Lebanese University
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الجامعة اللبنانية
كلية العلوم، الفرع 3
طرابلس – لبنان

Course: Classical Mechanics (P1100)

Part A: *Kinematics*

Exercise 1:

The acceleration of a particle is given by:

$$a = k \cdot \frac{V^{n+1}}{r^n}$$

where k is a constant, and n is an integer. Find the dimension of k.

Exercise 2:

The kinematic viscosity of a liquid is a physical quantity that is given by:

$$\mu = \frac{P \cdot t}{\rho}$$

where P denotes the liquid's pressure, t the time of flow, and ρ the liquid's density.

Find the dimensions of:

a) The pressure P.
b) The kinematic viscosity μ.

Exercise 3:

In 1900, Max Planck supposed that the energy of an oscillator is quantized as:

$$E = n \cdot h \cdot \nu \text{ or } E = n \cdot h \cdot \frac{c}{\lambda}$$

with n being a dimension-less integer, h the constant of Planck, ν the frequency corresponding to the wavelength λ, and c the speed of light. Find the dimension of h.

Exercise 4:

The gravitational force of attraction of two particles of respective masses m and m', and distant by r, is given by $F = G \cdot \frac{m \cdot m'}{r^2}$ where G denotes the universal gravitation constant.

Find the dimension, as well as the unit of G in SI.

Exercise 5:

The parametric equations that define the position of a particle are:

$$x = 3t^2, y = 4t + 2 \text{ and } z = 6t^3 - 8.$$

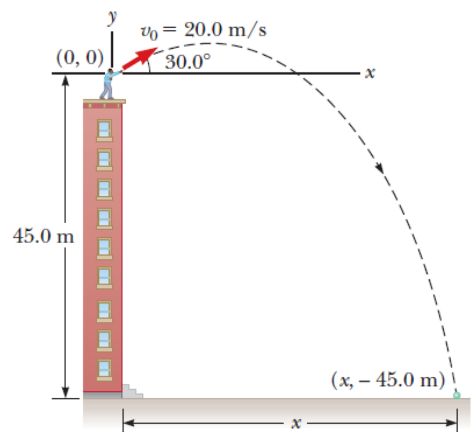
Determine at time $t = 2s$:

a) The vectors of velocity and acceleration.
b) The direction cosines of the tangent to the trajectory.

Exercise 6:

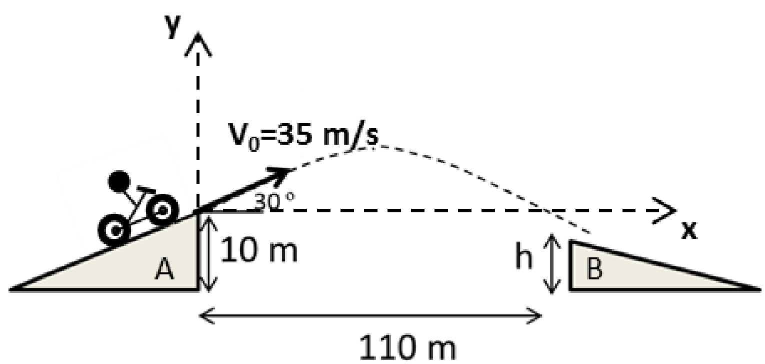
A ball is thrown upward from the top of a building at an angle of 30° to the horizontal and with an initial speed of $20\ m/s$. The point of release is $45\ m$ above the ground.

a) How long does it take for the ball to hit the ground?
b) Find the ball's speed at impact.
c) Find the horizontal distance (x) of the stone from the building to the point on the ground where the stone lands.



Exercise 7:

A motorbike rider had a speed of 35 m/s when he jumped off a racing track of 30°-slope (see figure).



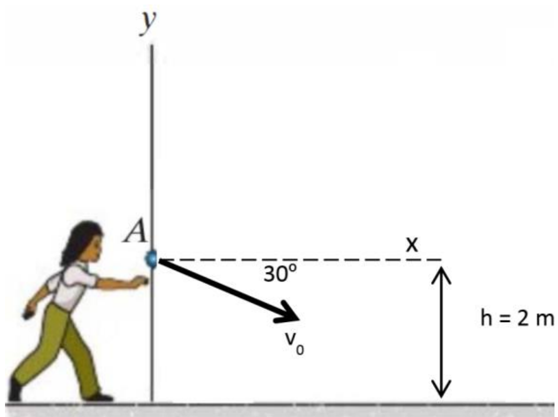
a) Give vectors of velocity and acceleration of the motorbike at time t = 0.
b) Give vectors of velocity and acceleration of the motorbike at time t.
c) Deduce the height h of ramp B necessary for the motorcycle to land safely.
d) At what time is the speed minimum?

Exercise 8:

A ball is thrown dowward at an angle of 30º to the horizontal at speed $\boldsymbol{v_0 = 8m/s}$.

a) Calculate its speed and tangential acceleration at $\boldsymbol{t = 0.25s}$.

b) If the ball is thrown from point A of height h = 2 m. Calculate its normal acceleration just when the ball hits the ground. ($g = 10\ m/s^2$)

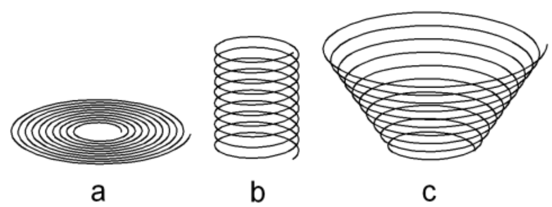


Exercise 9:

A particle moves along a curve whose parametric equations are:

$$x = t\sin\omega t, y = t\cos\omega t, z = t.$$

Choose from the shown curves the trajectory of this particle.



Exercise 10:

A moving object has a constant speed v_0 = 2 m/s at t = 0; it then undergoes an acceleration $a = 3v$ (rectilinear movement). We give: at t = 0, $x_0 = 0$.

- Calculate the speed after 3 seconds.
- Determine the instantaneous position.

Exercise 11:

The acceleration of a point in motion along the axis Ox is $a = 6x + 2$. For $x = 0$, we have $v_0 = 10$ cm/s. Find v as a function of x.

Exercise 12:

It is assumed that the speed of a particle along a straight and horizontal road is defined by $v = \frac{5}{4+x}$.

a) What is the nature of the motion?

b) Calculate the rate of change of its speed at time $t = 4s$.

Exercise 13:

A ball is thrown from the ground with an initial speed $v_{01} = 25\ m/sec$; at the same instant, another ball is dropped from a building 15 m high. After how long will the balls be at the same height?

Exercise 14:

A small object is released from rest in a tank of oil. The downward acceleration of the object is $g - hv$, where g is the constant acceleration due to gravity, h is a constant which depends on the viscosity of the oil and shape of the object. Derive expressions for the velocity v and vertical drop y as function of the time t after release.

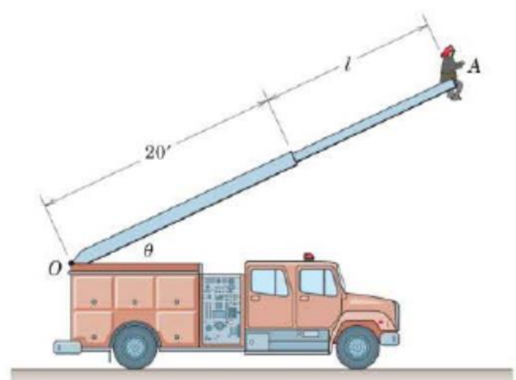
Exercise 15:

The polar coordinates of a particle are given by: $r = 2e^{\omega t}$ et $\theta = \omega t$.

a) Find its speed.
b) Deduce the path length S after 2 s. We give $\omega = 2\ rd/s$.

Exercise 16:

The ladder of a fire truck is designed to be extended at the constant rate $\dot{L} = 0.5\ m/s$ and to be elevated at the constant rate $\dot{\theta} = 2\ deg/s$. As the position $\theta = 50\ °$ and $L = 15\ m$ is reached, determine the magnitudes of the velocity v and the acceleration a of the fireman at A.

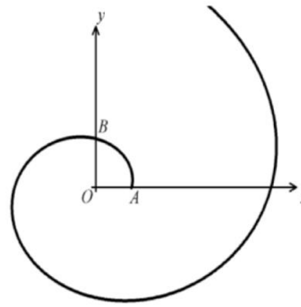


Exercise 17:

A point M moves on a spiral polar equation:

$$r = r_0 e^{\theta}$$

with $\theta = \omega t$ is the direction with the x-axis; r_0 and ω are constants.



a) What are the cartesian coordinates of A and B?
b) Knowing that the polar unit vectors are $\overrightarrow{e_r}$ and $\overrightarrow{e_\theta}$;
   $(\overrightarrow{e_r}\,, \overrightarrow{e_\theta}\,) = \frac{\pi}{2}$ and $\overrightarrow{OM} = r\overrightarrow{e_r}$.
   Calculate, in terms of ω and r, the radial component V_r and transverse component V_θ , and the magnitude of the velocity vector $\vec{V}$.
c) Determine the angle between $\vec{V}$ and $\overrightarrow{e_r}$.
d) Find the radial component a_r and transverse component a_θ, and the magnitude of the acceleration vector $\vec{a}$.

Exercise 18:

A car travels along the level curved road with a speed which is decreasing at the constant rate of 0.6 m/s each second. The speed of the car as it passes point A is 16 m/s.

Calculate the magnitude of the total acceleration of the car as it passes point B which is 120 m along the road from A. The radius of curvature of the road at B is 60 m.

Exercise 19:

The motion of a particle M is described by its position vector: $\overrightarrow{OM} = t\overrightarrow{e_x} + \frac{t^2}{2}\overrightarrow{e_y} + t\overrightarrow{e_z}$.

a) Determine the velocity and acceleration vectors of M.
b) Give the expressions of the tangential and normal components of the acceleration.
c) Deduce the expression of the radius of curvature of the trajectory.

Exercise 20:

A particle moving on a parabolic curve of equation $y = 4t^2$ where $x = 2t$.

a) Determine the velocity and the acceleration of the particle at an instant t in the (Ox, Oy) system.
b) Determine the normal $\vec{a}_n$ and tangential $\vec{a}_t$ component of the acceleration
c) Deduce the radius of curvature of the trajectory at any instant t.

Exercise 21:

The position of a particle M is specified in polar coordinate by:

$$\begin{cases} r(t) = 2\sin t \\ \theta(t) = t \end{cases}$$

a) Find the components radial (v_r) and orthoradial (v_θ) of the speed of M.
b) Deduce the magnitude of the instantaneous speed.
c) Determine the components radial (a_r) and the orthoradial (a_θ) of the acceleration of M.
d) Deduce the magnitude of the vector acceleration $\vec{a}$.
e) Calculate the tangential acceleration a_t.
f) Deduce the normal acceleration a_n.
g) Calculate the radius of curvature ρ.

Exercise 22:

The position of a particle is given by the vector

$$\vec{r} = R(\cos\omega t)\hat{\imath} + R(\sin\omega t)\hat{\jmath} + (ct)\hat{k}$$

where R, ω *and* c are constant.

a) Find a_n *and* a_t.
b) Find the radius of curvature of its trajectory

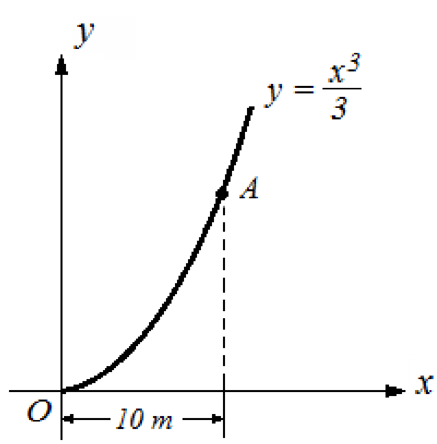
Exercise 23:

A car moves in a vertical plane on a path of equation $y = \frac{x^3}{60}$. It passes passes by a point A of abscissa $x_A = 10\,m$ at a constant speed of *10 m/s*.

a- Find the radius of curvature of its path at point A.

b- Determine the acceleration vector (modulus and direction).

(We give $\rho = \left| \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{d^2y/dx^2} \right|$)



Exercise 24:

A point M moves along a circle of radius R = 2m whose angular motion equation with respect to a given origin, can be written as: $\theta = 2t^2 + t$.

a) Determine the angular velocity of M as a function of time. Deduce its velocity $\vec{v}(t)$.

b) Determine the angular acceleration of M. What is then the nature of the motion of M?

c) Find the value of the tangential acceleration a_t of M.

d) Determine the acceleration vector $\vec{a}$ of M at time t = 1 s.

Exercise 25:

A particle M released without initial speed from a height h falls by performing a uniformly accelerated rectilinear motion of acceleration g along the vertical axis o_1y_1.

a) Determine the position vector of this particle with respect to a car that is moving at constant speed $\vec{v} = V\hat{\imath_1}$ along the axis o_1x_1. We recall: $\vec{r_a} = \vec{r_e} + \vec{r_r}$.

b) Deduce its relative trajectory.

Exercise 26:

Two cars A and B have a two-dimensional uniform motion, such as $v_A = v_B = 10\ m/s$.

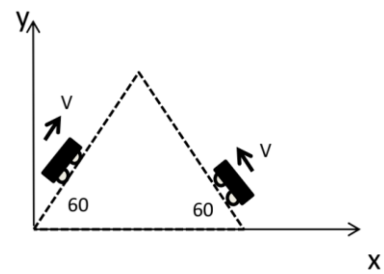
We give at t = 0: $x_A = y_A = 0$; $x_B = 25m$, $y_B = 0$.

Determine at t = 2s:

a) The position vectors $\vec{r}_A$ and $\vec{r}_B$.

b) The position vector ($\vec{r}_{A/B}$) of car A with respect to car B.

c) The velocity ($\vec{v}_{A/B}$) of car A with respect to car B.



Exercise 27:

A swimmer departed from point A and moves at a constant speed V = 4m/s with respect to the water of a river of width d = 50m and whose waters are animated by a current of constant speed u = 2m/s . Find the time needed to get to point B directly opposite to point A.