Math 152 Second Midterm Review Sheet

4/10/2023

Here is an itemized list of the material that the second midterm is based on. Make sure you study them carefully. For each topic, you can review your lecture notes, the worked-out examples in the book, and the homework problems and solutions available on WebAssign.

- Numerical integration, comparing the approximations L_n, R_n, T_n, M_n with the true value of the integral
- Improper integrals
- Application of integration in finding areas and volumes: Suppose we have two functions f, g such that $0 \leq g(x) \leq f(x)$ for $a \leq x \leq b$. Let R be the region bounded from above by the curve $y = f(x)$, from below by the curve $y = g(x)$, on the left by the line $x = a$ and on the right by the line $x = b$. Then

  □ The area of R is

  $$A = \int_a^b (f(x) - g(x))\, dx.$$

  □ The solid obtained by revolving R about the x-axis has volume

  $$V = \pi \int_a^b (f(x)^2 - g(x)^2)\, dx. \qquad \text{(washer method)}$$

  In the special case $g(x) = 0$, the region R is bounded from below by the x-axis, and the volume formula simplifies to

  $$V = \pi \int_a^b f(x)^2\, dx. \qquad \text{(disk method)}$$

  □ Assuming $0 \leq a < b$, the solid obtained by revolving R about the y-axis has volume

  $$V = 2\pi \int_a^b x\,(f(x) - g(x))\, dx. \qquad \text{(cylindrical shells method)}$$

  Again, if $g(x) = 0$ the formula simplifies to

  $$V = 2\pi \int_a^b x\, f(x)\, dx.$$

- In general, the volume of a solid (not necessarily of revolution type) is the integral of its cross-sectional area function:

  $$V = \int_a^b A(x)\, dx.$$

- The arc-length formula: If f is continuously differentiable, the length of the curve $y = f(x)$ between $x = a$ and $x = b$ is

$$L = \int_a^b \sqrt{1 + f'(x)^2}\, dx.$$

- Solving first order differential equations by separating variables, Newton's law of cooling
- Sequences and their limits, the Squeeze Theorem (aka Sandwich Lemma), increasing and decreasing sequences, bounded monotonic sequences are convergent.

Practice Problems

1. Suppose the function f is increasing and concave down on the interval $[0, 1]$. We divide the interval $[0, 1]$ into 50 equal pieces and compute the approximations $L_{50}, R_{50}, T_{50}, M_{50}$ to the integral $I = \int_0^1 f(x)\, dx$. The results are

$$2.63375, 2.63497, 2.63548, 2.63619.$$

Which number corresponds to which method? What is the best estimate for I based on these results?

2. Let p be a fixed number greater than 1. Verify that the improper integral

$$\int_2^\infty \frac{dx}{x(\ln x)^p}$$

is convergent and find its value.

3. Let R be the region in the plane bounded by the curves $y = e^x$ and $y = e^{-x}$, and the line $x = 3$.

(i) Sketch R and find its area.
(ii) Find the volume of the solid generated by rotating R about the x-axis.
(iii) Find the volume of the solid generated by rotating R about the y-axis.

4. Compute the length of the piece of the curve $y = 2x^{3/2}$ between the points $(1, 2)$ and $(4, 16)$.

5. Find the general solution to the differential equation

$$\frac{dy}{dx} = xe^{x-y}.$$

Then find the solution that satisfies the initial condition $y(0) = 1$.

6. In each case, find the limit of the given sequence as $n \to \infty$ or show that it does not exist:

$$1 + 2(-1)^n \qquad \frac{1 + 2(-1)^n}{\sqrt{n}} \qquad \frac{n^2}{(5n-1)(n+3)} \qquad \left(1 + \frac{a}{n}\right)^n$$