Boolean Algebra

Saturday, 26 December 2020 5:33 PM

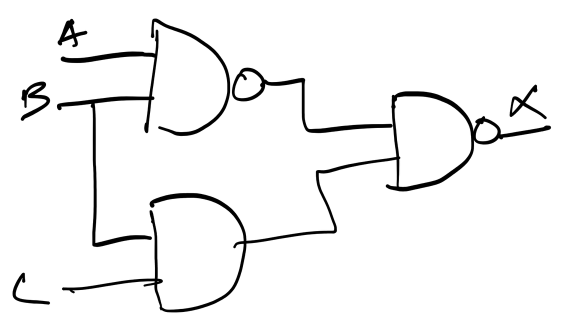
Bitwise operators:

NOT	$\bar{A}$
AND	$\cdot$
OR	$+$
XOR	$\oplus$
NAND	$\overline{AB}$
NOR	$\overline{A+B}$

OPERATIONS and EXPRESSIONS:

- Variable A
- Complement $\bar{A}$
- Literal
- SUM Term $(A+B+C+D)$
- PRODUCT Term $(ABCD)$

Describing Logic Circuits Algebrically:

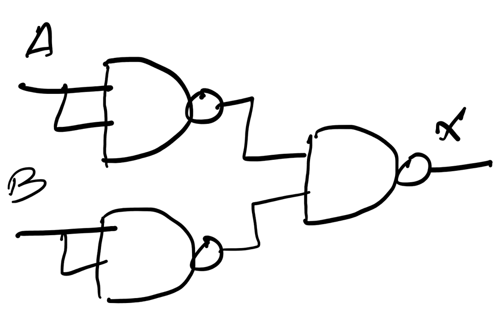


$X = \overline{\overline{AB}\cdot BC}$

Laws:

Postulates of algebra

1. $1.1 = 1$ $\quad 0+0=0$
2. $1.0 = 0.1 = 0$ $\quad 0+1 = 1+0 = 1$
3. $0.0 = 0$ $\quad 1+1 = 1$
4. $\bar{1} = 0$ $\quad \bar{0} = 1$



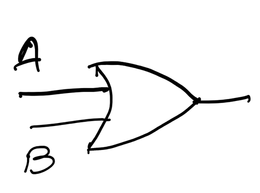
$X = \overline{\overline{AA}\cdot\overline{BB}} = \overline{\overline{AA}\cdot\overline{BB}}$

$\Rightarrow \overline{\bar{A}+\bar{A}\cdot\bar{B}+\bar{B}}$

$\Rightarrow \overline{\bar{A}\cdot\bar{B}}$

$\Rightarrow \bar{\bar{A}}+\bar{\bar{B}}$

$\Rightarrow A+B$



Theorems of algebra

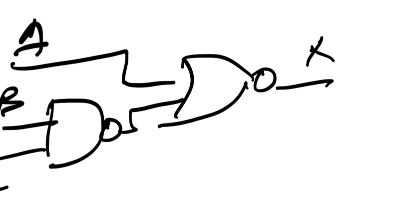
1. Commutative Law
   - $X+Y = Y+X$ (Addition)
   - $XY = Y\cdot X$ (multiplication)
2. Associative Law
   - $X+(Y+Z) = Y+(X+Z) = Z+(X+Y)$ For addition
   - $X\cdot(YZ) = Y\cdot(XZ) = Z\cdot(XY)$ For multiplication
3. Distributive Law
   - $X\cdot(Y+Z) = X\cdot Y + X\cdot Z$
   - $(X\cdot Y)+(X\cdot Z) = X\cdot(Y+Z)$.
4. Operations with '0' & '1'.

   Rules:
   1. $0+X = X$
   2. $1+X = 1$
   3. $0\cdot X = 0$
   4. $1\cdot X = X$
   5. $A+AB = A$
   6. $A+\bar{A}B = A+B$
   7. $(A+B)\cdot(A+C) = AA+AC+BA+BC$
      $= A+BC$
5. Idempotent Law / Identity Law.
   1. $X\cdot X\cdot X\cdot X \ldots\ldots X = X$
   2. $X+X+X+X \ldots\ldots X = X$ ✓
6. Complement / Complementation law
   1. $\bar{X}\cdot X = 0$
   2. $\bar{X}+X = 1$
7. Involution law:
   1. $\bar{\bar{X}} = X$
8. Absorbtion Laws:
   1. $X+XY = X$
   2. $X\cdot\bar{Y}+Y = X+Y$
   3. $(X+Y)\cdot(X+Z) = X+ZY$
9. DeMorgan's Theorem:
   1. $\overline{X_1+X_2+X_3+X_4+\ldots X_n} = \bar{X}_1\cdot\bar{X}_2\cdot\bar{X}_3\cdot\bar{X}_4\cdot\ldots\bar{X}_n$

      $\boxed{\overline{A+B} = \bar{A}\cdot\bar{B}}$ ✓
   2. $\overline{X_1\cdot X_2\cdot X_3\cdot X_4\cdot\ldots X_n} = \bar{X}_1+\bar{X}_2+\bar{X}_3+\bar{X}_4+\ldots+\bar{X}_n$

      $\boxed{\overline{A\cdot B} = \bar{A}+\bar{B}}$

Example Solutions:



(1) $X = (A+(BC)')'$

$\Rightarrow \overline{A+\overline{BC}}$ ✓

$\Rightarrow \bar{A}\cdot\overline{\overline{BC}}$

$\Rightarrow \bar{A}\cdot BC$

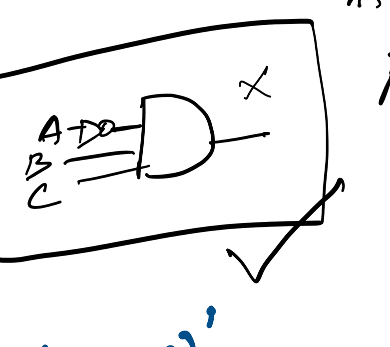
$\Rightarrow \bar{A}BC$ ✓

$\overline{A+\overline{BC}}$

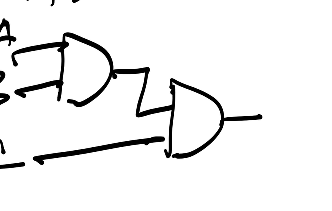
$\bar{A}\cdot\overline{\overline{BC}}$

$\bar{A}\cdot BC$

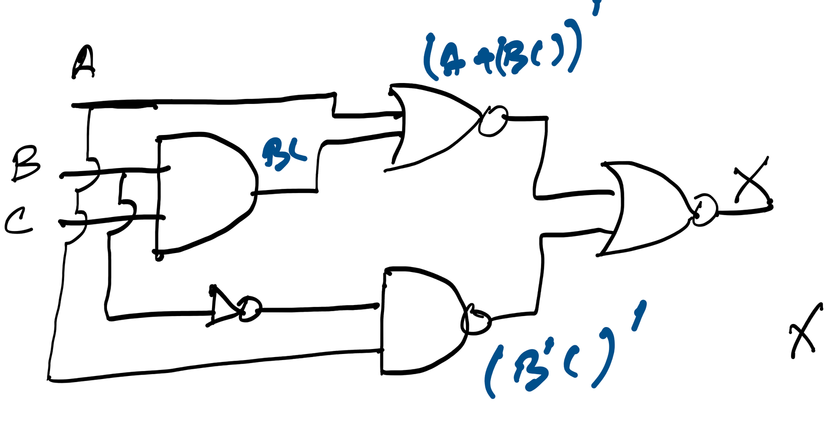
$\bar{A}BC$



ABC



(2)



$((A+(BC))'+(B'C)')'$

$\overline{\overline{A+BC}+\overline{\bar{B}C}}$

$X = ((A+(BC))' + (AB'))'$ ✓

$\overline{\overline{A+BC}+\overline{\bar{B}C}}$ ✓

$\Rightarrow \left(\overline{\overline{A+BC}}\right)\cdot\left(\overline{\overline{\bar{B}C}}\right)$ ✓

$\Rightarrow A+BC\cdot\bar{B}C$

$\Rightarrow A+(BC\cdot\bar{B}C)$

$\Rightarrow A+C(B\cdot\bar{B})$

$\Rightarrow A+C\cdot 0$

$\Rightarrow A+0$

$\Rightarrow A$ ✗

$A\bar{B}C + BC\bar{B}C$

$A\bar{B}C + B\bar{B}CC$

$A\bar{B}C + 0\cdot C$

$A\bar{B}C + 0$

$A\bar{B}C$.

$A\bar{B}C$	A ✓	B	C	B'	BC ✓	(B'C)'	(A+(BC))'	X
0	0	0	0	1	0	1	1	0
0	0	0	1	1	0	0	1	0
0	0	1	0	0	0	1	1	0
0	0	1	1	0	1	1	0	0
0	1	0	0	1	0	1	0	0
1	1	0	1	1	0	0	0	1
0	1	1	0	0	0	1	0	0
0	1	1	1	0	1	1	0	0