Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

MATHEMATICS 9709/32

Paper 3 Pure Mathematics 3

February/March 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 20 pages.

It is given that $x = \ln(2y - 3) - \ln(y + 4)$.	
Express y in terms of x .	[3]

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(a) On an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $-\frac{1}{3}\pi \le \arg(z-1-2\mathrm{i}) \le \frac{1}{3}\pi$ and $\mathrm{Re}\,z \le 3$. [3]

))	Calculate the least value of $\arg z$ for points in the region from (a). Give your answer in radians correct to 3 decimal places. [2]
•)	Calculate the least value of arg z for points in the region from (a). Give your answer in radians correct to 3 decimal places. [2]
)	Calculate the least value of arg z for points in the region from (a). Give your answer in radians correct to 3 decimal places. [2]
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4	Solve	the	equ	ation

$\frac{5z}{1+2i} - zz^* + 30 + 10i = 0,$	
giving your answers in the form $x + iy$, where x and y are real.	[5]
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5	The	parametric	equations	of	a curve	are

$$x = te^{2t},$$
 $y = t^2 + t + 3.$

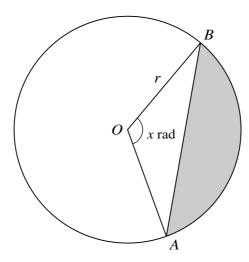
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Hence show that the normal to the curve, where $t = -1$, passes through the point $\left(0, 3 - \frac{1}{e}\right)$

-/	Express $5 \sin \theta + 12 \cos \theta$ in the form $R \cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.	
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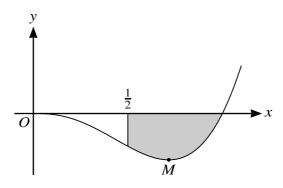
Hence solve the equation $5 \sin 2x + 12 \cos 2x = 6$ for $0 \le x \le \pi$.	[-
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The diagram shows a circle with centre O and radius r. The angle of the **minor** sector AOB of the circle is x radians. The area of the **major** sector of the circle is 3 times the area of the shaded region.

(a)	Show that $x = \frac{3}{4}\sin x + \frac{1}{2}\pi$.	[4]

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The diagram shows the curve $y = x^3 \ln x$, for x > 0, and its minimum point M.

(a)	Find the exact coordinates of M .	[4]

	ind the exact area of the shaded region bounded by the curve, the x -axis and the line $x = \frac{1}{2}$	2
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9 The variables x and y satisfy the differential equation
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$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{3y}\sin^2 2x.$$

It is given that y = 0 when x = 0.

Solve the differential equation and find the value of y when $x = \frac{1}{2}$.	[7]
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10	With respect to the origi	n O. the	points A, B ,	C and D have	position vectors	s given by
	With respect to the origin	,	points 11, 2,	C and D marc	position rector.	3 51 (CII C)

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}, \qquad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, \qquad \overrightarrow{OC} = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 5 \\ -6 \\ 11 \end{pmatrix}.$$

(a)	Find the obtuse angle between the vectors \overrightarrow{OA} and \overrightarrow{OB} .	[3]
The	line l passes through the points A and B .	
(b)	Find a vector equation for the line l .	[2]

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Find the position C and D .	•			•	[4
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11	Let $f(x) =$	$5x^2 + x + 11$
11	Let $I(x) =$	$\frac{5x^2 + x + 11}{(4+x^2)(1+x)}$

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