Boolean Algebra

Monday, 16 January 2023 7:14 PM

$\overline{A} \Rightarrow$ NOT A
$AB \Rightarrow$ A AND B
$A+B \Rightarrow$ A OR B

LAWS:

1. Commutative: $A+B = B+A, \quad AB = BA$
2. Associative: $A+(B+C) = (A+B)+C, \quad A(BC) = (AB)C$
3. Distributive: $A\cdot(B+C) = (AB)+(AC)$
   $A+(BC) = (A+B)\cdot(A+C)$
4. Idempotent: $A+A+A+A+\cdots+A = A$
   $A\cdot A\cdot A\cdot A\cdot\cdots\cdot A = A$
5. Identity: $1\cdot A = A, \quad 0+A = A$
6. Null: $0\cdot A = 0, \quad 1+A = 1$
7. Inverse: $A\cdot\overline{A} = 0, \quad A+\overline{A} = 1$
8. Absorption: $A\cdot(A+B) = A$
   $A+(AB) = A$
   $\left.\begin{array}{l} A+\overline{A}B = A+B \\ A\cdot(\overline{A}+B) = AB \end{array}\right\}$ Redundancy law
9. Double Complement: $\overline{\overline{A}} = A, \quad \overline{\overline{AB}} = AB, \quad \overline{\overline{A+B}} = A+B$
10. De-Morgan Theoram: $\overline{AB} = \overline{A}+\overline{B}$
    $\overline{A+B} = \overline{A}\cdot\overline{B}$

Absorption law: (Proofs)

- $A+AB = A$

  $A+AB \Rightarrow A\cdot 1 + AB$
  $A\cdot(1+B)$
  $A\cdot 1$
  A

- $A\cdot(A+B) = A$

  $A\cdot(A+B)$
  $AA + AB$
  $A + AB$
  $A\cdot(1+B)$
  $A\cdot 1$
  A

- $A+\overline{A}B = A+B$
  $(A+\overline{A})(A+B)$
  $1\cdot(A+B)$
  $A+B$

- $A\cdot(\overline{A}+B) = AB$
  $A\overline{A} + AB$
  $0 + AB$
  AB

Q1. $A+B+\overline{A}+\overline{B}$ Associative
$(A+\overline{A})+(B+\overline{B})$
$1 + 1$
1

Q2.
a) $ABC + \overline{A}BC + A\overline{B}C + AB\overline{C}$ Distributive
$BC(A+\overline{A}) + A(\overline{B}C+B\overline{C})$ Inverse
$BC\cdot 1 + A(\overline{B}C+B\overline{C})$ Identity
$BC + A(\overline{B}C+B\overline{C})$.

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b) $ABC + \overline{A}BC + A\overline{B}C + AB\overline{C}$ Associative
$ABC + (\overline{A}BC + A\overline{B}C + AB\overline{C})$ Distributive
$(ABC+\overline{A}BC) + (ABC+A\overline{B}C) + (ABC+AB\overline{C})$ Distributive
$BC(A+\overline{A}) + AC(B+\overline{B}) + AB(C+\overline{C})$ Inverse (Null
$BC + AC + AB$

Q3. $\overline{A}A + \overline{A}B + AB + B\overline{B} + AAA + AA\overline{B}$ Inverse
$0 + \overline{A}B + AB + 0 + AAA + AA\overline{B}$ Identity
$\overline{A}B + AB + AAA + AA\overline{B}$ Idempotent
$\overline{A}B + AB + A + A\overline{B}$
$B\cdot\underbrace{(\overline{A}+A)}_{\text{Inverse}} + A\cdot\underbrace{(1+\overline{B})}_{\text{Null}} \Rightarrow B\cdot 1 + A\cdot 1$ identity law
$B+A$ Commutative
$A+B$

Homework:

1. O/N 18, P32, Q3 (9608)
   $A\overline{B}\overline{C} + AB\overline{C} + ABC$

2. M/J 19, P33, Q3(c) 9608
   $\overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}\overline{C}D + \overline{A}\overline{B}CD + \overline{A}\overline{B}C\overline{D} + \overline{A}B\overline{C}\overline{D}$