Measures of Central Tendency

Your Instructor's Name

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Introduction

Welcome to today's lecture on **Measures of Central Tendency**! Imagine you're the captain of a ship navigating through the vast sea of data. Measures of central tendency are your compass, helping you find the central point around which your data clusters. We'll explore three essential measures: the **Arithmetic Mean**, the **Median**, and the **Mode**. Let's set sail!

1 Arithmetic Mean

The **Arithmetic Mean**, often simply called the **mean**, is the most commonly used measure of central tendency. Think of it as the average value of your data set.

1.1 Formula 1: Arithmetic Mean

$$\bar{X} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

Where:

- $\bar{X}$ is the arithmetic mean.
- $x_1, x_2, x_3, \ldots, x_n$ are the data points.
- n is the number of data points.

1.2 Example 1: Calculating the Mean

Find the mean of the following data:

$$8,\ 3,\ 5,\ 12,\ 10$$

Solution:

$$\bar{X} = \frac{8 + 3 + 5 + 12 + 10}{5} = \frac{38}{5} = 7.6$$

$$\boxed{\text{Mean} = 7.6}$$

1.3 Example 2: Mean of Student Scores

A student has the following degrees in six exams:

$$84,\ 91,\ 72,\ 68,\ 87,\ 78$$

Find the mean score.
Solution:

$$\bar{X} = \frac{84 + 91 + 72 + 68 + 87 + 78}{6} = \frac{480}{6} = 80$$

$$\boxed{\text{Mean} = 80}$$

Just like averaging the effort of a team to find the overall performance, the arithmetic mean gives us a single value representing the central point of the data.

2 Median

While the mean provides an average, the **Median** offers the middle value of a data set when it's ordered. It's particularly useful when your data contains outliers that might skew the mean.

2.1 Understanding the Median

- **Odd Number of Data Points:** The median is the middle number.
- **Even Number of Data Points:** The median is the average of the two middle numbers.

2.2 Example 3: Finding the Median with an Odd Number of Data Points

Find the median of the following data:

$$10,\ 9,\ 12,\ 20,\ 4,\ 15,\ 7,\ 7,\ 6,\ 20,\ 20$$

Solution:

1. **Arrange in Ascending Order:**

$$4,\ 6,\ 7,\ 7,\ 7,\ 9,\ 10,\ 12,\ 15,\ 20,\ 20$$

2. **Determine the Position:**

   Number of data points (n) = 11 (odd)

$$\text{Median Position} = \frac{n+1}{2} = \frac{11+1}{2} = 6$$

3. **Identify the Median:**

   The 6th number in the ordered list is **9**.

$$\boxed{\text{Median} = 9}$$

2.3 Example 4: Finding the Median with an Even Number of Data Points

Find the median of the following data:

$$8,\ 3,\ 4,\ 12,\ 10,\ 7,\ 12,\ 1$$

Solution:

1. **Arrange in Descending Order:**

$$12,\ 10,\ 8,\ 7,\ 4,\ 3,\ 1$$

2. **Determine the Positions:**

   Number of data points (n) = 8 (even)

$$\text{Median Positions} = \frac{n}{2} = 4 \quad \text{and} \quad 5$$

3. **Identify the Median:**

   The 4th and 5th numbers are **7** and **4**.

$$\text{Median} = \frac{7+4}{2} = 5.5$$

$$\boxed{\text{Median} = 5.5}$$

Think of the median as the "middle seat" in a theater—no matter how the audience arrives, the middle seat remains consistently central.

3 Mode

The **Mode** is the data point that appears most frequently in a data set. It's the trend-setter, the most popular choice among your data points.

3.1 Understanding the Mode

- **Single Mode:** One data point repeats most frequently.
- **Multiple Modes:** Two or more data points share the highest frequency.
- **No Mode:** All data points are unique.

3.2 Example 5: Single Mode

Find the mode of the following data:

$$7,\ 6,\ 9,\ 14,\ 6,\ 5,\ 5$$

Solution:

- **Frequency Count:**
  - 5 appears **2** times.
  - 6 appears **2** times.
  - 7, 9, 14 appear **once**.

$$\boxed{\text{Mode} = 5 \text{ and } 6}$$

In this case, both 5 and 6 are equally popular, each appearing twice.

3.3 Example 6: Multiple Modes

Find the mode of the following data:

$$108,\ 106,\ 100,\ 90,\ 100,\ 108,\ 95$$

Solution:

- **Frequency Count:**
  - 100 appears **2** times.
  - 108 appears **2** times.
  - 106, 90, 95 appear **once**.

$$\boxed{\text{Modes} = 100 \text{ and } 108}$$

3.4 Example 7: Three Modes

Find the mode of the following data:

$$20,\ 9,\ 7,\ 11,\ 7,\ 33,\ 20,\ 11,\ 33,\ 11,\ 20,\ 33$$

Solution:

- **Frequency Count:**
  - 7 appears **2** times.
  - 9 appears **1** time.
  - 11 appears **3** times.
  - 20 appears **3** times.
  - 33 appears **3** times.

$$\boxed{\text{Modes} = 11, 20, \text{ and } 33}$$

3.5 Example 8: No Mode

Find the mode of the following data:

$$16,\ 15,\ 3,\ 0,\ 2,\ 43$$

Solution:

- **Frequency Count:**
  - All data points appear **once**.

$$\boxed{\text{No Mode}}$$

Imagine a classroom where every student has a unique favorite color—there's no mode because no color repeats.

4 Assignment

Apply what you've learned by solving the following problem:

Find the mode, median, and mean of the following data:

$$7,\ 9,\ 6,\ 12,\ 15,\ 9,\ 10,\ 4,\ 7$$

4.1 Steps to Solve

1. **Arithmetic Mean:**

$$\bar{X} = \frac{7+9+6+12+15+9+10+4+7}{9} = \frac{79}{9} \approx 8.78$$

$$\boxed{\text{Mean} \approx 8.78}$$

2. **Median:**

   - **Arrange in Ascending Order:**

   $$4,\ 6,\ 7,\ 7,\ 9,\ 9,\ 10,\ 12,\ 15$$

   - **Find the Middle Value:**
     The 5th number is **9**.

   $$\boxed{\text{Median} = 9}$$

3. **Mode:**

   - **Frequency Count:**
     - 7 appears **2** times.
     - 9 appears **2** times.
     - 4, 6, 10, 12, 15 appear **once**.

   $$\boxed{\text{Modes} = 7 \text{ and } 9}$$

Remember, practice makes perfect! Share your solutions in the WhatsApp group, and let's navigate these concepts together.

Conclusion

Thank you for joining today's exploration of measures of central tendency. Keep charting your data seas with mean, median, and mode as your trusty navigational tools. Until next time, happy analyzing!