Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER			

4961494717

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **February/March 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

1 A line has equation $y = 3x - 2k$ and a curve has equation $y = x^2 - kx + 2$, where k is a constant.

Show that the line and the curve meet for all values of k. [4]

2 A function f is defined by $f(x) = x^2 - 2x + 5$ for $x \in \mathbb{R}$. A sequence of transformations is applied in the following order to the graph of $y = f(x)$ to give the graph of $y = g(x)$.

Stretch parallel to the x-axis with scale factor $\frac{1}{2}$

Reflection in the y-axis

Stretch parallel to the y-axis with scale factor 3

Find $g(x)$, giving your answer in the form $ax^2 + bx + c$, where a, b and c are constants. [4]

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[Turn over

3 A curve has equation $y = \frac{1}{60}(3x + 1)^2$ and a point is moving along the curve.

Find the x-coordinate of the point on the curve at which the x- and y-coordinates are increasing at the same rate. [4]

4 The circumference round the trunk of a large tree is measured and found to be 5.00 m. After one year the circumference is measured again and found to be 5.02 m.

(a) Given that the circumferences at yearly intervals form an arithmetic progression, find the circumference 20 years after the first measurement. [2]

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(b) Given instead that the circumferences at yearly intervals form a geometric progression, find the circumference 20 years after the first measurement. [3]

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5 Points A (7, 12) and B lie on a circle with centre (−2, 5). The line AB has equation $y = -2x + 26$.

Find the coordinates of B. [6]

6 In the expansion of $\left(\frac{x}{a} + \frac{a}{x^2}\right)^7$, it is given that

$$\frac{\text{the coefficient of } x^4}{\text{the coefficient of } x} = 3.$$

Find the possible values of the constant a. [6]

7 **(a)** By first obtaining a quadratic equation in $\cos\theta$, solve the equation

$$\tan\theta \sin\theta = 1$$

for $0° < \theta < 360°$. [5]

(b) Show that $\frac{\tan\theta}{\sin\theta} - \frac{\sin\theta}{\tan\theta} \equiv \tan\theta\sin\theta$. [3]

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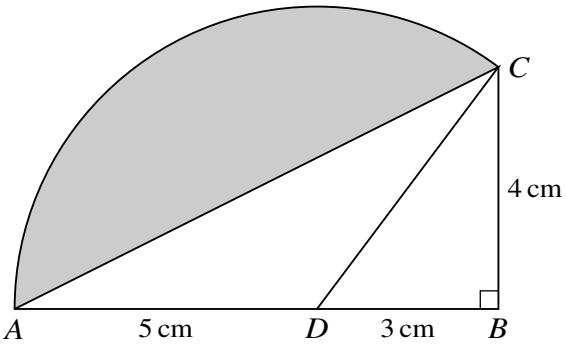

The diagram shows triangle ABC in which angle B is a right angle. The length of AB is 8 cm and the length of BC is 4 cm. The point D on AB is such that $AD = 5$ cm. The sector DAC is part of a circle with centre D.

(a) Find the perimeter of the shaded region. [5]

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(b) Find the area of the shaded region. [3]

9 The function f is defined by $f(x) = -3x^2 + 2$ for $x \leqslant -1$.

(a) State the range of f. [1]

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(b) Find an expression for $f^{-1}(x)$. [3]

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The function g is defined by $g(x) = -x^2 - 1$ for $x \leqslant -1$.

(c) Solve the equation $fg(x) - gf(x) + 8 = 0$. [5]

10 At the point $(4, -1)$ on a curve, the gradient of the curve is $-\frac{3}{2}$. It is given that $\frac{\mathrm{d}y}{\mathrm{d}x} = x^{-\frac{1}{2}} + k$, where k is a constant.

(a) Show that $k = -2$. [1]

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(b) Find the equation of the curve. [4]

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(c) Find the coordinates of the stationary point. [3]

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(d) Determine the nature of the stationary point. [2]

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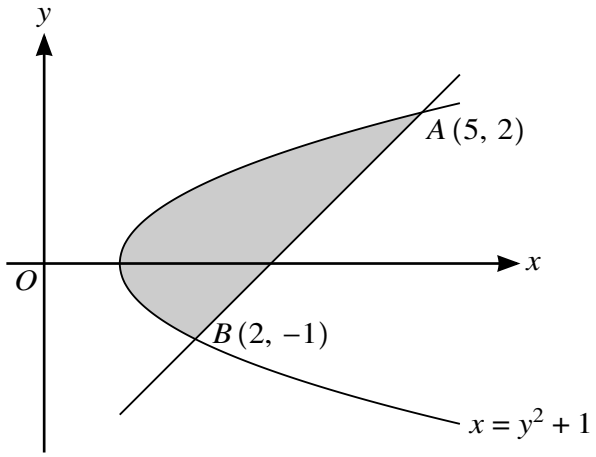

The diagram shows the curve with equation $x = y^2 + 1$. The points $A\,(5,\ 2)$ and $B\,(2,\ -1)$ lie on the curve.

(a) Find an equation of the line AB. [2]

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(b) Find the volume of revolution when the region between the curve and the line AB is rotated through 360° about the ***y*-axis**. [9]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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