ENSAE - Computational Statistics

Professor Christian P. Robert

Zakarya Ali

December 15, 2017

Problem 9.2. Write a Gibbs sampler to generate standard bivariate normal random variables (with mean 0, variance 1 and correlation ρ).
For $\rho = 0.3$, use the generated random variables to estimate the density of $X^2 + Y^2$ and calculate $P(X^2 + Y^2 > 2)$.

Let's consider the following Gibbs sampler :

$$(X, Y) \sim N\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}1 & \rho\\ \rho & 1\end{pmatrix}\right)$$

With marginal distributions :

$$\begin{cases} X|Y = y \sim N(\rho y, 1 - \rho^2) \\ Y|X = x \sim N(\rho x, 1 - \rho^2) \end{cases}$$

We generate both X and Y, we compute $X^2 + Y^2$, then we plot the generated histogram. The results we show, come from 10^6 iterations of the slice sampler.
Figure 1 demonstrates a good repartition of our generated points (X, Y) and Figure 2 shows a histogram of the sampler and how much $X^2 + Y^2 > 2$ we get.

$$P(X^2 + Y^2 > 2) = 0.364$$

```
# Initialization
set.seed(1)
rho = 0.3
iterations = 10**6

# X and Y generation function
generator ← function (n, x0){
   X = c(x0)
   Y = NULL
   for (i in seq (n)){
```

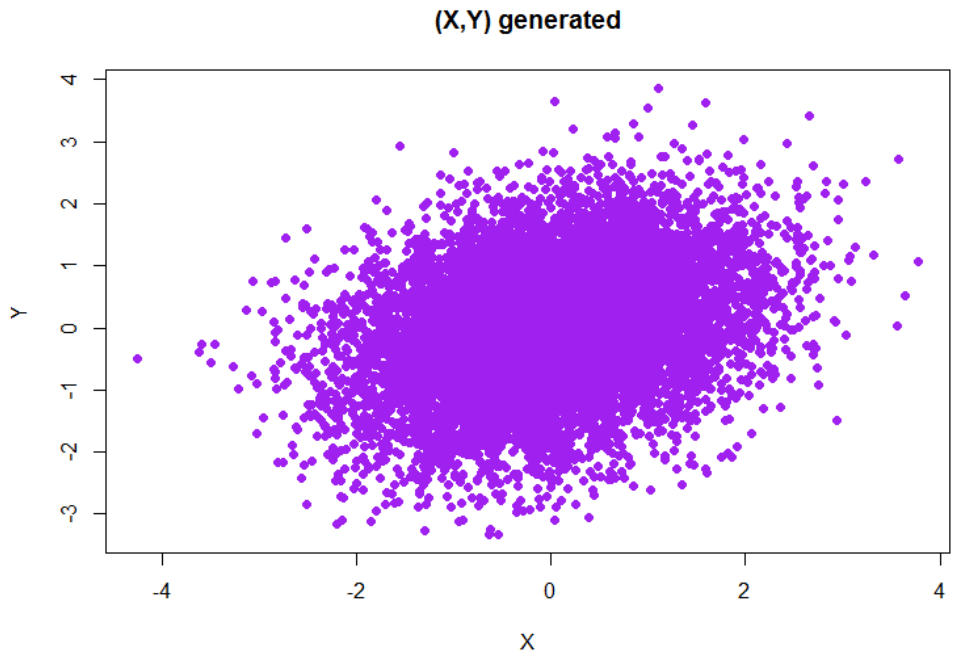

Figure 1: X and Y generated with Gibbs Sampler

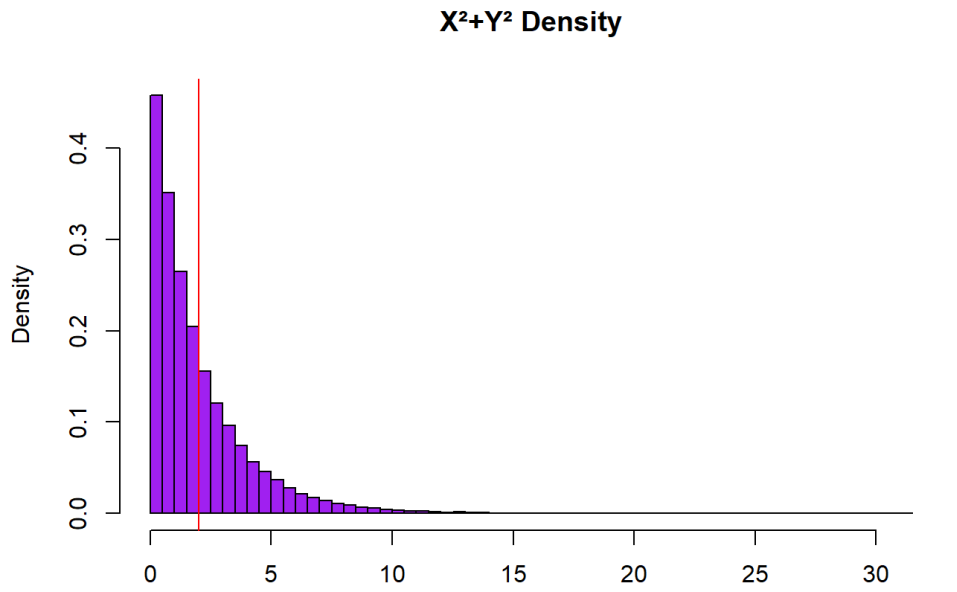

Figure 2: Sampler histogram

```
11        Xi = X[i]
12        Yi = rnorm (1, rho*Xi, sqrt(1 - rho**2))
13        Xi = rnorm (1, rho*Yi, sqrt(1 - rho**2))
14        X = c(X, Xi)
15        Y = c(Y, Yi)
16     }
17     m = matrix(c(X[ -1] ,Y), ncol =2)
18     colnames (m) = c("X","Y")
19     return (m)
20 }
21
22 # density of X^2 + Y^2
23 gibbs_density <- function (n, rho) {
24     X_Y_generated = generator(n, rho)
25     X2_Y2 = X_Y_generated[,"X"]**2 + X_Y_generated[,"Y"]**2
26     return (X2_Y2)
27 }
28
29 # We simulate the sampler and plot its distribution
30 X2_Y2 = gibbs_density (iterations,rho)
31 h1 = hist (X2_Y2 , freq = FALSE , breaks = 50 , col ="purple", main =
       "X^2+Y^2 Density", xlab="")
32 abline(v=2,col="red")
33
34 #Get the probability
35 event = X2_Y2>2
36 proba = mean(event)
```