DATA 468 Homework 4

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Instructions: Please write or type your solutions clearly and show all relevant steps. Once you are done, please upload your solutions to Gradescope. Ifyou need to scan your solutions, please use a free scanning app like CamScanner instead of sending photographs. Please submit your solutions within the prescribed time, as late submissions will be not considered.

Electric bikes on the campus move between stations A, B and C according to a Time homogeneous Markov chain with the following transition probability matrix P. Where $S=\{A,B,C\}=\{1,2,3\}$ and initial probability distribution is $\pi^{(0)}=[0.5 \quad 0.25 \quad 0.25]$

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{ccc} 1 & 2 & 3 \\ \begin{bmatrix} 0.1 & 0.4 & 0.5 \\ 0.6 & 0.05 & 0.35 \\ 0.7 & 0.2 & 0.1 \end{bmatrix} \end{array}$$

Q1.Find the following probabilities

a). $P(X_1=A|X_0=C)$
b). $P(X_{73}=A|X_{72}=C)$
c). $P(X_{73}=A|X_{72}=C, X_{51}=B)$
d). $P(X_1=B)$
e). $P(X_2=B, X_1=A, X_0=A)$

a. $\pi^{(0)} = [0.5, 0.25, 0.25]$

$S = \{A, B, C\} = \{1, 2, 3\}$

$P(X_1=A|X_0=C) = P_{31} = 0.7$

b. $P(X_{73}=A|X_{72}=C) = P_{31} = 0.7$

c. $P(X_{73}=A|X_{72}=C, X_{51}=B)$

$= P(X_{73}=A|X_{72}=C)$

$= P_{31} = 0.7$

d, $\pi(0) = [0.5 \quad 0.25 \quad 0.25]$

$\pi(1) = \pi(0) \cdot P$

$= [0.5 \quad 0.25 \quad 0.25] \cdot \begin{bmatrix} 0.1 & 0.4 & 0.5 \\ 0.6 & 0.05 & 0.35 \\ 0.7 & 0.2 & 0.1 \end{bmatrix}$

$= [0.375, 0.2625, 0.3625]$

$P(X_1 = B) = 0.2625$

e, $P(X_0 = A) = 0.5$

$P(X_1 = A | X_0 = A) = P_{11} = 0.1$

$P(X_2 = B | X_1 = A) = P_{12} = 0.4$

$P(X_2 = B, X_1 = A, X_0 = A)$

$= P(X_0 = A) \cdot P(X_1 = A | X_0 = A) \cdot P(X_2 = B | X_1 = A)$

$= 0.5 \times 0.1 \times 0.4$

$= 0.02$