Lebanese University – Faculty of Sciences 5 | Date: 21/11/2017

P1100 - Mechanics - Partial | Time: 60 min

Problem 1: *Polar coordinates.* (9pts/30)

A particle moves outward along a spiral starting from the origin at t = 0. Its trajectory is given by $r = b\theta$, where θ increases in time according to $\theta = ct^2$. Take b and c as positive constants.

a) Give the units of b and c.

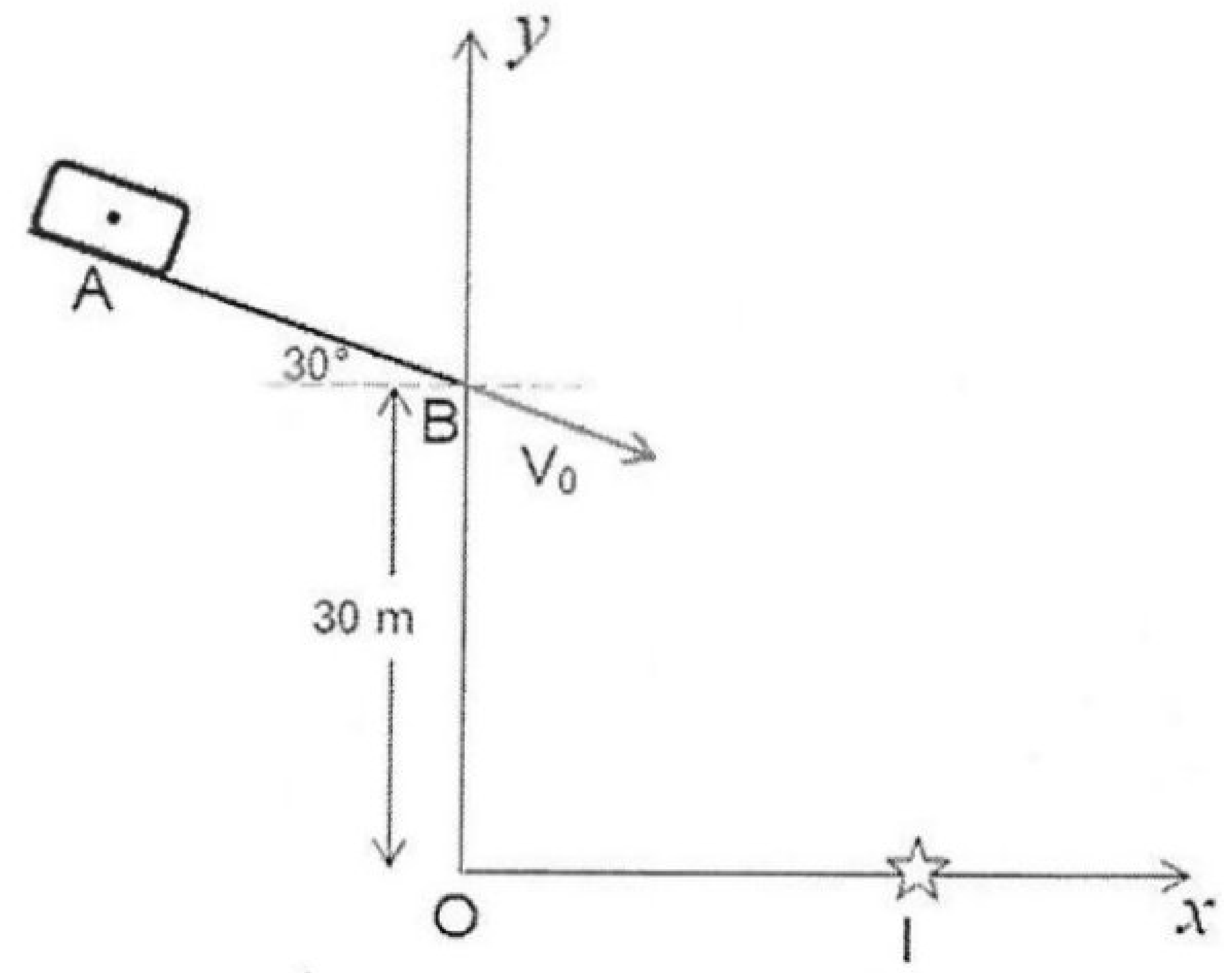
b) Express the acceleration vector as a function of time.

c) At what instant t and for what angle θ will the radial acceleration a_r becomes zero.

d) If c = 2.5, then determine the time at which the radial a_r and transverse a_θ accelerations have equal magnitude.

Problem 2: *Second law of motion - Projectile* (14pts/30)

A block of mass m = 100 kg placed at point A start moving from rest along a ramp of length AB = d = 50 m. The ramp makes an angle α = 30° with the horizontal and the coefficient of kinetic friction between the block and the ramp is: $\mu_K = 0.346$. Take $g = 10\ m/s^2$.



1) Draw the free body diagram showing the forces acting on the block between A and B.
2) Calculate the acceleration of the block along the ramp AB and it's velocity at point B
3) Assume that the speed of the block at B is $v_B = v_0 \approx 14\ m/s$. Beyond B, the block continues as a projectile and we ignore the air resistance: Write the position time equations (x(t),y(t)) of the block beyond point B,
4) Deduce the equation of trajectory and its nature.
5) The block hits the ground at point I. Calculate
   a. The time taken by the block to reach I.
   b. The horizontal range OI and the velocity (magnitude and direction) of the block at I.

Problem 3: *Drag force* (7pts/30)

A motorboat is moving across a lake at a speed $v_0 = 4m/s$ when suddenly its engine stops. The motorboat then slows down due to the drag force $\vec{f}_D = -b\vec{V}$ where b is a constant. Suppose that the motion of the motorboat is restricted along x-axis (one dimension).

1) Find the expressions of the velocity and position of the motorboat as functions of time knowing that at the moment when the engine stopped, taken as *t =0,* the motor boat is found at the origin O.

2) If the motorboat slows down from $V_0 = 4.0$ m/s to 1.0 m/s in 10 s, then calculate the distance traveled by the motorboat during this 10 s.

Good reflection

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Solution

Problem 1: Polar coordinates (9 pts/30)

1) - Unit of b : $m.rad^{-1}$ $\quad(\mathbf{0.25\ pt})$

- Unit of c : $rad.s^{-1}$ $(\mathbf{0.25\ pt})$

2) Polar coordinates.

$$\vec{a} = a_r\vec{e}_r + a_\theta\overrightarrow{e_\theta}$$

$$*\ a_r = \ddot{r} - r\dot{\theta}^2;\ (\mathbf{0.5\ pt})$$

$$r = b\theta = bct^2 \Rightarrow \dot{r} = 2bct\ \&\ \ddot{r} = 2bc\ (\mathbf{1\ pt})$$

$$\theta = ct^2 \Rightarrow \dot{\theta} = 2ct \quad \& \quad \ddot{\theta} = 2c\ (\mathbf{1\ pt})$$

Therefore:

$$a_r = 2bc - bct^2(4c^2t^2) = \mathbf{2bc - 4bc^3t^4 = a_r}\ \ (\mathbf{1\ pt})$$

$$*\ a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta}\ (\mathbf{0.5\ pt})$$

$$\Rightarrow a_\theta = 2(2bct)(2ct) + bct^2(2c) = 8bc^2t^2 + 2\,bc^2t^2(\mathbf{1\ pt}) \Rightarrow \boldsymbol{a_\theta = 10bc^2t^2}(\mathbf{0.5\ pt})$$

Finally:

$$\boldsymbol{\vec{a} = (2bc - 4bc^3t^4)\vec{e}_r + (10bc^2t^2)\overrightarrow{e_\theta}}\ (\mathbf{0.5\ pt})$$

3)

$$a_r = 2bc - 4bc^3t^4 = 0 \Rightarrow 2bc(1 - 2c^2t^4) = 0 \Rightarrow t^4 = \frac{1}{2c^2} \Rightarrow \boldsymbol{t = \left(\frac{1}{2c^2}\right)^{\frac{1}{4}} s}\ (\mathbf{1\ pt})$$

$$\theta = c \cdot t^2 = c\left(\frac{1}{2c^2}\right)^{\frac{1}{2}} = \boldsymbol{\frac{1}{\sqrt{2}} rad = \theta}\ (\mathbf{0.5\ pt})$$

4)

$$\boldsymbol{a_r = a_\theta} \Rightarrow 2bc - 4bc^3t^4 = 10\ bc^2t^2 \Rightarrow 1 - 2c^2t^4 = 5ct^2;\ c = 2{,}5$$

$$\Rightarrow 1 - 12.5t^4 = 12.5t^2 \Rightarrow 12.5t^4 + 12.5t^2 - 1 = 0\ (\mathbf{0.5\ pt})$$

$$\Rightarrow t^2 = \frac{-5+\sqrt{33}}{10}\ accepted \Rightarrow \boldsymbol{t} = \sqrt{\frac{\mathbf{-5+\sqrt{33}}}{\mathbf{10}}} \approx \mathbf{0.2729\ s}\ (\mathbf{0.5\ pt})$$

$$or\ t^2 = \frac{-5-\sqrt{33}}{10}\ rejected$$

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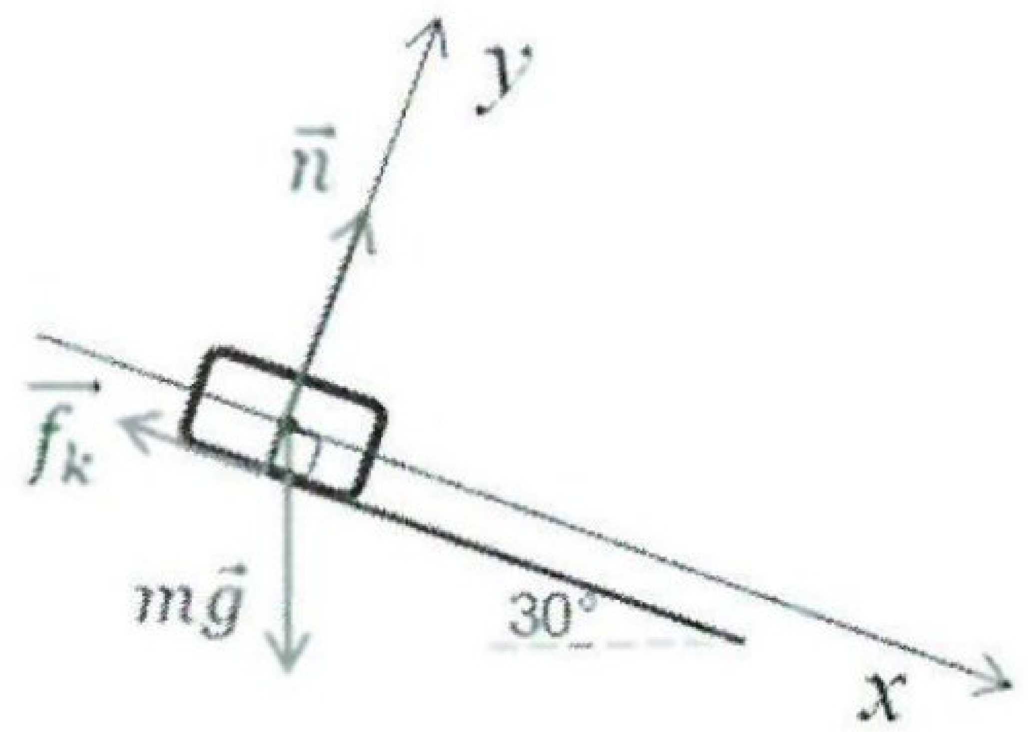
Problem 2: Application to Newton's 2^{nd} law (14/30)

1) Free body diagram: $(\mathbf{0.75}\ \boldsymbol{pt})$



2) System: block; mass $m = 100\ kg$.

Newton's 2^{nd} law:

$$\sum \vec{F}_{ext} = m.\vec{a} \Rightarrow \vec{w} + \vec{n} + \vec{f_k} = m\vec{a}\ (\mathbf{0.5}\ \boldsymbol{pt})$$

Projection along the x-direction

$$m \cdot g \cdot \sin 30 + 0 - \mu_k n = ma \Longrightarrow 100(10)\left(\frac{1}{2}\right) - 0.346n = 100a$$

$$\Longrightarrow a = 5 - 3.46 \times 10^{-3} n\ \ (1)\ (\mathbf{1}\ \boldsymbol{pt})$$

Projection along the y-direction

$$-m \cdot g \cdot \cos 30 + n = 0 \Longrightarrow \boldsymbol{n} \cong \mathbf{866}(\boldsymbol{N})(\mathbf{0.75}\ \boldsymbol{pt}) \longrightarrow (1): a = 5 - 3.46 \times 10^{-3}(866)$$

$$\Longrightarrow \boldsymbol{a} \cong \mathbf{2}\ \boldsymbol{m/s^2}(\mathbf{0.5}\ \boldsymbol{pt})$$

$$vdv = ads = 2ds \Rightarrow \int_{v_A}^{v_B} vdv = 2\int_{s_A}^{s_B} ds \Rightarrow \frac{v_B^2 - v_A^2}{2} = 2(s_B - s_A)(\mathbf{1.25}\ \boldsymbol{pt})$$

$$\Longrightarrow v_B^2 - 0 = 4AB = 200 \Longrightarrow \boldsymbol{v_B} = \mathbf{14.142}\ \boldsymbol{m/s}(\mathbf{0.5}\ \boldsymbol{pt})$$

$$\mathbf{3)} \sum \vec{F}_{ext} = m.\vec{a} \Rightarrow \vec{w} = m\vec{a} \Rightarrow m\vec{g} = m\vec{a} \Rightarrow \vec{a} = \vec{g} = -10\vec{j}\ \ (\mathbf{0.5}\ \boldsymbol{pt})$$

$$\Rightarrow \vec{V} = -10t\vec{j} + \vec{V_0} = -10t\vec{j} + (14cos30\vec{i} - 14\ sin30\vec{j}) = 7\sqrt{3}\,\vec{i} + (-10t - 7)\vec{j}$$

$$\vec{r} = 7\sqrt{3}t\vec{i} + (-5t^2 - 7t)\vec{j} + \vec{r_0}\ ;\ \ \vec{r_0} = 30\,\vec{j}\ \Longrightarrow$$

$$\boldsymbol{x(t) = 7\sqrt{3}t}\quad (2)\ (\mathbf{1.25}\ \boldsymbol{pt})$$

$$\boldsymbol{y(t) = -5t^2 - 7 \cdot t + 30}\quad (3)\ (\mathbf{1.5}\ \boldsymbol{pt})$$

$$\mathbf{4)}\ (2): x(t) = 7\sqrt{3}t \Longrightarrow t = \frac{x}{7\sqrt{3}} \longrightarrow (3): y = -5\left(\frac{x}{7\sqrt{3}}\right)^2 - 7 \cdot \left(\frac{x}{7\sqrt{3}}\right) + 30$$

$$\Longrightarrow\ \ \boldsymbol{y = -\frac{5}{147}x^2 - \frac{1}{\sqrt{3}}x + 30}\ (\mathbf{1}\ \boldsymbol{pt}): \boldsymbol{parabola}\ (\mathbf{0.5}\ \boldsymbol{pt})$$

5) a. At I : $y_I = 0 \longrightarrow (2): -5t^2 - 7 \cdot t + 30 = 0 \Longrightarrow \boldsymbol{t} \cong \mathbf{1.85}(\boldsymbol{s})\ or\ t \cong$ $-3.25\ s\ rejected\ (\mathbf{1}\ \boldsymbol{pt})$

b. $\boldsymbol{t} \cong \mathbf{1.85}(\boldsymbol{s}) \longrightarrow (1): x_I = 7\sqrt{3}(1.85) = 22.43\ m \Longrightarrow \boldsymbol{OI} \cong \mathbf{22.43}\ \boldsymbol{m}\ (\mathbf{1}\ \boldsymbol{pt})$

$$V_{x_I} = V_{0x} = 7\sqrt{3}\ m/s\ \ \ (\mathbf{0.5}\ \boldsymbol{pt})$$

$$V_{y_I} = -10(1.85) - 7 = -25.5\ m/s\ \ (\mathbf{0.5}\ \boldsymbol{pt})$$

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Magnitude :

$$V_I = \sqrt{\left(7\sqrt{3}\right)^2 + (-25.5)^2} = \mathbf{28.236\ m/s = V_I\ (0.5\ pt)}$$

Direction :

$$\theta_I = \tan^{-1}\left(-\frac{25.5}{7\sqrt{3}}\right) = \mathbf{-64.57° = \theta_I\ \ (0.5\ pt)}$$

Problem 3: Newton's 2nd law-Fluid friction (7/30)

1) System: boat; mass m .

Newton's 2nd law:

$$\sum \vec{F}_{ext} = m.\vec{a} \Rightarrow \vec{w} + \vec{B} + \vec{f} = m\vec{a}\ \ \mathbf{(0.5\ pt)}$$

Projection along the x-direction

$$\Rightarrow -bV_x\vec{\imath} = ma_x\vec{\imath}\mathbf{(0.5\ pt)} \Rightarrow m\frac{dV_x}{dt} = -bV_x \Rightarrow \frac{dV_x}{V_x} = -\frac{b}{m}dt$$

$$\Rightarrow \int_{V_0=4}^{V_x} \frac{dV_x}{V_x} = -\int_0^t \frac{b}{m}dt\ \ \mathbf{(1\ pt)}$$

$$\Rightarrow \ln V_x - \ln 4 = -\frac{b}{m}t \Rightarrow \ln\left(\frac{V_x}{4}\right) = -\frac{b}{m}t \Rightarrow \mathbf{V_x = 4e^{-\frac{b}{m}t}\ (1)\ \ (1\ pt)}$$

$$* V_x = \frac{dx}{dt} \Rightarrow dx = 4e^{-\frac{b}{m}t}dt\mathbf{(0.5\ pt)} \Rightarrow \int_0^x dx = \int_0^t 4e^{-\frac{b}{m}t}dt \Rightarrow x = -\frac{m}{b}\left(4e^{-\frac{b}{m}t} - 4\right)\mathbf{(1\ pt)}$$

$$\Rightarrow \mathbf{x = \frac{4m}{b}\left(1 - e^{-\frac{b}{m}t}\right)\ (2)(0.5\ pt)}$$

2) $V_x = V_0e^{-\frac{b}{m}t} \Rightarrow 1 = 4e^{-\frac{b}{m}(10)} \Rightarrow e^{-\frac{b}{m}(10)} = \frac{1}{4} \Rightarrow -10\left(\frac{b}{m}\right) = -\ln 4 \Rightarrow \mathbf{\frac{m}{b} = \frac{10}{\ln 4}\ \ (1\ pt)}$

$$\longrightarrow (2): x(t = 10\ s) = 4\left(\frac{10}{\ln 4}\right)\left(1 - \frac{1}{4}\right) = 21.64\ m\ \mathbf{(0.5\ pt)}$$

The covered distance is:

$$d = x(t = 10\ s) - x_0 = \mathbf{21.64\ m = d.\ (0.5\ pt)}$$

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