# CSCI 3202: Intro to Artificial Intelligence

# Lecture 13: Decision Making

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Wednesday - Review Day
- lecture slides + quizzes on Moodle
- algorithms - pseudocode

HW3 - due next Wed Mar. 4

Problem 3 - you may minimize or maximize.

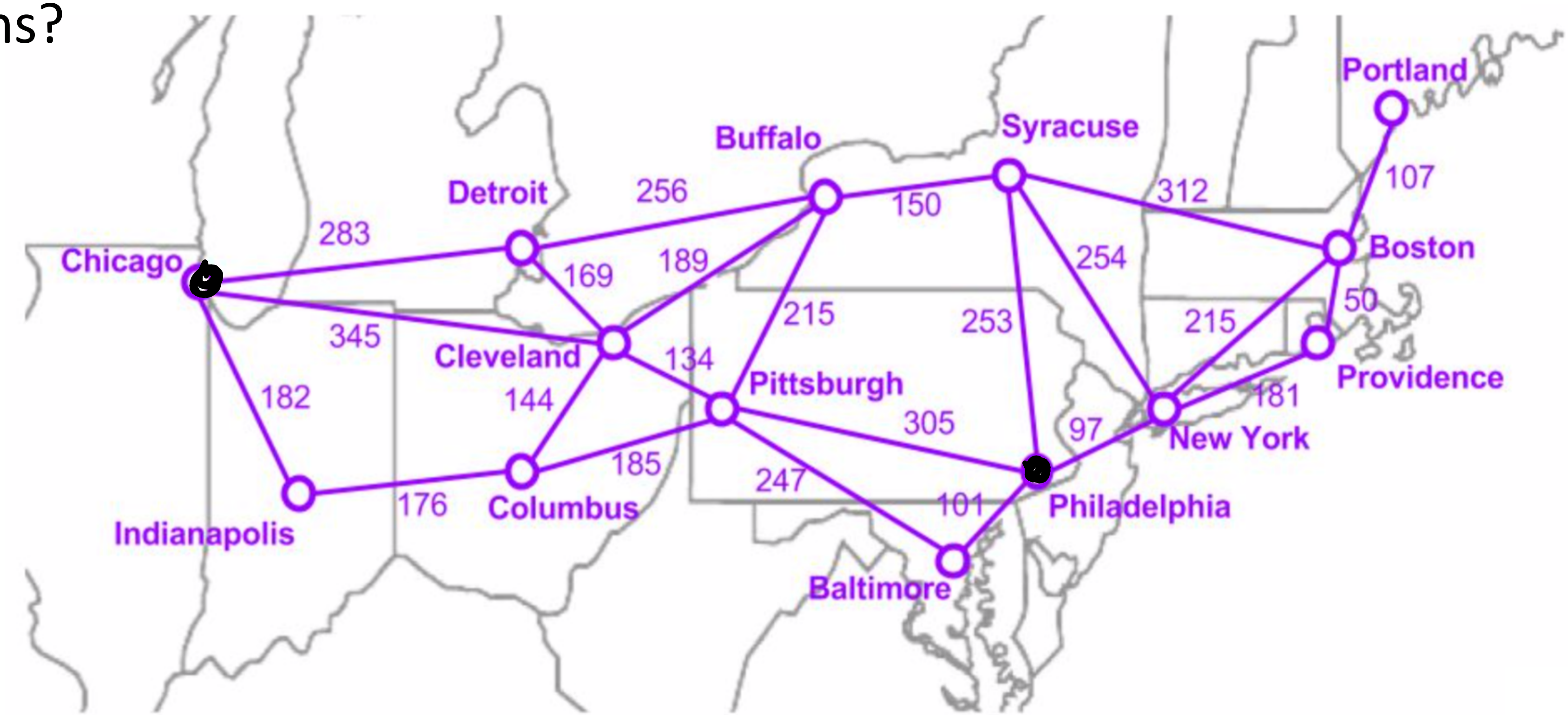
# Decision Making

Traveling in the US northeast

How do we decide which route to take?

What affects out decisions?

Decisions must be made in the face of uncertainty.



# Decision Making

What affects out decisions?

Decisions are affected by many things

- What we value
- Uncertainty

we may not know the consequence of our action

→ stochastic nature of our environment

→ unpredictable opponent.

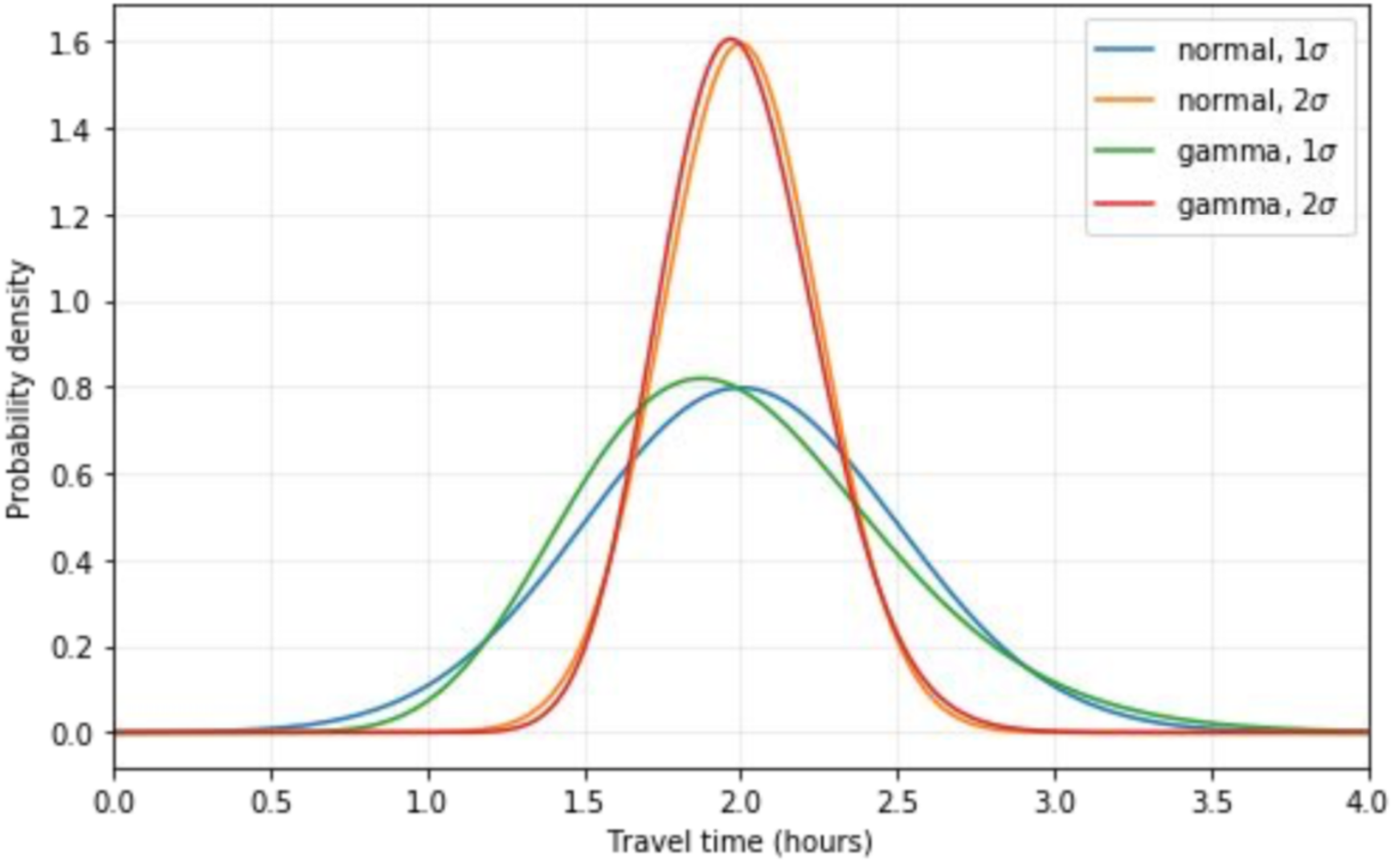
# Uncertainty meets Probability Theory

**Ontological commitments**

- 0 or 1
- e.g. propositional logic

**Epistemological commitments**

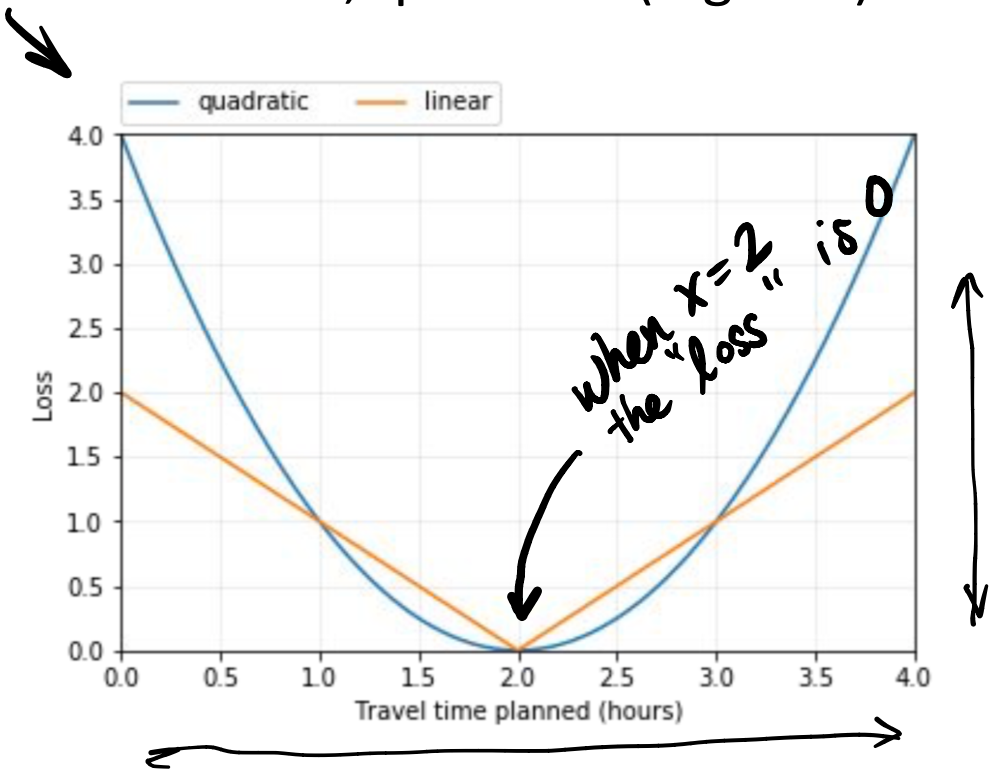
- between 0 and 1
- quantifies rigorously our degree
- of belief in all of the possible outcomes

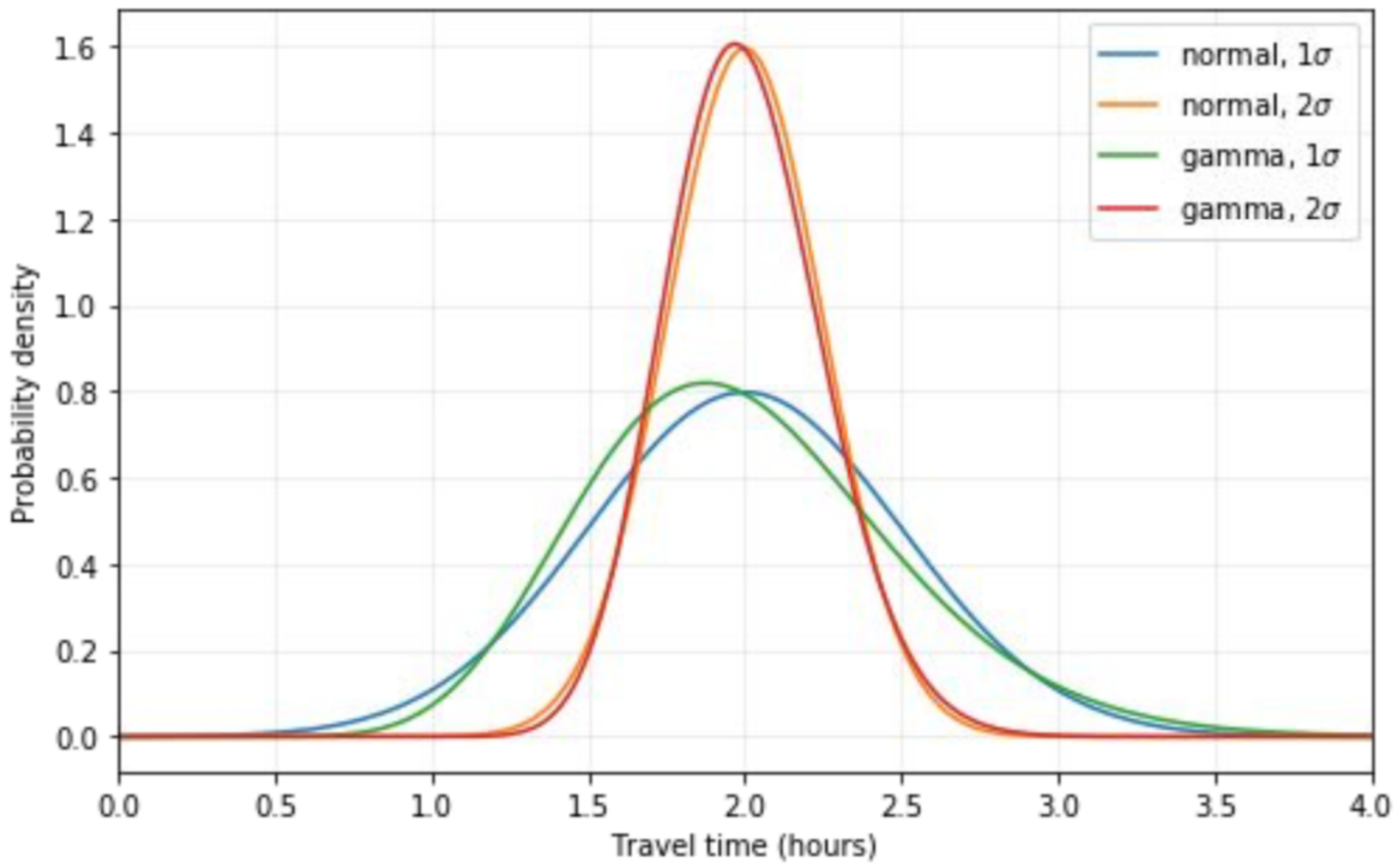


# Decision–making

## Loss functions

- Can be viewed inversely as utility
- Common choices:
  - Linear, quadratic (e.g. SSE)



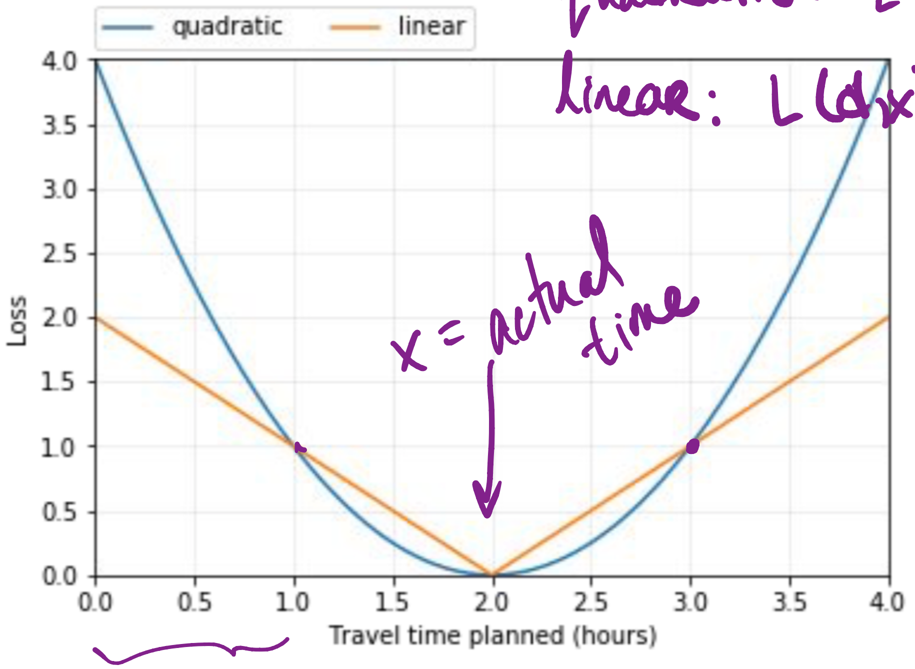



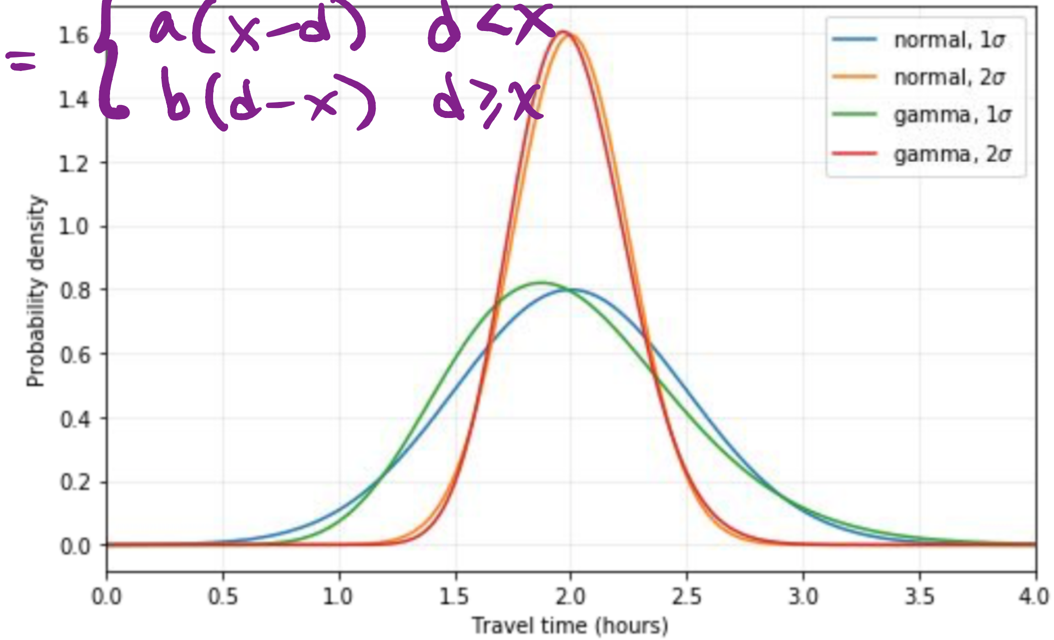
# Decision–making

The Bayes decision, $d_{Bayes}$, is the decision that minimizes the expected loss.

quadratic: $L(d,x) = K(d-x)^2$

linear: $L(d,x) = \begin{cases} a(x-d) & d<x \\ b(d-x) & d \geq x \end{cases}$




- x = state variable
- d = decision you make
- L(d, x) = loss when we make decision d in state x
- f(x) = probability density/distribution function for states x

# Decision–making

Recall $E[x] = \int_{-\infty}^{\infty} x f(x)\, dx$

The **Expected Value of Including Uncertainty** in your decision-making is:

$E[g(x)] = \int_{-\infty}^{\infty} g(x) f(x)\, dx$

$$EVIU \ = \int_x \left[L(d_{iu}, x) - L\left(d_{Bayes}, x\right)\right] f(x)\, dx$$

$$= \int L(d_{iu}, x) f(x)\, dx - \int L(d_{Bayes}, x) f(x)\, dx$$

$$= \underbrace{E[L(d_{iu}, x)] - E[L(d_{Bayes}, x)]}$$

tells you the average saving by taking uncertainty into account.

- $x$ = state variable
- $d_{iu}$ = decision you make by ignoring uncertainty
- $L(d_i, x) =$ loss when we make decision *d* in state $x$
- $f(x) =$ probability density/distribution function for states x

HW: $E[g(x)]$ – hint

# Decision–making

The **Expected Value of Perfect Information** in your decision-making is:

$$EVPI = \int_x \left[L\left(d_{Bayes}, x\right) - L\left(d_{pi}, x\right)\right] f(x)\, dx$$

$$= E\left[L\left(d_{Bayes}, x\right)\right] - E\left[L\left(d_{pi}, x\right)\right]$$

- $x$ = state variable
- $d_{pi}$ = decision you make with perfect information information regarding $x$
- $L\left(d_{pi}, x\right) =$ loss when we make decision *d* in state $x$
- $f(x) =$ probability density/distribution function for states x

# Decision–making

Recall: Expected Value is linear. $E[aX+b] = aE[x] + b$

The EVIU with a quadratic loss function is 0. $L(d,x) = k(d-x)^2$ $k>0$

$$\star\ EVIU = \underbrace{E_x[L(d_{iu},x)] - E_x[L(d_{Bayes},x)]}$$

In general: $E_x[k(d-x)^2] = E_x[k(d^2-2xd+x^2)]$

$$= k\,E_x[d^2-2xd+x^2]$$

$$= k\left(E_x[d^2] - 2E_x[xd] + E_x[x^2]\right)$$

$$= k\left(d^2 - 2d\,E_x[x] + E_x[x^2]\right)$$

Bayes' decision <u>minimizes</u> the expected loss.

$$\therefore\ \frac{d}{d(d)}E[k(d-x)^2] = 2kd - 2kE_x[x] \overset{\heartsuit}{=} 0$$

# Decision–making

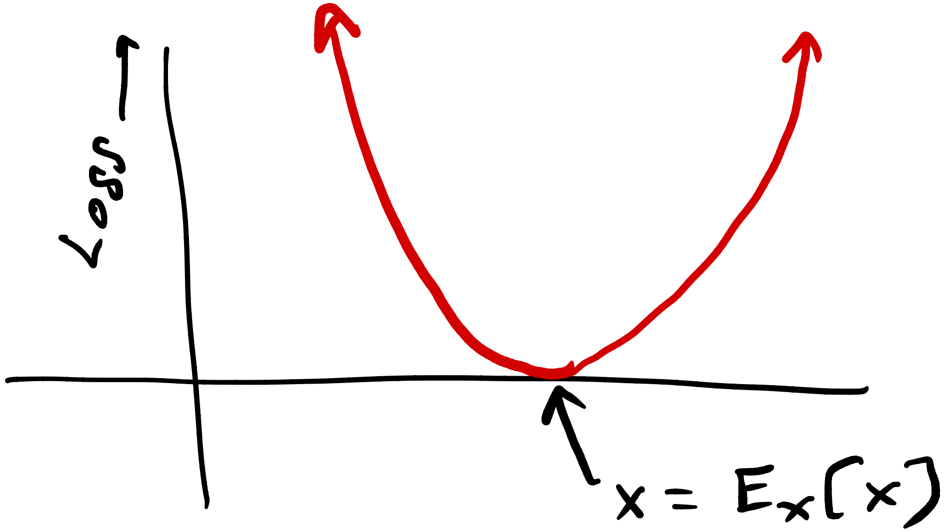
The EVIU with a quadratic loss function is 0.

... continued

$$\frac{d\, E_x[L(d,x)]}{dd} = 2kd - 2k E_x[x] = 0$$

$$2kd = 2k E_x[x]$$

$$d = E_x[x]$$




It turns out, by symmetry

$$d_{iu} = d_{Bayes} = E_x[x]$$

$$EVIU = E_x[x] - E_x[x]$$
$$= 0$$

# Decision–making

The EVIU with a quadratic loss function is 0.

- ❖ This is the expected value of including uncertainty.

- ❖ We assumed that, neglecting uncertainty, the “best” estimate of $x$ is $E_x[x]$.
  - ➢ If risk-averse and/or asymmetric loss, might make a different assumption.

# Decision–making

The EVPI with a $\boxed{\text{quadratic loss function}}$ is (probably) not 0.

$$EVPI = \underbrace{E_x[L(d_{Bayes}, x)]}_{E_x[x]} - \underbrace{E[L(d_{PI}, x)]}_{0}$$

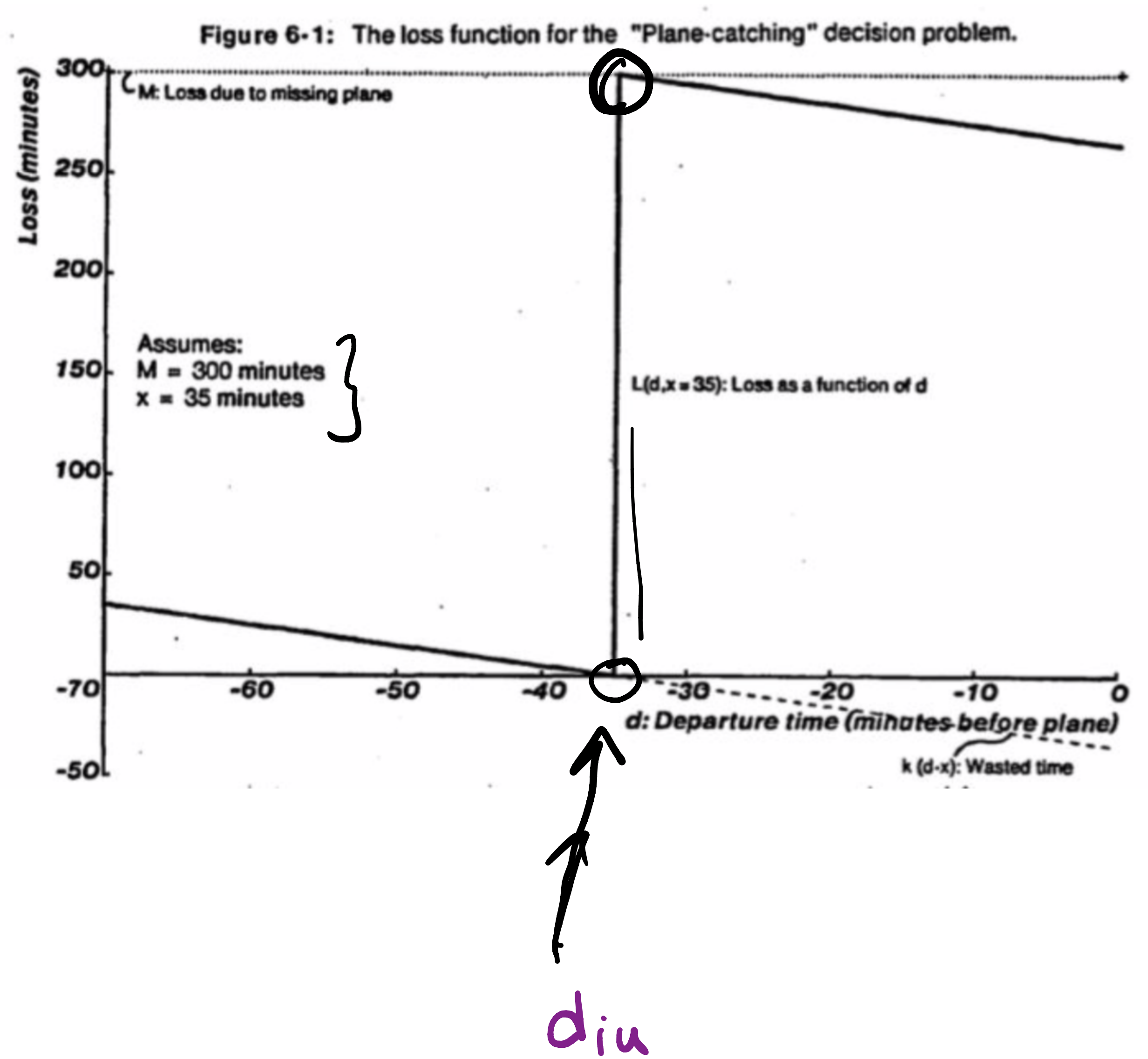
unlikely this is 0.

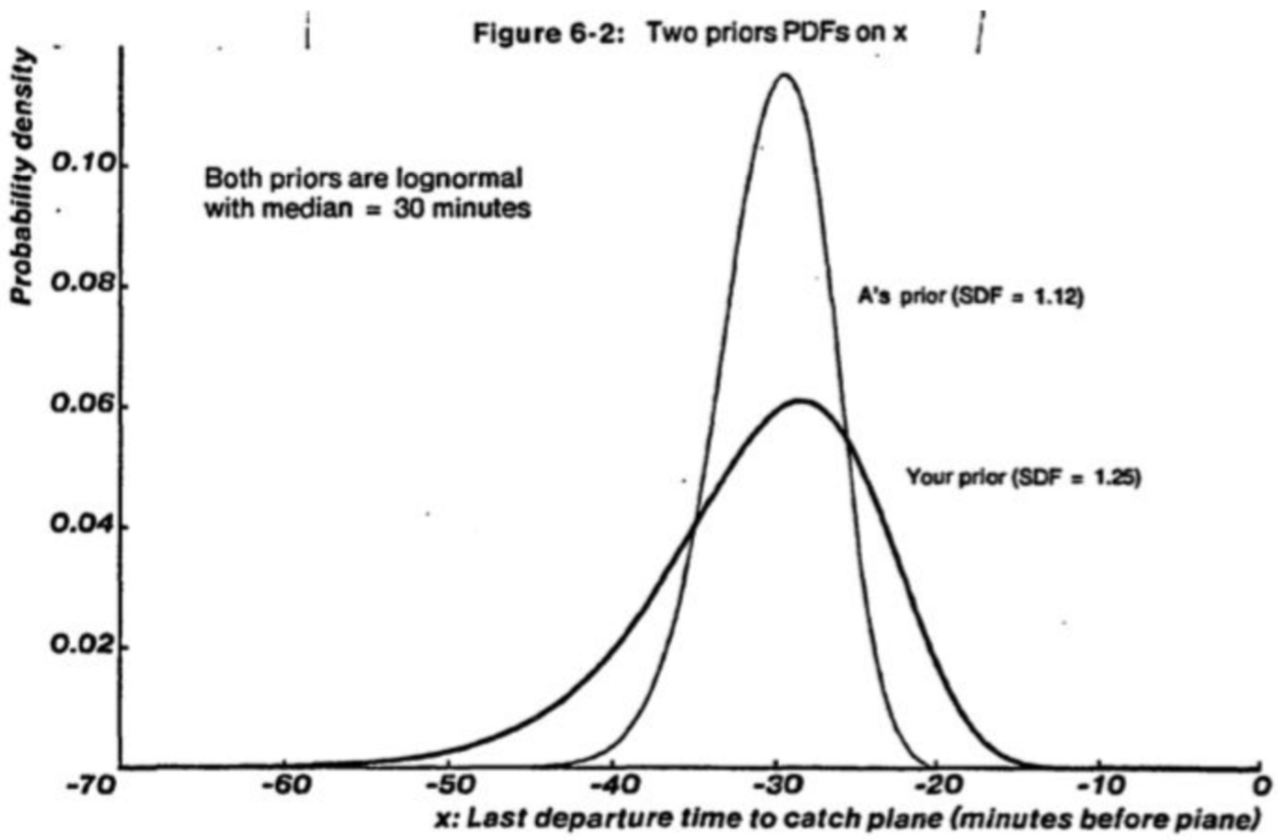
# Decision–making

**<u>Example</u>**: The airport problem. You need to drive to the airport to catch a plane.

- The decision $d$ represents how long you allow for the drive.
- The state variable $x$ represents how long it will actually take you to drive there.
- Loss function?
- Prior distribution of $x$?

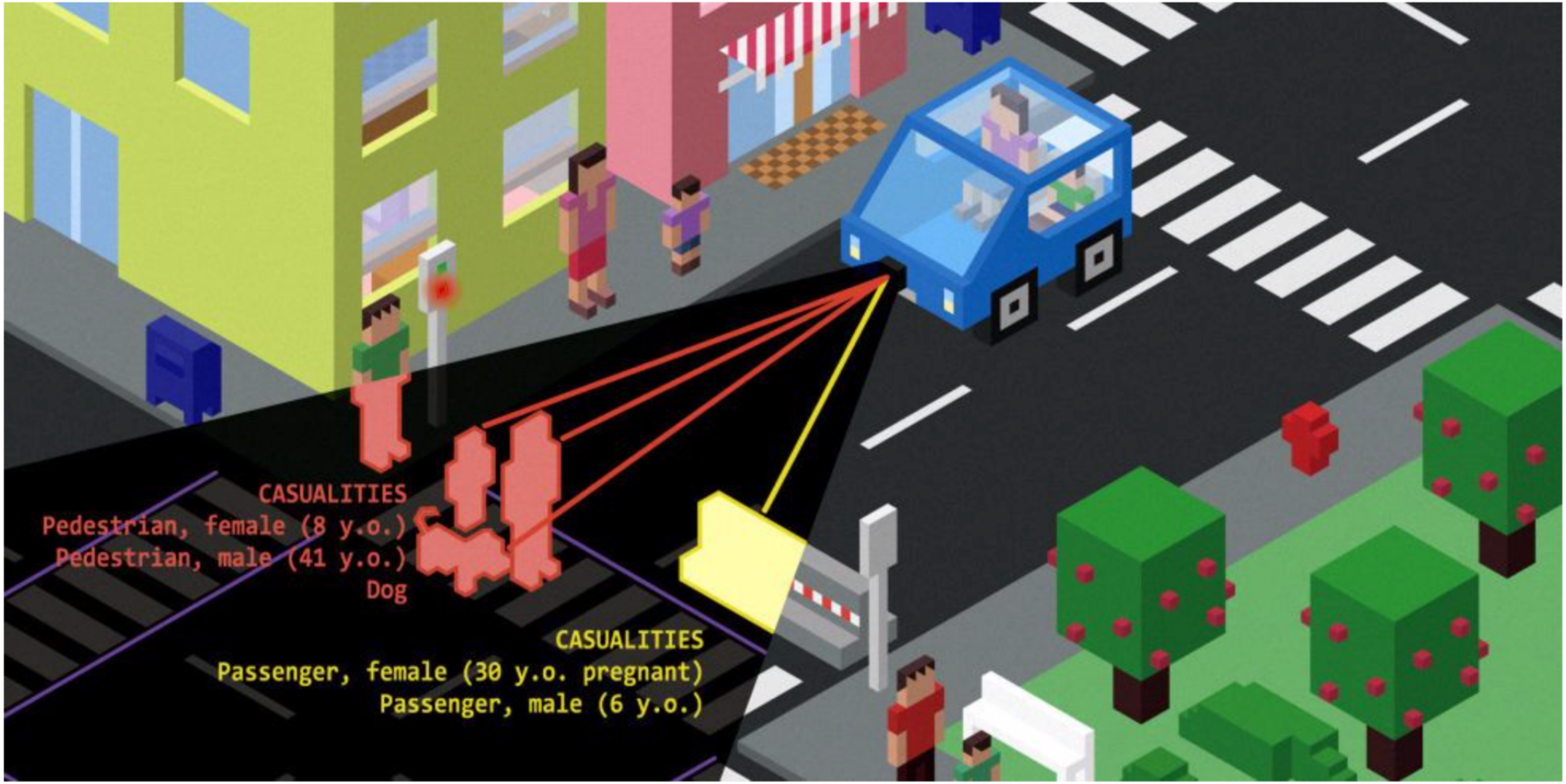
# Decision–making



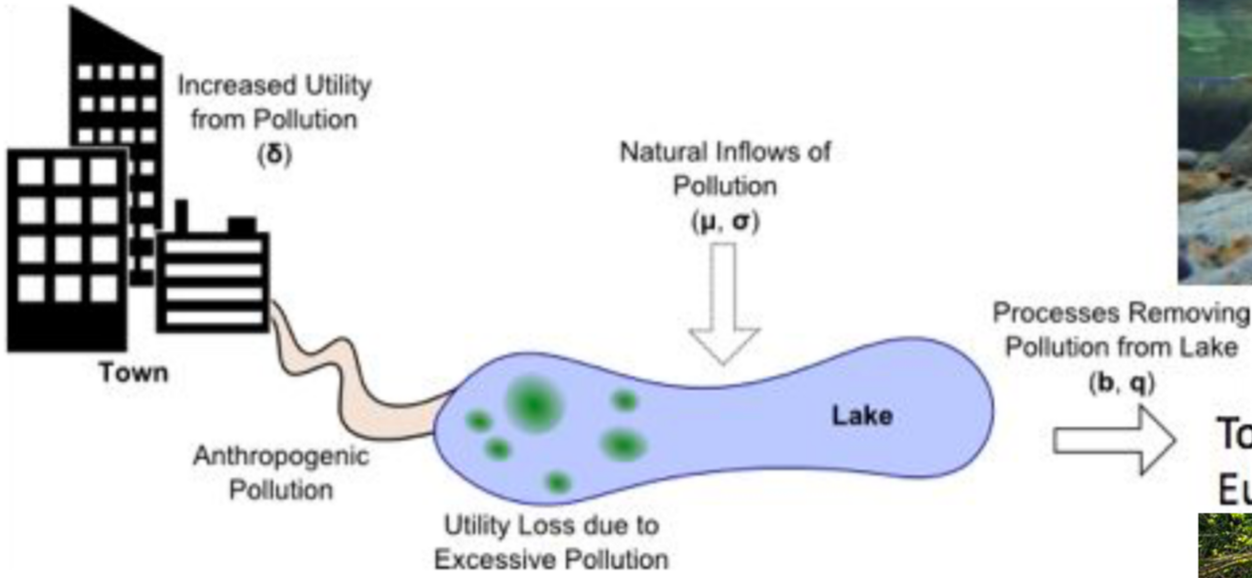

Figures from *Henrion, 1982: The Value of Knowing How Little You Know*

# Ethics & Decision–making



# Ethics & Decision–making

Phosphorous concentration below threshold: Healthy lake




Too much phosphorous: Eutrophication

More phosphorous discharge supports economic growth

*Figure adapted from Hadka et al., 2015*

# Next Time

- *Review for Midterm*