Tutorials on Measures of Central Tendency from Frequency Tables

1 Introduction

Welcome to our series of tutorials on calculating the **Median**, **Mode**, and **Arithmetic Mean** from frequency distribution tables. Each tutorial is designed to reinforce your understanding through modified examples and detailed, step-by-step solutions. Let's embark on this mathematical adventure together!

2 Tutorial 1: Finding the Median from a Frequency Distribution Table

Bismillah Ar-Rahman Ar-Raheem.

Imagine you're organizing a community bake sale and want to find the median number of pastries sold per day. Let's see how we can calculate this using a frequency distribution table.

2.1 Problem

Find the median from the following frequency table:

Classes (C)	Frequency (f)
30 – 39	5
40 – 49	3
50 – 59	8
60 – 69	4
70 – 79	2
Sum	**22**

2.2 Solution

1. **Calculate $\frac{\sum F}{2}$:**

$$\frac{\sum F}{2} = \frac{22}{2} = 11$$

2. **Determine the Median Class:**

   We need to identify the class where the cumulative frequency just exceeds 11.

3. **Create the Cumulative Frequency Table:**

C	f	Upper C	F.C.A
30 – 39	5	39	5
40 – 49	3	49	8
50 – 59	8	59	16
60 – 69	4	69	20
70 – 79	2	79	22

   - The cumulative frequency reaches **16** in the **50 − 59** class, which exceeds **11**. Thus, **50 − 59** is our median class.

4. **Identify Parameters:**

   - $\mathbf{A} = 50$ (Lower limit of the median class)
   - $\mathbf{f}_1 = 8$ (Cumulative frequency before the median class)
   - $\mathbf{f}_2 = 16$ (Cumulative frequency of the median class)
   - $\mathbf{L} = 10$ (Class interval length: 50 - 40 = 10)

5. **Apply the Median Formula (Formula 1):**

$$\text{Median} = A + \left(\frac{\frac{\sum F}{2} - f_1}{f_2 - f_1} \right) \times L$$

   Plugging in the values:

$$\text{Median} = 50 + \left(\frac{11 - 8}{16 - 8} \right) \times 10 = 50 + \left(\frac{3}{8} \right) \times 10 = 50 + 3.75 = 53.75$$

   Therefore, the Median is 53.75.

Note: Ensure all parameters are accurately identified to achieve precise results.

3 Tutorial 2: Finding the Median with a Different Frequency Table

Bismillah Ar-Rahman Ar-Raheem.

Let's tackle another scenario. Suppose you're analyzing the test scores of students to find the median score.

3.1 Problem

Find the median from the following frequency table:

Classes (C)	Frequency (f)
0 – 9	2
10 – 19	5
20 – 29	7
30 – 39	4
40 – 49	2
Sum	**20**

3.2 Solution

1. **Calculate $\frac{\sum F}{2}$:**

$$\frac{\sum F}{2} = \frac{20}{2} = 10$$

2. **Determine the Median Class:**

   Identify the class where the cumulative frequency just exceeds 10.

3. **Create the Cumulative Frequency Table:**

C	f	Upper C	F.C.A
0 – 9	2	9	2
10 – 19	5	19	7
20 – 29	7	29	14
30 – 39	4	39	18
40 – 49	2	49	20

   - The cumulative frequency reaches **14** in the **20 − 29** class, which exceeds **10**. Thus, **20 − 29** is our median class.

4. **Identify Parameters:**

   - $\mathbf{A} = 20$ (Lower limit of the median class)
   - $\mathbf{f}_1 = 7$ (Cumulative frequency before the median class)
   - $\mathbf{f}_2 = 14$ (Cumulative frequency of the median class)
   - $\mathbf{L} = 10$ (Class interval length: 20 - 10 = 10)

5. **Apply the Median Formula (Formula 1):**

$$\text{Median} = 20 + \left(\frac{10-7}{14-7}\right) \times 10 = 20 + \left(\frac{3}{7}\right) \times 10 \approx 20 + 4.2857 = 24.2857$$

Therefore, the Median is approximately 24.29.

Remember to round off the median as per the context required.

4 Tutorial 3: Finding the Mode from a Frequency Distribution Table

Bismillah Ar-Rahman Ar-Raheem.

Now, let's switch gears to finding the **Mode**, the most frequently occurring value in our dataset. Imagine you're analyzing the number of books sold by different bookstores.

4.1 Problem

Find the mode from the following frequency table:

C	**f**	**M**
5 – 9	6	7
10 – 14	9	12
15 – 19	12	17
20 – 24	8	22
25 – 29	4	27
Sum	**39**	

4.2 Solution

1. **Identify the Modal Class:**

   The modal class is the one with the highest frequency. Here, **15 – 19** has the highest frequency of **12**.

2. **Determine Parameters:**

   - **A** = 15 (Lower limit of the modal class)
   - $\mathbf{f}_1$ = 9 (Frequency of the class preceding the modal class)
   - $\mathbf{f}_2$ = 12 (Frequency of the modal class)
   - **L** = 5 (Class interval length: 15 - 10 = 5)

3. **Apply the Mode Formula (Formula 2):**

$$\text{Mode} = A + \left(\frac{f_2}{f_1 + f_2}\right) \times L$$

   Plugging in the values:

$$\text{Mode} = 15 + \left(\frac{12}{9 + 12}\right) \times 5 = 15 + \left(\frac{12}{21}\right) \times 5 \approx 15 + 2.8571 = 17.8571$$

   Therefore, the Mode is approximately 17.86.

In practice, it's common to round the mode to a suitable number of decimal places based on context.

5 Tutorial 4: Finding the Mode with a Different Frequency Table

Bismillah Ar-Rahman Ar-Raheem.

Let's delve into another example of finding the mode. Suppose you're analyzing the ages of participants in a marathon.

5.1 Problem

Find the mode from the following frequency table:

C	f	M
18 – 22	4	20
23 – 27	7	25
28 – 32	10	30
33 – 37	6	35
38 – 42	3	40
Sum	**30**	

5.2 Solution

1. **Identify the Modal Class:**

   The modal class is **28 − 32** with the highest frequency of **10**.

2. **Determine Parameters:**

   - $\mathbf{A} = 28$ (Lower limit of the modal class)
   - $\mathbf{f}_1 = 7$ (Frequency of the class preceding the modal class)
   - $\mathbf{f}_2 = 10$ (Frequency of the modal class)
   - $\mathbf{L} = 5$ (Class interval length: 28 - 23 = 5)

3. **Apply the Mode Formula (Formula 2):**

$$\text{Mode} = 28 + \left(\frac{10}{7+10}\right) \times 5 = 28 + \left(\frac{10}{17}\right) \times 5 \approx 28 + 2.9412 = 30.9412$$

Therefore, the Mode is approximately 30.94.

Ensure to round the mode appropriately based on the context.

6 Tutorial 5: Calculating the Arithmetic Mean from a Frequency Distribution Table

Bismillah Ar-Rahman Ar-Raheem.

Let's explore how to calculate the **Arithmetic Mean**, the average value of our dataset. Suppose you're analyzing the daily sales of a bookstore.

6.1 Problem

Calculate the arithmetic mean from the following frequency table:

C	f
100 – 109	5
110 – 119	8
120 – 129	10
130 – 139	7
140 – 149	3
Sum	**33**

6.2 Solution

1. **Determine Midpoints (m) for Each Class:**

$$m = \frac{\text{Lower Limit} + \text{Upper Limit}}{2}$$

- **100 – 109:** $m = \frac{100+109}{2} = 104.5$
- **110 – 119:** $m = \frac{110+119}{2} = 114.5$
- **120 – 129:** $m = \frac{120+129}{2} = 124.5$
- **130 – 139:** $m = \frac{130+139}{2} = 134.5$
- **140 – 149:** $m = \frac{140+149}{2} = 144.5$

2. **Create an Extended Frequency Table:**

C	f	m	m × f
100 – 109	5	104.5	522.5
110 – 119	8	114.5	916
120 – 129	10	124.5	1245
130 – 139	7	134.5	941.5
140 – 149	3	144.5	433.5
Sum	**33**		**4068.5**

3. **Apply the Arithmetic Mean Formula (Formula 3):**

$$\bar{X} = \frac{\sum(m \times f)}{\sum f}$$

Plugging in the values:

$$\bar{X} = \frac{4068.5}{33} \approx 123.318$$

Therefore, the Arithmetic Mean is approximately 123.32.

Ensure precision in your calculations for an accurate mean.

7 Tutorial 6: Calculating the Arithmetic Mean with a Different Frequency Distribution Table

Bismillah Ar-Rahman Ar-Raheem.

Let's dive into another example of calculating the arithmetic mean. Suppose you're analyzing the number of hours students spend studying each week.

7.1 Problem

Calculate the arithmetic mean from the following frequency table:

C	f
5 – 9	4
10 – 14	6
15 – 19	9
20 – 24	5
25 – 29	2
Sum	**26**

7.2 Solution

1. **Determine Midpoints (m) for Each Class:**

$$m = \frac{\text{Lower Limit} + \text{Upper Limit}}{2}$$

- **5 – 9:** $m = \frac{5+9}{2} = 7$
- **10 – 14:** $m = \frac{10+14}{2} = 12$
- **15 – 19:** $m = \frac{15+19}{2} = 17$
- **20 – 24:** $m = \frac{20+24}{2} = 22$
- **25 – 29:** $m = \frac{25+29}{2} = 27$

2. **Create an Extended Frequency Table:**

C	f	m	m × f
5 – 9	4	7	28
10 – 14	6	12	72
15 – 19	9	17	153
20 – 24	5	22	110
25 – 29	2	27	54
Sum	**26**		**417**

3. **Apply the Arithmetic Mean Formula (Formula 3):**

$$\bar{X} = \frac{\sum(m \times f)}{\sum f} = \frac{417}{26} \approx 16.0385$$

Therefore, the Arithmetic Mean is approximately 16.04 hours per week.

Always ensure your frequency table is accurate to compute the mean correctly.

8 Tutorial 7: Finding the Median in a Real-World Scenario

Bismillah Ar-Rahman Ar-Raheem.

Let's apply our knowledge to a real-world situation. Suppose you're analyzing the ages of participants in a local sports club.

8.1 Problem

Find the median from the following frequency table:

Classes (C)	**Frequency (f)**
15 – 19	3
20 – 24	7
25 – 29	12
30 – 34	5
35 – 39	3
Sum	**30**

8.2 Solution

1. **Calculate $\frac{\sum F}{2}$:**

$$\frac{\sum F}{2} = \frac{30}{2} = 15$$

2. **Determine the Median Class:**

   Identify the class where the cumulative frequency just exceeds 15.

3. **Create the Cumulative Frequency Table:**

C	**f**	**Upper C**	**F.C.A**
15 – 19	3	19	3
20 – 24	7	24	10
25 – 29	12	29	22
30 – 34	5	34	27
35 – 39	3	39	30

   - The cumulative frequency reaches **22** in the **25 − 29** class, which exceeds **15**. Thus, **25 − 29** is our median class.

4. **Identify Parameters:**

   - $\mathbf{A} = 25$ (Lower limit of the median class)
   - $\mathbf{f}_1 = 10$ (Cumulative frequency before the median class)
   - $\mathbf{f}_2 = 22$ (Cumulative frequency of the median class)
   - $\mathbf{L} = 5$ (Class interval length: 25 - 20 = 5)

5. **Apply the Median Formula (Formula 1):**

$$\text{Median} = 25 + \left(\frac{15-10}{22-10}\right) \times 5 = 25 + \left(\frac{5}{12}\right) \times 5 \approx 25 + 2.0833 = 27.0833$$

Therefore, the Median is approximately 27.08.

Ensure consistency in class interval lengths for accurate calculations.

9 Tutorial 8: Finding the Mode in a Real-World Scenario

Bismillah Ar-Rahman Ar-Raheem.

Let's apply our mode calculation skills to another real-world example. Suppose you're analyzing the number of visitors to a museum each month.

9.1 Problem

Find the mode from the following frequency table:

C	f	M
100 – 149	5	125
150 – 199	8	175
200 – 249	12	225
250 – 299	6	275
300 – 349	4	325
Sum	**35**	

9.2 Solution

1. **Identify the Modal Class:**

   The modal class is **200 – 249** with the highest frequency of **12**.

2. **Determine Parameters:**

   - $\mathbf{A} = 200$ (Lower limit of the modal class)
   - $\mathbf{f}_1 = 8$ (Frequency of the class preceding the modal class)
   - $\mathbf{f}_2 = 12$ (Frequency of the modal class)
   - $\mathbf{L} = 50$ (Class interval length: 200 - 150 = 50)

3. **Apply the Mode Formula (Formula 2):**

$$\text{Mode} = 200 + \left(\frac{12}{8+12}\right) \times 50 = 200 + \left(\frac{12}{20}\right) \times 50 = 200 + 30 = 230$$

Therefore, the Mode is 230 visitors per month.

The mode is an exact value in this context, so no rounding is necessary.

10 Tutorial 9: Calculating the Arithmetic Mean in a Real-World Scenario

Bismillah Ar-Rahman Ar-Raheem.

Let's conclude with another example of calculating the arithmetic mean. Suppose you're analyzing the weekly hours spent by employees in a company.

10.1 Problem

Calculate the arithmetic mean from the following frequency table:

C	f
30 – 34	3
35 – 39	5
40 – 44	10
45 – 49	7
50 – 54	2
Sum	**27**

10.2 Solution

1. **Determine Midpoints (m) for Each Class:**

$$m = \frac{\text{Lower Limit} + \text{Upper Limit}}{2}$$

- **30 – 34:** $m = \frac{30+34}{2} = 32$
- **35 – 39:** $m = \frac{35+39}{2} = 37$
- **40 – 44:** $m = \frac{40+44}{2} = 42$
- **45 – 49:** $m = \frac{45+49}{2} = 47$
- **50 – 54:** $m = \frac{50+54}{2} = 52$

2. **Create an Extended Frequency Table:**

C	f	m	m × f
30 – 34	3	32	96
35 – 39	5	37	185
40 – 44	10	42	420
45 – 49	7	47	329
50 – 54	2	52	104
Sum	**27**		**1134**

3. **Apply the Arithmetic Mean Formula (Formula 3):**

$$\bar{X} = \frac{\sum(m \times f)}{\sum f} = \frac{1134}{27} \approx 42$$

Therefore, the Arithmetic Mean is 42 hours per week.

Ensure accurate calculations to determine the correct mean.