Tutorial Series: Understanding T-Tests and F-Ratios

Welcome to our tutorial series on **T-Tests** and **F-Ratios**! Each tutorial is designed to deepen your understanding through modified examples and step-by-step solutions. Let's embark on this statistical journey together, using engaging scenarios and relatable analogies.

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2 Tutorial 1: One Sample T-Test – Assessing Average Study Hours

2.1 Problem

Imagine you're a teacher who believes that students in your class study an average of **5 hours** per week. To verify this, you collect data from a sample of **12 students** and find that the average study time is **5.8 hours** with a standard deviation of **1.2 hours**. Determine if the observed difference is statistically significant using a **One Sample T-Test**.

2.2 Solution

To determine if the average study time of your sample significantly differs from the hypothesized population mean, we'll perform a **One Sample T-Test**.

2.2.1 Step 1: State the Hypotheses

- **Null Hypothesis** (H_0): $\mu = 5$ hours (The average study time is 5 hours.)
- **Alternative Hypothesis** (H_1): $\mu \neq 5$ hours (The average study time is not 5 hours.)

2.2.2 Step 2: Identify the Given Values

$$\bar{X} = 5.8 \text{ hours}$$
$$\mu = 5 \text{ hours}$$
$$S = 1.2 \text{ hours}$$
$$n = 12 \text{ students}$$

2.2.3 Step 3: Apply Formula 1 – One Sample T-Test

$$T = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \tag{1}$$

2.2.4 Step 4: Plug in the Values

$$T = \frac{5.8 - 5}{\frac{1.2}{\sqrt{12}}} = \frac{0.8}{\frac{1.2}{3.464}} = \frac{0.8}{0.346} \approx 2.31$$

2.2.5 Step 5: Interpret the Result

A **T-value of 2.31** indicates that the sample mean is **2.31 standard errors** above the hypothesized population mean. To determine significance, compare this T-value against the critical T-value from the T-distribution table at your chosen significance level (e.g., 0.05). If the calculated T-value exceeds the critical value, the difference is statistically significant.

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3 Tutorial 2: One Sample T-Test – Evaluating Average Test Scores

3.1 Problem

A school claims that the average math test score for its students is **75**. To verify this claim, you randomly select **15 students** and find that their average score is **78** with a standard deviation of **5**. Use a **One Sample T-Test** to determine if the school's claim holds true.

3.2 Solution

We'll perform a **One Sample T-Test** to assess whether the observed average score significantly differs from the claimed population mean.

3.2.1 Step 1: State the Hypotheses

- **Null Hypothesis** (H_0): $\mu = 75$ (The average test score is 75.)
- **Alternative Hypothesis** (H_1): $\mu \neq 75$ (The average test score is not 75.)

3.2.2 Step 2: Identify the Given Values

$$\begin{aligned}\bar{X} &= 78\\ \mu &= 75\\ S &= 5\\ n &= 15\end{aligned}$$

3.2.3 Step 3: Apply Formula 1 – One Sample T-Test

$$T = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \tag{2}$$

3.2.4 Step 4: Plug in the Values

$$T = \frac{78 - 75}{\frac{5}{\sqrt{15}}} = \frac{3}{\frac{5}{3.873}} = \frac{3}{1.291} \approx 2.33$$

3.2.5 Step 5: Interpret the Result

A **T-value of 2.33** suggests that the sample mean is **2.33 standard errors** above the population mean. Comparing this to the critical T-value at a 0.05 significance level, if 2.33 exceeds the critical value, we reject the null hypothesis, indicating a significant difference.

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4 Tutorial 3: Independent Sample T-Test – Comparing Two Teaching Methods

4.1 Problem

A researcher wants to compare the effectiveness of two different teaching methods on student performance. **Group A** (using Method 1) has an average score of **82** with a standard deviation of **6** based on **20 students**. **Group B** (using Method 2) has an average score of **78** with a standard deviation of **5** based on **22 students**. Perform an **Independent Sample T-Test** to determine if there's a significant difference between the two methods.

4.2 Solution

We'll conduct an **Independent Sample T-Test** to compare the means of two independent groups.

4.2.1 Step 1: State the Hypotheses

- **Null Hypothesis** (H_0): $\mu_1 = \mu_2$ (No difference in means.)
- **Alternative Hypothesis** (H_1): $\mu_1 \neq \mu_2$ (There is a difference in means.)

4.2.2 Step 2: Identify the Given Values

$$\bar{X}_1 = 82$$
$$S_1 = 6$$
$$n_1 = 20$$
$$\bar{X}_2 = 78$$
$$S_2 = 5$$
$$n_2 = 22$$

4.2.3 Step 3: Apply Formula 2 – Independent Sample T-Test

$$T = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\left(\frac{n_1 S_1^2 + n_2 S_2^2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \tag{3}$$

4.2.4 Step 4: Calculate the Pooled Variance

$$\text{Pooled Variance} = \frac{20 \times 6^2 + 22 \times 5^2}{20 + 22 - 2} = \frac{20 \times 36 + 22 \times 25}{40} = \frac{720 + 550}{40} = \frac{1270}{40} = 31.75$$

4.2.5 Step 5: Compute the Standard Error

$$\text{Standard Error} = \sqrt{31.75 \times \left(\frac{1}{20} + \frac{1}{22}\right)} = \sqrt{31.75 \times 0.095} \approx \sqrt{3.016} \approx 1.737$$

4.2.6 Step 6: Calculate the T-Value

$$T = \frac{82 - 78}{1.737} = \frac{4}{1.737} \approx 2.30$$

4.2.7 Step 7: Interpret the Result

A **T-value of 2.30** indicates that the difference between the two group means is **2.30 standard errors** apart. Comparing this to the critical T-value at a 0.05 significance level, if 2.30 exceeds the critical value, we reject the null hypothesis, suggesting a significant difference between the teaching methods.

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5 Tutorial 4: Independent Sample T-Test – Comparing Two Marketing Strategies

5.1 Problem

A company employs two different marketing strategies to promote a new product. **Strategy X** results in an average of **150** sales with a standard deviation of **20** from **25 campaigns**. **Strategy Y** leads to an average of **140** sales with a standard deviation of **15** from **30 campaigns**. Use an **Independent Sample T-Test** to determine if there's a significant difference in sales between the two strategies.

5.2 Solution

We'll perform an **Independent Sample T-Test** to assess whether the difference in average sales between the two strategies is statistically significant.

5.2.1 Step 1: State the Hypotheses

- **Null Hypothesis** (H_0): $\mu_X = \mu_Y$ (No difference in average sales.)
- **Alternative Hypothesis** (H_1): $\mu_X \neq \mu_Y$ (There is a difference in average sales.)

5.2.2 Step 2: Identify the Given Values

$$\begin{aligned} \bar{X}_X &= 150 \\ S_X &= 20 \\ n_X &= 25 \\ \bar{X}_Y &= 140 \\ S_Y &= 15 \\ n_Y &= 30 \end{aligned}$$

5.2.3 Step 3: Apply Formula 2 – Independent Sample T-Test

$$T = \frac{\bar{X}_X - \bar{X}_Y}{\sqrt{\left(\frac{n_X S_X^2 + n_Y S_Y^2}{n_X + n_Y - 2}\right)\left(\frac{1}{n_X} + \frac{1}{n_Y}\right)}} \tag{4}$$

5.2.4 Step 4: Calculate the Pooled Variance

$$\text{Pooled Variance} = \frac{25 \times 20^2 + 30 \times 15^2}{25 + 30 - 2} = \frac{25 \times 400 + 30 \times 225}{53} = \frac{10,000 + 6,750}{53} \approx \frac{16,750}{53} \approx 316.04$$

5.2.5 Step 5: Compute the Standard Error

$$\text{Standard Error} = \sqrt{316.04 \times \left(\frac{1}{25} + \frac{1}{30}\right)} = \sqrt{316.04 \times 0.0767} \approx \sqrt{24.22} \approx 4.92$$

5.2.6 Step 6: Calculate the T-Value

$$T = \frac{150 - 140}{4.92} = \frac{10}{4.92} \approx 2.03$$

5.2.7 Step 7: Interpret the Result

A **T-value of 2.03** suggests that the average sales from **Strategy X** are **2.03 standard errors** higher than those from **Strategy Y**. Comparing this to the critical T-value at a 0.05 significance level, if 2.03 exceeds the critical value, we reject the null hypothesis, indicating a significant difference between the marketing strategies.

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6 Tutorial 5: Calculating F-Ratio – Comparing Variances in Two Groups

6.1 Problem

A psychologist wants to compare the variability in stress levels between **employed** and **unemployed** individuals. The **employed** group has a standard deviation of **8** with a sample size of **40**, and the **unemployed** group has a standard deviation of **6** with a sample size of **35**. Calculate the **F-Ratio** to determine if there's a significant difference in variances.

6.2 Solution

We'll calculate the **F-Ratio** to compare the variances of two independent groups.

6.2.1 Step 1: Identify the Given Values

$$\begin{aligned} S_{\text{Employed}} &= 8 \\ n_{\text{Employed}} &= 40 \\ S_{\text{Unemployed}} &= 6 \\ n_{\text{Unemployed}} &= 35 \end{aligned}$$

6.2.2 Step 2: Calculate the Variances

$$\begin{aligned} S^2_{\text{Employed}} &= 8^2 = 64 \\ S^2_{\text{Unemployed}} &= 6^2 = 36 \end{aligned}$$

6.2.3 Step 3: Apply Formula 3 – F-Ratio

$$F = \frac{\text{Larger Variance}}{\text{Smaller Variance}} = \frac{64}{36} \approx 1.78 \tag{5}$$

6.2.4 Step 4: Interpret the Result

An **F-Ratio of 1.78** indicates that the variance of the employed group is **1.78 times** that of the unemployed group. To determine significance, compare this F-value against the critical F-value from the F-distribution table at your chosen significance level (e.g., 0.05). If the calculated F-value exceeds the critical value, the difference in variances is statistically significant.

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7 Tutorial 6: Calculating F-Ratio – Comparing Variability in Two Teaching Methods

7.1 Problem

An educator assesses the consistency of student performances under two different teaching methods. **Method A** has a standard deviation of **4** with **25 students**, and **Method B** has a standard deviation of **5** with **30 students**. Compute the **F-Ratio** to evaluate if there's a significant difference in variability between the two methods.

7.2 Solution

We'll compute the **F-Ratio** to compare the variances of the two teaching methods.

7.2.1 Step 1: Identify the Given Values

$$S_A = 4$$
$$n_A = 25$$
$$S_B = 5$$
$$n_B = 30$$

7.2.2 Step 2: Calculate the Variances

$$S_A^2 = 4^2 = 16$$
$$S_B^2 = 5^2 = 25$$

7.2.3 Step 3: Apply Formula 3 – F-Ratio

$$F = \frac{\text{Larger Variance}}{\text{Smaller Variance}} = \frac{25}{16} = 1.5625 \tag{6}$$

7.2.4 Step 4: Interpret the Result

An **F-Ratio of 1.5625** suggests that the variance in **Method B** is **1.56 times** that of **Method A**. Comparing this to the critical F-value at a 0.05 significance level, if 1.5625 exceeds the critical value, we reject the null hypothesis, indicating a significant difference in variances between the two teaching methods.

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8 Tutorial 7: Assignment – Comparing Online and Offline Learning Platforms

8.1 Problem

In a study to compare **Online Learning** and **Offline Learning** platforms, you need to find the **T-Test** and **F-Ratio**. The data is as follows:

	$\bar{X}$	n	S	S^2
Online Learning	85	20	5	25
Offline Learning	80	18	7	49

8.2 Solution

We'll calculate both the **Independent Sample T-Test** and the **F-Ratio** to compare the means and variances between Online and Offline Learning platforms.

8.2.1 Part 1: Independent Sample T-Test

Step 1: State the Hypotheses

- **Null Hypothesis** (H_0): $\mu_{\text{Online}} = \mu_{\text{Offline}}$ (No difference in average scores.)
- **Alternative Hypothesis** (H_1): $\mu_{\text{Online}} \neq \mu_{\text{Offline}}$ (There is a difference in average scores.)

Step 2: Identify the Given Values

$$\begin{aligned}
\bar{X}_{\text{Online}} &= 85 \\
S_{\text{Online}} &= 5 \\
n_{\text{Online}} &= 20 \\
\bar{X}_{\text{Offline}} &= 80 \\
S_{\text{Offline}} &= 7 \\
n_{\text{Offline}} &= 18
\end{aligned}$$

Step 3: Apply Formula 2 – Independent Sample T-Test

$$T = \frac{\bar{X}_{\text{Online}} - \bar{X}_{\text{Offline}}}{\sqrt{\left(\frac{n_{\text{Online}} S^2_{\text{Online}} + n_{\text{Offline}} S^2_{\text{Offline}}}{n_{\text{Online}} + n_{\text{Offline}} - 2}\right)\left(\frac{1}{n_{\text{Online}}} + \frac{1}{n_{\text{Offline}}}\right)}} \tag{7}$$

Step 4: Calculate the Pooled Variance

$$\text{Pooled Variance} = \frac{20 \times 25 + 18 \times 49}{20 + 18 - 2} = \frac{500 + 882}{36} = \frac{1382}{36} \approx 38.389$$

Step 5: Compute the Standard Error

$$\text{Standard Error} = \sqrt{38.389 \times \left(\frac{1}{20} + \frac{1}{18}\right)} = \sqrt{38.389 \times 0.1056} \approx \sqrt{4.06} \approx 2.016$$

Step 6: Calculate the T-Value

$$T = \frac{85 - 80}{2.016} = \frac{5}{2.016} \approx 2.48$$

Step 7: Interpret the T-Test Result A **T-value of 2.48** indicates that the average score of **Online Learning** is **2.48 standard errors** higher than that of **Offline Learning**. Comparing this to the critical T-value at a 0.05 significance level, if 2.48 exceeds the critical value, we reject the null hypothesis, suggesting a significant difference in average scores.

8.2.2 Part 2: Calculating F-Ratio

Step 1: Apply Formula 3 – F-Ratio

$$F = \frac{\text{Larger Variance}}{\text{Smaller Variance}} = \frac{49}{25} = 1.96 \tag{8}$$

Step 2: Interpret the F-Ratio Result An **F-Ratio of 1.96** indicates that the variance in **Offline Learning** is **1.96 times** that of **Online Learning**. Comparing this to the critical F-value at a 0.05 significance level, if 1.96 exceeds the critical value, we reject the null hypothesis, indicating a significant difference in variances between the two learning platforms.

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9 Tutorial 8: Assignment – Comparing Morning and Evening Study Sessions

9.1 Problem

In a research study comparing **Morning Study Sessions** and **Evening Study Sessions**, determine the **T-Test** and **F-Ratio** using the following data:

	$\bar{X}$	n	S	S^2
Morning	3.5	12	0.8	0.64
Evening	3.0	10	1.2	1.44

9.2 Solution

We'll calculate both the **Independent Sample T-Test** and the **F-Ratio** to compare the means and variances between Morning and Evening Study Sessions.

9.2.1 Part 1: Independent Sample T-Test

Step 1: State the Hypotheses

- **Null Hypothesis** (H_0): $\mu_{\text{Morning}} = \mu_{\text{Evening}}$ (No difference in average study effectiveness.)
- **Alternative Hypothesis** (H_1): $\mu_{\text{Morning}} \neq \mu_{\text{Evening}}$ (There is a difference in average study effectiveness.)

Step 2: Identify the Given Values

$$\begin{aligned}
\bar{X}_{\text{Morning}} &= 3.5 \\
S_{\text{Morning}} &= 0.8 \\
n_{\text{Morning}} &= 12 \\
\bar{X}_{\text{Evening}} &= 3.0 \\
S_{\text{Evening}} &= 1.2 \\
n_{\text{Evening}} &= 10
\end{aligned}$$

Step 3: Apply Formula 2 – Independent Sample T-Test

$$T = \frac{\bar{X}_{\text{Morning}} - \bar{X}_{\text{Evening}}}{\sqrt{\left(\frac{n_{\text{Morning}} S^2_{\text{Morning}} + n_{\text{Evening}} S^2_{\text{Evening}}}{n_{\text{Morning}} + n_{\text{Evening}} - 2}\right)\left(\frac{1}{n_{\text{Morning}}} + \frac{1}{n_{\text{Evening}}}\right)}} \tag{9}$$

Step 4: Calculate the Pooled Variance

$$\text{Pooled Variance} = \frac{12 \times 0.64 + 10 \times 1.44}{12 + 10 - 2} = \frac{7.68 + 14.4}{20} = \frac{22.08}{20} = 1.104$$

Step 5: Compute the Standard Error

$$\text{Standard Error} = \sqrt{1.104 \times \left(\frac{1}{12} + \frac{1}{10}\right)} = \sqrt{1.104 \times 0.1833} \approx \sqrt{0.202} \approx 0.449$$

Step 6: Calculate the T-Value

$$T = \frac{3.5 - 3.0}{0.449} = \frac{0.5}{0.449} \approx 1.12$$

Step 7: Interpret the T-Test Result A **T-value of 1.12** suggests that the average study effectiveness in **Morning Sessions** is **1.12 standard errors** higher than in **Evening Sessions**. Comparing this to the critical T-value at a 0.05 significance level, if 1.12 is less than the critical value, we fail to reject the null hypothesis, indicating no significant difference in average study effectiveness.

9.2.2 Part 2: Calculating F-Ratio

Step 1: Apply Formula 3 – F-Ratio

$$F = \frac{\text{Larger Variance}}{\text{Smaller Variance}} = \frac{1.44}{0.64} = 2.25 \tag{10}$$

Step 2: Interpret the F-Ratio Result An **F-Ratio of 2.25** indicates that the variance in **Evening Study Sessions** is **2.25 times** that of **Morning Study Sessions**. Comparing this to the critical F-value at a 0.05 significance level, if 2.25 exceeds the critical value, we reject the null hypothesis, suggesting a significant difference in variances between the two study session times.

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10 Tutorial 9: Assignment – Comparing Two Diet Plans

10.1 Problem

A nutritionist is comparing the effectiveness of **Diet Plan A** and **Diet Plan B** on weight loss. The data collected is as follows:

	$\bar{X}$	n	S	S^2
Diet Plan A	10	15	2	4
Diet Plan B	8	12	3	9

Determine the **T-Test** and **F-Ratio** to assess the effectiveness and variability of the two diet plans.