Chapter 4

Bus Re-assignment Optimization

4.1 Introduction

...yet to be written.

4.2 Problem Description

The *bus re-assignment from low-demand trips to overcrowded trips problem* can be defined as follows. For a given set of bus trips where the in-vehicle crowd exceeds the capacity threshold at one or multiple stops and a set of bus trips where the demand is (very) low, given a fixed planning horizon with departure and arrival times, find bus trip(s) to cancel and re-assign before overcrowded ones. It is feasible if a single vehicle executes timetabled sequence of trips within the network. The notion is to increase bus services where the in-vehicle crowd exceeds the capacity threshold due to disruptions like extreme weather conditions or other events discussed in the previous chapter. As a result, the overcrowding problem during disrupted conditions could be encountered without requiring spare buses from the depot. However, cancelling a trip from a bus line and re-assigning its bus to another line could be very challenging. First, when a bus trip is cancelled, its passengers have to wait for the next bus or travel by other means of transport. Even for a small number of passengers, this could cause dissatisfaction. Depending on how trips are cancelled, their impact could propagate through the network due to the sequential execution of timetabled trips. A bus, for instance, is assigned to operate several trips one after another; thus, cancelling a single trip could result in cancelling several trips operated by the same bus. Second, the re-assigned buses are associated with additional operation costs, for example, deadhead driving.

Figure 4.1 represents a typical example of *bus re-assignment problem* with two lines and ten trips. For instance, trip t_{10} dispatching from stop 2 of line 2 is expected to be *overcrowded* at several segments and, therefore, requires another bus carry some of its passengers. Suppose, we want to re-assign the bus that operates trip t_3 of line 1. This bus should deadhead to $stop_{2,2}$ and start its new trip (t_{new}) in line 2. As a result, trip t_3 and t_4 will be cancelled. After completing the re-assigned trip, the bus must operate its next timetabled trip (t_5).

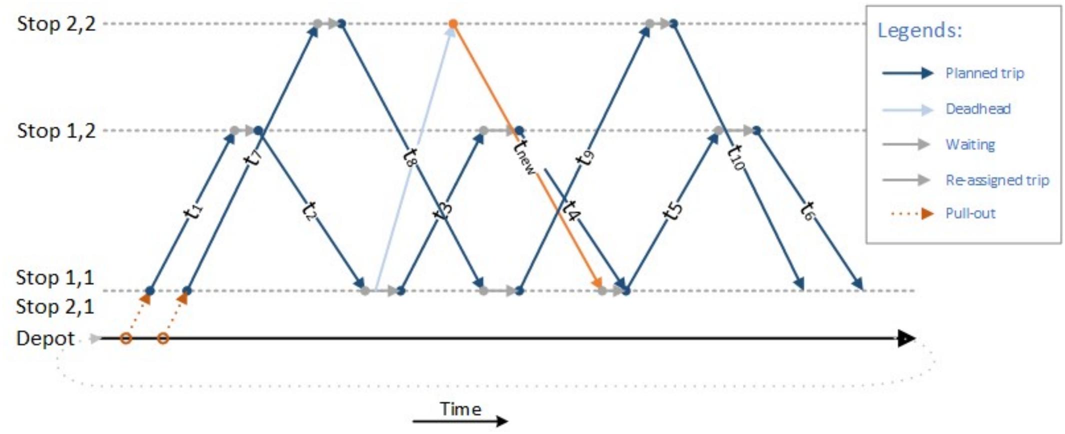

FIGURE 4.1: Time-space network trips

Before further proceeding with the problem formulation, the following assumptions are made:

- Buses operating in a line do not have to go to the depot at the end of every trip. This implies that the driving time from (to) the depot is not included in the problem formulation.
- The first and the last bus trips cannot be cancelled.
- The timetabled trips (also drivers' schedules) of bus lines where the new buses are assigned remain as planned.
- The number of daily trips and their departure times are already determined in the planning stage.
- All buses have equal capacity. Based on the predictive modeling results, when the in-vehicle crowd at least at one stop during a bus trip exceeds the capacity threshold, the trip is identified as *overcrowded*.

Notice that this study focuses on the inner-city bus lines. Due to inflexibility of the company's scheduling system, it is not possible to combine both inner-city and regional bus lines in a single optimization problem. A bus might operate multiple regional trips one after another, and, therefore, a single trip cancellation could propagate through several regions in the network. As a result, the costs of re-assignment will outweigh its benefits. Figure 4.2 represents our case study area, the city of Enschede (the Netherlands). In total, nine bus lines are operating throughout the city. Out of which, line 802 operates only on Saturdays and Sundays, connecting a P+R parking in the south to the central station. Thus, trips on this bus line are excluded from further analysis. Even though line 9 is not an inner-city bus line, it is included in our model because it is one of the most overcrowded lines connecting Hengelo to Enschede. However, we can only assign buses to this line, and no trips should be cancelled from this line.

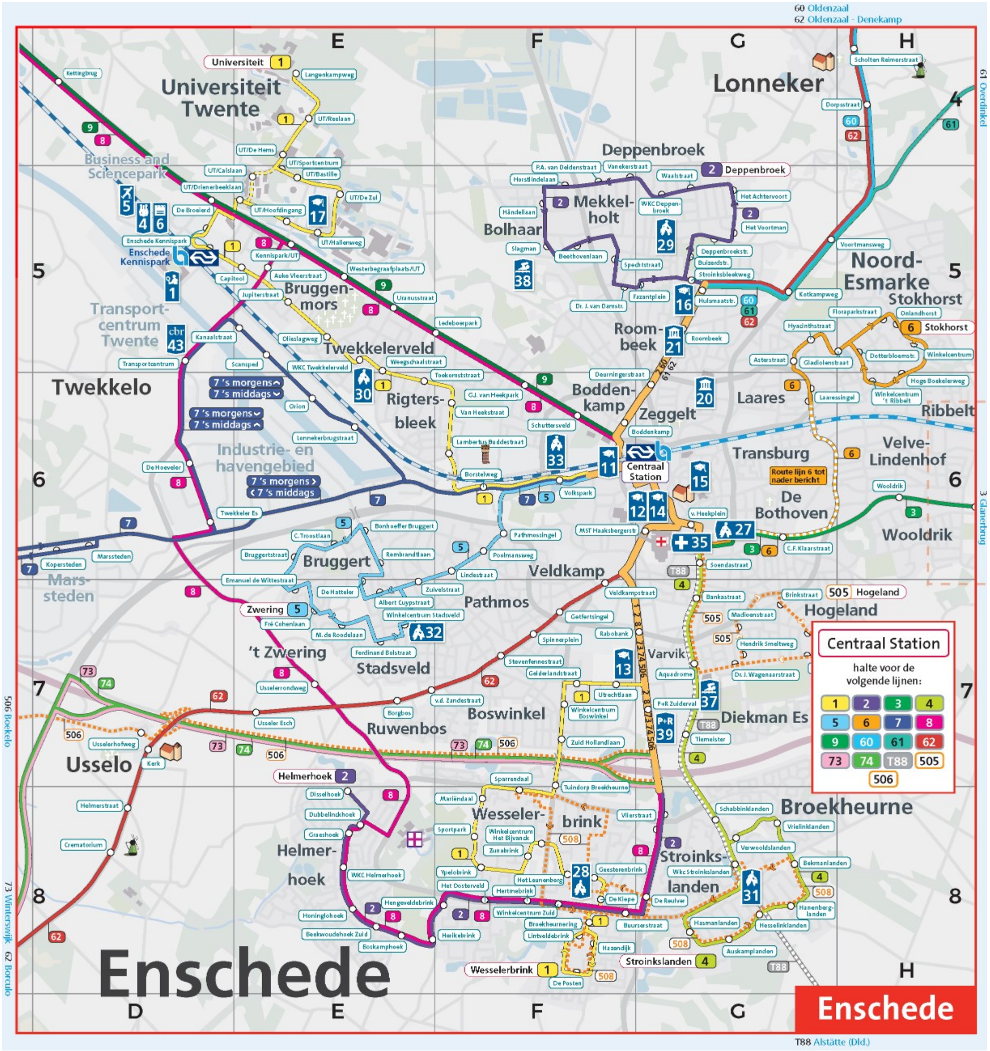

FIGURE 4.2: Enschede bus network

4.3 Literature review

... yet to be written.

4.4 Mathematical formulation

In this section, the mathematical formulation of the problem is described based on the assumption made in the preceding section. Let $T = \{1, 2, ..., T\}$ denote the set of daily timetabled trips within the network and $S_t = \{1, 2, ..., m_t\}$ be the set of stops served by trip $t \in T$. In order to differentiate between overcrowded trips and trips that are op-feasible for re-assignment, we divide the set T into two subsets: T^a and T^r. $T^a \subset T$ is the subset of trips where the in-vehicle crowd exceeds that capacity threshold at least at one stop and $T^r \subset T$ is the subset of trips that could potentially be cancelled and re-assigned to other lines. Let $L = \{(i, j) | i \in T^r, j \in T^a\}$ be the list of paired trips, where i is the potential trip that could be cancelled and re-assigned before j. Because there could be several *overcrowded* trips during the time window of disruptive events, the decision should be made on which bus trip $i \in T^r$ should be re-assigned before each overcrowded trip $j \in T^a$. The following preconditions should hold when generating these subsets:

- For any trip included in T^a, the in-vehicle crowd exceeds the capacity threshold at least at one stop during the trip.
- For any trip included in T^r, the in-vehicle crowd of the following trip on this line should not exceed the capacity threshold at any stop. This precondition prevents cancelling trips from a bus line where the following trip could be overcrowded. The reason is that passengers from a cancelled trip might take the following bus, and adding these passengers to an already *overcrowded* trip will automatically create another overcrowding problem.
- Any trip from the line 9 should be excluded from subset T^r since trips of this line cannot be cancelled.

Problem objectives

Ideally, the planned bus services can fulfill the expected demand and there is no need to re-assign any buses, so:

$$\vartheta_s^t \leq \eta, \forall t \in T, \forall s \in S_t \tag{4.1}$$

where ϑ_s^t is the in-vehicle crowd at stop s during trip t and η is the capacity threshold. However, disruptive events could cause an increase in the in-vehicle crowd, which may lead to overcrowding issues. To encounter such issues, we have to cancel some of the planned trips so that their buses could be re-assigned to other lines. Nevertheless, this is a two-folded problem. On the one side, trip cancellations mean additional waiting time for passengers whose bus trips got cancelled. On the other side, more passengers can be served if these buses operate the re-assigned trips instead of executing their timetabled trips. This leads to our first objective, which is minimizing the waiting time of passengers who cannot board buses due to overcrowding or trip cancellation (hereafter stranded passengers). We introduce two decision variables $x_{i,j}$ and $y_{\hat{i}}$. The re-assignment variable $x_{i,j}$ indicates whether trip i is cancelled and re-assigned before trip j. $x_{i,j} = 1$ if i is cancelled and re-assigned before j and 0 otherwise. $y_{\hat{i}}$ is the imposed cancellation variable, which refers to trip cancellations as a consequence of re-assignment. For instance, the bus that is supposed to execute a timetabled trip i cannot return on time to operate its next timetabled trip. $y_{\hat{i}} = 1$ if $\hat{i}$ is cancelled and 0 otherwise. Let $F_i = \{1, 2, ..., f_i\}$ be the set of following trips for trip i operated by the same bus, the objective function can be stated as follows:

$$\begin{aligned} \min f_1(x, y) = & \sum_{(i,j)\in L} \sum_{s\in S_t} \frac{1}{2}\zeta_s^j \cdot w_s^j \cdot x_{i,j} + \sum_{(i,j)\in L} \sum_{s\in S_t} 3\zeta_s^j \cdot w_s^j \cdot \left(1 - x_{i,j}\right) \\ & + \sum_{(i,j)\in L} \sum_{s\in S_t} 2\vartheta_s^i \cdot w_s^i \cdot x_{i,j} + \sum_{\hat{i}\in F_i} \sum_{s\in S_t} 2\vartheta_s^{\hat{i}} \cdot w_s^{\hat{i}} \cdot y_{\hat{i}} \end{aligned} \tag{4.2}$$

where w_s^j, w_s^i and $w_s^{\hat{i}}$ represent the average waiting time of passengers during trip j, i, and $\hat{i}$, respectively. ζ_s^j is the number of in-vehicle crowds exceeding the capacity threshold during trip j at stop s. ϑ_s^i and $\vartheta_s^{\hat{i}}$ are the number of expected in-vehicle crowds during trip i and $\hat{i}$ at stop s. The first part of Eq. 4.2 refers to reducing the total waiting time of stranded passengers for an overcrowded trip. If an additional bus trip is assigned before trip j, the current waiting time of stranded passengers for this trip will be reduced by half. This is based on the assumption that one additional trip before each overcrowded trip is enough to carry the stranded passengers. One could argue that it is not always true, and a single bus trip may not be enough to carry the stranded passengers of an overcrowded trip. Since this is not part of problem formulation of this study, we will investigate it later if, in fact, there is a scenario where this assumption does not hold. However, if no additional bus is assigned before trip j, its leftover passengers have to wait for the next bus. Assuming that they have no previous knowledge of the overcrowded situation, they might already be at bus stops. So, their waiting time will be increased by as much as the headway between two consecutive buses on that line or three times the normal waiting time, as stated in the second part of Eq. 4.2. The third part of the equation refers to the immediate impact of trip cancellation on the waiting time of passengers whose bus trips got cancelled. Because the headway between two consecutive trips will be doubled in case of trip cancellation, the passenger waiting time is also doubled. The fourth part of Eq. 4.2 denotes the imposed impact of trip cancellation. If a re-assigned bus cannot operate its next timetabled trips ($y_{\hat{\imath}} = 1$), the waiting time of these passengers will also be doubled. Depending on how many bus trips are cancelled as a consequence of re-assignment, the value of this part varies accordingly.

The second objective is to minimize the deadhead cost associated with the bus re-assignment. This cost refers to the deadhead driving from the last stop of trip i to the first stop of its re-assigned trip j and from the last stop of j to the first stop of its next timetabled trip $\hat{\imath}$. If both trips start from the same stop or end at the same stop (e.g., Enschede central), the deadhead time is typically very short, so the deadhead time is set to zero. This objective function is stated as follows:

$$\min f_2(x, y) = \sum_{(i,j)\in L} x_{i,j} \cdot \delta_{i,j} \cdot C + \sum_{\hat{\imath}\in F_i} y_{\hat{\imath}} \cdot \left(1 - y_{\hat{\imath}+1}\right) \cdot \delta_{j,\hat{\imath}} \cdot C \tag{4.3}$$

where $\delta_{i,j}$ is the deadhead time between the last stop of cancelled trip i to the first stop of re-assigned trip j and $\delta_{j,\hat{\imath}}$ is the deadhead time between the last stop of re-assigned trip j and the first stop of next timetabled trip $\hat{\imath}$ operated by the re-assigned bus. Note that if the bus cannot arrive on time to start its next trip $\hat{\imath}$, the model considers its deadhead time to the first stop of its next next trip ($\hat{\imath} + 1$), and so on. C is the driving cost of buses per unit of time. This objective is automatically zero if no bus is being re-assigned.

Based on the assumptions that (1) passengers arrive at stops uniformly and (2) arrivals of two consecutive buses are independent, the general formula for calculating the average passenger waiting time could be as follows (Osuna & Newell, 1972):

$$w = \frac{E(H^2)}{2E(H)} = \frac{E(H)}{2}(1 + \frac{Var(H)}{(E(H))^2} \tag{4.4}$$

where $E(H)$ and $Var(H)$ represent the headway mean and variance, respectively. Since the bus frequencies changes during the year, day of the week, and time of the day, these values also vary accordingly.

Constraints

Bus arrival: The bus operating trip i should be able to arrive on time for the re-assigned trip j. It means that the bus should drive from the last stop of its timetabled trip to the first stop of its re-assigned trip and arrive there before departure time of the overcrowded trip. This constraint guarantees that a re-assigned is available on time to start its re-assigned trip. So:

$$a_i + \delta_{i,j} + \epsilon \leq d_j, \forall i \in L \tag{4.5}$$

where a_i is the bus arrival time to the first stop of trip i and $\delta_{i,j}$ is the deadhead time between the last stop of trip i and the first stop of trip j. d_j is departure time of trip j from its first stop and ϵ is average boarding time (e.g., 2 minutes) at the first stop of the re-assigned trip.
Re-assignment: A bus trip can only be cancelled once during the time-window of optimization. Likewise, only one bus trip can be re-assigned before an overcrowded trip. These constraints can be stated as follow:

$$\sum_{j \in L} x_{i,j} \leq 1, \forall j \in L \tag{4.6}$$

$$\sum_{i \in L} x_{i,j} \leq 1, \forall i \in L \tag{4.7}$$

The above constraints ensure that a single bus trip is not re-assigned before multiple overcrowded trips or several cancelled trips are not re-assigned before a single overcrowded trip.

Imposed cancellation: If a re-assigned bus arrives at the first stop of its following timetabled trip $\hat{i}$ on-time to operate this trip, then $y_{\hat{i}} = 0$ and this equation is true:

$$a_i + \big(\delta_{i,j} + \lambda_i + \delta_{j,\hat{i}} + \epsilon\big) x_{i,j} \leq d_{\hat{i}}, \forall \hat{i} \in F_i, \forall i \in L \tag{4.8}$$

where λ_i is the average travel time from the first stop to the last stop of trip i and $d_{\hat{i}}$ is departure time of trip $\hat{i}$ from its first stop. $\delta_{j,\hat{i}}$ is the deadhead time when going from the last stop of trip j to the first stop trip $\hat{i}$. The above constraint cannot hold if the re-assigned bus cannot return on time for its next trip(s). To meet this constraint in the case of imposed cancellation, a large number M is added to the equation. So:

$$a_i + \big(\delta_{i,j} + \lambda_i + \delta_{i,\hat{i}} + \epsilon\big) x_{i,j} - M y_{\hat{i}} \leq d_{\hat{i}}, \forall i \in L, \forall \hat{i} \in F_i \tag{4.9}$$

Maximum imposed cancellations: To prevent the model from cancelling too many trips, up to a maximum number of following trips of a bus can be cancelled. This constraint limits propagation impact of imposed cancellations on the rest of the network. Also, it prevents early cancellation of trips when not necessary. In this study, we limit this value to two, so that up to three consecutive trips operated by a single bus could be cancelled in total.

$$\sum_{\hat{i} \in F_i} y_{\hat{i}} \leq 2, \forall \hat{i} \in F_i \tag{4.10}$$

Considering the aforementioned objective functions and constraints, the *bus re-assignment problem* can be formulated as follows:

$$\begin{aligned} \min f_1(x, y) = & \sum_{(i,j) \in L} \sum_{s \in S_t} \frac{1}{2} \zeta_s^j \cdot w_s^j \cdot x_{i,j} + \sum_{(i,j) \in L} \sum_{s \in S_t} 3 \zeta_s^j \cdot w_s^j \cdot \big(1 - x_{i,j}\big) \\ & + \sum_{(i,j) \in L} \sum_{s \in S_t} 2 \vartheta_s^i \cdot w_s^i \cdot x_{i,j} + \sum_{\hat{i} \in F_i} \sum_{s \in S_t} 2 \vartheta_s^{\hat{i}} \cdot w_s^{\hat{i}} \cdot y_{\hat{i}} \end{aligned} \tag{4.11}$$

$$\min f_2(x,y) = \sum_{(i,j)\in L} x_{i,j} \cdot \delta_{i,j} \cdot C + \sum_{\hat{i}\in F_i} y_{\hat{i}} \cdot \left(1 - y_{\hat{i}+1}\right) \cdot \delta_{j,\hat{i}} \cdot C \tag{4.12}$$

Subject to:

$$a_i + \delta_{i,j} + \epsilon \leq d_j, \forall i \in L \tag{4.13}$$

$$a_i + \left(\delta_{i,j} + \lambda_i + \delta_{i,\hat{i}} + \epsilon\right) x_{i,j} - M y_{\hat{i}} \leq d_{\hat{i}}, \forall i \in L, \forall \hat{i} \in F_i \tag{4.14}$$

$$\sum_{j\in L} x_{i,j} \leq 1, \forall j \in L \tag{4.15}$$

$$\sum_{i\in L} x_{i,j} \leq 1, \forall i \in L \tag{4.16}$$

$$\sum_{\hat{i}\in F_i} y_{\hat{i}} \leq 2, \forall \hat{i} \in F_i \tag{4.17}$$

$$x_{i,j} \in \{0,1\}, \forall (i,j) \in L \tag{4.18}$$

$$y_{\hat{i}} \in \{0,1\}, \forall \hat{i} \in F_i \tag{4.19}$$

Notice that the bus re-assignment time window depends on the intensity of disruptions. The starting time (lower bound) is set as the starting time of disruption occurrence. The length of the time window varies depending on the duration of a disrupted condition. However, the length of the time window cannot be more than one operating day, between 5:00 and 23:00. It is desirable that all buses execute their timetabled trips before the end of the time window. If a feasible solution cannot be found within the time window, it is advised to drive spare bus(es) from the depot.

4.4.1 Algorithms

The bus re-assignment model could be solved by a commercial solver, e.g. Gurobi, Pymoo or CPLEX. For the sake of this project, solvers must have the ability to integrate with our decision support tool (DSS). Therefore, we use Gurobi and Pymoo.