DATA 468 Homework 5

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Submission Date: May 25, 2025 (11:00 PM) (Beijing Time)

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Instructions: Please write or type your solutions clearly and show all relevant steps. Once you are done, please upload your solutions to Gradescope. If you need to scan your solutions, please use a free scanning app like CamScanner instead of sending photographs. Please submit your solutions within the prescribed time, as late submissions will be not considered.

1). Determine if the Chain in Fig 1, is an absorbing Markov chain.

2). Let's define a_i as the absorption probability in state 1 if we start from state i, calculate the following abortion probabilities

i). a1=P(absorption in 1|X0=1),

ii). a2=P(absorption in 1|X0=2),

iii). a3=P(absorption in 1|X0=3),

iv). a4=P(absorption in 1|X0=4).

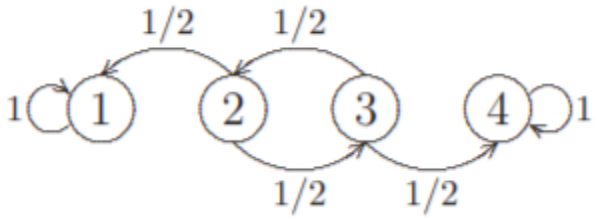

Fig 1. Markov Chain

3). Let's define t_i as the number of steps needed until the chain hits state 1 or state 4, given that X_0=i

i). Find the hitting times (t_1 and t_2).

① state 1 and state 4 are absorption states, because their self-transition probabilies are 1. state 2 and state 3 are non-absorption states. So this Markov chain is an absorbing Markov chain.

② $$P=\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{cases} a_1=1 \\ a_2=\frac{1}{2}a_1+\frac{1}{2}a_3 \\ a_3=\frac{1}{2}a_2+\frac{1}{2}a_4 \\ a_4=0 \end{cases} \Rightarrow \begin{matrix} a_2=\frac{1}{2}\times 1+\frac{1}{2}a_3 \\ a_3=\frac{1}{2}a_2+\frac{1}{2}\times 0=\frac{1}{2}a_2 \end{matrix} \Rightarrow \begin{matrix} a_2=\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}a_2 \\ a_2=\frac{2}{3} \end{matrix}$$

$$\Rightarrow a_3=\frac{1}{2}\times\frac{2}{3}=\frac{1}{3}$$

$$\Rightarrow \begin{cases} a_1=1 \\ a_2=\frac{2}{3} \\ a_3=\frac{1}{3} \\ a_4=0 \end{cases}$$

③ $\begin{cases} t_1 = 0 \\ t_2 = 1 + \frac{1}{2}t_1 + \frac{1}{2}t_3 \\ t_3 = 1 + \frac{1}{2}t_2 + \frac{1}{2}t_4 \\ t_4 = 0 \end{cases}$

$\Rightarrow t_2 = 1 + \frac{1}{2} \times 0 + \frac{1}{2}t_3 = 1 + \frac{1}{2}t_3$

$t_3 = 1 + \frac{1}{2}t_2 + \frac{1}{2} \times 0 = 1 + \frac{1}{2}t_2$

$\Rightarrow t_2 = 1 + \frac{1}{2} \times (1 + \frac{1}{2}t_2)$

$t_2 = 2$

$\Rightarrow t_3 = 1 + \frac{1}{2} \times 2 = 2$

$\Rightarrow \begin{cases} t_1 = 0 \\ t_2 = 2 \\ t_3 = 2 \\ t_4 = 0 \end{cases}$

$\Rightarrow t_1 = 0 \quad t_2 = 2$