High Dimensional Dynamic Stochastic Copulas Models

Creal, D. D., & Tsay, R. S. (2015)

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HMM & Sequential Monte Carlo

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Introduction

The paper by Drew D. Creal and Ruey S. Tsay proposes a class of dynamic stochastic copula models to analyze the time dependence between many financial variables. These models use a factor structure to make high-dimensional computation possible, while incorporating flexible dynamics and distributions that capture the fat tails of financial data. They use Bayesian estimation via particle Gibbs sampling. Application to a 200-dimensional panel including equities and CDS for 100 US Corporations shows that the grouped stochastic Student's copula model is preferred over other models.

Based on this, the implementation tasks of our project revolve around:

1. Developing a bootstrap filter for the proposed model and testing it on simulated data, focusing on the behavior of the likelihood estimator under varying parameters.
2. Implement the Gibbs sampler, particularly the simulation of $\Lambda_{1:t}$, which utilizes the CSMC algorithm.
3. Applying the Gibbs sampler to real-world data such as S&P 500

Model Implementation

The model studied in Section 4 of the paper is defined as:

$$y_t^i = W_t^i \cdot \beta_y^i + \gamma_y^i \cdot \delta_t^i + \sqrt{\delta_t^i} \cdot \exp\left(\frac{h_t^i}{2}\right) \cdot \epsilon_{y,t}^i$$

$$h_{t+1}^i = \mu_h^i + \phi_h^i \cdot \left(h_t^i - \mu_h^i\right) + \sigma_h^i \cdot \epsilon_{h,t}^i$$

where:

$$\epsilon_{\cdot,t}^i \sim \mathcal{N}(0,1)$$

$$\delta_t^i \sim \text{Inv-Gamma}\left(\frac{\nu_y^i}{2}, \frac{\nu_y^i}{2}\right)$$

$$W_t^i = \left(1, y_{t-1}^i, \ldots, y_{t-k}^i\right)$$

We assume, moreover, that:

$$\text{Corr}\left(\epsilon_{y,t}^i, \epsilon_{h,t}^i\right) = \rho^i$$

The different indices are defined as:

- i: the index of the time series.
- t: the date of observation of the stochastic process.

- h and y: whether the variable is associated with the state h or the observation y.

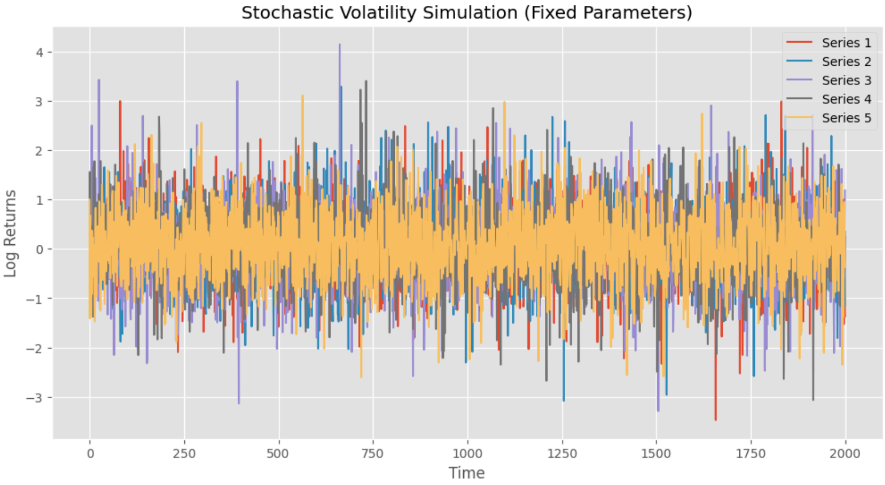


Figure: Stochastic Volatility Simulation

Boostrap Filter Results

- We simulate synthetic data from the uni-variate marginal model and we filter it with the bootstrap algorithm in order to estimate the log volatility.

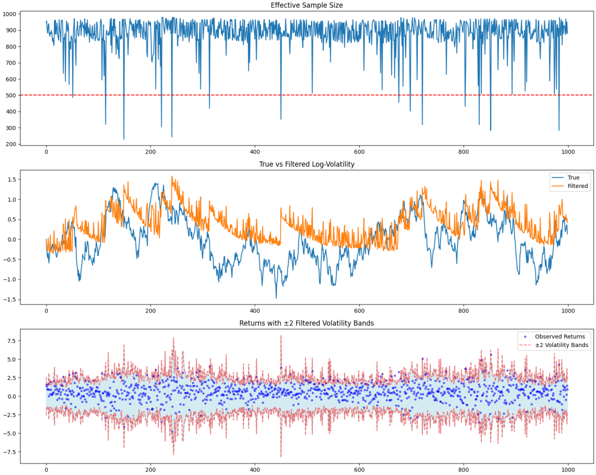


Figure: Bootstrap filter Diagnosis on Simulated data

Tests on S&P 500 data

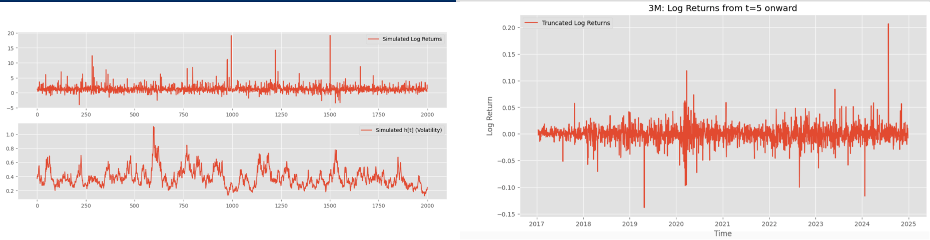


Figure: Simulated Log Returns .

1. The first plot displays two key components of the simulated stochastic volatility (SV) model:Simulated log-returns and Simulated volatility. The simulation illustrates how the SV model captures both return dynamics and changing volatility.

2. The second graph shows the truncated log returns of a selected S&P 500 company, starting at $t = 5$ due to the lag-based covariate construction. It visualizes the dynamics of log returns over time, providing a basis for analyzing and modeling the firm's stochastic behavior.

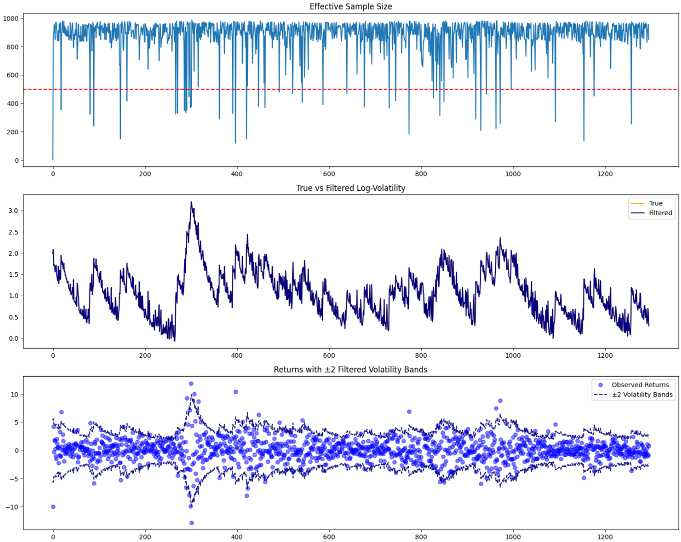


Figure: Bootstrap filter Diagnosis on real data

$\Lambda_{1:t}$ simulation with Particle Gibbs

Copula model

$$u_t \sim p\left(u_t \middle| \Lambda_t, X_t, \theta\right), \quad t = 1, \ldots, T$$
$$\Lambda_{t+1} = \mu + \Phi_\lambda\left(\Lambda_t - \mu\right) + \eta_t, \quad \eta_t \sim \mathrm{N}\left(0, \Sigma\right)$$

Figure: Copula model

Copula model

The copula with the high performance is the grouped Student' t copula

$$
\begin{aligned}
u_{it} &= T\left(x_{it} | \nu_j\right), \quad i = 1, \ldots, n, \; t = 1, \ldots, T \\
x_{it} &= \sqrt{\zeta_{t,j}^{i}} \left[\tilde{\lambda}_{it}' \tilde{z}_{1t} + \left(\beta_i \odot \tilde{X}_{it}\right)' \tilde{z}_{2t} + \sigma_{it} \tilde{\varepsilon}_{it} \right], \\
\tilde{z}_t &\sim \mathrm{N}\left(0, \mathrm{I}_{p+k}\right), \; \tilde{\varepsilon}_{it} \sim \mathrm{N}(0, 1), \\
\zeta_{t,j} &\sim \text{Inv-Gamma}\left(\frac{\nu_j}{2}, \frac{\nu_j}{2}\right), \quad j = 1, \ldots, G.
\end{aligned}
$$

Figure: GroupedStudent copula

Bayesian Estimation

Key Components:

- Prior $p(\theta)$
- Posterior distribution:

$$\mathsf{p}(\theta, z_{1:T}, \Lambda_{1:T}, \zeta_{1:T} | u_{1:T}, X_{1:T}) \\ \propto \\ \mathsf{p}(\mathsf{u}_{1:T} | X_{1:T}, z_{1:T}, \zeta_{1:T}, \Lambda_{1:T}, \theta) p(z_{1:T}, \Lambda_{1:T}, \zeta_{1:T} | \theta) p(\theta)$$

Improving MCMC Performance

Principles:

- Condition on as few parameters as possible.
- Draw parameters/state variables in large blocks to enhance mixing and convergence.

Advantages:

- Better mixing of Markov chains.
- Faster convergence.

Drawing Factor Loadings in Blocks

Particle Gibbs (PG):

At $t = T$, draw a particle $\lambda_{i,T}^{*} = \lambda_{i,T}^{(m)}$ with probability $\hat{w}_T^{(m)}$.
For $t = T-1, \ldots, 1$, run:

- For $m = 1, \ldots, M$, calculate the backwards weights: $w_{t|T}^{(m)} \propto \hat{w}_t^{(m)} p\left(\lambda_{i,t+1}^{*} | \lambda_{i,t}^{(m)}, \theta\right)$
- For $m = 1, \ldots, M$, normalize the weights: $\hat{w}_{t|T}^{(m)} = \frac{w_{t|T}^{(m)}}{\sum_{m=1}^{M} w_{t|T}^{(m)}}$.
- Draw a particle $\lambda_{i,t}^{*} = \lambda_{i,t}^{(m)}$ with probability $\hat{w}_{t|T}^{(m)}$.

Figure: PG Algorithm

Drawing Factor Loadings in Blocks

Particle Gibbs (PG) Sampler:

- PF.
- Draws paths $\lambda_{i,1:T}$ for $i = 1, \ldots, np$ in large blocks.

Steps:

1. Draw from the proposal distribution.
2. Calculate importance weights.
3. Normalize weights and resample particles.

Drawing Factor Loadings in Blocks

BackwardSampling:

At $t = T$, draw a particle $\lambda^*_{i,T} = \lambda^{(m)}_{i,T}$ with probability $\hat{w}^{(m)}_T$.
For $t = T-1, \ldots, 1$, run:

- For $m = 1, \ldots, M$, calculate the backwards weights: $w^{(m)}_{t|T} \propto \hat{w}^{(m)}_t p\left(\lambda^*_{i,t+1} | \lambda^{(m)}_{i,t}, \theta\right)$
- For $m = 1, \ldots, M$, normalize the weights: $\hat{w}^{(m)}_{t|T} = \frac{w^{(m)}_{t|T}}{\sum_{m=1}^{M} w^{(m)}_{t|T}}$.
- Draw a particle $\lambda^*_{i,t} = \lambda^{(m)}_{i,t}$ with probability $\hat{w}^{(m)}_{t|T}$.

Figure: The additional Backward sampling

Simulation Results

Simulation Results:

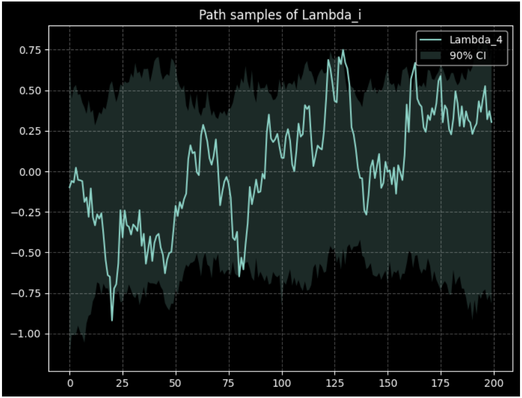
- Implementation of Particle Gibbs Sampler + Simulation
- 

Figure: Drawn path for λ_4 with confidence Interval

Application on Real S&P500 data

Simulation Results:

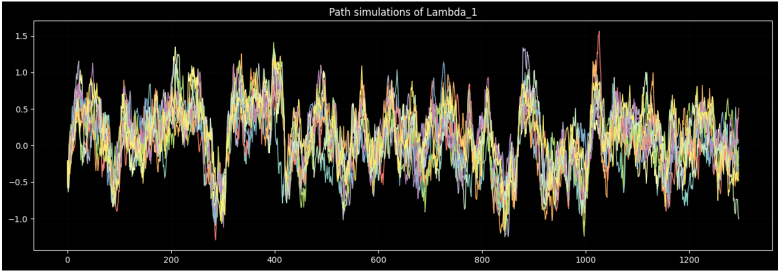
- Implementation of Particle Gibbs Sampler + Simulation
- 

Figure: Multiple paths simulation for λ_1

Bibliography

[1] Creal, D. D., & Tsay, R. S. (2015). *High dimensional dynamic stochastic copula models*. Journal of Econometrics, 189(2), 335–345. http://refhub.elsevier.com/S0169-2070(21)00175-8/sb477

[2] Chopin, N., Singh, S.S., 2013. *On the particle Gibbs sampler*. Working paper, ENSAE. http://arxiv.org/abs/1304.1887

Thank you!