DATA 468: Applied Stochastic Process

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Instructions

1. Please leave all personal belongings at the front of the classroom. Do not begin the exam until instructed to do so.

2. Talking or looking at other students' exams is strictly prohibited. If you require assistance, please raise your hand and speak with the instructor.

3. The use of digital devices, including phones and computers, is not allowed during the exam.

4. Ensure that you attempt all questions, clearly encircle or tick your selected answer.

Name:——————————-Class——————————

Student ID:——————————————————————

Date:——————————————————————-

1. Choose the right answer. (15 points)

i). In Fig 1, which ones are transient state(s)?

(a) 1, 2, 5 (b) 1, 4, 7 (c) 1 (d) None of these

ii). In Fig 1, which ones are recurrent state(s)?

(a) 2, 3, 4 (b) 5, 6, 7 (c) All except 1 (d) None of them

iii). A random variable is a..... that maps the outcome of a random experiment into a real number.

(a) State (b) Function (c) Markov China (d) none of these

iv). To predict the future state of a Markov chain, it is only important to know the ..... state.

(a) Future (b) Present (c) Fast (d) none of these

v). If it is possible to go from state i to state j, we say that state j is ..... from i.

(a) Communicate (b) Accessible (c) Periodic (d) Closed

2. Mark each statement as True or False. (15 points)

i). A Markov chain is a type of stochastic chain, but not all stochastic chains are Markov.

(a) True (b) False

ii). Each row of a transition matrix (P) should sum to 1.

(a) True (b) False

iii). $P_{ij}^{(n)}$ represents the probability of making a transition from state i to state j in a single step.

(a) True (b) False

iv). A state is called transient if, starting from state i, there is a 100% probability of returning to state i.

(a) True (b) False

v). Each state in a Markov Chain has a period.

(a) True (b) False

3. In radio communications, the carrier signal is often modeled as a sinusoid with a

random phase. Consider the stochastic process X(t)=cos(2πt + θ), where the random phase shift θ has a Uniform (0, 2π) distribution. (10 points)

i). Find the mean function.

ii). Find X(2).

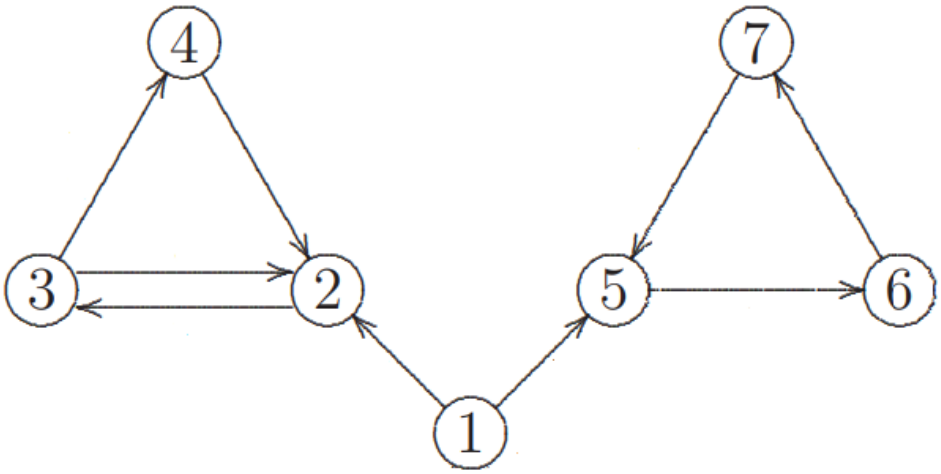

图 1: Transition Probabilities.

4. A Markov chain X_0, X_1, X_2, . . . has the transition probability matrix (for the states {0, 1, 2})(30 points)

$$P = \begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.5 & 0.1 & 0.4 \\ 0.5 & 0.2 & 0.3 \end{bmatrix}$$

and initial distribution: $\pi = [0.5, 0.5, 0]$.

i). Draw a transition diagram of P

ii). Find $P(X_2 = 1|X_1 = 0, X_0 = 2)$.

iii). Find $P(X_1 = 1)$.

iv). Find $P(X_{45} = 1|X_{44} = 0)$.

v). Find $P(X_1 = 0)$.

5. There are 3 types of weather in a particular area: {sunny, cloudy, and rainy}={1, 2, 3}. A news Media channel wants to broadcast its prediction about next week's weather. The news channel hires a weather forecasting company to find out next week's weather. (30 points)

	Sunny	**Cloudy**	**Rainy**
Sunny	70%	20%	10%
Cloudy	20%	50%	30%
Rainy	50%	25%	25%

表 1: Weather transition probabilities

Note: Currently, there is sunny weather in that area.

i). Write the transition matrix and draw the transition diagram for the above problem.

ii). Compute the chances of different weather conditions (sunny, cloudy, and rainy) next week.

iii). What is the probability that the weather will be sunny next week?

iv). What is the probability that the weather will be cloudy next week?

v). What is the probability that the weather will be rainy next week?

Important Formulas

i). Uniform distribution function

$f(x) = \frac{1}{b-a}$**, where** $a \leq x \leq b$

ii) The expected value of X is given by

$\mathbb{E}[g(X)] = \int_{-\infty}^{\infty} g(x) f(x)\, dx$

ii) The integral of $\cos(2\pi t + \theta)$ **is**

$\int \cos(2\pi t + \theta)\, dt = \frac{1}{2\pi} \sin(2\pi t + \theta)$