Math 310 Problem Set 9

11/13/2025

1. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a continuous function that is *decreasing* (in the sense that $x < x' \implies f(x) > f(x')$). Use the intermediate value theorem to show that f has a unique fixed point, i.e., there is a unique $c \in \mathbb{R}$ such that $f(c) = c$.

2. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ -x+2 & \text{if } x > 1 \end{cases}$$

Prove that f is not differentiable at $x = 1$.

3. Let $f(x) = x|x|$. At what x does $f'(x)$ exist?

4. Let $f(x) = x - \lfloor x \rfloor$ (as usual, $\lfloor x \rfloor$ is the integer part of x). At what x does $f'(x)$ exist?

5. Let $f, g, h : \mathbb{R} \to \mathbb{R}$ be three functions that are differentiable everywhere. Find formulas for the derivative of the product fgh and the composition $f \circ g \circ h$. Can you generalize your formulas to the products and compositions of n functions?