Technical Summary of the Spar Optimization Project

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1 Introduction

The objective of this project is to optimize the design of an aircraft wing spar to minimize its mass while satisfying structural constraints on stress and deflection. This involves applying **Multidisciplinary Design Optimization (MDO)** techniques, utilizing Python programming, and leveraging the **OpenMDAO** library for efficient computation.

2 Problem Description

The spar is a critical structural component of the wing, responsible for supporting bending loads due to aerodynamic forces. The goal is to determine the optimal spar geometry that results in the minimum possible mass without exceeding allowable stress and deflection limits.

3 Wing Geometry Modeling

The wing has an elliptical planform, and the chord length $c(y)$ varies along the spanwise position y according to:

$$c(y) = c_{\text{root}} \cdot \sqrt{1 - \left(\frac{2y}{b}\right)^2} \tag{1}$$

where:

- c_{root} is the chord length at the root, calculated as:

$$c_{\text{root}} = \frac{4S}{\pi b} \tag{2}$$

- S is the wing area.
- b is the wingspan.

4 Lift Load Distribution

The lift load per unit span $L'(y)$ is distributed elliptically along the wing:

$$L'(y) = \frac{4L}{b\pi} \cdot \sqrt{1 - \left(\frac{2y}{b}\right)^2} \tag{3}$$

where $L = mg$ is the total lift force (equal to the aircraft's weight).

5 Spar Geometry

The spar is modeled using hollow circular sections. The following equations define the outer dimension $D(y)$, wall thickness $t(y)$, inner dimension $d(y)$, cross-sectional area $A_{\text{boom}}(y)$, and moment of inertia $I(y)$ at each spanwise position y.

- **Outer Dimension,** $D(y)$**:**

$$D(y) = D_{\text{root}} + (D_{\text{tip}} - D_{\text{root}}) \frac{y}{b/2} \tag{4}$$

- **Wall Thickness,** $t(y)$**:**

$$t(y) = t_{\text{root}} + (t_{\text{tip}} - t_{\text{root}}) \frac{y}{b/2} \tag{5}$$

- **Inner Dimension, $d(y)$:**

$$d(y) = D(y) - 2 \times t(y) \tag{6}$$

- **Cross-Sectional Area of the Spar, $A_{\text{boom}}(y)$:**

$$A_{\text{boom}}(y) = D^2(y) - d^2(y) \tag{7}$$

- **Moment of Inertia of the Spar, $I_{\text{boom}}(y)$:**

$$I_{\text{boom}}(y) = \frac{1}{12}\left(D^4(y) - d^4(y)\right) \tag{8}$$

- **Distance from the Neutral Axis to the Spar Center, $h(y)$:**

$$h(y) = \frac{H \cdot c(y) - D(y)}{2} \tag{9}$$

  where $c(y)$ is the chord length at spanwise position y and H is the fraction of the chord where the spar is located.

- **Total Moment of Inertia, $I(y)$:**

$$I(y) = 2\left(I_{\text{boom}}(y) + A_{\text{boom}}(y) \cdot h^2(y)\right) \tag{10}$$

6 Structural Analysis

Using Euler-Bernoulli beam theory, the following quantities are calculated along the span:

6.1 Shear Force $V(y)$

$$V(y) = -A_y + \int_0^y L'(s)\,ds \tag{11}$$

where A_y is the reaction force at the root.

6.2 Bending Moment $M(y)$

$$M(y) = M_A + \int_0^y V(s)\,ds \tag{12}$$

where M_A is the reaction moment at the root.

6.3 Deflection Angle $\theta(y)$

$$\theta(y) = \int_0^y \frac{M(s)}{EI(s)}\,ds \tag{13}$$

6.4 Vertical Deflection $w(y)$

$$w(y) = \int_0^y \theta(s)\,ds \tag{14}$$

6.5 Stress $\sigma(y)$

$$\sigma(y) = \frac{M(y)H(y)c(y)}{2I(y)} \tag{15}$$

where:

- E is Young's modulus of the spar material.
- $c(y)$ is the chord length at position y.

7 Optimization Problem

The optimization aims to minimize the total mass of the spar while satisfying stress and deflection constraints.

7.1 Objective Function

$$\text{Minimize } m = 2 \int_0^{b/2} \rho_{\text{material}} \cdot A_{\text{boom}}(y)\, dy \tag{16}$$

where:

- ρ_{material} is the material density.
- The factor of 2 accounts for both wings.

7.2 Design Variables

- Outer dimension at root D_{root}
- Outer dimension at tip D_{tip}
- Wall thickness at root t_{root}
- Wall thickness at tip t_{tip}

The outer dimension and wall thickness vary linearly along the span:

$$D(y) = D_{\text{root}} + (D_{\text{tip}} - D_{\text{root}}) \left(\frac{2y}{b} \right) \tag{17}$$

$$t(y) = t_{\text{root}} + (t_{\text{tip}} - t_{\text{root}}) \left(\frac{2y}{b} \right) \tag{18}$$

7.3 Constraints

Stress Constraint

$$\sigma(y) \leq \sigma_{\text{allowable}} \tag{19}$$

Deflection Constraint

$$w \left(\frac{b}{2} \right) \leq w_{\text{allowable}} \tag{20}$$

8 Input Parameters

The following input parameters were used in the optimization:

- Wingspan, $b = 11.23$ m
- Wing area, $S = 22.48$ m^2
- Aircraft mass, $m = 3000$ kg
- Total lift force, $L = mg = 3000 \times 9.81$ N
- Young's modulus, $E = 100$ GPa
- Spar location fraction, $H = 0.13$
- Material density, $\rho_{\text{material}} = 2700$ kg/m^3
- Allowable stress, $\sigma_{\text{allowable}} = 250$ MPa
- Allowable deflection, $w_{\text{allowable}} = 0.1$ m

9 Implementation with OpenMDAO

The problem is implemented in Python using the **OpenMDAO** framework, which facilitates MDO by providing tools for defining components, connecting them, and performing optimization.

9.1 Components

1. **WingGeometry**: Calculates chord lengths and spanwise positions.
2. **LiftDistribution**: Computes lift load distribution along the wing.
3. **SparGeometry**: Defines spar dimensions along the span.
4. **StructuralAnalysis**: Performs calculations of shear force, bending moment, deflection, and stress.
5. **SparMass**: Computes the total mass of the spar.

9.2 Optimization Setup

- **Design Variables**:
  - D_{root}, D_{tip}, t_{root}, t_{tip}
- **Objective**:
  - Minimize spar mass.
- **Constraints**:
  - Stress at all spanwise positions must not exceed $\sigma_{\text{allowable}} = 250$ MPa.
  - Tip deflection must not exceed $w_{\text{allowable}} = 0.1$ m.

9.3 OpenMDAO Code Structure

Defining Components Each component is defined as a class inheriting from `om.ExplicitComponent`, with inputs and outputs specified.

```
class WingGeometry(om.ExplicitComponent):
    def setup(self):
        self.add_input('b', val=11.23)
        self.add_input('S', val=22.48)
        self.add_output('x', val=np.zeros(n))
        self.add_output('cChord', val=np.zeros(n))
```

Connecting Components Components are connected within the `SparOptimization` group.

```
self.connect('wing_geom.x', ['lift_dist.x', 'spar_geom.x', 'struct_analysis.x'])
self.connect('wing_geom.cChord', ['spar_geom.cChord', 'struct_analysis.cChord'])
```

```
self.add_design_var('spar_geom.D_root', lower=0.05, upper=0.5)
self.add_design_var('spar_geom.D_tip', lower=0.05, upper=0.5)
self.add_design_var('spar_geom.t_root', lower=0.005, upper=0.1)
self.add_design_var('spar_geom.t_tip', lower=0.005, upper=0.1)
self.add_objective('spar_mass.mass')
self.add_constraint('struct_analysis.sigma', upper=250e6)
self.add_constraint('struct_analysis.w', upper=0.1)
```

10 Results

After running the optimization, the following results were obtained:

10.1 Optimized Dimensions

- Optimized outer dimension at root: $D_{\text{root}} = 0.0500$ m
- Optimized outer dimension at tip: $D_{\text{tip}} = 0.0500$ m
- Optimized wall thickness at root: $t_{\text{root}} = 0.0050$ m
- Optimized wall thickness at tip: $t_{\text{tip}} = 0.0050$ m

10.2 Minimum Spar Mass

- Minimum mass of spar: $m = 27.29$ kg

10.3 Optimization Output

The optimization process yielded the following output:

```
Optimization terminated successfully    (Exit mode 0)
            Current function value: 27.288900000000012
            Iterations: 3
            Function evaluations: 2
            Gradient evaluations: 2
Optimization Complete
```

10.4 Stress and Deflection Profiles

The optimized design satisfies all constraints:

- The maximum stress along the span is below the allowable stress of 250 MPa.
- The tip deflection is less than the allowable deflection of 0.1 m.

11 Comparison with MATLAB Results

For validation, the optimization was also performed using MATLAB, yielding similar results:

- Optimized outer dimension at root: $D_{\text{root}} = 0.0500$ m
- Optimized outer dimension at tip: $D_{\text{tip}} = 0.0500$ m
- Optimized wall thickness at root: $_{\text{root}} = 0.0050$ m
- Optimized wall thickness at tip: $t_{\text{tip}} = 0.0050$ m
- Minimum mass of spar: $m = 27.29$ kg

11.1 MATLAB Optimization Output

```
 Iter  Func-count            Fval   Feasibility   Step Length       Norm of   First-order
                                                                       step    optimality
    0           5    3.456613e+02     0.000e+00     1.000e+00     0.000e+00     1.112e+04
    1          10    2.728890e+01     1.388e-17     1.000e+00     2.242e-01     8.692e+03
    2          11    2.728890e+01     1.388e-17     7.000e-01     1.377e-17     1.388e-17
```

A Variables and Constants Used

- b: Wingspan (m)
- S: Wing area (m^2)
- $c(y)$: Chord length at spanwise position y (m)
- c_{root}: Root chord length (m)
- L: Total lift force (N)
- $L'(y)$: Lift load per unit span at position y (N/m)
- $D(y)$: Outer dimension of the spar at position y (m)
- $d(y)$: Inner dimension of the spar at position y (m)
- $t(y)$: Wall thickness of the spar at position y (m)
- $H(y)$: Spar height at position y (m)
- $I(y)$: Moment of inertia at position y (m^4)
- $M(y)$: Bending moment at position y (Nm)
- $V(y)$: Shear force at position y (N)
- $\theta(y)$: Deflection angle at position y (rad)
- $w(y)$: Vertical deflection at position y (m)
- $\sigma(y)$: Stress at position y (Pa)
- E: Young's modulus (Pa)
- ρ_{material}: Material density (kg/m^3)
- $\sigma_{\text{allowable}}$: Allowable stress (Pa)
- $w_{\text{allowable}}$: Allowable deflection (m)

B Assumptions

- The wing is modeled as an idealized structure with an elliptical lift distribution.
- Material properties are homogeneous and isotropic.
- The spar dimensions vary linearly from root to tip.
- The analysis uses Euler-Bernoulli beam theory, assuming small deflections.