

DATA 468 Homework 2

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Instructions: Please write or type your solutions clearly and show all relevant steps. Once you are done, please upload your solutions to Gradescope. If you need to scan your solutions, please use a free scanning app like CamScanner instead of sending photographs. Please submit your solutions within the prescribed time, as late submissions will be not considered.

Let $\{X(t), t \in [0, \infty)\}$ be defined as $X(t) = A + Bt$, for all $t \in [0, \infty)$, where A and B are independent normal $N(2, 2)$ random variables. continuous

1. Find all possible sample functions for this random process.
2. Write down the normal distribution of Z and Y
3. Define the random variable $Z = X(2)$, Find the PDF of Y
4. Let also $Y = X(3)$, Find $E[YZ]$

$$A \sim N(2, 2)$$

$$B \sim N(2, 2)$$

| |
μ σ²

$$1. X(t) = A + Bt$$

$$\text{Let } A = a, B = b$$

$$X(t, \omega_i) = a + bt$$

$$2. Z = X(2) = A + 2B$$

$$E(Z) = E(A + 2B)$$

$$= E(A) + 2E(B)$$

$$= 2 + 2 \times 2 = 6$$

$$\text{Var}(Z) = \text{Var}(A + 2B)$$

$$= \text{Var}(A) + 4\text{Var}(B)$$

$$= 2 + 4 \times 2 = 10$$

$$Z \sim N(6, 10)$$

$$Y = X(3) = A + 3B$$

$$E(Y) = E(A + 3B)$$

$$= E(A) + 3E(B)$$

$$= 2 + 3 \times 2$$

$$= 8$$

$$\text{Var}(Y) = \text{Var}(A + 3B)$$

$$= \text{Var}(A) + 9\text{Var}(B)$$

$$= 2 + 9 \times 2$$

$$= 20$$

$$Y \sim N(8, 20)$$

3. PDF of Y

since $Y \sim N(8, 20)$

$$f_Y(y) = \frac{1}{\sqrt{20\pi}} e^{-\frac{(y-8)^2}{20}}$$

$$= \frac{1}{2\sqrt{5}\pi} e^{-\frac{(y-8)^2}{20}}$$

4. $Yz = (A+3B)(A+2B) = A^2 + 5AB + 6B^2$

$$E(Yz)$$

$$= E(A^2 + 5AB + 6B^2)$$

$$= E(A^2) + 5E(AB) + 6E(B^2)$$

$$= (\text{Var}(A) + E(A)^2) + 5E(A) \cdot E(B) + 6(\text{Var}(B) + E(B)^2)$$

$$= (2 + 2^2) + 5 \times 2 \times 2 + 6(2 + 2^2)$$

$$= 62$$