

# DATA 468 Homework 1

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**Instructions:** Please write or type your solutions clearly and show all relevant steps. Once you are done, please upload your solutions to Gradescope. If you need to scan your solutions, please use a free scanning app like CamScanner instead of sending photographs. Please submit your solutions within the prescribed time, as late submissions will be not considered.

1. A probability space is defined as a triple  $(\Omega, \mathcal{F}, P)$ . Given the following:

- The sample space  $\Omega = \{H, T\}$ , where H represents heads and T represents tails.
- The  $\sigma$ -algebra is  $\mathcal{F} = \{\emptyset, \{H\}, \{T\}, \{H, T\}\}$ , a collection of subsets of  $\Omega$ .
- $P$  is a probability measure such that  $P\{H\} = p$ ,  $P\{T\} = 1 - p$ .

(a) Show that  $\mathcal{F}$  satisfies the properties of  $\sigma$ -algebra.

(b) Verify that  $P$  satisfies the axioms of a probability measure.

(c) If  $p = 0.5$ , compute  $P(\{H, T\})$  and  $P(\emptyset)$ .

Note:  $\emptyset$  (empty set)