Lecture on Measures of Central Tendency from Frequency Tables

1 Section 1: The Median from the Frequency Distribution Table

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Welcome back, students! In our previous lecture, we delved into the **Arithmetic Mean** derived from a frequency table. Today, we're going to explore two other pivotal measures of central tendency: the **Median** and the **Mode** from a frequency table. Let's embark on this mathematical journey together.

1.1 Understanding the Median

The **Median** is the value that separates a dataset into two equal halves. When dealing with a frequency distribution table, calculating the median involves a specific formula tailored to grouped data.

1.1.1 Formula 1: Median Calculation

$$\text{Median} = A + \left(\frac{\frac{\sum F}{2} - f_1}{f_2 - f_1} \right) \times L$$

Where:

- **A** = Lower limit of the median class
- $\sum F$ = Sum of all frequencies
- f_1 = Cumulative frequency before the median class
- f_2 = Cumulative frequency of the median class
- **L** = Class interval length

Let's break down these components to ensure clarity.

1.2 Key Components Explained

1. **A (Lower Limit of the Median Class):**
   - This is the starting value of the class interval where the median lies.
2. $\sum F$ **(Total Frequency):**
   - Sum of all the frequencies in the table.
3. f_1 **(Cumulative Frequency Before Median Class):**
   - Total frequency accumulated before reaching the median class.
4. f_2 **(Cumulative Frequency of Median Class):**
   - Total frequency up to and including the median class.
5. **L (Class Interval Length):**
   - The difference between the upper and lower limits of the class interval.

1.3 Example 1: Finding the Median

Let's apply our understanding to an example.

1.3.1 Problem:

Find the median from the following frequency table:

Classes (C)	Frequency (f)
60 – 69	4
70 – 79	2
80 – 89	5
90 – 99	6
100 – 109	3
Sum	**20**

1.3.2 Solution:

1. **Calculate $\sum F/2$:**

$$\frac{\sum F}{2} = \frac{20}{2} = 10$$

2. **Determine the Median Class:**
   - We need to find the class where the cumulative frequency just exceeds 10.
3. **Cumulative Frequency Table:**

C	f	Upper C	F.C.A
60 – 69	4	69	4
70 – 79	2	79	6
80 – 89	5	89	11
90 – 99	6	99	17
100 – 109	3	109	20

   - The cumulative frequency reaches **11** in the **80 − 89** class, which exceeds **10**. Thus, **80 − 89** is our median class.
4. **Identify Parameters:**
   - $\mathbf{A} = 80$ (Lower limit of the median class)
   - $\mathbf{f}_1 = 6$ (Cumulative frequency before the median class)
   - $\mathbf{f}_2 = 11$ (Cumulative frequency of the median class)
   - $\mathbf{L} = 10$ (Class interval length: 80 - 70 = 10)
5. **Apply Formula 1:**

$$\text{Median} = 80 + \left(\frac{10-6}{11-6}\right) \times 10 = 80 + \left(\frac{4}{5}\right) \times 10 = 80 + 8 = 88$$

Therefore, the Median is 88.

Note: There seems to be a minor discrepancy in the final value based on the provided data. Ensure all parameters are correctly identified to achieve accurate results.

2 Section 2: The Mode from the Frequency Distribution Table

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Moving forward, let's uncover the **Mode**, another essential measure of central tendency. The **Mode** represents the most frequently occurring value in a dataset.

2.1 Understanding the Mode

When dealing with grouped data in a frequency table, the **Mode** can be calculated using the following formula.

2.1.1 Formula 2: Mode Calculation

$$\text{Mode} = A + \left(\frac{f_2}{f_1 + f_2}\right) \times L$$

Where:

- **A** = Lower limit of the modal class
- $\mathbf{f_1}$ = Frequency of the class preceding the modal class
- $\mathbf{f_2}$ = Frequency of the modal class
- **L** = Class interval length

2.2 Example 2: Finding the Mode

Let's apply this formula to an example.

2.2.1 Problem:

Find the mode from the following frequency table:

C	f	M
10 – 14	3	12
15 – 19	4	17
20 – 24	7	22
25 – 29	2	27
30 – 34	4	32
Sum	**20**	

2.2.2 Solution:

1. **Identify the Modal Class:**
   - The modal class is the one with the highest frequency, which is **20 − 24** with a frequency of **7**.
2. **Determine Parameters:**
   - **A** = 20 (Lower limit of the modal class)
   - $\mathbf{f_1}$ = 4 (Frequency of the class preceding the modal class)
   - $\mathbf{f_2}$ = 7 (Frequency of the modal class)
   - **L** = 5 (Class interval length: 20 - 15 = 5)
3. **Apply Formula 2:**

$$\text{Mode} = 20 + \left(\frac{7}{4+7}\right) \times 5 = 20 + \left(\frac{7}{11}\right) \times 5 \approx 20 + 3.18 = 23.18$$

Therefore, the Mode is approximately 23.2.

In practice, it's common to round the mode to a suitable number of decimal places based on context.

3 Section 3: The Arithmetic Mean from the Frequency Distribution Table

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Now, let's revisit the **Arithmetic Mean**, a measure we touched upon earlier but will explore in greater depth with frequency tables.

3.1 Understanding the Arithmetic Mean

The **Arithmetic Mean** is the average of all data points, calculated by dividing the sum of all values by the number of values.

3.1.1 Formula 3: Arithmetic Mean Calculation

$$\bar{X} = \frac{\sum(m \times f)}{\sum f}$$

Where:

- **m** = Midpoint of the class
- **f** = Frequency of the class

3.2 Example 3: Calculating the Arithmetic Mean

Let's apply this to an example.

3.2.1 Problem:

Calculate the arithmetic mean from the following frequency table:

C	f
2 – 4	4
4 – 6	6
6 – 8	5
8 – 10	8
10 – 12	7
Sum	**30**

3.2.2 Solution:

1. **Determine Midpoints (m) for Each Class:**

$$m = \frac{\text{Lower Limit} + \text{Upper Limit}}{2}$$

- **2 – 4:** $m = \frac{2+4}{2} = 3$
- **4 – 6:** $m = \frac{4+6}{2} = 5$
- **6 – 8:** $m = \frac{6+8}{2} = 7$
- **8 – 10:** $m = \frac{8+10}{2} = 9$
- **10 – 12:** $m = \frac{10+12}{2} = 11$

2. **Create an Extended Frequency Table:**

C	f	m	m × f
2 – 4	4	3	12
4 – 6	6	5	30
6 – 8	5	7	35
8 – 10	8	9	72
10 – 12	7	11	77
Sum	**30**		**226**

3. **Apply Formula 3:**

$$\bar{X} = \frac{226}{30} \approx 7.5333$$

Therefore, the Arithmetic Mean is approximately 7.5.

4 Section 4: Confirming Calculations

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It's crucial to verify our calculations to ensure accuracy. Let's confirm our findings for the **Mode**.

4.1 Verification of Mode Calculation

Recall from **Example 2**:

$$\text{Mode} \approx 23.18 \text{ (rounded to 23.2)}$$

However, based on the visual content provided, the calculation resulted in:

$$\text{Mode} \approx 21.333 \text{ (rounded to 22)}$$

4.2 Steps to Confirm:

1. **Re-examining the Formula and Parameters:**
   - $\mathbf{A} = 20$
   - $\mathbf{f}_1 = 4$
   - $\mathbf{f}_2 = 2$
   - $\mathbf{L} = 4$
2. **Reapplying Formula 2:**

$$\text{Mode} = 20 + \left(\frac{2}{4+2}\right) \times 4 \approx 20 + 1.333 = 21.333$$

Therefore, the correct Mode is approximately 21.3, rounded to 22.

This discrepancy highlights the importance of accurately identifying the modal class and its preceding frequency.

5 Section 5: Creating an Ascending Frequency Table

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To effectively utilize frequency tables for calculating measures like the median and mode, it's essential to organize data in ascending order.

5.1 Steps to Create an Ascending Frequency Table

1. **List the Data Points:**

$$2, 3, 4, 4, 5, 5, 5, 6, 6, 6, 9, 6, 7, 7, 8, 8, 8, 8, 9, 9, 10, 11, 14, 15, 16, 18, 19, 20, 22$$

2. **Determine the Range:**

$$\text{Range} = \text{Highest Value} - \text{Lowest Value} = 22 - 2 = 20$$

3. **Decide the Number of Classes:**

$$\text{Number of Classes} = 5$$

4. **Calculate Class Interval Length (L):**

$$L = \frac{\text{Range}}{\text{Number of Classes}} = \frac{20}{5} = 4$$

5. **Establish Class Limits:**

   - **First Class:** 2 – 5
   - **Subsequent Classes:** Add the class interval length to the previous class's lower limit.

6. **Create the Frequency Table:**

C	f	fi
2 – 5	8	||| |||| | | |
6 – 9	14	||| |||| ||| | | | | | | | | |
10 – 13	2	| |
14 – 17	3	|| |
18 – 21	3	|| |
Sum	**30**	

Here, "fi" represents tally marks for visual clarity.

5.2 Application in Calculations

Having an ascending frequency table simplifies the identification of the median and modal classes, ensuring accurate computations.

6 Conclusion

In today's lecture, we've thoroughly examined how to calculate the **Median**, **Mode**, and **Arithmetic Mean** from frequency distribution tables. By understanding the underlying formulas and meticulously organizing data, you can derive these essential statistical measures with confidence.

Remember:

- Always ensure your frequency table is in ascending order.
- Carefully identify the median and modal classes based on cumulative frequencies.
- Double-check your calculations to maintain accuracy.

Keep practicing with various examples to solidify your understanding. Until next time, keep exploring the fascinating world of statistics!