CSCI 3202: Intro to Artificial Intelligence
Lecture 7: A* Search and Heuristics

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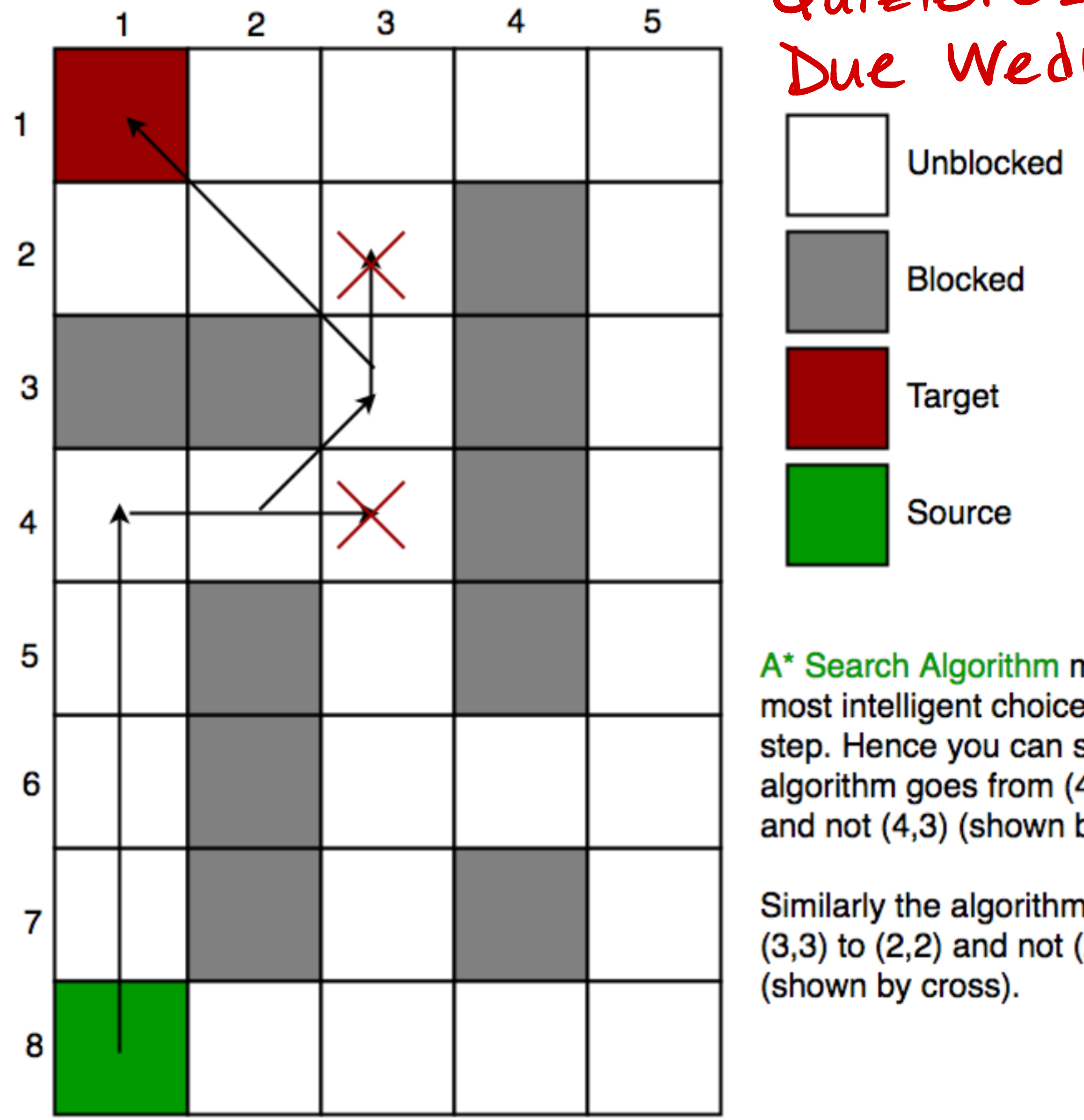
• Quizlet02 Due Wednesday



A* Search Algorithm makes the most intelligent choice at each step. Hence you can see that algorithm goes from (4,2) to (3,3) and not (4,3) (shown by cross).

Similarly the algorithm goes from (3,3) to (2,2) and not (2,3) (shown by cross).

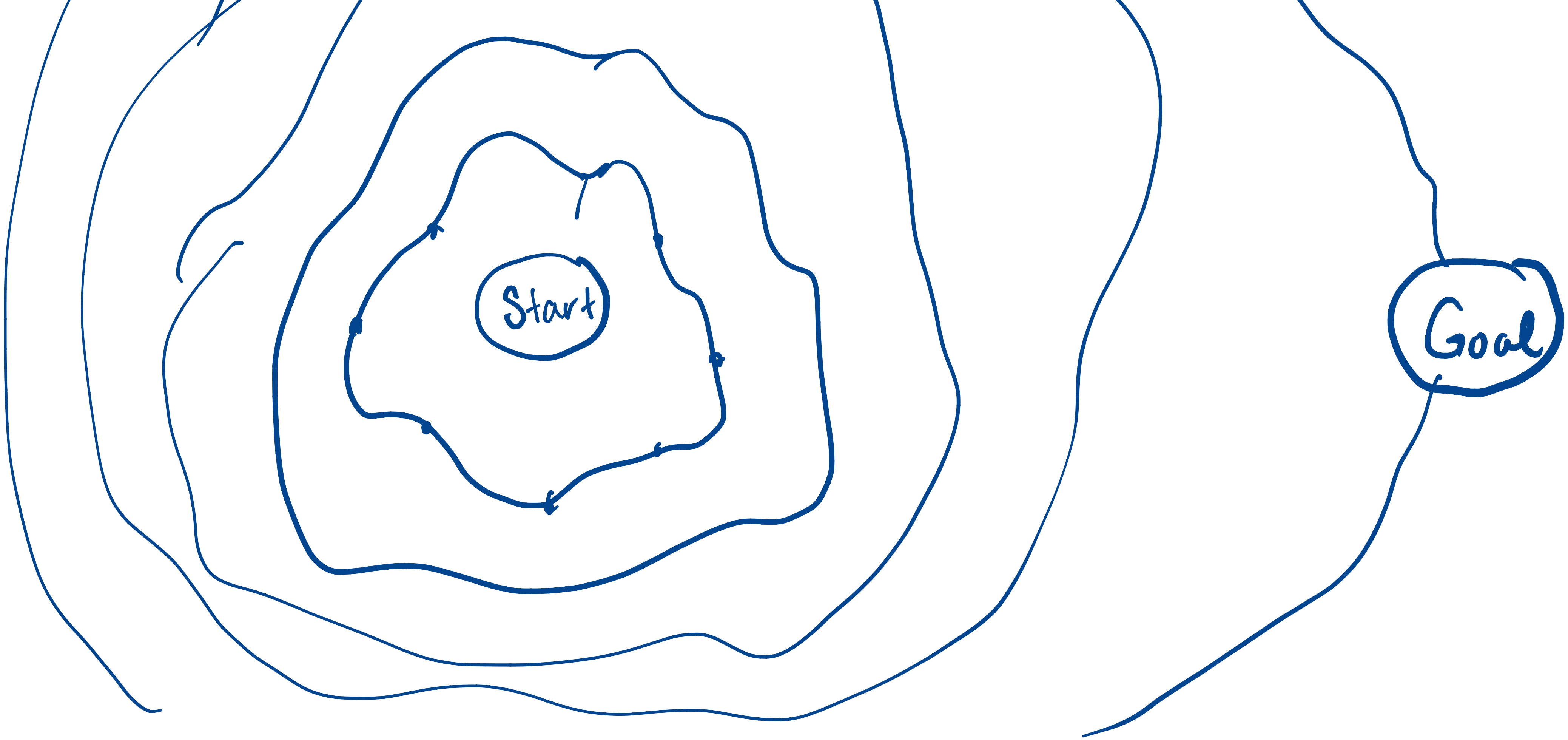
Source

• HW2 posted on Moodle

Review: Uniform-cost Search (UCS)

- Expand out in contours, where least cost dictates which nodes we explore.

- Eventually, we will find a path to the goal - but the search is not directed



Search Algorithms

❖ Search algorithms are fundamentally the same except for their frontier strategies.

Uninformed Search: e.g. Uniform Cost Search

- the good: UCS is complete and optimal → if a solution exists, it will find it with the least cost path
- the bad: explores in every direction

Informed Search: include information about where the goal is

- what do we need to have? A heuristic.

heuristic: A function that estimates how close a state is to a goal.

Greedy best–first search

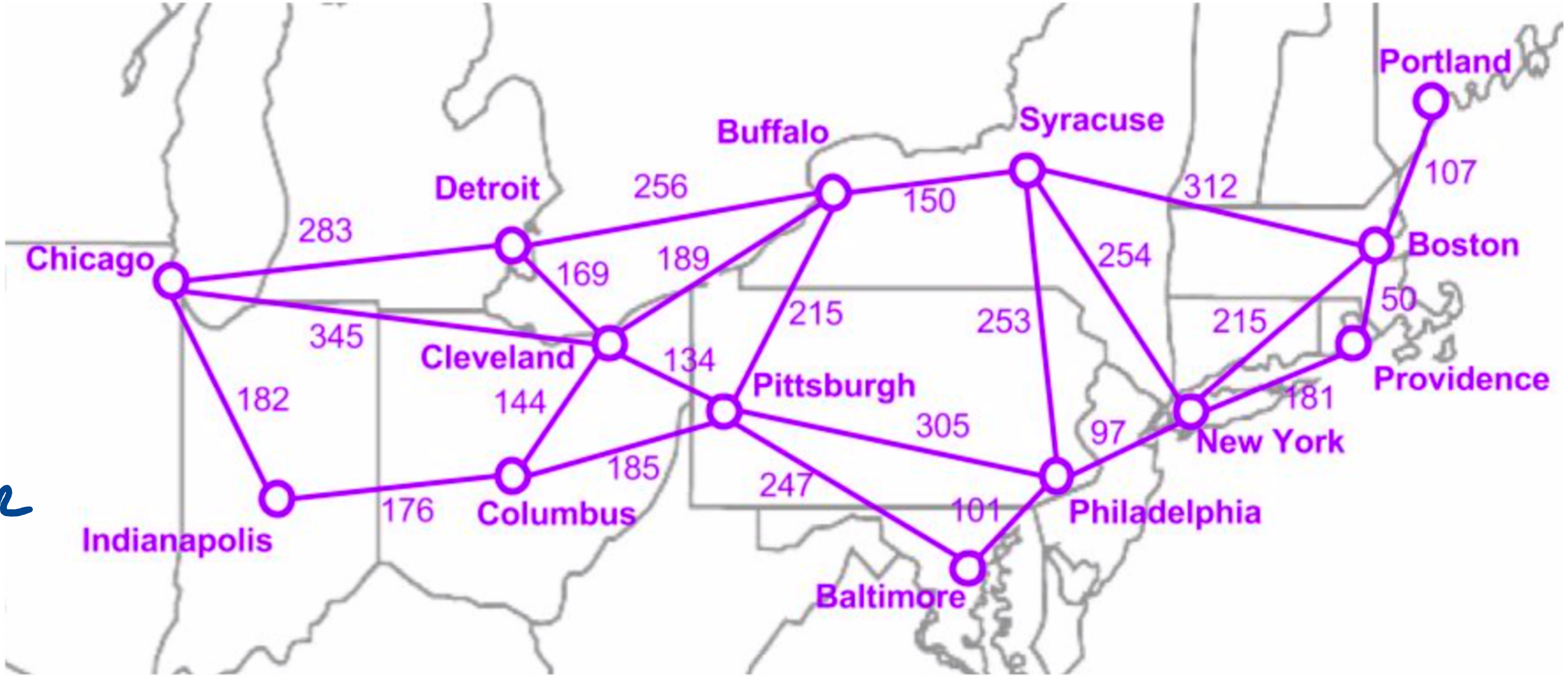
❖ First expand the path that's closest to the goal.

To determine what's closest to the goal, we need to define a heuristic function.

Example: For the traveling in the northeast problem, let's estimate the distance to the goal as the straight-line distance between city and the goal city.

this is our heuristic function

Step costs: miles between cities along major highways



Greedy best–first search

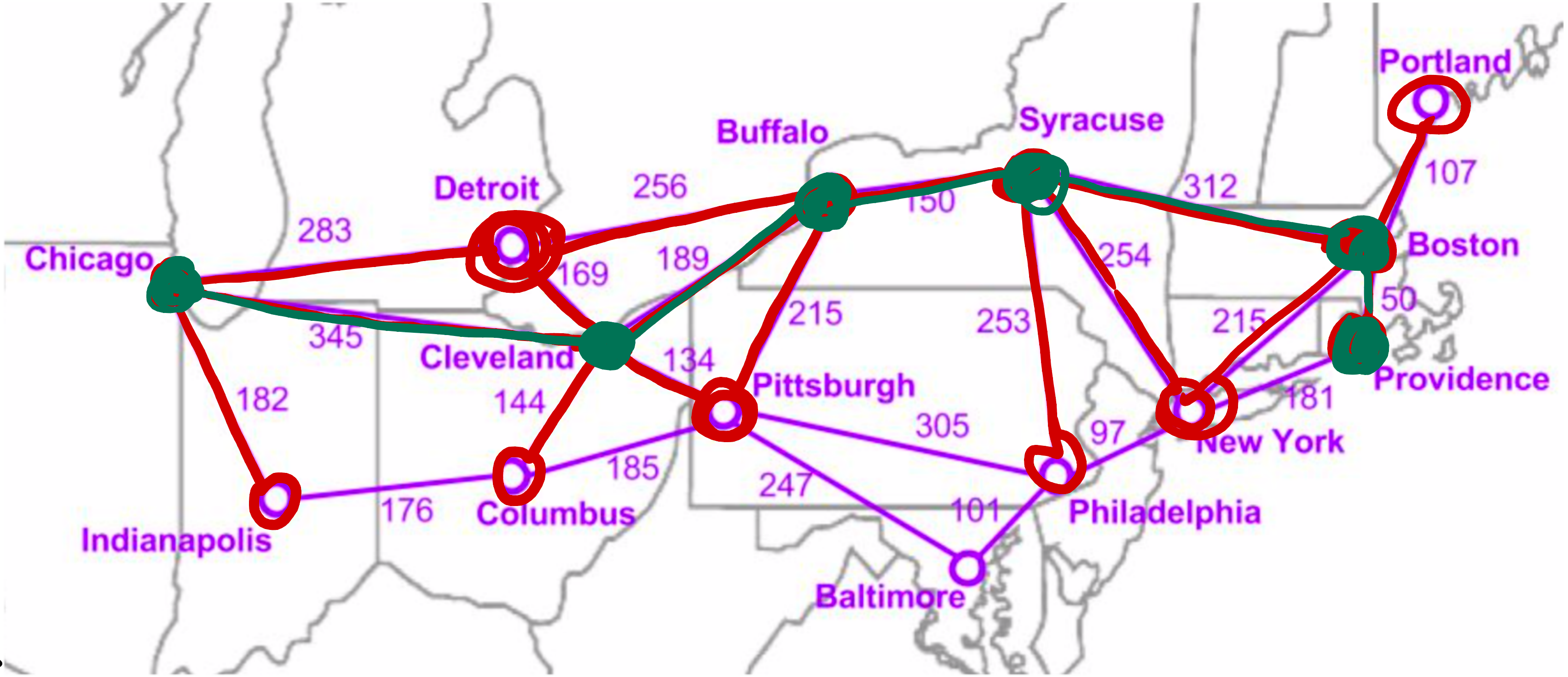
Start Goal

Example: Use the greedy best-first search to find a route from Chicago to Providence.

Chicago → Providence 833 miles
Detroit → Providence 597 miles
Indianapolis → Providence 783 miles
Cleveland → Providence 530 miles
Columbus → Providence 618 miles
Buffalo → Providence 388 miles
Pittsburgh → Providence 456 miles
Baltimore → Providence 324 miles
Syracuse → Providence 256 miles
Philadelphia → Providence 235 miles
New York → Providence 155 miles
Portland → Providence 140 miles
Boston → Providence 41 miles

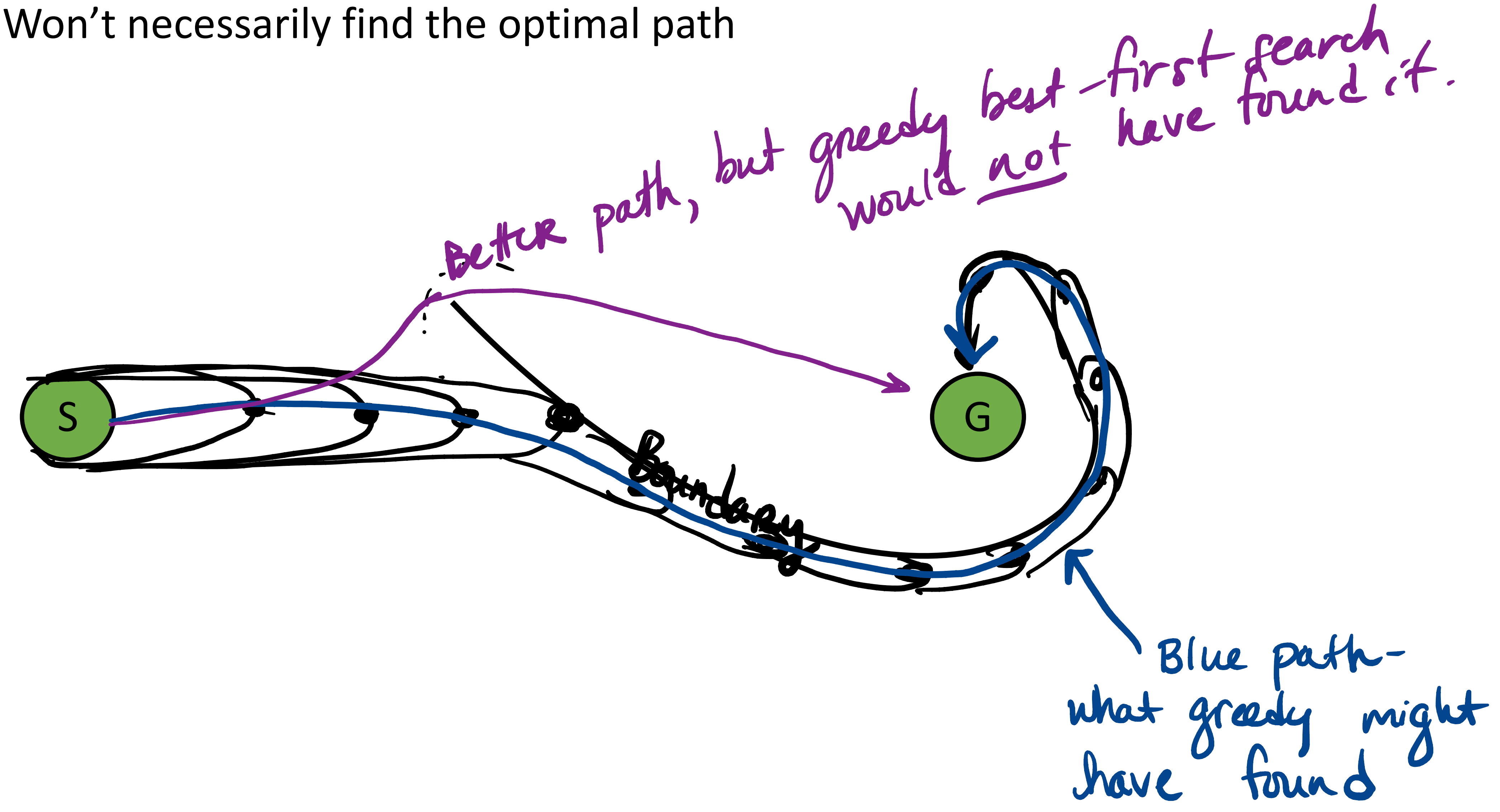
"heuristic function"

Heuristic: $h(n)$ = straight-line distance to Providence



Greedy best–first search

Possible Issue: Won't necessarily find the optimal path



A* Search

Uniform-cost search:

$f(n) = g(n)$ (cost to get to *n*)

Greedy:

$f(n) = h(n)$ (estimated cost to get from *n* to goal)

A*:

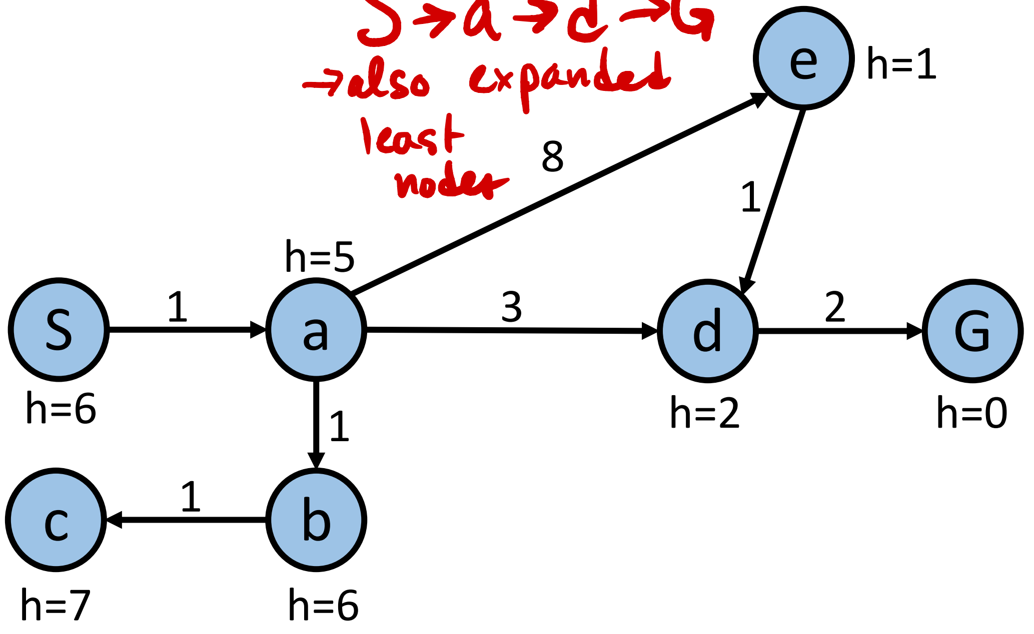
$f(n) = g(n) + h(n)$ (estimated total cost of cheapest solution through *n*)

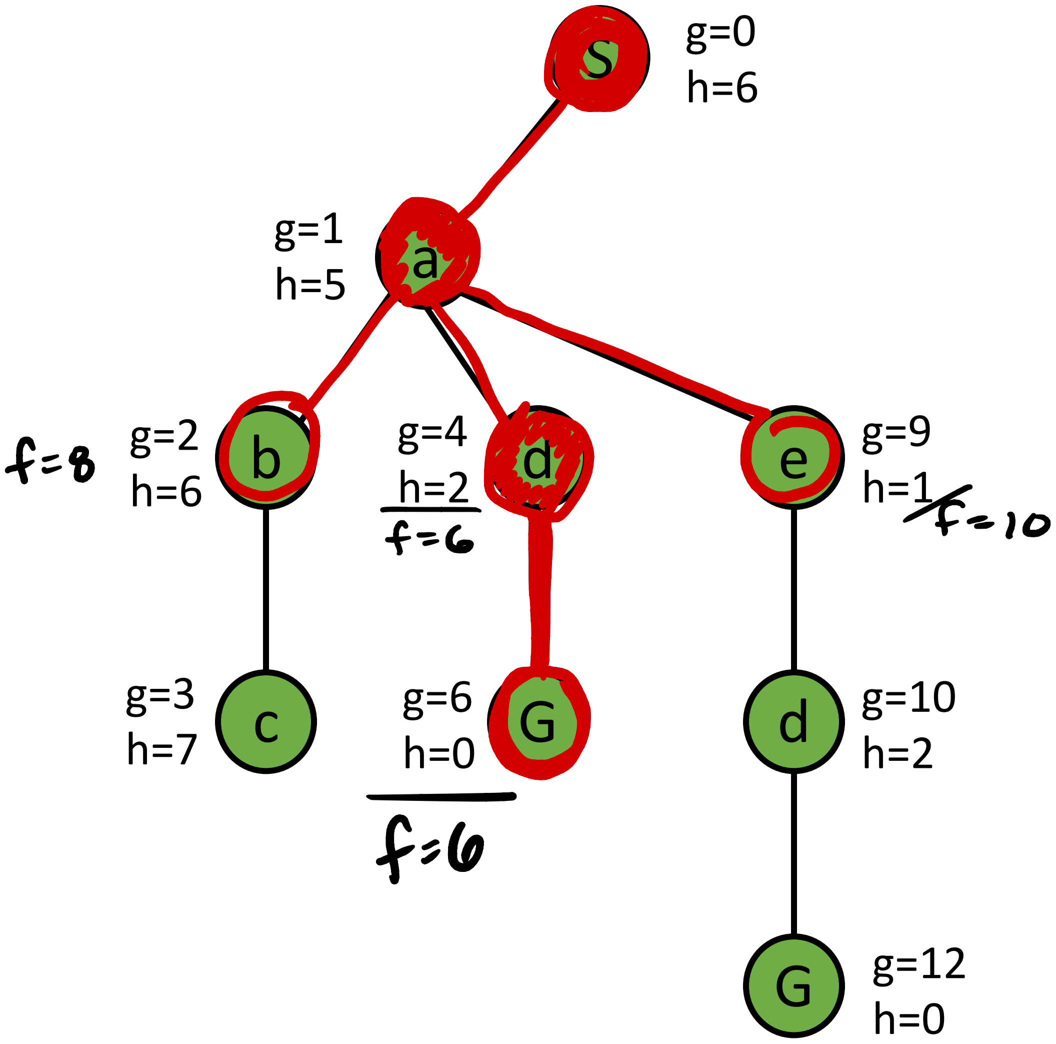
A* Search

S→a→d→G

Example: Compare Uniform Cost, Greedy S→a→e→d→G Search, and A* on the graph below.

S→a→d→G
→also expanded least nodes

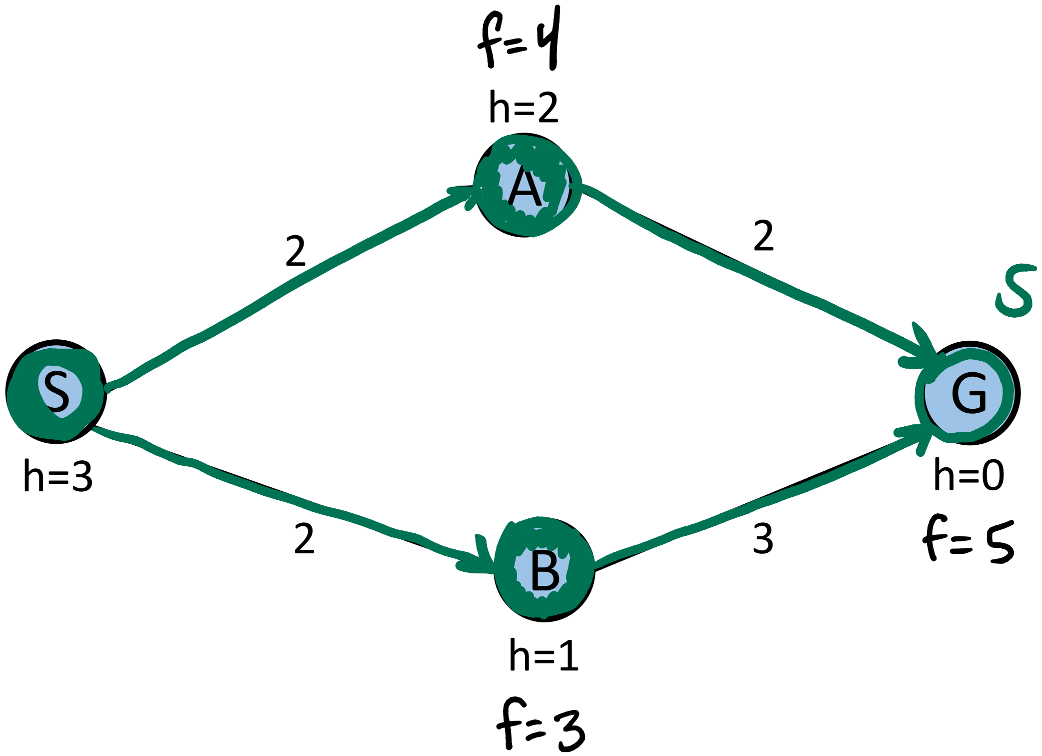




A* Search

Example: When should A* search terminate?

When you "dequeue" the goal.

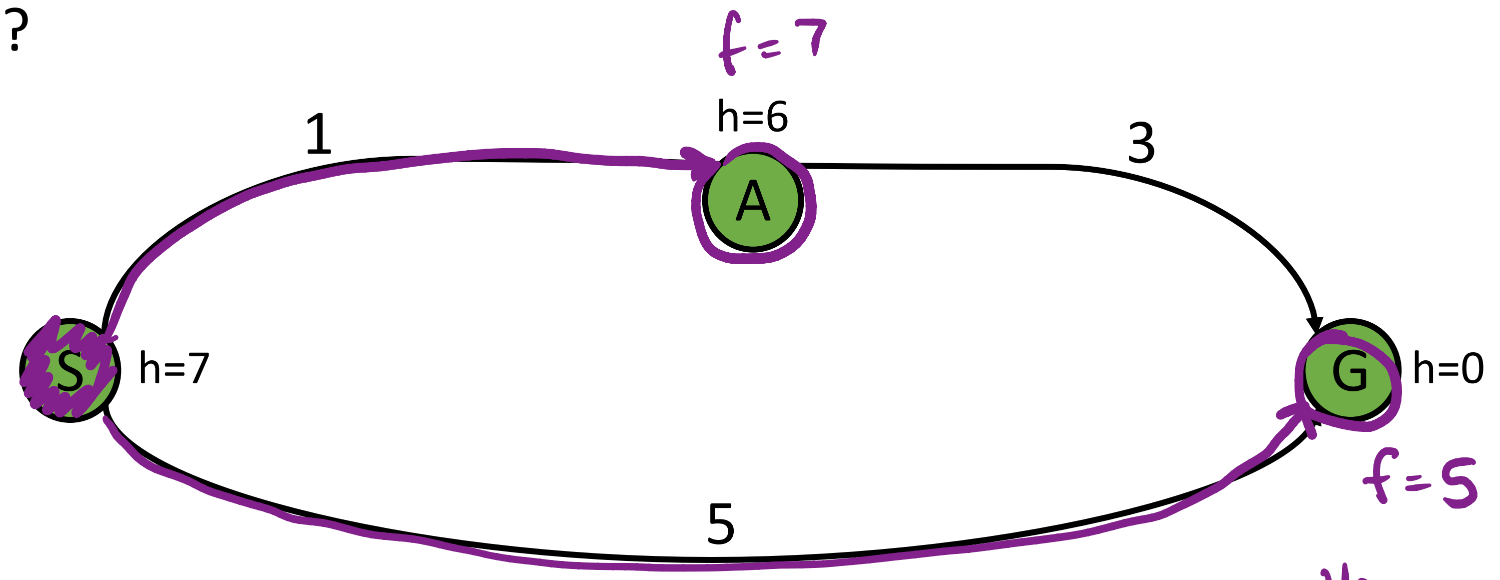


S → A → G path cost = 4

S → B → G path cost = 5

A* Search

Is A* optimal?

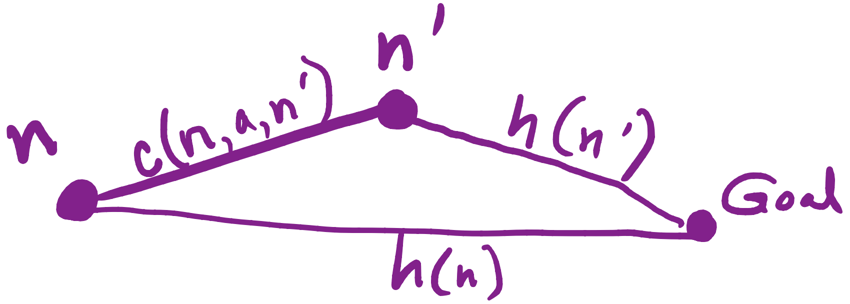


Here we would expand G.
· We would terminate.
· solution path S → G

* Why did A* fail?
poorly chosen heuristic at node A.
Heuristics must $\boxed{\text{admissible.}}$

A* Search

Consistent: for every node n and successor n’ of n, generated by some action a, the estimated cost of reaching the goal from n is no greater than the step cost from n to n’, plus the estimated cost of reaching the goal from n’



- That is: $h(n) \leq c(n, a, n') + h(n')$

- General **triangle inequality** between n, n’, and the goal

A heuristic h is **admissible** (optimistic) if $0 \leq h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to the nearest goal.

A* Search

Search only works when:

- domain is fully observable
- domain must be known
- domain must be deterministic
- domain must be static

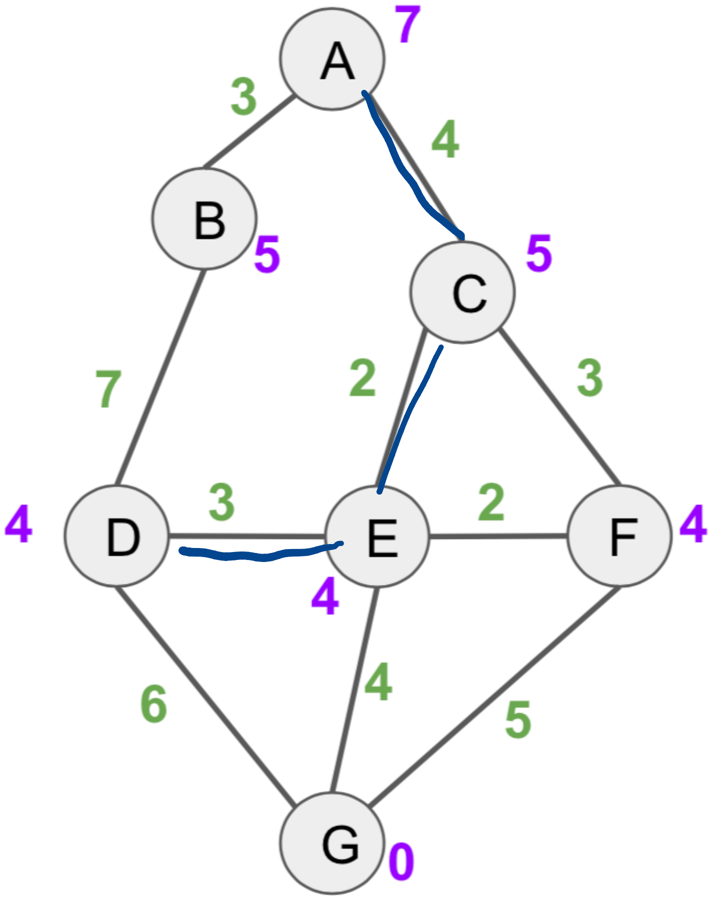
implementation: use a **node**

- state - indicates state at end of path
- action - action taken to get here
- cost - total cost
- parent - pointer to another node

A* Search

A* Search:

- Find the cheapest path from A to G
- $h(n)$ values are given in **purple**
- Step costs are given in **green**



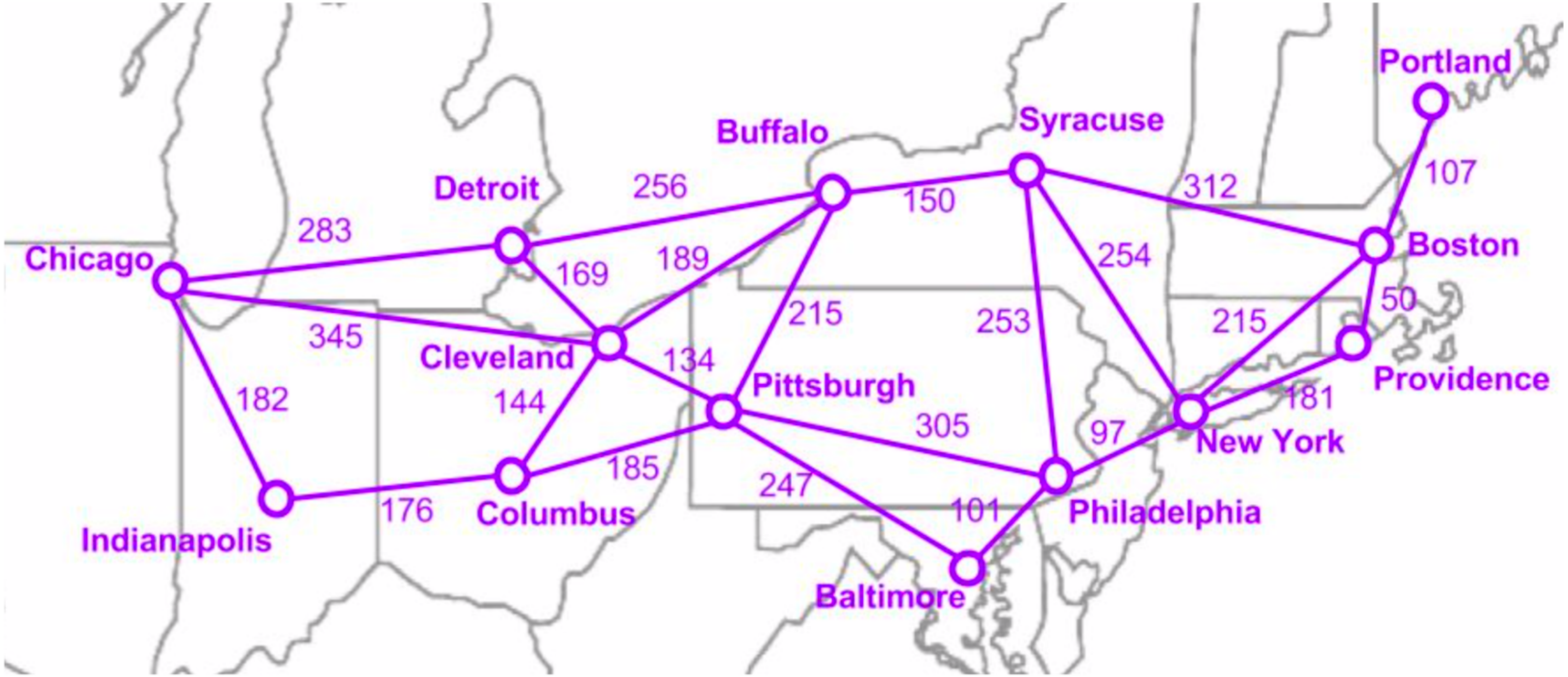
Explored	Frontier
{}	A $g=0$, $f=7$
{A}	B $g=3$, $h=5$, $f=8$; C $g=4$, $h=5$, $f=9$
{A, B}	C $g=4$, $h=5$, $f=9$; D $g=10$, $h=4$, $f=14$
{A, B, C}	D $g=10$, $h=4$, $f=14$; E $g=6$, $h=4$, $f=10$; F $g=7$, $h=4$, $f=11$
{A, B, C, E}	~~D $g=10$, $h=4$, $f=14$~~; D $g=9$, $h=4$, $f=13$; F $g=7$, $h=4$, $f=11$; ~~F $g=8$, $h=4$, $f=12$~~; G $g=10$, $h=0$, $f=10$
{A, B, C, E, G} goal test	$\boxed{A \rightarrow C \rightarrow E \rightarrow G}$

A* Search

Example: Use A* search to find a route from Chicago to Providence.

practice at home

$h(n)$ = straight-line distance to Providence
$g(n)$ = Path cost so far



A* Search

consistent: $h(n) \leq c(n,a,n') + h(n')$

admissible: $h(n) \leq h^*(n)$

Any consistent heuristic is also admissible (but not the other way around).

Example: Prove the above statement by induction.

Proof: Base Case: Assume that a heuristic $h(n)$ is consistent for $K=1$ nodes along the shortest path to the goal node.

$h(n) \leq c(n,a,n') + h(n')$ by consistent property.

Let n' be the goal node.

$\Rightarrow h(n') = 0$

$\Rightarrow h(n) \leq c(n,a,n') + 0$

$= h^*(n)$

Thus $h(n) \leq h^*(n)$ and $h(n)$ is admissible.

A* Search

Any consistent heuristic is also admissible (but not the other way around).

<u>Example</u>: Prove the above statement by induction.

Next Time

Optimality and Variants of A*