,km/s$$

$$\boxed{T = \frac{2\pi}{\omega}} = \frac{2\pi}{v/(R+h)} = \frac{2\pi(R+h)}{v} = \frac{2\pi(R+h)}{\sqrt{\frac{g_0R^2}{R+h}}} \qquad \vec{v} = \vec{\omega} \wedge \vec{r}$$

$$= \frac{2\pi(R+h)^{3/2}}{\sqrt{g_0R^2}} = 1^h\,40^{min}$$

7

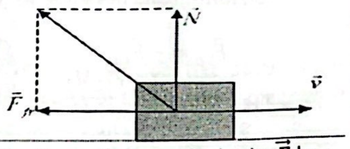
system : satellite
Newton's 2nd law :
$\sum \vec{F}_{ext} = m\vec{a}$
$\vec{F}_G = m\vec{a}$

$\frac{G.mM}{r^2}.\hat{n} = m(a_n \hat{n} + a_t \hat{\tau})$
$r = R + h$
Proj along $\hat{n}$: $\frac{GmM}{(R+h)^2} = m\,a_n = m\frac{v^2}{(R+h)}$

$\frac{GM}{R+h} = v^2$
$v = \sqrt{\frac{GM}{R+h}}$
• Let's express GM in terms of g, at the surface of Earth: h = 0 : $F_G = mg$ → (1)

2.3 Friction forces

Friction represents the action of a rigid surface on a solid, an action that opposes the motion of the solid relative to the surface. $\vec{F}$



$\vec{F}$ is the force exerted by the surface on the body. $\vec{F}$ has two components: vertical (the normal force) and horizontal (the frictional force).

STATIC friction OR KINETIC friction?? no motion on the surface

- As long as the external force is less than or equal to f_s^{max} ($F_{ext} \le f_s^{max}$), the object remains at **rest**: static friction takes place: $f_s \le f_s^{max} = \mu_s N$. projection
- Once the object is in **motion**, static friction is no longer acting. Instead, **kinetic friction** takes over, which is generally lower than static friction: $f_k = \mu_k N$.

حاجة تجاوز توقف الحركة

| if $F_s > F_{s\,max}$ → there is kinetic frict |

Example 1:

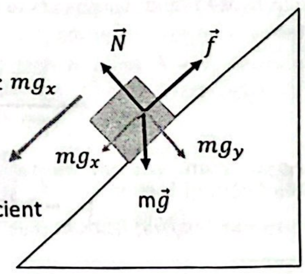
$\sum \vec{F}_{ext} = m\vec{a}$; *Proj. on x'ox:*

$$mg_x - f = ma_x$$

1. System at rest: $f = f_s \ge mg_x$

$$\boxed{\vec{f}_s^{max} = \mu_s \vec{N}}$$

μ_s is called the static coefficient of friction



2. Once the **object** starts **moving**, the frictional force f is slightly less than f_s but remains approximately proportional to N.

$$\boxed{\vec{f}_k = \mu_k \vec{N}}$$

μ_k is is called the coefficient of kinetic friction
($\mu_k < \mu_s$)

→ static frict : for no motion on the surface
$F_s \le F_{s\,max} = \mu_s.N$
static coeff of frict
→ kinetic frict: $f_K = \mu_K.N$ → kinetic coeff of fri

frictionلما بيكون عنا

proj لازم نحل

2 dimensionsال

equilibrium كان ال a=0

Example 2:

An object of mass m is in equilibrium on an inclined plane with an angle α.
Determine the minimum value of the coefficient of static friction μ_s for the object to remain in equilibrium.

Solution

Step 1: identify the system: m

Equilibrium → static frict

System : masse m

$\sum \vec{F}_{ext} = m\vec{a}$

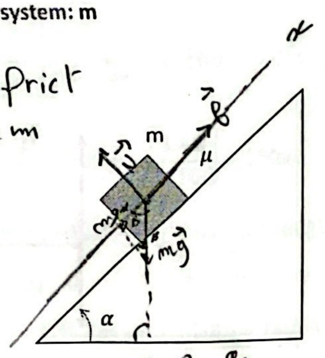
$m\vec{g} + \vec{N} + \vec{f} = m\vec{a}$

proj on x :

$mg_x + 0 + f_s = m\cancel{a_x} = 0$

$-mg\cos\beta + f_s = 0$

$-mg\sin\alpha + f_s = 0$

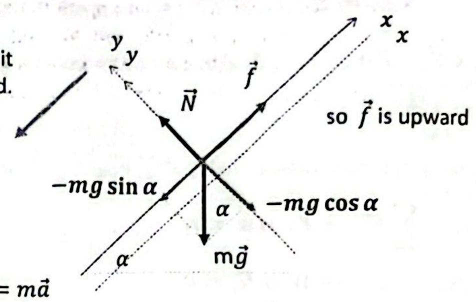


$\alpha + \beta = 90$

$\cos\alpha = \sin\beta$; $\sin\alpha = \cos\beta$

Free-body diagram

If there is movement, it is downward.



$$\sum \vec{F}_{ext} = m\vec{a}$$

system at equilibrium, $a = 0$, $m\vec{g} + \vec{N} + \vec{f} = \vec{0}$

Let's choose a reference frame:: (xOy)

Projection $x'Ox$: $-mg\sin\alpha + 0 + f_s = 0 \rightarrow f_s = mg\sin\alpha$

$y'Oy$: $-mg\cos\alpha + N + 0 = 0 \rightarrow N = mg\cos\alpha$

$$f_s \le f_s^{max} = \mu_s N$$

$$mg\sin\alpha \le \mu_s mg\cos\alpha$$

$$\mu_s \ge \frac{mg\sin\alpha}{mg\cos\alpha} = \tan\alpha \quad \Rightarrow \mu_s^{min} = \tan\alpha$$

proj on y : $mg_y + N + 0 = m\cancel{a_y} = 0$

$-mg\cos\alpha + N = 0$

$N = mg\cos\alpha$

Stat frict : $f_s \le f_{s\,max}$

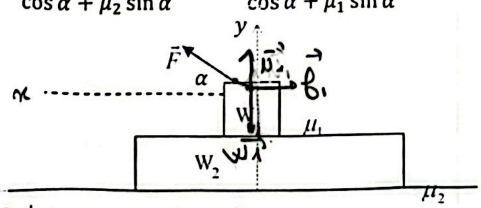
$mg\sin\alpha \le \mu_s . mg\cos\alpha$

$\tan\alpha \le \mu_s$

$\mu_{s\,min} = \tan\alpha$

Application II-3- In the figure below, two blocks of weight W_1 and W_2 are superimposed on a horizontal plane. The coefficient of friction between W_1 and W_2 is μ_1 and that between W_2 and the plane is μ_2.
We exert a force F on W_1 at an angle α to the horizontal. If $\cot\alpha \ge \mu_1 \ge \mu_2$, demonstrate that the necessary and sufficient condition for W_2 to move relative to the plane *without* W_1 *moving relative to* W_2 is as follows:

$$\frac{\mu_2(W_1 + W_2)}{\cos\alpha + \mu_2\sin\alpha} < F < \frac{\mu_1 W_1}{\cos\alpha + \mu_1\sin\alpha}$$



→ W_2 moves relative to the plane

→ W_1 doesn't move relative to W_2

System : W_1

$\sum \vec{F}_{ext} = m_1\vec{a}$

$\vec{F} + \vec{N_1} + \vec{W_1} + \vec{f_1} = m_1\vec{a}$

Proj on x : $+F\cos\alpha - f_1 = m_1 a$

$F\cos\alpha \le f_1$

Proj on y

: $F\sin\alpha + N_1 - W_1 = 0$

$N_1 = W_1 - F\sin\alpha$

$\Rightarrow F\cos\alpha \le \mu_1(W_1 - F\sin\alpha)$

$F(\cos\alpha + \mu_1\sin\alpha) \le \mu_1 W_1$

$F \le \dfrac{\mu_1 W_1}{\cos\alpha + \mu_1\sin\alpha}$

Newton's second law on the system of the two blocs:

$$\underbrace{\vec{W_1} + \vec{W_2} + \vec{N} + \vec{F} + \vec{f_2}}_{\sum \vec{F'}} = (m_1 + m_2)\vec{a}$$

The system $W_1 + W_2$ moves if : $\sum F'_x > f_2 = \mu_2 N$

? ?

Projection

$x'Ox$: $\sum F'_x = F\cos\alpha > \mu_2 N$

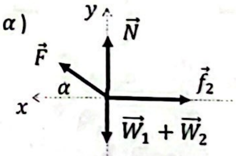
$y'Oy$: $-W_1 - W_2 + N + F\sin\alpha = 0$

$$N = W_1 + W_2 - F\sin\alpha$$

$\rightarrow f_2 = \mu_2(W_1 + W_2 - F\sin\alpha)$

$\rightarrow F\cos\alpha > \mu_2(W_1 + W_2 - F\sin\alpha)$

$$F > \frac{\mu_2(W_1 + W_2)}{(\cos\alpha + \mu_2\sin\alpha)}$$



Let's take the system : weight W_1

$$\underbrace{\vec{W_1} + \vec{N}_1 + \vec{F} + \vec{f}_1}_{\sum \vec{F''}} = m_1\vec{a}$$

W_1 *does not move relative to* W_2 if: $\sum F''_x < f_1 = \mu_1 N_1$

$x'Ox$: $\sum F''_x = F\cos\alpha < f_1$

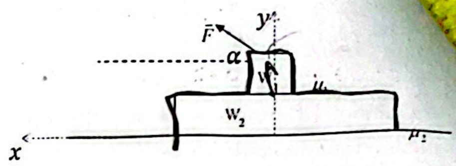
$y'Oy$: $F\sin\alpha - W_1 + N_1 = 0$

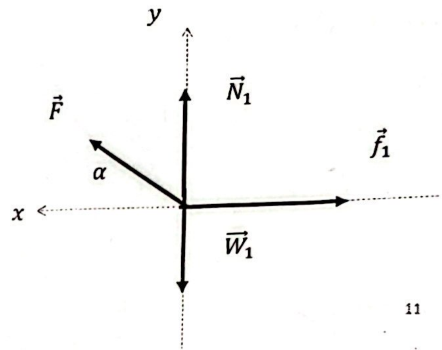
$N_1 = W_1 - F\sin\alpha$

$\rightarrow F\cos\alpha < \mu_1(W_1 - F\sin\alpha)$

$\rightarrow F < \dfrac{\mu_1 W_1}{(\cos\alpha + \mu_1 \sin\alpha)}$

$$\frac{\mu_2(W_1+W_2)}{(\cos\alpha + \mu_2\sin\alpha)} < F < \frac{\mu_1 W_1}{(\cos\alpha + \mu_1\sin\alpha)}$$





System: $W_1 + W_2$

$\sum \vec{F}_{ext} = (m_1 + m_2)\vec{a}$

$\vec{W}_1 + \vec{W}_2 + \vec{N}_2 + \vec{F} + \vec{f}_2 = (m_1 + m_2)\vec{a}$

Proj on x: $F\cos\alpha - f_2 = (m_1 + m_2)a$

W_2 ~~[illegible]~~ moves relative to the plane: $F\cos\alpha > f_2 = \mu_2 N_2$

Proj. on y: $-W_1 - W_2 + N_2 + F\sin\alpha = 0 \Rightarrow$

$N_2 = W_1 + W_2 - F\sin\alpha$

$F\cos\alpha > \mu_2(W_1 + W_2 - F\sin\alpha)$

$F(\cos\alpha + \mu_2\sin\alpha) > \mu_2(W_1 + W_2)$

$F > \dfrac{\mu_2(W_1+W_2)}{\cos\alpha + \mu_2\sin\alpha}$

2.4 Equation of motion

الي بتخبرنا كيف

ال system ع يتحلو مع الوقت

- Equation of motion in cartesian coordinates:

$$\sum \vec{F} = m\vec{a} = \sum F_x\hat{\imath} + \sum F_y\hat{\jmath} + \sum F_z\hat{k} = m(a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k})$$

By identification

$$\sum F_x = ma_x, \quad \sum F_y = ma_y, \sum F_z = ma_z$$

- Equation of motion in normal and tangential coordinates:

$$\sum F_t\hat{u}_t + \sum F_n\hat{u}_n = m(a_t\hat{u}_t + a_n\hat{u}_n)$$

$$\sum F_t = ma_t \quad \text{et} \quad \sum F_n = ma_n$$

$a_t = \dfrac{dv}{dt}$ $\quad a_n = \dfrac{v^2}{\rho}$ $\quad \rho = \left|\dfrac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}{\dfrac{d^2y}{dx^2}}\right|$

$a_t \underbrace{ds}_{arc} = v\,dv$ $\quad a_t(\theta) \leftrightarrow v(\theta)$

$a\,dx = v\,dv$ $\quad a(x) \leftrightarrow v(x)$

إذا كان ال $a(\theta)$ او ال $v(\theta)$ لازم نستخدم عبالنا

$a_t ds = v\,dv$

$\mu \Rightarrow$ يعني fi friction

Application II-5- The equation of the trajectory of a ski slope is given by $y = \frac{1}{200}x^2$-200. The skier with a mass of m = 150 kg passes through the point A (0,−200 m) with a speed of v= 65 m/s.

$\vec{N} = \vec{R} = $ Normal

Determine the reaction of the slope and the skier's acceleration at point A.

Solution

$a_n = \frac{v^2}{\rho}$

$$m\vec{g} - \vec{R} = m(\vec{a}_n + \vec{a}_t) \Rightarrow \begin{vmatrix} \hat{n}: -mg + R = ma_n \\ \hat{\tau}: 0 + 0 = ma_t \end{vmatrix} \qquad R = m(g + a_n) = m(g + v^2/\rho)$$

$R_A = ?$

$a_A = ??$

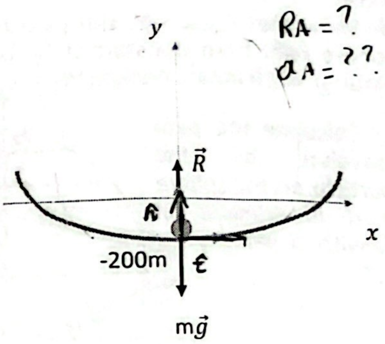
$$\rho = \left| \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{d^2y/dx^2} \right| \quad , \quad \frac{dy}{dx} = x/100 \quad , \quad d^2y/dx^2 = 1/100 \; ,$$

$$\begin{matrix} x = 0 \\ y = -200 \end{matrix} \Rightarrow dy/dx = 0 \Rightarrow \rho = \left| \frac{1+0}{1/100} \right| = 100m \, .$$

$\cancel{R = m(g + v^2/\rho) = 150(9.81 + 65^2/100) = 1569N}$

$\vec{a} = \vec{a}_n + \vec{a}_t$ avec $a_t = 0 \Rightarrow a = a_n = 42.2 m/s^2$.

$R = m(g + v^2/\rho) = 150(9.81 + 65^2/100) = 7809N$



Proj on $\hat{\tau}$: $0 + 0 = ma_t \Rightarrow a_t = 0$

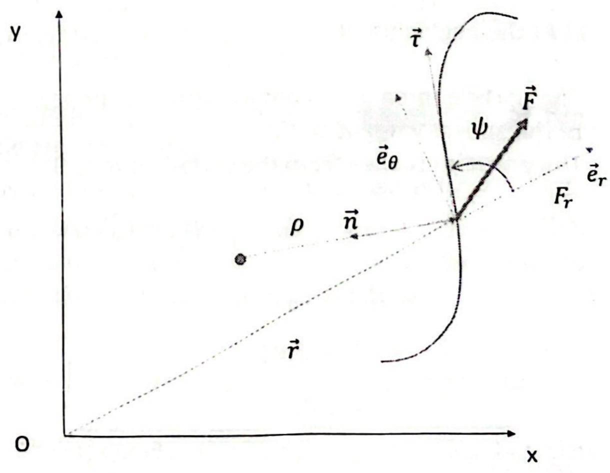
$\vec{a} = \vec{a}_n = 42,2 \, \hat{n} \; (m/s^2)$

– Equation of motion in polar coordinates :

When the particle is required to move in a plane (ρ; θ), it may be more practical to express the FRD using the two unit vectors ($\vec{e}_\rho, \vec{e}_\theta$). The equations of motion will then be:

$$\sum F_r\vec{e}_r + \sum F_\theta\vec{e}_\theta = m(a_r\vec{e}_r + a_\theta\vec{e}_\theta)$$

$$\sum F_r = m(r'' - r\theta'^2) \qquad \sum F_\theta = m(r\theta'' + 2r'\theta')$$



We could determine F_n and F_τ relative to the polar components by the determination of the angle ψ:

$$\psi = (\vec{r}, \vec{\tau})$$

$$\tan\psi = \frac{r}{dr/d\theta}$$

System: m

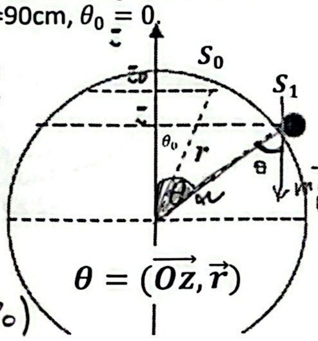
كثير مهم

II-6- A particle of mass m is launched from the point S_0 (with an elevation ($z_0 = r\cos\theta_0$) on a sphere with center O and radius r with an initial velocity v_0 (tangent to the sphere and in the vertical plane passing through O); it slides without friction on the sphere and then takes off, leaving the sphere at a point S_1 . Let g denote the acceleration due to gravity.

a- Express the reaction R of the support on the particle as a function of its elevation $z = r\cos\theta$ at any given moment, and the parameters m, r, g, v_0, and Z_0.

b- Show that if $v_0 > V$, the particle leaves the sphere right from the start at S_0. Determine V. Note: $g = 10\,m/s^2$, r=90cm, $\theta_0 = 0$.

c- Calculate the path traveled by the particle on the sphere if it is released at S_0 with a velocity $v_0 = \frac{V}{2}$.



$\theta = (\overrightarrow{Oz}, \vec{r})$

$R = R(z, m, g, v_0, z_0)$

$$\sum \vec{F}_{ext} = m\vec{a} \rightarrow m\vec{g} + \vec{R} = m(\vec{a}_t + \vec{a}_n)$$

$$\begin{cases} \vec{n} : mg\cos\theta - R = ma_n = m\frac{v^2}{r} \rightarrow R = m\left(g\cos\theta - \frac{v^2}{r}\right) \\ \vec{\tau} : \cancel{m}g\sin\theta = \cancel{m}a_t = \cancel{m}\frac{dv}{dt} \end{cases}$$

$v_t = a_t(\theta)$

$$v\,dv = a_t\,ds = g\sin\theta\, r\,d\theta$$

$$\int_{v_0}^{v} v\,dv = \int_{\theta_0}^{\theta} g\,r\sin\theta\, d\theta$$

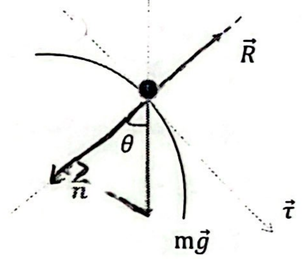
$$\frac{1}{2}(v^2 - v_0^2) = -gr(\cos\theta - \cos\theta_0)$$ We have to eliminate θ

$\cos\theta = z/r$, $\cos\theta_0 = z_0/r$,

$$R = m\left[g\frac{z}{r} - \frac{1}{r}\left(-2gr\left(\frac{z}{r} - \frac{z_0}{r}\right) + v_0^2\right)\right]$$

$$R = \frac{3mgz}{r} - \frac{2mgz_0}{r} - \frac{mv_0^2}{r}$$

$$R = \frac{m}{r}(3gz - 2gz_0 - v_0^2)$$



$$R = \frac{m}{r}(3gz - 2gz_0 - v_0^2)$$

if $v_0 > V$

b) At the beginning at $s_0, z = z_0$ $\quad R_0 = \frac{m}{r}(gz_0 - v_0^2)$

The particle remains in contact with the surface of the sphere when $R > 0$;
The particle escapes from the surface if $R \leq 0$,

$$R_0 = \frac{m}{r}(gz_0 - v_0^2) \leq 0$$

$$gz_0 - v_0^2 \leq 0$$

$$gz_0 \leq v_0^2$$

$$v_0^2 \geq gz_0 = V^2$$

So for $v_0 \geq \sqrt{gz_0}$ the particle leaves the sphere at the beginning at S_0 . The lower limit of v_0 is

$$V = \sqrt{gz_0} = 3\,m/s$$

c) Path travelled before leaving = Arc S_0S_1

The traveled distance is: Arc $S_0S_1 = r(\theta_1 - \theta_0)$

At point S_1 ($z = z_1$), the particle leaves the sphere if $R = 0$

If $v_0 = \frac{V}{2} = \frac{\sqrt{gz_0}}{2}$, $\quad R = \frac{3mgz_1}{r} - \frac{2mgz_0}{r} - \frac{mgz_0}{4r}$

$$R = \frac{3mg}{r}\left(z_1 - \frac{3}{4}z_0\right) = 0$$

$$z_1 = \frac{3z_0}{4} = r\cos\theta_1 \rightarrow \theta_1 = \cos^{-1}\left(\frac{3z_0}{4r}\right)$$

The traveled distance is
Arc $S_0S_1 = r(\theta_1 - \theta_0)$

$$= r\left(\cos^{-1}\left(\frac{3z_0}{4r}\right) - \cos^{-1}\left(\frac{z_0}{r}\right)\right)$$