Math 152 First Midterm Review Sheet

2/26/2023

Here is an itemized list of the material that the first midterm is based on. Make sure you study them carefully. For each topic, you can review your lecture notes, the worked-out examples in the book, and the homework problems and solutions available on WebAssign.

- Inverse functions, derivative of the inverse function

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} \qquad \text{or} \qquad \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

- The natural logarithm $\ln x$ and its basic properties, the derivative formula

$$(\ln x)' = \frac{1}{x} \qquad \text{and more generally} \qquad (\ln u)' = \frac{u'}{u}$$

- The natural exponential function e^x and its basic properties, the derivative formula

$$(e^x)' = e^x \qquad \text{and more generally} \qquad (e^u)' = e^u \cdot u'$$

- General exponential and logarithmic functions a^x and $\log_a x$ and their basic properties, the derivative formulas

$$(a^x)' = a^x \cdot \ln a \qquad \text{and} \qquad (\log_a x)' = \frac{1}{x \ln a}$$

- Logarithmic differentiation
- Exponential growth and decay, doubling time and half-life
- Inverse trigonometric functions $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$ and their basic properties, the derivative formulas

$$(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}, \quad (\cos^{-1} x)' = \frac{-1}{\sqrt{1-x^2}}, \quad (\tan^{-1} x)' = \frac{1}{1+x^2}$$

- L'Hospital's rule, its applications in finding limits of indeterminate forms
- The fundamental theorem of calculus, basic integration formulas (for example, review the formulas 1-8, 10, 12, 14, 16, 17 in the table of integrals on reference page 6 in the back of the book)
- Integration by substitution
- Integration by parts
- Trigonometric integrals and trigonometric substitutions
- Partial fractions and integration of rational functions

Practice Problems

1. Show that the function $f(x) = x^3 + 2x$ is strictly increasing and therefore one-to-one. Then find the derivative $(f^{-1})'(-3)$.

2. In each case, find the derivative $y' = dy/dx$:

- $y = \ln(x + \sqrt{x})$
- $y = e^{-x} \tan^{-1}(x^2) + \cos^{-1}(5x)$
- $y = (1 - x)^{(x^2)}$

3. Use L'Hospital's rule to find $\lim_{x \to 0} \dfrac{x - \sin x}{x^3}$.

4. The number of bacteria in a lab sample grows exponentially. If $5,000$ bacteria were initially present and $8,000$ were present 10 hours later, how long will it take for the number of bacteria to double?

5. Integrate:

- $\displaystyle\int_0^{2\pi} x^2 \cos x \, dx$ [Hint: Integration by parts twice]
- $\displaystyle\int \left(\sin^2 x + \frac{1}{\sqrt{3 - x^2}} \right) dx$
- $\displaystyle\int \frac{x^3}{\sqrt{x^2 + 4}} \, dx$ [Hint: Trigonometric substitution]
- $\displaystyle\int \frac{x - 4}{x^2 - 5x + 6} \, dx$ [Hint: Partial fractions]

6. Find the area of the region under the curve $y = \dfrac{1}{x^3 + 2x}$ from $x = 1$ to $x = 2$.